Rock Mech. Rock Engng. (2007) 40 (5), 453–475
DOI 10.1007/s00603-006-0107-6
Printed in The Netherlands

Rock Mechanics
and Rock Engineering

Experimental Calibration of ISRM Suggested Fracture Toughness Measurement Techniques in Selected Brittle Rocks

By

M. J. Iqbal and **B. Mohanty**

Department of Civil Engineering and Lassonde Institute,
University of Toronto, Toronto, Canada

Received September 9, 2005; accepted June 29, 2006

Summary

A wide variety of specimen types and methods are employed in fracture toughness measurement of rocks, which result in scattered values for the same rock type. In order to provide some consistency to the values, the International Society for Rock Mechanics (ISRM) recommended three suggested methods using core based specimens, the Chevron Bend (CB) test, the Short Rod (SR) test and the Cracked Chevron Notch Brazilian Disc (CCNBD) test. This standardization helped obtain more consistent values but still a variation of 20–30% was observed in the values of fracture toughness obtained with the CB and SR methods. The values obtained with the CCNBD method were found to be consistently lower (30–50%) than those of the other two methods (CB and SR). Many reasons have been offered to explain this deviation. These include size of the specimen, anisotropy of rock, a dimensionless parameter in the fracture toughness calculation equation for the CCNBD test, etc. A comprehensive test program was initiated to identify the cause of these discrepancies between the CB and CCNBD methods. Three brittle rock types were selected for the study and more than 200 tests were conducted to measure the values of fracture toughness.

A rigorous statistical analysis was carried out to determine the confidence level and find the significance of the test results. It was found that the CB and CCNBD methods were very comparable provided the correct equation for fracture toughness calculation was used for the CCNBD method and the size of the specimens was selected carefully. The error in the ISRM 1995 formula of fracture toughness for the CCNBD method could be the major factor responsible for the consistently lower values obtained with the method.

Keywords: Fracture toughness, test methods, rock fragmentation.

1. Introduction

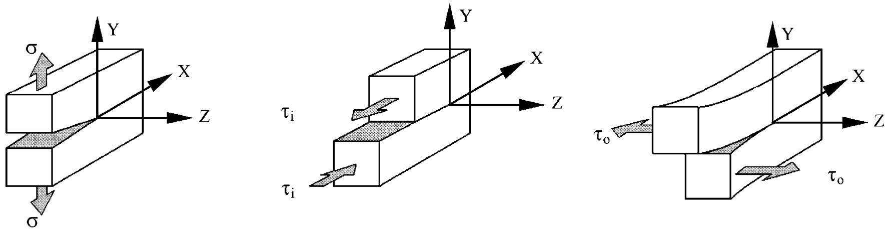
Analysis of properties of fracture resistance is essential to the understanding of rock fragmentation processes like drilling, blasting, tunnel boring, cutting and crushing.

The basic material parameter in fracture mechanics is called the fracture toughness. It is considered to be an intrinsic material property that describes when, where and why fracture in a material takes place, and is a function of the stress intensity factor (SIF) 'K' (ISRM, 1988).

The stress intensity factor 'K' characterizes the state of stress near the crack tip caused by a remote load. It can be related to the energy required to create a new surface in a material containing a pre-existing crack and subjected to an externally applied stress field. The magnitude of the stress intensity 'K' at the crack tip is much higher than the externally applied stress field, depending upon the degree of sharpness of the crack tip. When value of the stress intensity becomes greater than that of a critical stress intensity value, known as critical stress intensity factor 'Kc', fracture is presumed to initiate. The crack will continue to grow as long as $K \geqslant Kc$. Under certain conditions 'Kc' can be regarded as a material property and is a convenient measure of fracture toughness of a material.

The three basic modes of loading for a crack are: a normal stress 'σ', an in-plane shear stress 'τ_i', and an out-of-plane shear stress 'τ_o'. The corresponding modes of displacement for a crack surface are: an opening mode (mode-I), a sliding mode (mode-II), and a tearing mode (mode-III) as shown in Fig. 1. The critical stress intensity factors corresponding to the three basic modes of crack surface displacement are denoted by K_{IC}, K_{IIC} and K_{IIIC}, respectively. Of the three basic fracture modes, mode-I appears to be the most important. Although practical problems may be of mixed mode type, most of the work has been done on the analysis of mode-I, particularly concerning fracture mechanics of rock. This is partly due to the simplicity of its application, and largely due to the fact that it is the predominant loading condition of a crack in many practical situations.

