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TECHNICAL NOTE

Slope Equivalent Mohr–Coulomb Strength Parameters for Rock Masses Satisfying the Hoek–Brown Criterion

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List of symbols

c_t	Intercept of the straight line to τ-axes
D	Disturbance coefficient varying from 0.0 to 1.0
f_1–f_3	Non-dimensional functions
H	Height of inclined rock slope
k	Cohesion coefficient
L	Distance between the sliding surface at the top and the edge
m	Material constant
m_i	Material constant
n	Material constant
N_L	Stability factor for linear failure criterion
N_n	Stability factor for nonlinear failure criterion
R_0	The initial radius of the log-spiral in Fig. 1
s	Material constant
v_t	Velocity at velocity discontinuity
α	Angle of the slope in Fig. 1
β	Angle of the slope in Fig. 1
γ	Unit weight of the rock mass
θ_0	Angle related horizontal line to line OB in Fig. 1
θ_h	Angle related horizontal line to line OC in Fig. 1
φ_t	Tangent friction angle
σ_1	Maximum principal stress
σ_3	Minimum principal stress
σ_c	Uniaxial compressive stress
σ_n	Normal component of the failure surface
τ	Shear component of the failure surface

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1 Introduction

At present, slope stability analysis is often based on limit equilibrium methods. To determine the slope stability factors, the common technique used for design is the slice technique where the slope is divided into slices. In order to solve the stability problem, some assumptions regarding the location and inclination of forces between slices are made for static equilibrium equations of such slices to be solvable. Due to the arbitrary assumptions regarding the inter-slice forces, the solutions using the limit equilibrium method cannot be regarded as rigorous in a strict mechanical sense.

Recently, the upper bound theorem approach of limit analysis is widely used (Chen 1975; Donald and Chen 1997; Lysmer 1970; Wang and Yin 2002; Yu et al. 1998). The theorem states that the rate of work done by the actual forces is less than or equal to the rate of energy dissipation in any kinematically admissible velocity field. By constructing a kinematically admissible velocity field, it is possible to find out the upper bound to the true limit load, and, theoretically, by constructing various kinematically admissible velocity fields, the lowest possible upper bound solution can be found. A kinematically admissible velocity field is governed by the normality rule and is compatible with the velocities at the boundary of the rock mass. The solution obtained from the upper bound theorem is rigorous in that no additional assumption regarding the inter-slice forces is required.

As well known, the Mohr–Coulomb (MC) failure criterion is widely used in rock engineering. This is due to the fact that most computer codes, design practice and

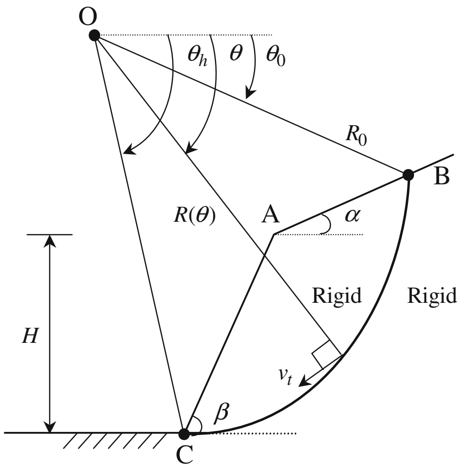

Fig. 1 Failure mechanism for a homogeneous slope

standards, which are currently employed for evaluating the stability of rock structures, are formulated in terms of a MC failure criterion. However, evidence shows that the nonlinear Hoek–Brown (HB) failure criterion is able to represent fairly satisfactorily the failure of almost all types of rocks (Agar et al. 1985; Cai et al. 2007; Cai 2007; Collins et al. 1988; Goodman 1989; Hobbs 1966; Hoek et al. 2002; Kennedy and Lindberg 1978). Consequently, a question, which arises in practice, is how to estimate the equivalent MC strength parameters when the failure criterion is nonlinear. In order to solve the problem, a method for evaluating the equivalent MC friction angle and cohesion have been proposed for tunnel stability when the rock mass follows the nonlinear HB failure criterion (Sofianos and Halakatevakis 2002; Sofianos 2003; Sofianos and Nomikos 2006). Yang and Yin (2006) calculated the equivalent MC friction angle and cohesion for a strip footing resting on a rock mass which follows the nonlinear HB criterion.

