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Factors Affecting Crack Initiation in Low Porosity Crystalline Rocks

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Abstract Crack initiation in uniaxial compressive loading of rocks occurs well before the peak strength is reached. The factors that may influence the onset of cracking and possible initiating mechanisms were explored using a discrete element numerical approach. The numerical approach was based on grain-based model that utilized the Voronoi tessellation scheme to represent low porosity crystalline rocks such as granite. The effect of grain size distribution (sorting coefficient ranging from 1.5 to 1.03), grain size (average grain size ranging from 0.75 to 2.25 mm), and the heterogeneities of different mineral grains (quartz, K-feldspar, plagioclase) on the onset of cracking were examined. The modelling revealed that crack initiation appears to be a tensile mechanism in low porosity rocks, and that shear cracking along grain boundaries is only a prominent mechanism near the peak strength. It was also shown that the heterogeneity introduced by the grain size distribution had the most significant effect on peak strength and crack initiation stress. The peak strength ranges from 140 to 208 MPa as the grain size distribution varies from heterogeneous to uniform, respectively. However, the ratio of crack initiation to peak stress showed only minor variation, as the heterogeneity decreases. The other factors investigated had only minor effects on crack initiation and peak strength, and crack initiation ratio.

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C. D. Martin e-mail: derek.martin@ualberta.ca **Keywords** Crack initiation stress · Discrete element methods · Grain-based models · Mineral-based models · Cracking model

1 Introduction

It is well known that the microstructure of a rock should be taken into account when determining its compressive strength. To minimize its influence on the test results, the ISRM "Suggested Methods for determining the uniaxial compressive strength (UCS) and deformability of rock materials states that the test specimen shall be right cylinders having a height to diameter ratio of 2.5-3.0 and a diameter preferably of not less than 54 mm". The method then states that the diameter of the specimen should be greater than ten times the largest mineral grain in the rock (Brown 1981). Presumably, the latter requirement ensures that the right-cylinder specimen statistically captures the flaws that are typically associated with grain boundaries. The effect of flaw size on compressive strength was examined by Cook (1963). Using the energy approach of Griffith (1921, 1924); Cook (1963) showed that the compressive strength was proportional to the flaw size and as the flaw size increased the compressive strength decreases. The effect of grain size on the strength of rocks has been studied using anhydrite, dolomite and granite by Skinner (1959); Olsson (1974) and Prikryl (2001), respectively. These authors found that as the grain size increased the UCS decreases, supporting the theory proposed by Cook (1963) and the requirements of the ISRM Suggested Methods, assuming that the grain boundary can be considered as a flaw. While, it appears that the compressive strength is influenced by grain size, it is not clear if grain size influences other properties associated with compression testing.

Brace (1964) and Bieniawski (1967) using laboratory tests that monitored the local axial and lateral strains carried out detailed analyses of the stress–strain response as the compression load was applied. They clearly showed that microcrack growth is initiated at stress magnitudes well below the peak compressive strength. The early work of Brace (1964) showed that in laboratory specimens loaded in compression, dilatancy occurred at approximately 50 % of the peak compressive load. This dilatancy stress, which is today referred as the crack initiation (CI) stress, was found by Brace (1964) to range between 1/3 and 2/3 of the peak strength for a wide range of rock types. More recently, Nicksiar and Martin (2013) showed that CI was observed in all 376 specimens of low porosity rocks tested in compression.

Figure 1 shows a summary of the Igneous, Sedimentary and Metamorphic data used by Nicksiar and Martin (2013) in their study. As shown in Fig. 1 the relationship of the CI stress to the UCS is linear, despite the wide range of UCS values from 15 to 370 MPa.

If grain size affects the peak strength, as discussed above, then it is also likely that grain size should affect the CI stress. The rocks used to establish (Fig. 1) had grain sizes ranging from microns to centimetre, despite these orders of magnitude range in grain size, the CI stress to uniaxial compressive strength ratio (CIR) remained well constrained at approximately 0.45 ± 0.06 , supporting the early findings of Brace (1964). It would appear that grain size might not be a significant factor in controlling CIR. Similarly, the effect of material and geometry heterogeneities would also appear not to significantly impact CIR. In this study, a grain-based discrete element numerical model is used to examine the effect of grain size, material and geometry heterogeneities on CI stress and CIR in



Fig. 1 Crack initiation (CI) stress versus uniaxial compressive strength (UCS) for low porosity igneous, sedimentary and metamorphic rocks. Regression line and its 95 % confidence interval are shown as *solid and dashed line*, respectively (after Nicksiar and Martin 2013)

uniaxial compression. The work of Stacey (1981); Read et al. (1998, 2004), Martin et al. (1999), Diederichs (2007), Andersson et al. (2009), and Martin and Christiansson (2009) showed that CI stress from laboratory tests is a relevant parameter when evaluating the potential for spalling around underground openings. Hence if this laboratory parameter is needed for assessing the stability of underground openings it is important to understand the factors that affect its magnitude.