The measurement of fracture toughness of rock has been done with a wide variety of specimen types and methods. As a result the values of fracture toughness were not generally comparable for the same rock type (Ouchterlony, 1982; Barton, 1982). Later on, the International Society for Rock Mechanics (ISRM, 1988) recommended two methods for determining the fracture toughness of rock using core based specimens. These are Chevron Bend (CB) and Short Rod (SR) tests, as shown in Figs. 2 and 3, respectively. This standardization helped obtain more representative values, but still a variation of 20–30% was observed in the CB and SR test values, for the same rock type. Many reasons have been offered to explain this variation. These include size of



Mode-I: Opening mode **Mode-II:** Sliding mode **Mode-III:** Tearing mode

Fig. 1. Modes of displacement for a crack surface corresponding to three basic modes of loading for a crack (after Whittaker et al., 1992)

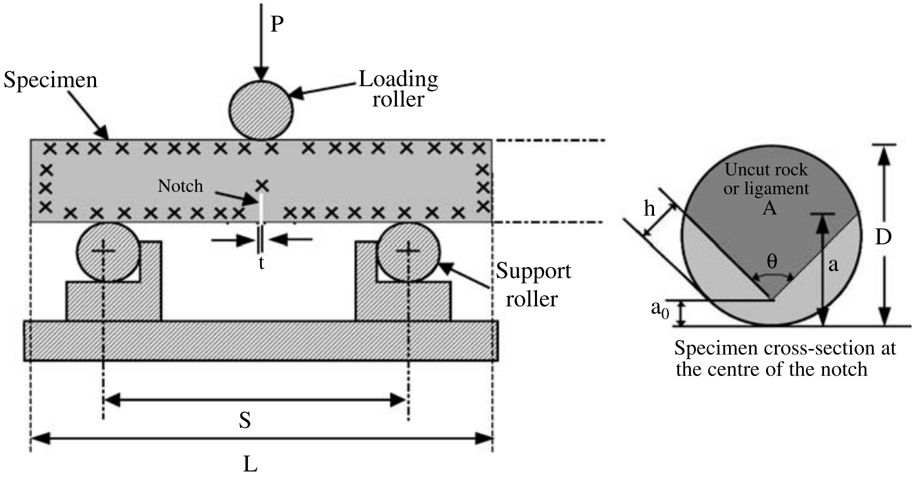


Fig. 2. Geometry of specimen, notations and setup used for CB test (after ISRM, 1988). *L* Length of Chevron bend specimen, 4D; *S* distance between support points, 3.33D; a_0 distance of Chevron tip from specimen surface, 0.15D; *a* length of crack; *D* diameter of Chevron bend specimen; *θ* Chevron angle, 90°; *h* depth of cut in notch flank; *A* projected ligament area; *t* width or thickness of notch

specimens, anisotropy of rock, size of fracture process zone near crack tip, micro- and macro-structure of rock, storage conditions of specimens (e.g. moisture), etc. The specimen size and anisotropy of rock were considered as major reasons for the variation in the values of fracture toughness, and a number of improvements had been suggested in this regard e.g. empirical relations for determining the required minimum diameter of the core specimens and estimation of the values of fracture toughness (Ouchterlony, 1989; Matsuki et al., 1991; Ouchterlony et al., 1991). In-spite of these efforts, several drawbacks of the methods could not be eliminated. These included relatively low loads required to initiate fracture (1–2 KN) and thereby less precision in measurements, large numbers of intact cores of rock required at the correct orientation, complicated fixtures for specimen installation and loading, and complex procedure of specimen preparation for the SR method (Fowell and Xu, 1993).

To overcome these disadvantages of the CB and SR methods, the ISRM recommended a third method (ISRM, 1995) for determining the fracture toughness of rock using Cracked Chevron Notched Brazilian Disc (CCNBD) specimen, as shown in Fig. 4. Even after standardization of the CCNBD method, most of the research on fracture toughness of rock employed other similar methods (e.g. Central Straight-through Crack Brazilian Disc in diametral compression (CSCBD)). The reason most commonly offered for this was the difficult notch making process in the disc specimens for the CCNBD test. Also, a comparison of all the ISRM suggested methods showed that the values of fracture toughness obtained with the CCNBD test were considerably lower (30–50%) than the values obtained with the CB and SR tests, for the same rock type (Dwivedi et al., 2000). Several reasons have been offered to explain this deviation. These include the critical dimensionless stress intensity factor (SIF) value for the disc specimens used in formula for the CCNBD test, the anisotropy of rock, and the micro- and macro-structure of rock. The critical dimensionless stress

