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TECHNICAL NOTE

Mechanical Behavior of Carbonate Rocks at Crack Damage Stress Equal to Uniaxial Compressive Strength

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1 Introduction

A number of researchers such as Brace et al. (1966), Bieniawski (1967), Brady and Brown (1993), Martin and Chandler (1994), Hatzor and Palchik (1997), Pettitt et al. (1998), Eberhardt et al. (1999), Katz and Reches (2004) and Cai et al. (2004) have investigated different stages of stress–strain behavior of brittle rocks during uniaxial compression. Two stress–strain diagrams in Fig. 1 obtained by the author for carbonate rocks, exhibit three main characteristic stress levels (crack initiation stress σ_{ci}, crack damage stress σ_{cd} and uniaxial compressive strength σ_c) which are represented by points *A*, *B* and *C*, respectively, over the total volumetric strain curve. The crack initiation stress (σ_{ci}, point *A*) is the stress level at which microfracturing begins. The point *A* is the end of the elastic stage (linear portion) of stress–strain curve. The crack damage stress (σ_{cd}, point *B*) is the stress at the onset of dilation: when σ_{cd} is attained, the rock volume begins to increase (Schock et al. 1973; Brace 1978; Paterson 1978; Palchik and Hatzor 2002). The crack damage stress (σ_{cd}, point *B*) is the stress level at which maximum total volumetric strain (ε_{cd}) is attained. The uniaxial compressive strength (σ_c, point *C*) is the maximum axial stress (at failure). In Fig. 1, strain ε_{cd} at σ_{cd} (point *B*) is the maximum total volumetric strain, and strain $\varepsilon_{a\ max}$ at σ_c is the maximum axial strain.

Martin and Chandler (1994), Eberhardt et al. (1999) and Palchik and Hatzor (2002) have shown that the crack damage stress (σ_{cd}) is defined as the point (see point *B* in Fig. 1a) where a total volumetric strain reversal occurs and unstable crack growth begins. In this case (see Fig. 1a), crack damage stress σ_{cd} is lower than the uniaxial compressive strength (σ_c). Indeed, Brace et al. (1966), Bieniawski (1967), Martin (1993), Pettitt et al. (1998), Eberhardt et al. (1999), Heo et al. (2001) and Katz and Reches (2004) have found that the crack damage stresses σ_{cd} of granites, sandstones and quartzite vary from $0.71\sigma_c$ to $0.84\sigma_c$. They have also shown that the ratios σ_{ci}/σ_c and σ_{ci}/σ_{cd} for above-mentioned rocks range from 0.39 to 0.6 and from 0.52 to 0.82, respectively.

Hatzor and Palchik (1997) and Palchik and Hatzor (2002) have shown that there exist total volumetric strain curves (in heterogeneous carbonate rocks) which do not have any point of reversal, with the maximum total volumetric strain (ε_{cd}) attained at the uniaxial compressive strength (σ_c). In this case, crack damage stress is equal to uniaxial compressive strength (i.e. $\sigma_{cd} = \sigma_c$, $B = C$ in Fig. 1b). Thus, there are two types of total volumetric strain curves in brittle rocks: type 1 (see Fig. 1a), with a point of reversal (*B*) in the total volumetric strain curve, and $\sigma_{cd} < \sigma_c$; and type 2 (see Fig. 1b), where the total volumetric strain curve has no reversal point and, therefore, $\sigma_{cd} = \sigma_c$.

Type 1 (Fig. 1a) curves have been studied by Brace et al. (1966), Bieniawski (1967), Pettitt et al. (1998), Eberhardt et al. (1999), Heo et al. (2001), Katz and Reches (2004), etc., whereas little attention has been paid to type 2 mechanical behavior of brittle rocks (Fig. 1b). In this paper, we intend to study relations between characteristic compressive stress levels, strains and mechanical properties of heterogeneous carbonate rocks exhibiting type 2 behavior of the total volumetric strain curve (i.e. at $\sigma_{cd} = \sigma_c$).