2 Crack Initiation Models in Compression Loading

The initiation of cracks in rocks associated with axial compressive loading has been studied extensively over the past several decades. One of the consistent findings from such studies is that the majority of the cracks are oriented within $\pm 15^{\circ}$ to the loading direction (Brace et al. 1966; Hallbauer et al. 1973; Lajtai 1974). Two basic models are often used to represent this observation (see Fig. 2): (1) the Sliding-crack model (Nemat-Nasser and Horii 1982; Nemat-Nasser and Obata 1988; Ashby and Hallam 1986), and (2) the Force-chain crack model (Potyondy and Cundall 2004; Cho et al. 2007). Inspection of the models in Fig. 2 illustrates an important fundamental difference in the concepts used by the two models. With the Slidingcrack model, a suitably oriented weak plane must exist and sliding along this weak plane must occur before the local tensile stresses are adequate to initiate and grow the axially aligned fractures. The Force-chain crack model, however, does not require this weak plane to exist or the initial sliding to occur. The heterogeneity of mechanical and mineralogical properties of the mineral assemblage in low porosity rocks in the Force-chain model will create local tensile stresses that are sufficient to form cracks. Clearly, these two models represent very different fundamental concepts for cracking in low porosity rocks. The Slidingcrack model implies that in compressive loading, rock is weaker in shear than in tension since sliding must occur first before tensile cracking can occur. The Force-chain crack model, however, implies that rock is weakest in tension, and as shown by Potyondy and Cundall (2004) and Cho et al. (2007), once a sufficient number of tensile cracks form a macroscopic shear fracture will naturally develop.

To investigate which crack model in Fig. 2 is more applicable for describing CI observed in low porosity rocks, a specimen of Lac du Bonnet granite was subjected to a number of monotonically load–unload cycles (Fig. 3a). The axial stress was increased from an initial of 175 to 198 MPa using four load steps with approximately ten cycles for each load step (Fig. 3b). This test followed the procedure given by Martin and Chandler (1994) where the axial and lateral strains from each unloaded cycle provided

(a) Response to applied axial force in low porosity rock



Fig. 2 Two models commonly used to explain crack initiation observed in laboratory tests

the permanent axial and lateral strain. The UCS of Lac du Bonnet granite is approximately 220 MPa, and the CI stress is approximately 120 MPa (Lau and Chandler 2004). Hence the initial load step of 175 MPa is well above the CI stress.

Figure 3c shows the accumulated permanent axial and lateral strain for each cycle. The accumulated axial strain shows essentially no increase for all 38 cycles despite the load increasing from 175 to 198 MPa. This is consistent with the axial stress versus axial strain response shown in Fig. 3a. The accumulated permanent lateral strain in Fig. 3b, however, shows a gradual increase in strain with each cycle. Figure 3c also shows that the accumulated lateral strain increases as the load is increased, particularly when the load step exceeds 190 MPa. These results show that permanent damage in the specimen results from initiating crack sub-parallel to the loading direction and the lateral strain is a good indicator of this damage.

Inspection of the initial stress–strain response in Fig. 3a shows that the specimen tested contains initial cracks, indicated by the nonlinear stress strain response in the early stages of loading. As noted by Martin and Stimpson (1994), this is a common phenomenon in low porosity rocks, and is attributed to naturally occurring and/or stress-induced microcracking. It is likely that slip could occur along some of these cracks, assuming they were optimally oriented. Despite the presence of these microcracks, which may cause the nonlinear axial strain response, no permanent



Fig. 3 Comparison of measured axial and lateral strain in a uniaxial compression tests subjected to 38 load–unload cycles. **a** *Axial strain* for 38 load–unload cycles; **b** *Lateral strain* for 38 load–unload cycles; and **c** permanent axial and lateral strain for 38 load–unload cycles

damage has been recorded in the axial strain recording (see Fig. 3c). Thus, the Sliding-crack model, which would require permanent axial strains, is not supported by these data.

Brace (1964) and Bieniawski (1967) were early researchers to suggest that the long-term strength of rock was the stress associated with volumetric strain reversal and coined the term "unstable fracture propagation"

(unstable crack growth in Fig. 4). This inflection point in the stress-strain curve marks the beginning of a significant increase in crack density (Hallbauer et al. 1973). Two load cycles were examined to assess the crack model likely associated with the unstable crack growth. The 183 and 190 MPa load cycles were selected as the formation of unstable crack growth initiated in the 183 MPa cycle and significantly propagated in the 190 MPa cycle (Fig. 4a). The volumetric strain plots in Fig. 4a have been purposely offset to clearly show both graphs. This offset does not affect the axial stress of 179 MPa associated with the unstable crack growth. The axial strain and lateral strain for the 190 MPa volumetric strain in Fig. 4a were analyzed separately to determine the incremental slope of axial strain and Lateral strain response (LSR). The slope was determined by carrying out a linear least squares fit and the results are shown in Fig. 4b. It is clear from Fig. 4b that there is no change in the axial strain response even at the initiation of the unstable crack growth and beyond. However, the lateral strain shows a marked change in response in the region of the stress associated with unstable crack growth. The slope of the lateral strain plot in Fig. 4b suggests that unstable crack growth initiated before the stress associated with volumetric strain reversal at an axial stress of 175 MPa. Comparing Fig. 4b with Fig. 3c, it can be observed that the volumetric strain reversal is also associated with the damage measured by the lateral strain. Again the data suggest that the Force-chain crack model best describes the damage mechanism associated with the initiation and propagation of unstable crack growth and not the Sliding-crack model. Again the data suggest that the damage mechanism associated with the initiation and propagation of unstable crack growth is dominated by axial cracking without any evidence for sliding-induced damage.