In order to avoid the difficulties resulting from nonlinearity, this paper applies the kinematical theorem of limit analysis to calculate the equivalent MC cohesion coefficient and friction angle for rock slopes when the rock failure is governed by a modified HB failure criterion. A logarithmic spiral failure mechanism is employed in upper bound computations. Equivalent MC cohesion coefficient and friction angle from HB failure criterion for five rock mass types are presented in a tabular form for practical use in rock engineering.

2 Hoek–Brown Failure Criterion

In rock engineering, the MC failure criterion is widely used. However, according to a large number of triaxial experiments on a variety of rock types with varying degrees of fracturing, Hoek and Brown proposed the HB failure criterion in 1980. This criterion in its revised form is written as (Hoek et al. 2002):

$$\sigma_1 - \sigma_3 = \sigma_c[m\sigma_3/\sigma_c + s]^n \tag{1}$$

where σ_1 is the major principal stress at failure, σ_3 is the minor principal stress at failure, σ_c is the rock uniaxial compressive strength. The material parameters m, n and s can be estimated on the basis of the geological strength index (GSI) which characterizes the quality of the rock mass. GSI depends on the rock mass structure and the surface condition of the joints. Hoek et al. (2002) revised the parameter determination using its generalized exponential form:

$$\frac{m}{m_i} = \exp\left(\frac{\mathrm{GSI} - 100}{28 - 14D}\right) \tag{2}$$

$$s = \exp\left(\frac{\mathrm{GSI} - 100}{9 - 3D}\right) \tag{3}$$

$$n = \frac{1}{2} + \frac{1}{6}\left[\exp\left(-\frac{\mathrm{GSI}}{15}\right) - \exp\left(-\frac{20}{3}\right)\right] \tag{4}$$

where D is a disturbance coefficient which varies from 0.0 for the undisturbed in situ rock masses to 1.0 for very disturbed rock masses. The parameter m_i can be obtained from triaxial testing of rock. If there are no test data available, the approximate values for five rock types can be estimated as follows Hoek (1990): $m_i \approx 7$, for carbonate rocks with well developed crystal cleavage (dolomite, limestone and marble); (b) $m_i \approx 10$, for lithified argillaceous rocks (mudstone, siltstone shale and slate); (c) $m_i \approx 15$, for arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone and quartzite); (d) $m_i \approx 17$, for fine-grained polymінerallic igneous crystalline rocks (andesite, dolerite, diabase and rhyolite); and (e) $m_i \approx 25$, for coarse-grained polymінerallic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite and quartz-diorite).

In the (σ_n, τ) stress space, the modified HB failure criterion can be drawn as a nonlinear curve. The tangent line to the curve is expressed as

$$\tau = c_t + \sigma_n \tan\varphi_t \tag{5}$$

where φ_t and c_t are the tangent friction angle and the intercept of the straight line to τ-axes, respectively. Normal stress σ_n and intercept c_t are functions of φ_t reflecting the location of tangency point, are given in the following form (Yang and Yin 2006):

$$\frac{\sigma_n}{\sigma_c} = \left(\frac{1}{m} + \frac{\sin\varphi_t}{mn}\right)\left[\frac{mn(1 - \sin\varphi_t)}{2\sin\varphi_t}\right]^{\left(\frac{1}{1-n}\right)} - \frac{s}{m} \tag{6}$$

$$\frac{c_t}{\sigma_c} = \frac{\cos\varphi_t}{2}\left[\frac{mn(1-\sin\varphi_t)}{2\sin\varphi_t}\right]^{\left(\frac{n}{1-n}\right)} - \frac{\tan\varphi_t}{m}\left(1+\frac{\sin\varphi_t}{n}\right) \times \left[\frac{mn(1-\sin\varphi_t)}{2\sin\varphi_t}\right]^{\left(\frac{1}{1-n}\right)} + \frac{s}{m}\tan\varphi_t \tag{7}$$

where c_t/σ_c is defined as cohesion coefficient $k = c_t/\sigma_c$. The modified HB failure criterion can be applied to intact rock, or to a rock mass where a sufficient number of closely spaced discontinuities is present so that an isotropic behavior involving failure on discontinuities can be assumed. The HB failure criterion cannot be applied to rock masses containing only a few discontinuities or characterized by an anisotropic behavior. Unless specially mentioned in this paper, the basic idea of isotropy and homogeneity is adopted for computing the magnitude of the equivalent MC strength parameters for rock slopes.