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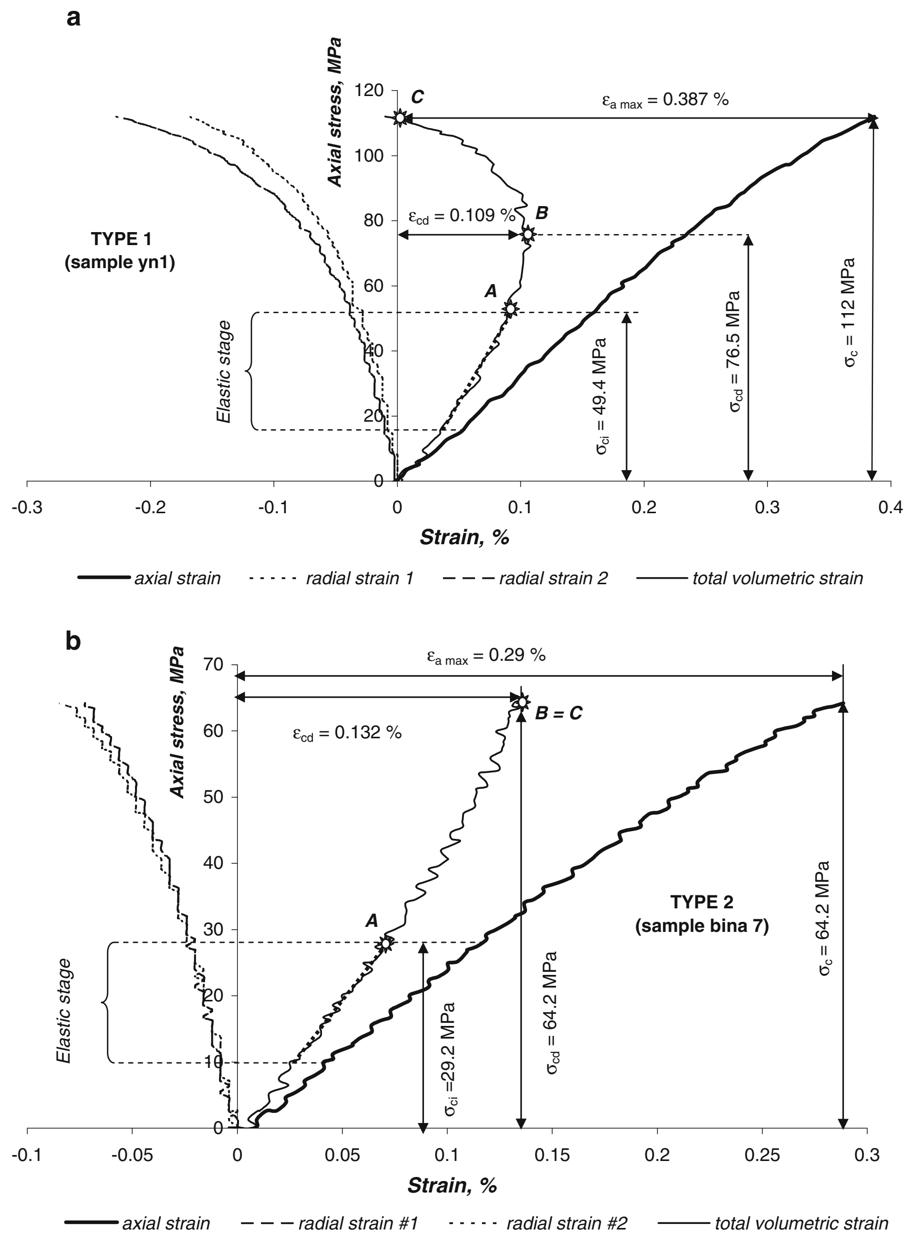