The measured stress–strain response of laboratory specimens subjected to compressive loading suggests that the Sliding-crack model is not an appropriate modelling approach for representing early stage cracking in low porosity rocks. The measured stress–strain response of the onset of CI to beyond unstable crack growth supports the concepts introduced by the Force-chain crack model, i.e. material heterogeneity alone is adequate to create tensile stresses sufficient to initiate cracking.

3 UDEC Modelling of Brittle Failure in Intact Rocks

In 1979, Cundall and Strack (1979) proposed a discrete element method for modelling discrete spheres of random size. This approach was used by Potyondy et al. (1996) to develop the bonded particle method (BPM) for modelling of intact rock. In the BPM, the intact behaviour is calculated by the mechanical response of distinct circular discs



Fig. 4 Comparison of volumetric strain reversal when applied stress is 183 and 190 MPa, and slope of the *axial* and *lateral strain* for 190 MPa. **a** Volumetric strain reversal, **b** Slope of axial and lateral strain for 190 MPa

or spheres, which incorporates the elements of the forcechain contact model. Cho et al. (2007) showed that the proposed BPM by Potyondy et al. (1996) could not capture the stress-strain response of intact rock without modification. They used a clumping technique to represent the polygonal structure/geometry commonly found in rocks and showed that with this polygonal structure the BPM could capture the behaviour of intact rock in compression. Since 2007; Lan et al. (2010) showed that an efficient means of representing the polygonal structure of intact mineral assemblages could be achieved using the discrete element formulation in the Universal distinct element code (UDEC; Cundall 1980). In UDEC, a Voronoi tessellation approach is used to partition the domain into polygons of pre-defined edge length and size distribution. These polygons represent individual mineral grains. It should be noted that Voronoi tessellation produces the linear contacts along the grains by subdividing the block into polygonal grains. Therefore, effects caused by serrated grain boundaries, such as transgranular cracking, have not been considered. Unlike the BPM, in UDEC the mineral grains represented by the individual polygons cannot break, i.e. all rupture

must follow polygonal boundaries. This UDEC limitation has implications for the post-peak response, but as noted by Tapponnier and Brace (1976) the pre-peak response occurs primarily at grain boundaries.

The UDEC methodology of force interaction of the blocks and the contacts is the same as with the BPM method, except that the contacts are not point contact, but both point and area contacts. The force–displacement of the contacts is determined by normal and shear stiffness along the contact. The contact tensile strength, cohesion and friction are defined according to the selected constitutive contact model and the contact fails in tension or shear when the stress exceeds the contacts' strength.

3.1 Model Setup

The primary focus of the UDEC modelling is to establish the effect of the various input parameters on CI during compressive loading. The initial model calibration was carried out using the approach described by Lan et al. (2010). The primary focus of the calibration was to establish the laboratory Young's Modulus and Poisson's ratio, and the onset of cracking. In UDEC, the initiation of microcracks occurs due to the rupture along the polygon boundaries, and hence tracking the rupture of these grain boundaries can be used to identify the onset of cracking (Lan et al. 2010). The typical laboratory properties of a medium-grained granite were used to calibrate the UDEC model, and these properties are essentially the same as those of Lac du Bonnet granite (Scholtès and Donzé 2013). In most models, one property was changed while other properties are held constant to find the effect of the changed parameter on the CI stress.

The UDEC models are $51 \text{ mm} \times 128 \text{ mm}$ and all models are bounded at the top and bottom by the steel loading platens. The Voronoi tessellation is then applied to discretize the model into a finite difference zone representing the mineral grains based on the target grain size. The mechanical behaviour of the model is controlled by parameters that can be divided into two groups (Christianson et al. 2006):

- 1. Parameters that control the specimen's unconfined compressive strength: cohesion and friction angle
- 2. Parameters that control the specimen's elastic constants: bulk and shear modulus of the grains as well as normal and shear stiffness of the polygonal contacts

The load is applied to the specimen by a uniform velocity in the y direction on the top platen with the bottom platen remaining fixed. Cundall and Strack (1979) examined the effect of various loading rates and recommended a loading rate that keeps the internal unbalanced forces low compared to the contact forces.

A sampling area (51 mm \times 10 mm) at the middle of the specimen where the stress distribution is expected to be relatively uniform was used to simulate strain gauges in laboratory testing (Fig. 5). The lateral and axial strains are calculated using the average displacement in the *x* and *y* directions at several locations in this sampling area to monitor the specimen response. The stress–strain responses of the blocks and contacts are calculated using UDEC built-in commands.