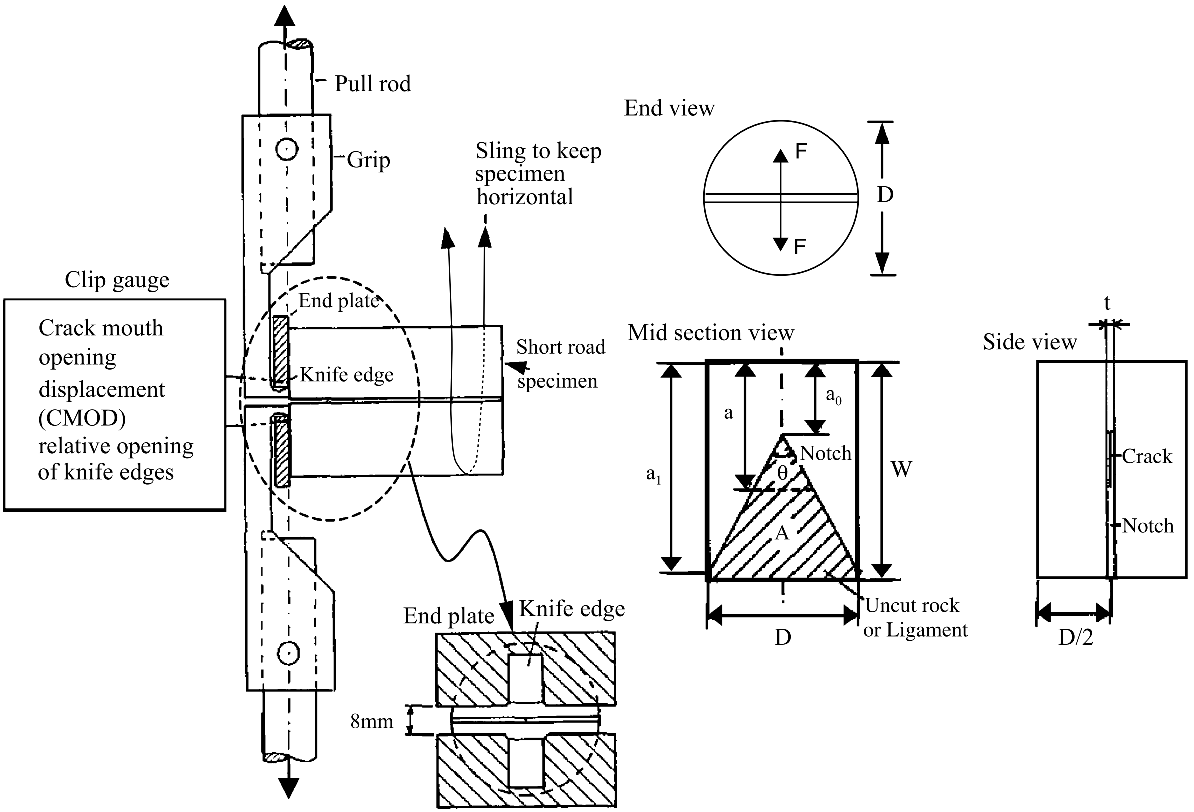


Fig. 3. Geometry of specimen, notations and setup used for SR test (after ISRM, 1988). *D* Diameter of short rod specimen; *W* length of specimen, 1.45D; θ Chevron angle, 54.6°; a_0 Chevron tip distance from loaded end, 0.48D; *a* crack length; a_1 maximum depth of Chevron flanks; *t* width or thickness of notch; *A* projected ligament area; *F* load on specimen

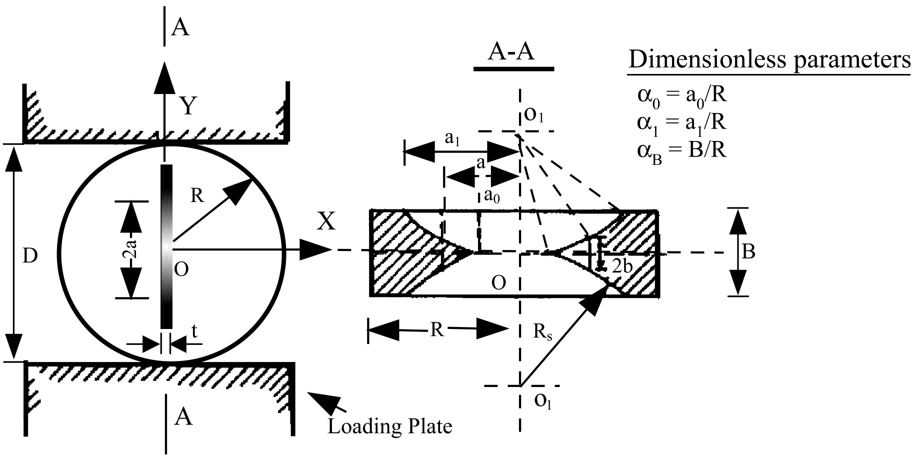


Fig. 4. Geometry of specimen, notations and setup used for CCNBD test (after ISRM, 1995). *R* Radius of disc; *B* thickness of disc; *D* diameter of disc; R_S radius of saw; *t* width or thickness of notch; *a* length of crack; a_0 initial half length of Chevron notch; a_1 final half length of Chevron notch