Fig. 1 Stress–strain behavior of brittle rocks during uniaxial compression. Crack initiation stress (σ_{ci}), crack damage stress (σ_{cd}) and uniaxial compressive strength (σ_c) are represented by *points* *A*, *B* and *C*, respectively. The total volumetric strain (ε_v) is calculated as a sum of the component strains: $\varepsilon_v = \varepsilon_a + \varepsilon_{R1} + \varepsilon_{R2}$, where ε_a is axial strain, and ε_{R1} and ε_{R2} are radial strains measured in orthogonal directions (Palchik and Hatzor 2002): **a** type 1—total volumetric strain curve has a reversal point (*B*) and $\sigma_{cd} < \sigma_c$; **b** type 2—there is no reversal point in total volumetric strain curve, and $\sigma_{cd} = \sigma_c$

2 Testing and test results

Mechanical properties of carbonate rock samples exhibiting type 2 behavior of the total volumetric strain curve are summarized in Table 1. These samples were collected from Adulam chalk, Aminadav dolomite, Bina limestone, Yarka limestone, Yagur dolomite, and Nekorot limestone formations. The NX ($d = 54$ mm) sized cylindrical rock samples having the ratio $L/d = 2$ (here L and d are the length and diameter of a sample, respectively) were prepared. The samples were ground to the planeness of 0.01 mm and cylinder perpendicularity within 0.05 radians. Prior to testing, rock samples were oven dried at the temperature of 110°C for 24 h.

Uniaxial compressive tests were performed at the Rock Mechanics Laboratory of the Ben-Gurion University. The tests were conducted using a load frame (TerraTek system, model FX-S-33090) at a constant strain rate of 10^{-5}/s. The load frame operates under a hydraulic closed-loop servo-control. The load frame stiffness and maximum axial force are 5×10^9 N/m and 1.4 MN, respectively. The axial strain cantilever set has a 10% strain range, and the radial strain cantilevers have a strain range limit of 7%, with the linearity of 1% over the full scale in both sets. The servo-controlled press description, physical and mechanical properties of the studied rock formations are presented in detail elsewhere (Hatzor and Palchik 1997; Palchik and Hatzor 2002, 2004).

Table 1 Observed values of compressive stress levels, elastic modulus, dry bulk density, Poisson's ratio, porosity, maximum axial strain and maximum total volumetric strain

Rock form	Sample	σ_{ci} (MPa)	$\sigma_{cd} = \sigma_c$ (MPa)	E (MPa)	ρ (g/cm^3)	v	n (%)	$\varepsilon_{a\ max}$ (%)	ε_{cd} (%)
AD	rc1	40.1	53.2	17,400	2.12	0.23	21.5	0.32	0.16
	rc3	29.3	51	16,000	2.07	0.26	23.3	0.41	0.229
	rc4	15	31.9	11,700	1.93	0.2	28.5	0.32	0.166
	rc8	30.9	60.3	17,300	2.11	0.2	21.9	0.37	0.218
	rc9	49.4	63.1	20,500	2.17	0.27	19.6	0.31	0.134
	st2a	29.6	52.3	14,300	2.14	0.2	20.7	0.4	0.186
	st2b	25.5	37.4	10,700	2.06	0.22	23.7	0.34	0.159
AM	ad 43	145	274	64,000	2.65	0.27	5.4	0.475	0.21
	ad 80	102	174	58,500	2.62	0.28	6.4	0.316	0.134
BIN	bina 2	41.7	77	34,400	2.36	0.25	15.7	0.317	0.136
	bina 5	49.7	80	38,700	2.41	0.24	13.9	0.22	0.1
	bina 7	29.2	64.2	25,000	2.4	0.2	14.3	0.29	0.132
	tb3-2	89.2	139	42,830	2.59	0.26	7.5	0.362	0.147
	b3	22.2	35	21,000	2.06	0.31	26.4	0.19	0.084
	b5	65.9	98	35,200	2.33	0.25	16.8	0.29	0.137
YR	yark 3	29.8	41	6,200	2.3	0.18	17.9	0.69	0.312
YG	yn 4	45.2	75	6,100	2.1	0.13	25	1.36	0.9
	ca3541	110	174	54,000	2.57	0.19	8.2	0.346	0.149
	ca5671	39.5	60	35,500	2.33	0.19	16.8	0.232	0.14
NK	gn2-1b	80	177	47,000	2.48	0.23	11.4	0.431	0.236
	gn2-4a	100	141	44,600	2.43	0.25	13.2	0.33	0.165
	gn2-5b	121.5	162	48,600	2.49	0.25	11.1	0.364	0.197
	gn3-2a	90.5	150	44,800	2.45	0.24	12.5	0.361	0.161
	gn3-2c	99	163	44,400	2.42	0.25	13.6	0.395	0.186