3.2 Block Constitutive Model

Crack initiation in laboratory specimens occurs well before peak stress is reached. At these low loads cracking is expected to occur along the grain boundaries while the mineral grains deform elastically. Consequently, an elastic constitutive block model is used to maintain the elastic behaviour of the grains. In the elastic constitutive model, the required parameters are density, bulk and shear modulus. The overall elastic constants of the model-specimen will be the average values of its rock-forming minerals calibrated with the laboratory stress–strain results.

3.3 Contact Constitutive Model

The contact behaviour used a Coulomb slip model with residual strength properties. The initial contact cohesion and friction values were based on values reported in laboratory tests reports (Glamheden et al. 2007). The contacts are assumed to instantly lose their cohesion and friction to their residual values once failure occurs. Moreover, the only model that captures the failed contacts in UDEC model is the Coulomb slip model with residual strength.

The normal and shear stiffness values for the contacts were obtained by calibrating to the laboratory test results. Different contact shear to normal stiffness ratios have



Fig. 5 Axial and lateral strains are recorded at the locations shown to monitor the specimen response

been reported by several authors using different DEM methods. Diederichs et al. (2004) used the ratio of 0.4 for Lac du Bonnet granite in BPM model. Lan et al. (2010) used the ratio of 0.67 for Lac du Bonnet in their UDEC model. In this study, the contact shear stiffness was set to 0.6 of the normal stiffness. Table 1 gives the contact and block properties used to establish the macroscale response of the UDEC specimen. It is worth noting that the values in Table 1 are the result of extensive calibration of the UDEC model to obtain agreement with the laboratory stress-strain data. Details of this calibration process are given by different authors (see Cundall 2001; Potyondy and Cundall 2004; Donzé 2009; Lan et al. 2010; Scholtès and Donzé 2012, 2013) and similar methodologies have been used by Cho et al. (2007), (2008), Shin et al. (2007), Lan et al. (2012) and Shin (2010). The recent paper by Scholtès and Donzé (2013) highlights the essential elements needed for calibrating DEM to simulate the "real rock" behaviour. They discovered and repeated what was demonstrated in Cho et al. (2007). Lan et al. (2010) demonstrated that all the issues associated with the bonded particle model and described in Cho et al. (2007) and Scholtès and Donzé (2013) were removed with the polygonal structure of the Voronoi tessellation in UDEC. This also meant that calibration became much simpler, relying on micro-scale properties that could be assessed readily.

3.4 Crack Initiation Stress

Crack initiation in laboratory compression tests is noted as the first stage of stress-induced damage in low porosity rocks (Fig. 6a). Nicksiar and Martin (2012) reviewed the various methods that have been proposed over the past 30 years for identifying CI in laboratory tests, and

Table 1 Material properties used in the UDEC models

concluded that statistically all methods identify acceptable crack initiation values.

The stress associated with CI in our grain-based model can be obtained in several ways. Figure 6b shows the three ways that were used in processing the models' results and are summarized below:

- 1. A plot of axial stress versus lateral strain in the grainbased model is identical to the axial stress versus lateral strain in a laboratory result. The initial change in the slope of the lateral strain versus axial stress is called CI and can be determined using the LSR technique described by Nicksiar and Martin (2012)
- 2. In the UDEC model, it is often convenient to track the macro lateral strain and axial strain to assess the change in Poisson's ratio as the stress is applied. The ratio remains constant until cracking occurs. The stress associated with the change in Poisson's ratio is also referred to as the CI stress (Diederichs et al. 2004). A plot of Poisson's ratio versus Lateral strain is also shown in Fig. 6b.
- 3. Cracking of the contacts in the grain-based model can be tracked as the stress is applied. The cracks are designated as either tensile or shear. The number of tensile cracks normalised to the total number of contacts is also plotted as function of lateral strain in Fig. 6b. Determining the stress associated with CI is not straight forward, as illustrated in Fig. 6b. In essence there are two constraints, no cracks when the applied stress is zero, and fully cracked when the peak stress is reached. The samples showing these conditions are also given in Fig. 6b.

Figure 6b illustrates the three methods that are used to establish the stress associated with CI in the grain-based models. It is clear from Fig. 6b that the stress obtained

Contact properties							
Normal stiffness (GPa/m)	Shear stiffness (GPa/m)	Cohesion (MPa)	Friction angle (deg)	Tensile strength (MPa)	Dilation angle (deg)	Residual cohesion (MPa)	Residual friction (deg)
43,428	26,056	42	57.5	18	15	2	30
Block properties							
Bulk modulus (GPa) Shear modulus (GPa)) Den	sity (kg/m ⁻³)				
45	30		2,63	0			
Platen properties							
Bulk modulus (GPa	a) Shear	modulus (GPa) Den	sity (kgm ⁻³)			
160	79		7,75	0			

The bulk and shear modulus as well as normal and shear stiffness are changed in the mineral-based models



Fig. 6 Comparison of the crack initiation stress in a laboratory compression test and the methods used to establish crack initiation stress in the UDEC Model. **a** Stress strain response and crack initiation in a laboratory test, **b** Crack initiation in the UDEC model (details discussed in the text)

from either the LSR or Poisson's ratio techniques is in reasonable agreement. However, it is also clear that relying solely on the density of tensile cracks is more problematic. For the analyses used in this paper, both the stress from the Poisson's ratio and the LSR are used to establish the CI stress.