3 Equivalent Parameters for Rock Slopes

In the present study, instead of the modified HB failure criterion, the MC failure criterion as given in Eq. 5, which is chosen to be tangent to the modified HB failure criterion, is employed to calculate the rate of external work and internal energy dissipation, although the location of the tangency point is not known.

Due to the use of the MC failure criterion, the kinematical admissibility condition requires the failure surface for a rigid collapse to be a log-spiral surface or a plane surface. Figure 1 shows a log-spiral failure mechanism with failure surface passing through the slope toe. In this failure mechanism, the log-spiral line BC separating the volume ABC from the rest of structure that is kept motionless, is a velocity discontinuity line. The volume ABC rotates about point O with an angular velocity. The velocity discontinuity at any point of BC makes an angle φ_t with the tangent line at the same point.

Since there is no deformation inside ABC (rigid-body motion), the internal energy is only dissipated along the velocity discontinuity BC. Equating the work rate of external force to the internal energy dissipation rate, the objective function is:

$$H = \frac{c_t}{\gamma}\cdot\frac{\sin\beta\{\exp[2(\theta_h-\theta_0)\tan\varphi_t]-1\}}{2\sin(\beta-\alpha)\tan\varphi_t(f_1-f_2-f)_3} \cdot\{\sin(\theta_h+\alpha)\exp[(\theta_h-\theta_0)\tan\varphi_t]-\sin(\theta_0+\alpha)\} \tag{8}$$

where c_t is a function of φ_t reflecting the location of tangency point, determined by Eq. 7, the functions f_1–f_3 depend on geometry parameters θ_h, θ_0 and tangent line angle, which are reported in the Appendix, and α and β are the geometrical parameters of slope, respectively.

The logarithmic spiral failure surface, as shown in Fig. 1, is controlled by three variables φ_t, θ_h and θ_0. Herein, the sequential quadratic programming is employed to optimize the objective function 8 with respect to φ_t, θ_h and θ_0, to get a least upper bound for the critical height of the inclined rock slope. When Eq. 8 is minimized, we can obtain the values of φ_t, θ_h and θ_0, and the failure surface is determined. With the friction angle obtained, the equivalent MC cohesion coefficient is calculated using Eq. 7.

With a linear MC failure criterion, the values of c_t and φ_t are known, and some researchers investigated the stability problem of soil and rock slopes with the limit analysis of plasticity (Chen 1975; Yu and Sloan 1997; Yu et al. 1998). Chen (1975) presented the mathematical development for computing the critical height. In the present analysis, the derivation of Eq. 8 is similar to Chen (1975). However, the values of c_t and φ_t are not specified.

4 Numerical Results

In practical engineering, the critical height of a rock slope is always invariable, whether the rock mass obeys the MC or HB failure criterion. The critical height computed from a linear MC failure criterion, which always circumscribes the actual nonlinear failure criterion, will be an upper bound value (Chen 1975). This is due to the fact that the strength of the circumscribing tangent line failure criterion is equal or larger than that of the actual nonlinear HB failure criterion.

In this paper, the critical height of the rock slope using the nonlinear HB failure criterion is equal to the critical height of the rock slope obtained by using the tangent line failure criterion. Therefore, the tangent line failure criterion is the equivalent failure criterion. The equivalent MC strength parameters are obtained by optimization. When Eq. 8 is minimized, the least critical height and the corresponding MC friction angle are obtained. With the MC friction angle obtained, the cohesion coefficient is calculated using Eq. 7.

In the following analysis, the equivalent MC cohesion coefficient and friction angle for rock slopes are calculated with the modified HB failure criterion proposed by Hoek et al. (2002).

4.1 Comparisons

Collins et al. (1988) proposed a technique to evaluate the nonlinear stability factors N_n by using the linear stability factors N_L previously given by Chen (1975). They related the stability factor N_n to N_L by the form: $N_n = N_L(\varphi_t)c_t(\varphi_t)/\left(s^{0.5}\sigma_c\right)$. The least upper bound solution is minimized with respect to φ_t, which appears in c_t (φ_t) and $N_L(\varphi_t)$ by Lagrangean polynomial approximation. Employing the stability factor N_L given by Chen (1975) and Collins et al.