AD Adulam chalk, *AM* Aminadav dolomite, *BIN* Bina limestone, *YR* Yarka limestone, *YG* Yagur dolomite, *NK* Nekorot limestone, σ_{ci} crack initiation stress, σ_{cd} crack damage stress, σ_c uniaxial compressive strength, *E* elastic modulus, ρ dry bulk density, v Poisson's ratio, *n* porosity, $\varepsilon_{a\ max}$ maximum axial strain (at σ_c), ε_{cd} maximum total volumetric strain (at σ_{cd})

The results of uniaxial tests are given in Table 1. The latter presents the values of crack initiation stress (σ_{ci}), $\sigma_{cd} = \sigma_c$ (here σ_{cd} is the crack damage stress and σ_c is the uniaxial compressive strength), elastic modulus (E), dry bulk density (ρ), Poisson's ratio (v), porosity (n), maximum axial strain $\varepsilon_{a\ max}$ (at σ_c) and maximum total volumetric strain ε_{cd} (at σ_{cd}) for each of 24 studied rock samples.

Porosity (n, %) was calculated from the measured values of dry bulk density (ρ in Table 1) and specific gravity of the solids ($G_s = 2.7$–2.8 g/cm^3): $n = [1 - (\rho/G_s)] \times 100\%$. The precision of the porosity estimation is 0.1%. The elastic modulus (E) and Poisson's ratio (v) were calculated using linear regressions along the linear portion (elastic stage) of the stress–strain curve. The total volumetric strains (ε_v) calculated as a sum of the component strains (ε_a, ε_{R1} and ε_{R2}) were plotted versus axial stresses for each of 24 studied rock samples. Figure 2 demonstrates examples of axial stress–total volumetric strain curves for five rock samples (ad 80, ca3541, gn3-2a, gn2-1b and rc9) exhibiting $\varepsilon_{cd} = 0.134\%$ at $\sigma_{cd} = \sigma_c = 174$ MPa, $\varepsilon_{cd} = 0.149\%$ at $\sigma_{cd} = \sigma_c = 174$ MPa, $\varepsilon_{cd} = 0.161\%$ at $\sigma_{cd} = \sigma_c = 150$ MPa,

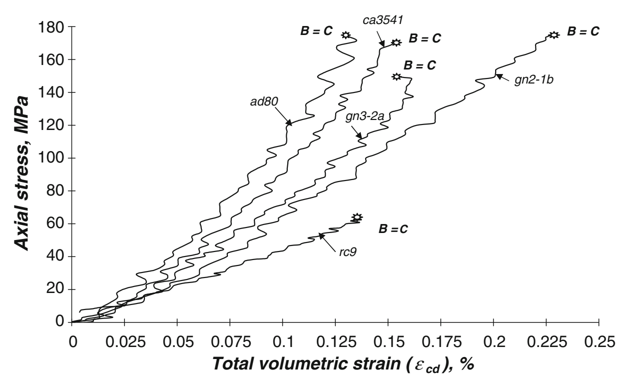

Fig. 2 Examples of observed axial stress–total volumetric strain curves (type 2) for five studied rock samples

$\varepsilon_{cd} = 0.236\%$ at $\sigma_{cd} = \sigma_c = 177$ MPa and $\varepsilon_{cd} = 0.134\%$ at $\sigma_{cd} = \sigma_c = 63.1$ MPa, respectively, in point the $B = C$.