4 Modelling Approach Used to Assess the Effect of Heterogeneity

The objective of the study was to establish the effect of: (1) grain size distribution; (2) average grain size and (3) mineralogy on initiation of cracking in the numerical specimens. Consequently, a total of 20 models were evaluated and CI and peak stress were recorded for each model.

4.1 Grain Size Distribution

The first group of models evaluated the effect of grain size distribution. In these models, the average grain size resembles fine-medium to medium-grained granite (1-2 mm) and the random seed number for the Voronoi tessellation for all three models remained constant to generate a similar distribution of grains. The sorting coefficient (So) introduced by Trask (1932) has been used to describe the degree of uniformity in the grain size distribution. The coefficient is defined as:

So =
$$\sqrt{Q_{25\%}}/Q_{75\%}$$

where $Q_{25\%}$ and $Q_{75\%}$ correspond to the diameter, for which 25 and 75% of the grains are coarser than this diameter on the grain size cumulative frequency plot. As the sorting coefficient approaches 1, the grain size distribution becomes more uniform. Four models were generated with So values ranging from 1.03 to 1.5 and these are illustrated in Fig. 7. While the models with So 1.03–1.14 were created using the built-in functionality within UDEC 5.0, the model with So = 1.5 was created using the methodology described by Lan et al. (2010). The grain size distribution for each model was calculated using the MATLAB image processing tools and the results are presented in Fig. 7. These models represent grain size distributions ranging from uniform (So = 1.03) to heterogeneous (So = 1.5).

The grain size distribution was changed from uniform to heterogeneous as to represent the variation in grain size distribution of Forsmark and Lac du Bonnet granite (Fig. 8).

4.2 Grain Size

Three numerical specimens each with different average grain size and constant grain size distribution were used to examine the effect of grain size on CI. Three models with fine (0.75 mm), fine to medium (1.5 mm) and medium-grained (2.25 mm) specimens were designed with So equals to 1.03. The computational time required to complete each model increased dramatically as the average grain size decreased. The average grain size was determined using MATLAB by fitting an ellipse to the mineral grain and calculating the average of minimum and maximum radius of that ellipse. Despite this increase in run times the overall dimensions of the models were unchanged.

4.3 Mineralogy

The last group of numerical models examined the effect of mineralogy. All the previous models have been created



Fig. 7 Grain size distribution of the models with different sorting coefficient. The uniformity of the grain size increases from a to d



Fig. 7 continued



Fig. 8 Comparison between the UDEC model (So = 1.5) and the grain size distribution of Forsmark and Lac du Bonnet granite (based on Lim et al. 2012)

 Table 2
 Elastic properties of igneous rock forming minerals (Bass 1995)

Grain type	Density (kgm ⁻³)	Shear modulus (GPa)	Bulk modulus (GPa)
Quartz	2,650	44.0	37.0
Plagioclase	2,630	29.3	50.8
K-feldspar	2,560	27.2	53.7

using the grain properties given in Table 1. However, in these groups of models the properties of different igneous rock forming minerals given in Table 2, i.e. quartz, K-feldspar and plagioclase, were assigned randomly to the grains. The percentage of quartz, K-feldspar and plagioclase was selected to simulate the composition of the rocks, illustrated in Fig. 9, as Model 1 through Model 6. All the geometric parameters are kept constant using the same uniform grain size distribution (So = 1.03) and the average grain size of 2.25 mm (medium grain size). The mineral heterogeneity was applied to the models in several steps:

- 1. The location of each grain centroid was determined using MATLAB image processing tool.
- 2. The mineral types were assigned randomly to meet the mineral content percentages shown in Fig. 9.
- 3. The mineral properties were imported into UDEC model via a FISH code that reads the location data file from Step 2

The contact properties were assigned according to the neighbouring material types. The contact stiffness values were calculated according to Hooke's law by defining the equivalent stiffness of a contact between two grains as illustrated in Fig. 10 using the properties presented in Table 3.

There is no clear methodology for estimating the stiffness of the contact properties for two adjoining mineral grains. In order to have a reasonable estimate of grain contact normal and shear stiffness, two separate UDEC models have been created representing rocks that are composed of specific minerals. For this example, the unconfined compression test results for syenite and diorite were used. These two specimens represent rock types with more than 90 % K-feldspar and Plagioclase, respectively. The normal and shear stiffness of the contacts between the minerals were adjusted until the elastic Young's modulus and Poisson's ratio and peak uniaxial strength of the UDEC specimen matched the laboratory data (Table 4). Normal and shear stiffness of quartz were later calibrated from the medium-grained granite properties (Table 1) knowing the contact stiffness of plagioclase and K-feldspar.