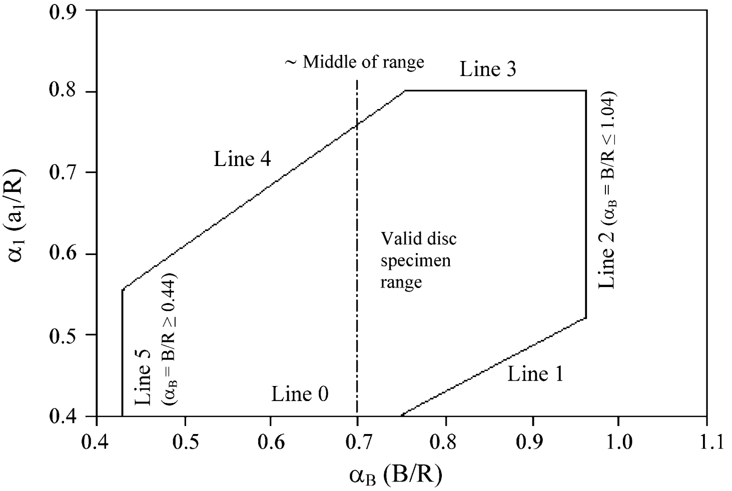


Fig. 5. Valid geometrical range of disc specimens for the CCNBD test (after ISRM, 1995). Prescribed dimensions of the disc specimens for the CCNBD test, showing the valid geometrical range for notch parameters

intensity factor value for the disc specimens is used to consider a wide range of valid specimen geometries that can be tested using this method as shown in Fig. 5. It corresponds to the critical state of the specimen when the crack front is somewhere between the initial and the final notch lengths, 'a_0 and a_1' of the disc specimens (refer to Fig. 4), and hence the critical crack length and the critical dimensionless SIF values are specimen geometry dependent only (ISRM, 1995; Fowell and Xu, 1993; Xu and Fowell, 1994). The critical dimensionless SIF value has been considered as the major factor for the lower values of fracture toughness obtained with the CCNBD method.

A review of literature showed serious differences among the researchers in the use of the equations employed to calculate the value of fracture toughness 'K_{IC}' with the CCNBD method. One of the reasons for the lower values obtained with the CCNBD method could be traced to this. The equation given in the ISRM suggested CCNBD method document (ISRM, 1995) is:

$$K_{IC} = \frac{P_{\max}}{B\sqrt{D}} Y^*_{\min} \tag{1}$$

where

$P_{\max}$ = Max. load at failure;
B = Thickness of the disc specimen;
D = Diameter of the disc specimen;
$Y^*_{\min}$ = Critical dimensionless stress intensity value for the disc specimen.

During the course of this investigation, it was discovered that there was an error in the above mentioned ISRM equation. The actual equation should have been as follows (Fowell and Xu, 1993; Xu and Fowell, 1994):

$$K_{IC} = \frac{P_{\max}}{B\sqrt{R}} Y^*_{\min} \tag{2}$$

where

R = Radius of the disc specimen.

In view of the problems mentioned above, a comprehensive testing program was devised to investigate the various standard techniques for the measurement of fracture toughness of rock (CB, SR and CCNBD, suggested by the ISRM). The objective of the study was to determine the consistency of the values of fracture toughness obtained with the standard ISRM suggested methods, and to determine the best method in terms of accuracy, precision and ease of conducting the experiments.

Of the three suggested methods, the SR method was excluded from the study due to the difficult procedure for preparation of specimens and complex fixture for installation of the specimen and loading. Three brittle rock types were selected for the study and more than 200 tests were conducted using the CB and CCNBD methods to measure the values of fracture toughness. A systematic statistical analysis was also carried out to determine the confidence level and significance of the test results.

2. Selected Rock Types and their Properties

Three granite rock types were selected for the study on account of their brittleness. They were Barre granite, Laurentian granite and Stanstead granite. The brittle rock types were selected to limit size of the fracture process zone ahead of the crack tip i.e. to minimize the size of the fracture process zone and remain within linear elastic fracture mechanics (LEFM) boundaries. The Barre granite was from the south-west region of Burlington in the state of Vermont, USA. Its grain size ranges from 0.2 to 3.0 mm (medium-fine grained) with an average grain size of 0.87 mm. The Laurentian granite was from Grenville province of the Precambrian Canadian shield, north of St. Lawrence and north-west of Quebec City, Canada. Its grain size ranges from 0.2 to 2.0 mm (fine grained) with an average grain size of 0.59 mm. The Stanstead granite was from the Beebe region of the Eastern Townships in Quebec, Canada. Its grain size ranges from 0.2 to 5.0 mm (coarse grained) with an average grain size of 1.08 mm (Nasseri et al., 2002).

All the selected rock types seemed fairly isotropic visually. The rock material was available in the form of cubic blocks measuring 35 cm on the side. Since anisotropy of rock and micro- and macro-structural effects were not included in the objectives of this phase of research, it was decided to core and notch all the specimens in such a way that the anisotropy and micro- and macro-structure effects could at least be indirectly accounted for. This was done by measuring seismic wave velocities (P- and S-wave) in the rock blocks along the three sides. The seismic wave velocities were used to adopt a uniform reference system depending upon the P-wave velocities of all the blocks of the selected rock types as follows:

- The maximum P-wave velocity direction was marked as z-axis.
- The minimum P-wave velocity direction was marked as y-axis.
- The inermediate P-wave velocity direction was marked as x-axis.