In Table 1, the observed values of crack initiation stress (σ_{ci}) and crack damage stress or uniaxial compressive strength ($\sigma_{cd} = \sigma_c$) vary from 15 to 145 MPa with the mean of 61.7 MPa and from 31.9 to 273.9 MPa with the mean of 101.4 MPa, respectively. The difference (D) between σ_c and σ_{ci} and the ratio $k = \sigma_{ci}/\sigma_c$ for each of the studied rock samples were calculated. Values of D and k range from 11.2 to 128.9 MPa ($D_{mean} = 39.7$ MPa) and from 0.45 to 0.78 ($k_{mean} = 0.62$), respectively, for all studied samples. Here, standard deviations (Δ) of the mean D and k values are significant—29.2 MPa and 0.09, respectively. Standard deviation of the mean has been calculated as follows:

$$\Delta = \sqrt{\frac{\sum_{i=1}^{n} (q_i - q_m)^2}{n-1}} \qquad (1)$$

where $i = 1, 2, \ldots, n$ is the number of observed sample ($n = 24$), q_i is the value of the observed parameter (D or k) in the ith sample, q_m is the arithmetic mean of parameters observed in n samples.

Note also that the values of D and k are not constant even for a single set of samples within the same rock formation: 11.9 MPa $< D <$ 29.4 MPa ($0.47 < k < 0.78$), 72.2 MPa $< D <$ 128.9 MPa ($0.53 < k < 0.59$), 12.8 MPa $< D <$ 49.8 MPa ($0.47 < k < 0.63$), 20.5 MPa $< D <$ 63.9 MPa ($0.6 < k < 0.66$) and 40.5 MPa $< D <$ 97 MPa ($0.45 < k < 0.75$) for Adulam chalk, Aminadav dolomite, Bina limestone, Yagur dolomite and Nekorot limestone, respectively. The standard deviation (Δ) of the mean k within the same rock formation ranges from 0.03 to 0.12. For example, Nekorot limestone has a large $\Delta k = 0.12$, whereas Yagur dolomite exhibits relatively a small $\Delta k = 0.03$.

Table 1 demonstrates the values of elastic modulus (E), porosity (n) and Poisson's ratio (v) for each of 24 studied rock samples. Here, 6,100 MPa $< E <$ 64,000 MPa, 5.4% $< n <$ 28.5% and $0.13 < v < 0.31$. Mean values of E, n and v are 31,600 MPa, 16.5% and 0.23, respectively, and standard deviations (Δ) of mean E, n and v values are 17,300 MPa, 6.5% and 0.04, respectively. The values of E, n and v in heterogeneous carbonate rocks are also non-constant even for samples within the same rock formation. For example, Bina limestone exhibits 21,000 MPa $< E <$ 42,830 MPa, 7.5% $< n <$ 26.4% and $0.2 < v < 0.31$.

In Table 1, maximum total volumetric strain (ε_{cd}) and maximum axial strain ($\varepsilon_{a\ max}$) range from 0.084 to 0.9% with the mean of 0.2% and from 0.19 to 1.36% with the mean of 0.39%, respectively. Thus, the maximum axial strain ($\varepsilon_{a\ max}$) is 1.5–2.5 times larger than the maximum total volumetric strain (ε_{cd}) in case where crack damage stress (σ_{cd}) is equal to uniaxial compressive strength (σ_c). For example, rock sample bina 7 exhibiting $\varepsilon_{cd} = 0.132\%$ at $\sigma_{cd} = 64.2$ MPa and $\varepsilon_{a\ max} = 0.29\%$ at $\sigma_c = 64.2$ MPa (see Fig. 1b), has the ratio $\varepsilon_{a\ max}/\varepsilon_{cd} = 0.29/0.132 = 2.2$.

3 Relations between mechanical properties and compressive stresses

3.1 Effect of elastic modulus, porosity and E/n ratio on σ_{ci} and $\sigma_{cd} = \sigma_c$

Figure 3a shows how the elastic modulus (E) influences the values of σ_{ci} and $\sigma_{cd} = \sigma_c$. Relations $\sigma_{ci} - E$ and $\sigma_c - E$ best follow a polynomial law with good squared regression coefficients $R^2 = 0.86$ and 0.91 for $\sigma_{ci} - E$ and $\sigma_c - E$, respectively. An increase in the elastic modulus (E) from 6,100 to 64,000 MPa leads to an increase in σ_{ci} and $\sigma_c = \sigma_c$ values from 15 to 145 MPa and from 31.9 to 273.9 MPa, respectively. Hence, an increase in the elastic modulus by a factor of 10.5 leads to an increase in the values of σ_{ci} and $\sigma_{cd} = \sigma_c$ by a factor of 9.7 and 8.6, respectively.