5 Results and Discussion

5.1 Crack Initiation Mechanism

The two cracking models proposed in "Sect. 2" and illustrated in Fig. 2 rely on either tensile failure or sliding (shear) failure. In laboratory compression testing, CI (tensile) is recorded early in the loading process when the applied stress reaches approximately 0.4–0.5 of the peak stress while the opportunity for crack sliding occurs when the applied stress reaches approximately 0.7–0.8 of the peak stress (Hallbauer et al. 1973). The Force-chain model in Fig. 2 will cause cracking, provided that the normal stress acting on the crack surface is equal to its tensile strength. However, the Sliding-crack model will be valid when the shear stress on the crack surface is equal to the shear strength of the cracks (τ) which is defined as:

$$\tau = C + \sigma_{\rm n} \tan \phi$$

where *C* and ϕ are cohesion and internal friction angle and σ_n is the normal stress acting on the crack surface.

The four models with So equal to 1.5, 1.14, 1.05 and 1.03 were analyzed to assess the mode of rupture (tensile or sliding) as cracking occurs during the loading process. The number of shear and tensile cracks were determined for every 5,000 time steps in the UDEC model which equates to approximately a 0.07 increment of the peak strength. Figure 11 shows the number of cracks normalized to the number of grain contacts for each loading increment versus the applied stress normalized to the peak strength. Figure 11 illustrates that all the cracking observed in the early stages of loading is tensile cracking. The more uniform grain size distributions with So ranging from 1.14 to 1.03 showed no shear cracking until the loads exceeded 0.7 of the peak strength, while the heterogeneous specimen with So = 1.5 showed shear cracking initiating at approximately 0.6 of the peak strength. There are only tensile cracks associated with the CI. However, for the crack







Quartz



Fig. 10 Equivalent mineral stiffness based on Hooke's law

Table 3 Contact normal and shear stiffness based on Hooke's law for different minerals (units are in GPa/m)

Grain type	Quartz	Plagioclase	K-Feldspar
Quartz	$k_{\rm n} = 43,428$	$k_{\rm n} = 37,939$	$k_{\rm n} = 26,146$
	$k_{\rm s} = 26,057$	$k_{\rm s} = 22,763$	$k_{\rm s} = 15,687$
Plagioclase	$k_{\rm n} = 37,939$	$k_{\rm n} = 32,450$	$k_{\rm n} = 20,657$
	$k_{\rm s} = 22,763$	$k_{\rm s} = 19,548$	$k_{\rm s} = 12,444$
K-Feldspar	$k_{\rm n} = 26,146$	$k_{\rm n} = 20,657$	$k_{\rm n} = 8,865$
	$k_{\rm s} = 15,\!687$	$k_{\rm s} = 12,444$	$k_{\rm s} = 5,349$

Table 4 Modelling properties of Syenite and Gabbro

Rock type	UCS specir	nen	UDEC model		
	<i>E</i> (GPa), <i>v</i>	Peak stress (MPa)	<i>E</i> (GPa), <i>v</i>	Peak stress (MPa)	
Syenite ^a Diorite ^c	51, 0.27 ^b 92, 0.24	222.6 210.0	60, 0.24 102, 0.24	241 234	

Keq=Kn1+Kn2

Plagioclase

Kn_{Qtz}+Kn_{Plg}

Kn_{Plg}

 $\frac{\text{Kn}_{\text{Plg}} + \text{Kn}_{\text{Kfel}}}{2}$

2

K-Feldspar

Kn_{Qtz}+Kn_{Kfel}

2 Kn_{Plg}+Kn_{Kfel}

2

Kn_{Kfel}

Quartz

Kn_{Qtz}

Kn_{Qtz}+K_{Plg}

2

2

Kn_{Qtz}+Kn_{Kfel}

^a Lan et al. (2010); Jian-An and Sijing (1985)

^b Poisson's ratio from D'yachkova et al. (1966)

^c Shimada et al. (1983)

Fig. 11 Relationship between the quantity of tensile and shear cracks, expressed as a percentage of the total contacts, for each approximate 0.07 load increment. The applied stress is normalized to the peak strength. *CI* refers to crack initiation and *CD* refers to crack damage (unstable crack growth)



damage (CD) both shear and tensile cracks are associated with this stress threshold and for So ranging from 1.14 to 1.03, the number of shear cracks is significantly greater than the number of tensile cracks as the peak strength is approached. All four models (i.e. different grain size distribution, different average grain size and two groups of models with different mineralogy) support the notion that tensile cracks are a precursor to the macro scale failure process, but near the peak strength both shear and tensile cracks contribute to the failure process.

In addition to the crack type, the direction of the crack is also an indicator of the loading model that is causing the cracking. Lajtai (1998) reported that the orientation of the majority of stress-induced cracks observed in laboratory tests is oriented in the direction of the applied load. The directions of the cracks were recorded from the starting stress to crack damage stress in all models and the results are summarized in Fig. 12 and Table 5. The results clearly illustrate that the majority of the cracks are within $\pm 10^{\circ}$ of the applied axial stress and that the mean orientation is within $\pm 1^{\circ}$ of the applied stress. These orientations are approximately parallel to the applied stress also support the notion that the Force-Chain model, resulting in the generation of tensile cracking, is the dominant mode of cracking until the stage of unstable crack growth.