A summary of test results of the P-wave velocity of the selected rock types is given in Table 1. It should however be noted that the seismic wave velocities measured above might not necessarily represent the actual maximum and minimum seismic wave velocities of the selected rock types, as the respective axes were defined by the pre-cut faces of the block samples as received.

In order to have better knowledge of the physical and strength properties of the selected rock types, experiments were conducted on the core specimens of 56 mm diameter using the ISRM suggested methods. A summary of test results of the physical and strength properties of the selected rock types is given in Tables 2 and 3, respectively.

Table 1. Summary of results of P-wave velocity tests

Rock type	Maximum P-wave velocity (m/s) z-axis	Intermediate P-wave velocity (m/s) x-axis	Minimum P-wave velocity (m/s) y-axis
Barre granite	4625	3940	3670
Laurentian granite	4645	4340	4055
Stanstead granite	3940	3680	2940

Table 2. Summary of results of physical property tests of the selected rock types

Rock type	Density 'ρ' (gm/cm^3)	Porosity 'n' (%)	Void ratio 'e'
Barre granite	2.66_3	0.59_3	0.006_3
Laurentian granite	2.63_3	0.64_3	0.006_3
Stanstead granite	2.65_3	0.62_3	0.006_3

All values in the table are average values of the number of tests shown in subscripts.

Table 3. Summary of results of strength property tests of the selected rock types

Rock type	UCS 'σ_c' (MPa)	Static Young's modulus 'E' (GPa)	Static Poison's ratio 'ν'	Brazilian tensile strength 'σ_{tB}' (MPa)	Point load strength $I_{S(50)}$ (MPa)
Barre granite	212_3	82_3	0.16_3	12.70_6	7.69_8
Laurentian granite	259_2	92_3	0.21_3	12.79_6	9.08_8
Stanstead granite	173_3	66_3	0.16_3	7.88_6	6.43_8

All values in the table are average values of the number of tests shown in subscripts.

3. Fracture Toughness Testing

3.1 Geometry of Specimens and Notations used

The geometry of specimens and notations used for the CB and CCNBD tests are shown in Figs. 2 and 4, respectively (ISRM, 1988, 1995). The valid geometrical range of disc specimens for the CCNBD test is shown in Fig. 5 (ISRM, 1995).

3.2 Preparation of Test Specimens

As discussed above, the selected rock material was available in the form of 35 cm side length cubic blocks. In order to have some idea about the required minimum diameter of core and disc specimens, for the determination of representative K_{IC} values, the empirical relations suggested by Matsuki et al. (1991), Ouchterlony et al. (1991) (Eqs. (3) and (4)) and ISRM (1995) (Eq. (5)) were used. The empirical relations are given by:

$$D_{\min} = 1.6\left(\frac{K_{IC}}{\sigma_t}\right)^2 \quad \text{or} \quad D_{\min} = 1.6\left(\frac{1.2K_{IC}^e}{\sigma_t}\right)^2 \tag{3}$$

and

$$K_{IC}^e = 0.23(E)^{0.65} \tag{4}$$

$$D_{\min} = 8.88 + 1.4744\left(\frac{K_{IC}}{\sigma_t}\right)^{-2} \tag{5}$$

where

$D_{\min}$ = Required minimum diameter of specimen;
K_{IC} = Mode-I fracture toughness;
K_{IC}^e = Estimated mode-I fracture toughness;
E = Young's modulus in GPa;
σ_t = Uniaxial tensile strength.

Using Eqs. (3) and (4), the required minimum diameters of the core specimens for the CB test were found to range from 200 to 300 mm for all the selected rock types. These diameters of core specimens were found to be unworkable due to very long cores required for the testing. Therefore, it was decided to follow the CB method's requirement of a minimum diameter of core specimens i.e. 10× largest grain size. In this case 32 mm diameter of core specimens was a reasonable selection for the Barre and

Laurentian granites, and 56 mm diameter for the Stanstead granite. Keeping in mind all the factors mentioned above and in order to investigate the effects of the size of specimen, it was decided to carry out tests with 32, 56 and 76 mm diameter core specimens for the CB method for all the selected rock types.

Using Eqs. (4) and (5) the required minimum diameters of the disc specimen for the CCNBD test were found to range from 20 to 35 mm for all the selected rock types. However, these diameters of the disc specimens were found to be unworkable due to the very small size of the notch required for these specimens, which would be very difficult to make. It was then decided to adopt the CCNBD method's requirement of a minimum diameter of the disc specimen i.e. 75 mm for all the selected rock types. In order to investigate the effects of the size of disc specimens (the effects of thickness of the disc specimens), it was decided to prepare the disc specimens approximately along line 5 (α_B $(B/R) \geqslant 0.44$), in the middle (α_B $(B/R) \cong 0.7$) and along line 2 (α_B $(B/R) \leqslant 1.04$) of the valid geometrical range for the CCNBD test, as shown in Fig. 5 (see also Fig. 4 for notations).