On the other hand, values of σ_{ci} decrease from 145 to 15 MPa and from 273.9 to 31.9 MPa, respectively, with porosity increase (n, Fig. 3b) from 5.4 to 28.5%. In Fig. 3b, polynomial correlations ($R^2 = 0.76$–0.79) between the porosity (n) and σ_{ci} and $\sigma_{cd} = \sigma_{ci}$ values are obtained.

An increase in the values of stresses σ_{ci} and $\sigma_{cd} = \sigma_c$ with increasing ratio E/n is shown in Fig. 4a. The latter demonstrates that σ_{ci} and $\sigma_{cd} = \sigma_c$ are well correlated ($R^2 = 0.82$–0.86) with the ratio E/n. It is not surprising, since the compressive stresses σ_{ci} and $\sigma_{cd} = \sigma_c$ depend on the elastic modulus E (Fig. 3a) and porosity n (Fig. 3b).

3.2 Effect of E/λ ratio on σ_{ci} and $\sigma_{cd} = \sigma_c$

Porosity n is a measure of void space (pores and open cracks) and represents a ratio between the void space (V_p) and bulk volume (V).

$$n = \frac{V_p}{V} \qquad (2)$$

The bulk volume (V) is the initial volume of a sample before loading, and therefore, it does not reflect the change in the volume due to compression. Palchik and Hatzor (2002) have proposed to use the ratio between the volume of voids and change in the bulk volume due to compression, since such ratio reflects the mechanical behavior of the rock matrix. This ratio can be represented as a ratio between the volume of voids (V_p) and the maximum compaction (V_c) of a rock sample:

$$\lambda = \frac{V_p}{V_c} \qquad (3)$$

where V_c is the maximum decrease in a sample volume (maximum compaction of a sample), which is attained at the maximum total volumetric strain ε_{cd} (at crack damage stress σ_{cd}).