5.2 Effect of Heterogeneity Induced by Grain Size Distribution

The effect of the heterogeneity, i.e. the grain size distributions illustrated in Fig. 7, on the CI stress and peak

strength is summarized in Fig. 13 and Table 6. According to Table 6, the peak uniaxial strength ranges from 140 to 208 MPa as the grain size distribution changes from heterogeneous to uniform, respectively. This 32 % increase in peak strength, as the heterogeneity decreases, is also accompanied by an increase in the stress required for CI from 72 to 101 MPa. In essence, both the CI and peak stresses increase as the specimen grain size distribution becomes more uniform. Consequently, the crack initiation ratio of CI to peak stress shows only minor variation: 0.51, 0.45, 0.47 and 0.49, as the heterogeneity decreases.

5.3 Effect of Grain Size

It is often reported that the strength of intact rock is a function of grain size (Brace 1961; Fredrich et al. 1990). The three models with average grain size of fine (0.75 mm), fine to medium (1.5 mm) and medium-grained (2.25 mm), and constant grain size distribution (So = 1.03) gave peak uniaxial strengths of 203, 200, and 208 MPa (Fig. 14; Table 7). The results suggest that grain size has no effect on the peak uniaxial strength provided the grain size distribution is held constant. The CI stresses were also found to be relatively insensitive to grain size, ranging from 97 MPa for fine grain, 94 MPa for finemedium grain to 101 MPa for medium grain (Table 7). The values given in Table 7 give ratios of CI to peak strength of 0.48, 0.47 and 0.49 for fine, fine to medium, and medium grained, respectively. Based on these results, it appears that the peak strength and CI stress are independent of average



Fig. 12 The direction of cracks measured in the four models (So = 1.5, 1.14, 1.05 and 1.03)

 Table 5
 Average crack orientation in models with different attributes

Avg. grain si	ze = 1-2 mn	1			
Models with	different So				
So = 1.50	So = 1.14	So = 1.05	So = 1.03		
-0.28°	-0.79°	1.09°	0.89°		
Sorting Coef	ficient $= 1.03$				
Models with	average grain	ı size			
0.75 mm	1.15 mm	2.25 mm			
0.14°	-0.65°	0.89°			
Sorting Coef	ficient $= 1.14$; Avg. grain	size $= 2.25$	mm	
Models with	different min	eralogy (Qtz/	Pl/K-fel %)		
55/10/35	55/35/10	35/10/55	35/55/10	10/35/55	10/55/35
-0.91°	-0.72°	-1.00°	-0.23°	0.38°	-0.41°
Sorting coeff	icient = 1.03	; Avg. grain s	ize = 2.25	mm	
Models with	different min	eralogy (Qtz/	Pl/K-fel %)		
55/10/35	55/35/10	35/10/55	35/55/10	10/35/55	10/55/35
0.27°	1.79°	-0.61°	-0.82°	0.75°	0.28°

Crack orientations are measured relative to the load direction and given in degrees

grain size for a rock with a uniform grain size distribution and constant mineralogy.

5.4 Effect of Mineralogy

The effect of mineralogy on CI stress was examined using uniform (So = 1.03) and non-uniform (So = 1.14) grain size distribution. It should be emphasized that the effect of mineralogy was evaluated using variation in the elastic constants for each mineral grain. Figure 15 and Table 8 summarize the results from the UDEC models and



Fig. 13 Crack initiation stress as a function of peak stress in models with different grain size distribution

illustrate that the peak uniaxial strength increases as the quartz content increases. From Table 3, it can be observed that the magnitude of the contact normal stiffness for the quartz mineral is the largest. In a Force-chain model, the stresses that develop are heterogeneous and a function of this contact stiffness. Lan et al. (2010) evaluated the effect of mineral heterogeneity on the stress distribution in a sample loaded in compression and concluded that the peak strength positively correlated to the mineral heterogeneity, i.e. the greater the heterogeneity, the lower the peak strength. This correlation occurred because the greater the heterogeneity the greater the crack density for the same stress magnitude. It appears that in our uniform and non-uniform samples the heterogeneity is captured by the percentage of quartz, with the higher the percentage the

 Table 6
 Crack initiation stress peak stress in models with different grain size distribution

So	CI— Poisson's ratio (MPa)	CI— LSR (MPa)	UCS (MPa)	Average CI (MPa) normalized cracks (%)	CI/ UCS
1.50	71	73	140	72, 0.9	0.51
1.14	82	80	179	81, 1.2	0.45
1.05	102	82	193	91, 1.4	0.47
1.03	105	97	208	101, 0.8	0.49



Fig. 14 Crack initiation stress as a function of average grain size when So=1.03

Table 7 Crack initiation stress and peak stress in models with different average grain size when So = 1.03

Average grain size	CI— Poisson's ratio (MPa)	CI— LSR (MPa)	UCS (MPa)	Average CI (MPa) and normalized cracks (%)	CI/ UCS
Fine (0.75 mm)	101	93	203	97, 0.8	0.48
Fine to medium (1.5 mm)	92	95	200	94, 0.7	0.47
Medium (2.25 mm)	105	97	208	101, 1.0	0.49

higher the peak strength, and presumably the lower the heterogeneity. Lajtai (1998), using laboratory data, also concluded the positive correlation of the quartz framework to strength of granites, although the heterogeneity of those granites is not known.