(1988) presented the stability factors using the original HB failure criterion proposed by Hoek and Brown in 1980.

In order to validate Eq. 8, the results obtained by using this equation and given by Collins et al. (1988) are shown in Table 1 corresponding to the same parameter values of n, α, β, s and m. From Table 1, also shown in Yang et al. (2004), it is found that the numerical results using Eq. 8 are almost equal to those of Collins et al. (1988). The maximum error is less than 2%. Therefore, Eq. 8 is an effective expression for evaluating the stability of rock slopes with the modified HB failure criterion.

4.2 Effects of Slope Angle α

Table 2 gives the equivalent MC friction angles and the equivalent MC cohesion coefficients with the parameter $\beta = 70°$, $D = 0$, $m_i = 15$, GSI $= 40$ and α being equal to 0, 5, 10, 15 and 20°. From Table 2, it is found that the slope angle α has a small influence on the equivalent MC strength parameters (Example One). In Table 2, for the case $\alpha = 0°$, the equivalent friction angle and cohesion are 53.59° and $8.89 \times 10^{-3}\sigma_c$, respectively, while the equivalent friction

Table 1 Comparison between present stability factors and solutions of Collins et al. (1988)

β (°)	Parameters s, m				
	$s = 1.0$, $m = 15.7$	$s = 0.1$, $m = 6.6388$	$s = 0.004$, $m = 1.7117$	$s = 0.0001$, $m = 0.2822$	$s = 0.00001$, $m = 0.0786$
90					
Present solutions	1.93	1.93	1.93	1.93	1.93
Collins et al.	1.93	1.93	1.94	1.94	1.93
80					
Present solutions	3.09	3.31	3.56	3.61	3.47
Collins et al.	3.09	3.31	3.56	3.60	3.45
70					
Present solutions	5.10	5.94	6.92	7.11	6.56
Collins et al.	5.11	5.95	6.90	7.05	6.48
60					
Present solutions	8.78	10.97	13.57	14.07	12.61
Collins et al.	8.80	10.97	13.49	13.89	12.38
50					
Present solutions	15.32	19.95	25.34	26.37	23.26
Collins et al.	15.36	19.93	25.18	26.00	22.89
45					
Present solutions	20.22	26.60	34.00	35.41	31.28
Collins et al.	20.28	26.64	33.89	35.01	30.73

$\alpha = 0°$ and $n = 0.5$

Table 2 Effects of slope angle α on equivalent MC strength parameters (Example One)

α	0°	5°	10°	15°	20°
Friction angles	53.59°	53.56°	53.52°	53.48°	53.42°
Cohesion coefficients	8.89×10^{-3}	8.91×10^{-3}	8.94×10^{-3}	8.97×10^{-3}	9.02×10^{-3}

$\beta = 70°$, $D = 0$, GSI $= 40$ and $m_i = 15$

Table 3 Effects of slope angle α on equivalent MC strength parameters (Example Two)

α	0°	5°	10°	15°	20°
Friction angles	45.26°	45.22°	45.16°	45.09°	44.99°
Cohesion coefficients	15.69×10^{-3}	15.74×10^{-3}	15.81×10^{-3}	15.89×10^{-3}	15.99×10^{-3}

$\beta = 65°$, $D = 0.2$, GSI $= 50$ and $m_i = 10$

Table 4 Effects of disturbance coefficient D on equivalent MC strength parameters

D	0.0	0.1	0.2	0.3	0.4	0.5
Friction angles	46.22°	46.21°	46.19°	46.11°	45.97°	45.74°
Cohesion coefficients	14.23×10^{-3}	12.91×10^{-3}	11.64×10^{-3}	10.42×10^{-3}	9.25×10^{-3}	8.14×10^{-3}
D	0.6	0.7	0.8	0.9	1.0	
Friction angles	45.39°	44.88°	44.14°	43.08°	41.54°	
Cohesion coefficients	7.09×10^{-3}	6.11×10^{-3}	5.21×10^{-3}	4.39×10^{-3}	3.66×10^{-3}	