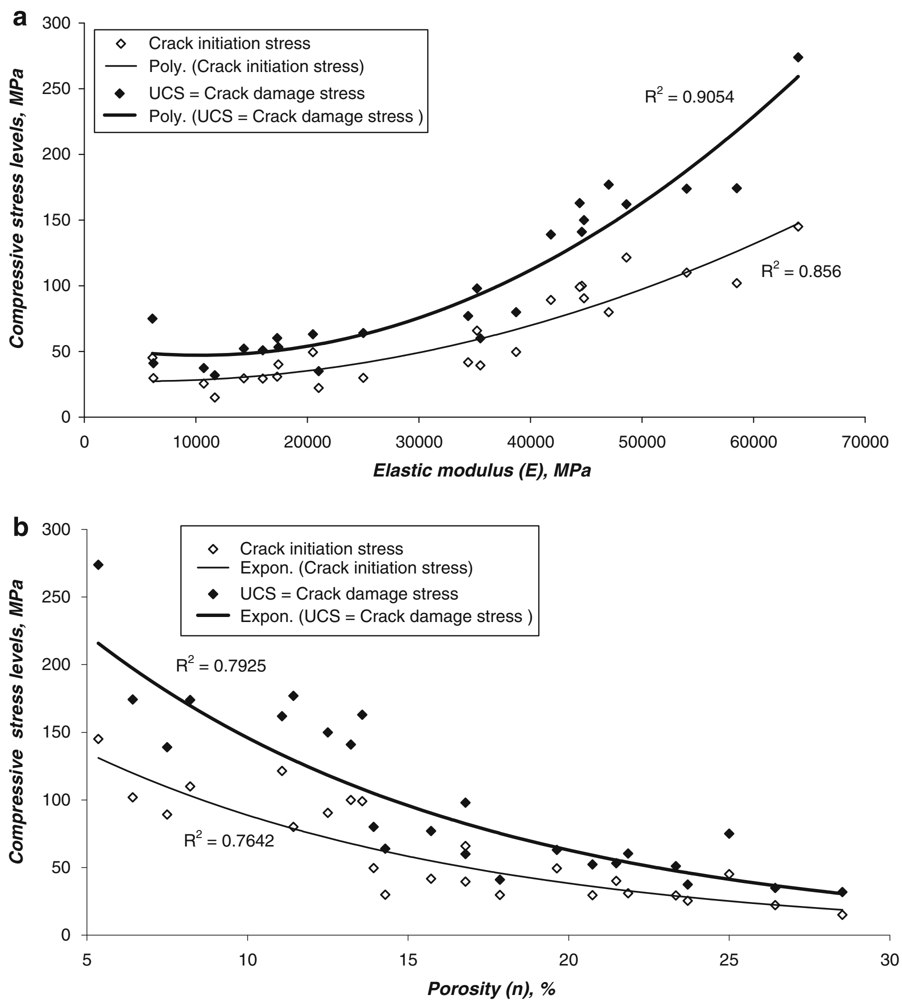

Fig. 3 Relations between compressive stress levels and **a** elastic modulus (E), and **b** porosity (n)

The parameter λ can be presented as

$$\lambda = \frac{V_p}{V_c} = \frac{V_p}{V} \Big/ \frac{V_c}{V} = \frac{n}{\varepsilon_{cd}} \tag{4}$$

where $V_c/V = \varepsilon_{cd}$, ε_{cd} is the maximum total volumetric strain at the crack damage stress σ_{cd}.

When we use $\lambda = V_p/V_c$ instead of $n = V_p/V$, the ratio E/n can be rewritten as E/λ. Relations between E/λ and compressive stress levels are presented in Fig. 4b. From Fig. 4b it is clear that the values of $\sigma_{cd} = \sigma_c$ and $D = \sigma_c - \sigma_{ci}$ are well correlated with E/λ. Here, power dependences between $\sigma_{cd} = \sigma_c$ ($R^2 = 0.96$), D ($R^2 = 0.87$) and E/λ with good squared regressions coefficients R^2 are obtained:

$$\sigma_{cd} = \sigma_c = a\left(\frac{E}{\lambda}\right)^b \tag{5}$$

$$D = c\left(\frac{E}{\lambda}\right)^d \tag{6}$$

where coefficients $a = 2.75$, $b = 0.6$, $c = 0.82$ and $d = 0.64$.

Note that the use of E/λ ratio (Fig. 4b) instead of E/n (Fig. 4a) versus the compressive stress $\sigma_{cd} = \sigma_c$ allows us to increase the value of R^2 from 0.86 to 0.96. Hence, the effect of the ratio E/λ on uniaxial compressive strength (σ_c) is more pronounced than the effect of E/n.

4 Conclusions

The mechanical behavior of heterogeneous carbonate rocks exhibiting total volumetric strain curves of type 2 was studied. Studied rock samples exhibiting a wide range of mechanical properties (31. 9 MPa $< \sigma_{cd} = \sigma_c <$ 273.9 MPa, 6,100 MPa $< E <$ 64,000 MPa and 5.4% $< n <$ 28.5%) were collected from different geological settings of Israel. From the results of this study it can be concluded that:

- Crack initiation (at the crack initiation stress σ_{ci}) for studied heterogeneous carbonate rocks occurs at $0.45\sigma_c/0.78\sigma_c$ (at significant standard deviations $\Delta D = 29.2$ MPa and $\Delta k = 0.09$ for all studied samples).