This trend is also observed in the stress required for CI, i.e. the higher the quartz content the higher stress required for CI, provided that the grain size distribution is uniform (So = 1.03). When the grain size distribution is more heterogeneous (So = 1.14) these trends are less obvious

suggesting that grain size distribution is likely more relevant than the proportions of an individual mineral.

The CI ratio for models with different mineralogy in various grain size distributions is also summarized in Table 8. In models with uniform grain size distribution, the CI stress ratio is ranging from 0.41 to 0.47 (average of 0.44 \pm 0.03). On the other hand, in models with non-uniform grain size distribution, the CI stress ratio ranges from 0.37 to 0.47 (average of 0.41 \pm 0.03).

5.5 Summary

The results from the numerical grain-based models have provided insights into the mechanism associated with the laboratory findings by Nicksiar and Martin (2012) and summarized in Fig. 1. For the low porosity rocks simulated, tensile cracking is the dominant mode of damage that occurs as compressive loads are applied to a heterogeneous assemblage of mineral grains. Figure 16 summarizes the results from the 20 grain-based models. For the range of factors investigated in these models, the heterogeneity created by grain size distributions has the greatest impact on peak strength and CI stress. However, despite the range in peak strength is relatively well constrained.

6 Conclusion

Crack initiation in the uniaxial compressive loading rocks occurs well before the peak strength is reached. The factors that may influence the onset of cracking and possible initiating mechanisms were explored using a discrete element numerical approach. The numerical approach was based on grain-based model that utilized a Voronoi tessellation scheme to represent low porosity crystalline rocks such as granite. The porosity of the models ranges between 0.1 and 1 % representing the porosity of many igneous rock properties. This approach enabled complete tracking of the failure process along the mineral grain boundaries.

The two possible models for CI observed in laboratory compression tests are as follows: (1) sliding (shear) crack model and (2) force-chain (tensile) crack model. The numerical models simulating the low porosity crystalline rock showed that CI is a phenomenon associated with tensile cracking. This tensile cracking dominates at the early stages of loading. Shear cracking was only observed after CI occurred and in most of the models was only prominent near the peak strength.

The effect of grain size distribution (sorting coefficient ranging from 1.5 to 1.03), grain size (ranging from 0.75 to 2.25 mm), and the heterogeneities of different mineral grains (quartz, K-Feldspar, plagioclase) were examined.

 Table 8
 Crack initiation stress

 and peak stress in models with
 different mineralogy and grain

size distributions



Fig. 15 Crack initiation stress and peak stress in models with different mineralogy and grain size distributions

Model (Qtz/Pl/K-Fel)	CI—Poisson's ratio (MPa)	CI—LSR (MPa)	UCS (MPa)	Average CI and	d normalized cracks (%)
So = 1.03					
1 (55/10/35)	93	102	207	98, 1.7	0.49
2 (55/35/10)	72	87	195	80, 1.2	0.45
3 (35/10/55)	79	95	206	87, 1.0	0.46
4 (35/55/10)	75	81	179	78, 1.2	0.45
5 (10/35/55)	65	72	159	69, 1.5	0.45
6 (10/55/35)	76	74	161	75, 0.9	0.46
So = 1.14					
1 (55/10/35)	63	63	158	63, 1.1	0.40
2 (55/35/10)	54	58	139	56, 1.2	0.42
3 (35/10/55)	75	64	167	70, 1.0	0.38
4 (35/55/10)	74	67	174	71, 1.2	0.39
5 (10/35/55)	64	61	133	63, 1.3	0.46
6 (10/55/35)	64	57	163	61, 1.5	0.35



Fig. 16 Relation between crack initiation stress (*CI*) and peak uniaxial compressive strength (*UCS*) found in the grain-based numerical models

Modelling revealed that the grain size distribution had the most significant effect on peak strength and CI stress. The peak strength ranges from 140 to 208 MPa as the grain size distribution varies from heterogeneous to uniform, respectively. However, the ratio of CI to peak stress showed only minor variation, as the heterogeneity decreases. The results suggest that grain size has no effect on the peak uniaxial strength provided the grain size distribution is held constant. The CI stresses were also found to be relatively insensitive to grain size, ranging from 97 MPa for fine grain, 94 MPa for fine-medium grain to 101 MPa for medium grain (Table 7). The other factors investigated had only minor effects on CI and peak strength, and CI ratio. The findings from these investigations suggest that CI is a consequence of the heterogeneity of the materials and that the geometrical heterogeneity has the greatest influence. The findings also help to explain why the ratio of CI stress to peak strength in laboratory uniaxial compression tests is usually found to be approximately 0.45 ± 0.05 for a wide range of low porosity rocks. In other words, the CI stress ratio is almost constant in models with different characteristics.

The application of our two dimensional models to simulate phenomena observed in cylindrical laboratory samples has limitations. All of our models were constructed to simulate low porosity granite-like rocks, and CI in these models was only compared to a Sliding-crack and Force-chain crack model. Other models not considered may be valid. Future research should examine, using a three dimensional model, the effect of heterogeneity on other rock types, including those with porosities higher than that found in our granite-like rock.

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