All the specimens were cored along the y-axis of the rock blocks. For the CB test specimens, the cores were cut to the required lengths ($L = 4D$) and the Chevron notches were then made along the z-axis of the cores. The width or thickness '*t*' of the notches was kept of approximately 1.0, 1.5 and 2.25 mm for the 32, 56 and 76 mm diameter core specimens, respectively (refer to Fig. 2). Similarly for the CCNBD test specimens, the 76 mm diameter cores were used to cut disc specimens of three thicknesses '*B*' ($\alpha_B \geqslant 0.44$, $\alpha_B \cong 0.7$ and $\alpha_B \leqslant 1.04$, refer to Fig. 5). The Chevron notches were then made in the disc specimens along the z-axis of the cores. The notch width or thickness '*t*' was kept at approximately 1 mm for all the disc specimens (refer to Fig. 4). In order to have some idea about the effects of length of the notch (e.g. the effects of variation of $2a_0$ and $2a_1$, refer to Fig. 4), disc specimens with varying lengths of notches were also prepared in the middle of the valid geometrical range ($\alpha_B \cong 0.7$) (Fig. 5). All other specifications for the preparation of specimens for the CB and CCNBD tests were also followed. Typical pictures of the core and disc specimens for the CB and CCNBD tests are shown in Fig. 6. Since water was used as lubricant during coring, cutting and notch making processes, the specimens were stored at room temperature for at least two to three weeks before testing, in order to allow them to return to normal moisture content conditions.

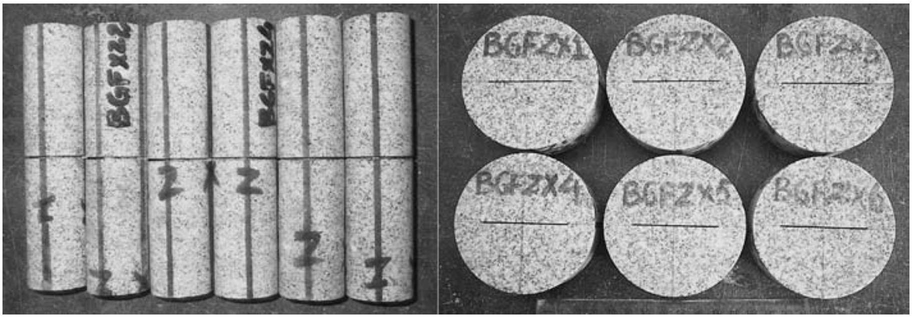


Fig. 6. Typical pictures of core and disc specimens for the CB and CCNBD tests

3.3 Details of the Tests

A MTS hydraulic servo-control testing system with MTS TestStar-II (digital controller) was used to conduct the tests and the MTS TestWare-SX application was used to write the procedures for tests. The fixtures for installation of the test specimens for the CB and CCNBD methods were prepared in the laboratory, and special alignment aid plates were used to facilitate accurate positioning of the notch of specimen for testing as shown in Figs. 7 and 8, respectively. The CB method offers two levels of the test.