$\beta = 70°$, GSI = 50, $\alpha = 0°$ and $m_i = 7$

Table 5 Equivalent MC friction angles for five types of rocks

m_i	β (°)	Friction angles (°)						
		GSI = 10	GSI = 20	GSI = 30	GSI = 40	GSI = 50	GSI = 60	GSI = 70
7	85	55.71	58.05	57.59	55.89	53.56	50.85	47.90
	80	54.18	56.10	55.48	53.79	51.54	48.96	46.16
	75	52.31	53.66	52.86	51.22	49.13	46.75	44.17
	70	49.94	50.56	49.59	48.06	46.22	44.13	41.84
	65	46.95	46.76	45.64	44.29	42.77	41.04	39.12
	60	43.30	42.37	41.15	40.01	38.83	37.51	36.01
	55	39.11	37.62	36.41	35.47	34.59	33.64	32.55
10	85	60.39	62.41	61.98	60.46	58.36	55.91	53.22
	80	58.65	60.19	59.58	58.05	56.05	53.73	51.18
	75	56.40	57.26	56.43	54.97	53.15	51.06	48.76
	70	53.42	53.43	52.41	51.08	49.55	47.81	45.85
	65	49.61	48.76	47.60	46.47	45.28	43.94	42.40
	60	45.08	43.60	42.38	41.43	40.54	39.59	38.48
	55	40.16	38.33	37.12	36.31	35.66	35.01	34.27
15	85	65.05	66.71	66.30	64.99	63.18	61.04	58.67
	80	63.03	64.14	63.52	62.20	60.49	58.50	56.29
	75	60.23	60.51	59.65	58.42	56.94	55.23	53.71
	70	56.39	55.72	54.67	53.59	52.44	51.12	49.60
	65	51.56	50.15	48.98	48.06	47.22	46.30	45.22
	60	46.19	44.35	43.15	42.35	41.72	41.11	40.40
	55	40.74	38.72	37.53	36.82	36.33	35.91	35.47
17	85	66.34	67.90	67.50	66.24	64.52	62.49	60.22
	80	64.23	65.20	64.58	63.32	61.71	59.83	57.73
	75	61.23	61.33	60.47	59.30	57.93	56.34	54.54
	70	57.09	56.23	55.18	54.17	53.13	51.94	50.57
	65	51.97	50.42	49.26	48.40	47.64	46.83	45.89
	60	46.40	44.49	43.29	42.53	41.96	41.42	40.83
	55	40.83	38.79	37.60	36.91	36.46	36.09	35.72
25	85	69.93	71.17	70.79	69.71	68.24	66.51	64.56
	80	67.48	68.02	67.34	66.33	64.99	63.44	61.69
	75	63.75	63.30	62.42	61.47	60.43	59.23	57.83
	70	58.64	57.33	56.28	55.47	54.72	53.91	52.96
	65	52.76	50.96	49.82	49.09	48.53	48.01	47.12
	60	46.77	44.75	43.57	42.88	42.43	42.08	41.72
	55	41.01	38.91	37.74	37.09	36.71	36.44	36.21

Table 6 Equivalent MC cohesion coefficients for five types of rocks ($\times 10^{-3}$)