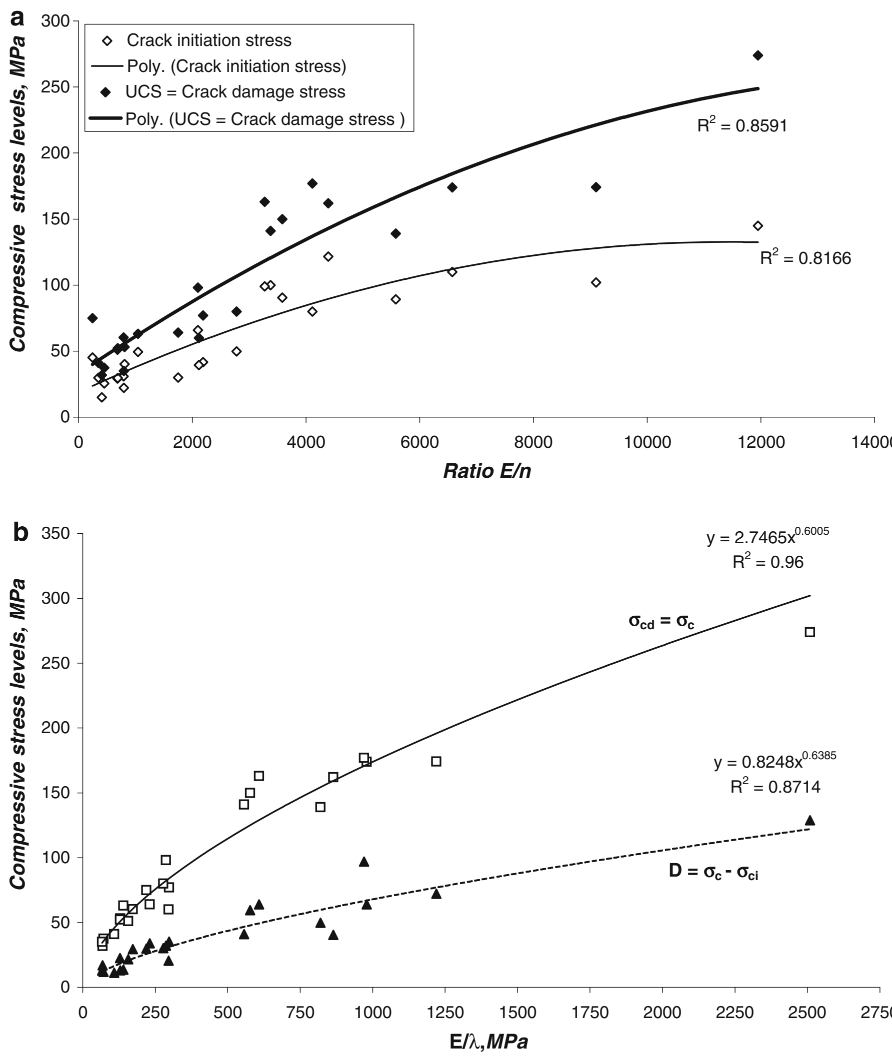

Fig. 4 Relations between compressive stress levels and ratios **a** E/n and **b** E/λ

- Values of the difference (D) between the uniaxial compressive strength and crack initiation stress, and the ratio (k) between the crack initiation stress and uniaxial compressive strength are not constant even for samples within the same rock formation. The standard deviation (Δ) of the mean k within the same rock formation varies between 0.03 and 0.12.
- Values of the maximum axial strain ($\varepsilon_{a\ max}$) and maximum volumetric strain (ε_{cd}) at $\sigma_{cd} = \sigma_c$ are 0.19/1.36% and 0.084/0.9%, respectively. At $\sigma_{cd} = \sigma_c$, the maximum axial strain ($\varepsilon_{a\ max}$) is 1.5–2.5 times the maximum volumetric strain (ε_{cd}).
- The ratio between the elastic modulus (E) and parameter λ strongly influences the values of $\sigma_{cd} = \sigma_c$ and $D = \sigma_c - \sigma_{ci}$. Power dependencies between $\sigma_{cd} = \sigma_c$, $\sigma_c - \sigma_{ci}$ and E/λ are obtained. Parameter λ is a ratio between the volume of voids (V_p) and the maximum compaction (V_c) of rock sample. Parameter λ is calculated as n/ε_{cd}, where ε_{cd} is the maximum total volumetric strain.

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