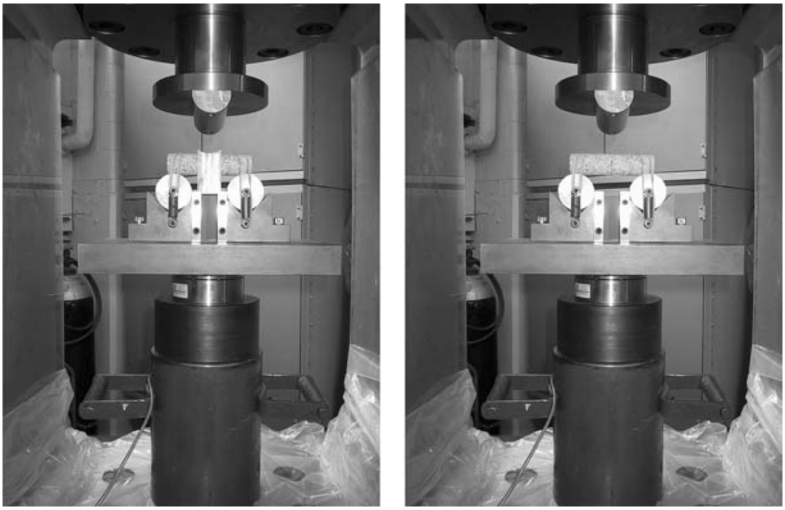


Fig. 7. Fixture for installation of specimen and notch alignment for the CB test

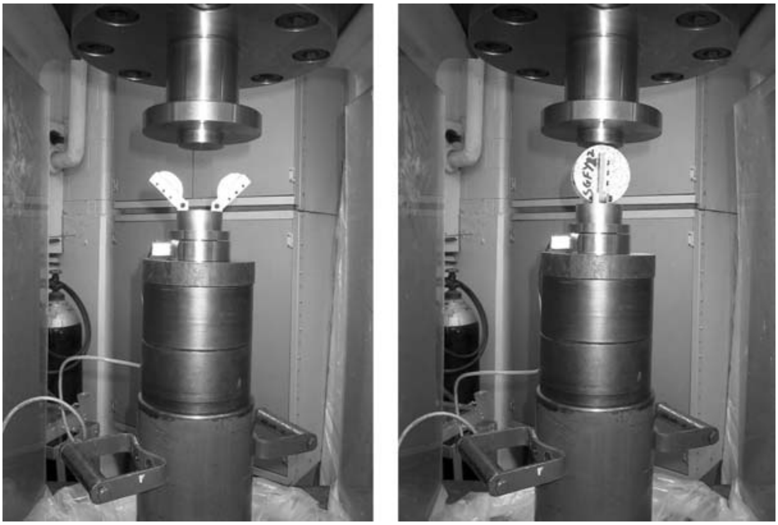


Fig. 8. Fixture for installation of specimen and notch alignment for the CCNBD test

Level-I testing requires only the measurement of maximum load during the test whereas the level-II testing requires the continuous measurements of load and displacement. In level-II testing, the fracture toughness values from level-I testing can be corrected for nonlinearity using a correction factor obtained from the load vs. displacement data. Since the rock types used in this study were considered to be fairly brittle, only level-I testing was conducted for the CB method. The load was applied at a constant rate of 0.002 mm/s for both the CB and CCNBD tests. The maximum failure load was measured using 25 and 50 KN load cells, which were calibrated prior to testing.

The values of fracture toughness for the CB tests were calculated using the following equation (ISRM, 1988):

$$K_{CB} = A_{\min} \frac{F_{\max}}{D^{1.5}} \tag{6}$$

where

K_{CB} = Fracture toughness from the CB method and for brittle material $K_{CB} = K_{IC}$ (fracture toughness in mode-I);

$F_{\max}$ = Maximum load at failure;

D = Diameter of the core specimen;

$$A_{\min} = \left[1.835 + 7.15\left(\frac{a_0}{D}\right) + 9.85\left(\frac{a_0}{D}\right)^2\right]\left(\frac{S}{D}\right);$$

S = Distance between support points, 3.33D (see Fig. 1);

a_0 = Distance of Chevron tip from specimen surface, 0.15D (see Fig. 2).

The plots of the values of fracture toughness for the Barre, Laurentian and Stanstead granites obtained with the CB tests, along with the standard deviation of the values