m_i	β (°)	Cohesion coefficients						
		GSI = 10	GSI = 20	GSI = 30	GSI = 40	GSI = 50	GSI = 60	GSI = 70
7	85	0.45	1.15	2.52	5.11	9.98	19.14	36.35
	80	0.48	1.26	2.78	5.61	10.84	20.54	38.61
	75	0.53	1.45	3.22	6.41	12.15	22.62	41.82
	70	0.61	1.78	3.96	7.72	14.23	25.77	46.54
	65	0.76	2.36	5.22	9.87	17.53	30.63	53.56
	60	1.03	3.35	7.30	13.32	22.72	38.08	64.07
	55	1.50	4.96	10.56	18.63	30.62	49.31	79.75
10	85	0.38	0.99	2.17	4.41	8.62	16.54	31.48
	80	0.42	1.13	2.49	5.01	9.64	18.21	34.16
	75	0.49	1.38	3.07	6.06	11.35	20.88	38.27
	70	0.61	1.87	4.15	7.93	14.27	25.26	44.72
	65	0.85	2.78	6.09	11.17	19.19	32.39	54.91
	60	1.31	4.36	9.30	16.46	27.08	43.69	70.77
	55	3.12	6.93	14.31	24.54	39.08	60.81	94.70
15	85	0.32	0.84	1.84	3.74	7.30	14.00	26.65
	80	0.37	1.01	2.24	4.48	8.56	16.05	29.92
	75	0.46	1.38	3.08	5.98	10.95	19.71	35.47
	70	0.67	2.18	4.80	8.89	15.41	26.27	44.97
	65	1.11	3.72	7.95	14.10	23.23	37.54	60.88
	60	1.95	6.39	13.19	22.57	35.85	55.60	86.28
	55	3.44	10.67	21.21	35.34	54.78	82.78	124.59
17	85	0.31	0.80	1.75	3.55	6.94	13.30	25.31
	80	0.36	0.98	2.19	4.36	8.29	15.49	28.80
	75	0.47	1.41	3.14	6.04	10.96	19.56	34.93
	70	0.71	2.34	5.13	9.40	16.08	27.05	45.71
	65	1.24	4.15	8.80	15.42	25.12	40.04	64.05
	60	2.26	7.28	14.85	25.18	39.64	60.87	93.36
	55	4.04	12.27	24.10	39.84	61.36	92.05	137.45
25	85	0.26	0.68	1.51	3.06	5.95	11.39	21.63
	80	0.33	0.93	2.08	4.10	7.68	14.14	25.96
	75	0.50	1.59	3.53	6.59	11.55	19.91	34.47
	70	0.93	3.12	6.68	11.82	19.43	31.32	50.69
	65	1.87	6.08	12.46	21.18	33.43	51.44	79.11
	60	3.66	11.13	21.87	36.13	55.63	83.40	124.35
	55	6.78	19.10	36.12	58.39	88.46	130.63	191.47

angle and cohesion are 53.42° and $9.02 \times 10^{-3}\ \sigma_c$ at $\alpha = 20°$. The absolute difference of the equivalent friction angle is 0.17°, and the absolute difference of the equivalent cohesion is $0.03 \times 10^{-3}\ \sigma_c$. The same phenomenon can also be found in Table 3 corresponding to $\beta = 65°$, $D = 0.2$, $m_i = 10$ and GSI = 50 (Example Two).

4.3 Effects of Disturbance Coefficient D

Table 4 gives the equivalent MC friction angles and cohesion coefficients corresponding to $m_i = 7$ and GSI = 50, with the disturbance coefficient D ranging from 0.0 to 1.0, with 0.1 interval. From Table 4, it is found that the equivalent MC strength parameters decrease as the disturbance coefficient D increases when parameters β, GSI, α and m_i are constant.

4.4 Design Tables

Based on the modified HB failure criterion, shown in Eqs. 1–4, the equivalent MC cohesion coefficient and friction angle are obtained when Eq. 8 is minimized. Tables 5 and 6 give the values of equivalent MC strength parameters for five rock types with the parameters $\alpha = 0°$, $D = 0$ and β varying from 85 to 55°. These values are presented to meet the demands of software, design practice and standards written in terms of the MC failure criterion.

5 Conclusions

The upper bound theorem of limit analysis is employed to calculate the equivalent MC cohesion coefficient and friction angle to meet the demands of software written in terms of the MC failure criterion when the rock mass failure follows the modified HB failure criterion for slopes in limit state. A MC failure criterion, which is tangent to the actual modified HB failure criterion, is used to calculate the rate of external work and internal energy dissipation. Equating the work rate of external forces to the internal energy dissipation rate, we can obtain the objective function. The equivalent MC friction angle is obtained when the objective function is minimized. The equivalent MC cohesion is calculated with Eq. 7. Based on the analysis above, the following conclusions are drawn:

(a) For rock slopes, the stability factors obtained from Eq. 8 are nearly the same as previously available, which shows that Eq. 8 is an effective expression for evaluating the equivalent MC strength parameters when rock mass failure follows the modified HB failure criterion. Equivalent MC cohesion coefficient and friction angle for five rock types are presented for practical use in rock engineering.

(b) The equivalent MC friction angles and cohesion depend not only on the nonlinear HB failure criterion but also on the rock slope angles, α and β.

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Appendix

$$f_1 = \frac{(3\tan\varphi_t\cos\theta_h + \sin\theta_h)\exp[3(\theta_h-\theta_0)\tan\varphi_t] - (3\tan\varphi_t\cos\theta_0 + \sin\theta_0)}{3(1+9\tan^2\varphi_t)} \tag{9}$$

$$f_2 = \frac{1}{6}\frac{L}{r_0}\left(2\cos\theta_0 - \frac{L}{r_0}\cos\alpha\right)\sin(\theta_0+\alpha) \tag{10}$$

$$f_3 = \frac{\exp[(\theta_h-\theta_0)\tan\varphi_t]}{6}\cdot\left[\sin(\theta_h-\theta_0) - \frac{L}{r_0}\sin(\theta_h+\alpha)\right] \times\left\{\cos\theta_0 - \frac{L}{r_0}\cos\alpha + \cos\theta_h\cdot\exp[(\theta_h-\theta_0)\tan\varphi_t]\right\} \tag{11}$$

$$\frac{L}{r_0} = \frac{\sin(\theta_h-\theta_0)}{\sin(\theta_h+\alpha)} - \frac{\sin(\theta_h+\beta)}{\sin(\theta_h+\alpha)\sin(\beta-\alpha)} \cdot\{\exp[(\theta_h-\theta_0)\tan\varphi_t]\sin(\theta_h+\alpha) - \sin(\theta_0+\alpha)\} \tag{12}$$

References

Agar JG, Morgenstern NR, Scott J (1985) Shear strength and stress–strain behaviour of Athabasca oil sand at elevated temperatures and pressure. Can Geotech J 24(1):1–10

Cai M (2007) Back-analysis of rock mass strength parameters using AE monitoring data. Int J Rock Mech Mining Sci 44(4):538–549

Cai M, Kaiser PK, Tasaka Y, Minami M (2007) Determination of residual strength of jointed rock masses using the GSI system. Int J Rock Mech Mining Sci 44(2):247–265

Chen WF (1975) Limit analysis and soil plasticity. Elsevier Science, Amsterdam

Collins IF, Gunn CI, Pender MJ, Yan W (1988) Slope stability analyses for materials with nonlinear failure envelope. Int J Numer Anal Methods Geomech 12(6):533–550

Donald IB, Chen ZY (1997) Slope stability analysis by the upper bound approach: fundamental and method. Can Geotech J 34(6):853–862

Goodman RE (1989) Introduction to rock mechanics, 2nd edn. Wiley, New York

Hobbs D (1966) A study of the behaviour of broken rock under triaxial compression and its application to mine roadways. Int J Rock Mech Mining Sci 3:11–43

Hoek E (1990) Estimating Mohr–Coulomb friction and cohesion values from the Hoek–Brown failure criterion. Int J Rock Mech Mining Sci 27(3):227–229

Hoek E, Carranze-Torres C, Corkum B (2002) Hoek–Brown failure criterion-2002 edition. In: Proceedings of the North American Rock Mechanics Society Meeting, pp 267–273

Kennedy TC, Lindberg HE (1978) Tunnel closure for nonlinear Mohr Coulomb functions. J Eng Mech Div 104(EM6):1313–1326

Lysmer J (1970) Limit analysis of plane problems in soil mechanics. ASCE J Soil Mech Found Div 96:1311–1334

Sofianos AI (2003) Tunnelling Mohr–Coulomb strength parameters for rock masses satisfying the generalized Hoek–Brown criterion. Int J Rock Mech Mining Sci 40(5):435–440

Sofianos AI, Halakatevakis N (2002) Equivalent tunnelling Mohr–Coulomb strength parameters from given Hoek–Brown ones. Int J Rock Mech Mining Sci 39(1):131–137

Sofianos AI, Nomikos PP (2006) Equivalent Mohr–Coulomb and generalized Hoek–Brown strength parameters for supported axisymmetric tunnels in plastic or brittle rock. Int J Rock Mech Mining Sci 43(5):683–704

Wang YJ, Yin JH (2002) Wedge stability analysis considering dilatancy of discontinuities. Rock Mech Rock Eng 25(2):127–137

Yang XL, Yin JH (2006) Linear Mohr–Coulomb strength parameters from the nonlinear Hoek–Brown rock masses. Int J Non-Linear Mech 41(8):1000–1005

Yang XL, Li L, Yin JH (2004) Seismic and static stability analysis for rock slopes by a kinematical approach. Geotechnique 54(8):543–549

Yu HS, Sloan SW (1997) Finite element limit analysis of reinforced soils. Comput Struct 63:567–577

Yu HS, Salgado R, Sloan SW (1998) Limit analysis versus limit equilibrium for slope stability. ASCE J Geotech Geoenviron Eng 124(3):265–276