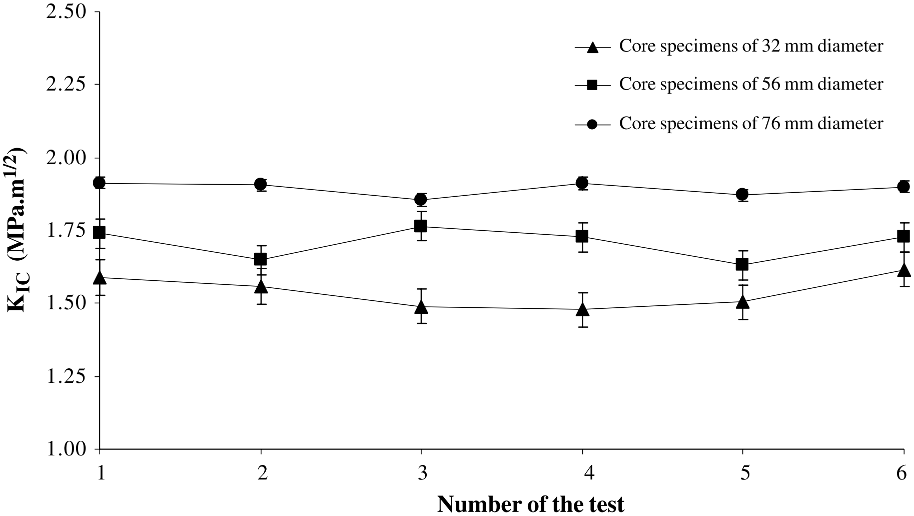


Fig. 9. Fracture toughness values of Barre granite obtained with CB tests

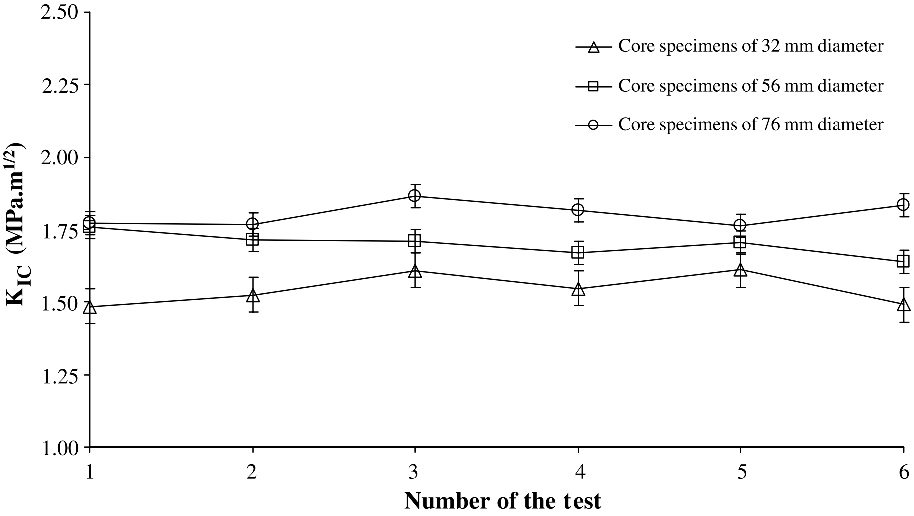


Fig. 10. Fracture toughness values of Laurentian granite obtained with CB tests

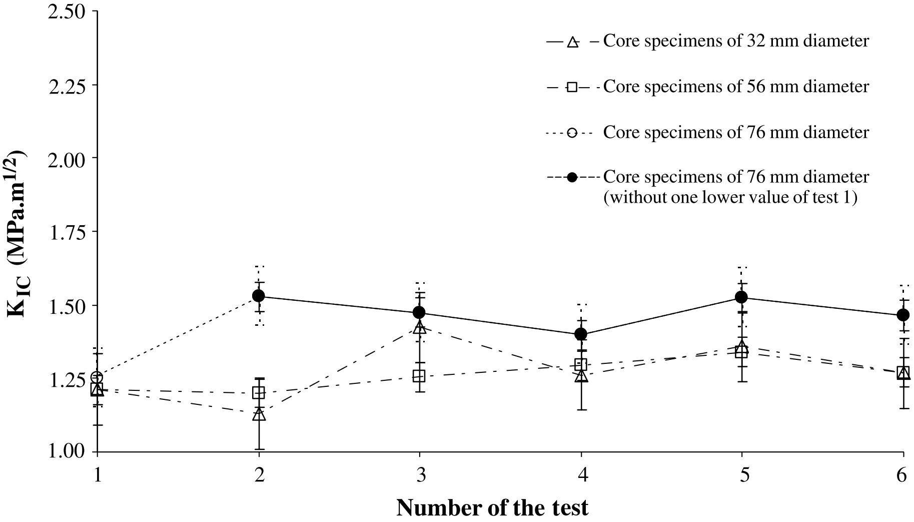


Fig. 11. Fracture toughness values of Stanstead granite obtained with CB tests

within each size of core diameter, are shown in Figs. 9–11, respectively. The plots show that, generally, the fracture toughness values increase with the increase in diameter of the core specimens.

The values of fracture toughness for the CCNBD tests were calculated using the following equation (Fowell and Xu, 1993; Xu and Fowell, 1994 i.e. Eq. (2)):

$$K_{IC} = \frac{P_{\max}}{B\sqrt{R}} Y^{*}_{\min} \tag{7}$$

where

K_{IC} = Fracture toughness (mode-I);
$P_{\max}$ = Maximum load at failure;
R = Radius of the disc specimen (see Fig. 4);
B = Thickness of the disc specimen (see Fig. 4);
$Y^*_{\min} = \mu e^{\upsilon\alpha_1}$ where 'μ' and 'ν' are constants determined from 'α_0' and 'α_B' where $\alpha_0 = a_0/R$ and $\alpha_B = B/R$ (refer to Fig. 4 and Table 2 (ISRM, 1995) for notations and values of the constants) for further details refer to ISRM (1995), Fowell and Xu (1993), Xu and Fowell (1994).

The plots of the values of fracture toughness for the Barre, Laurentian and Stanstead granites obtained with the CCNBD tests, along with the standard deviation

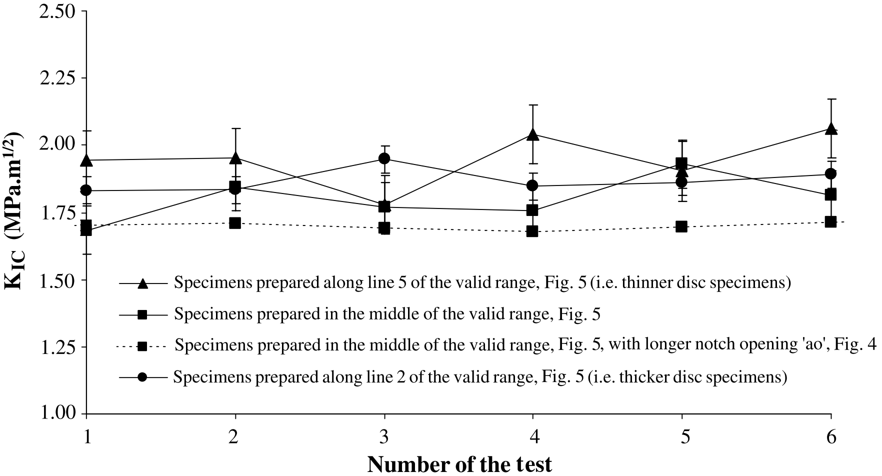


Fig. 12. Fracture toughness values of Barre granite obtained with CCNBD tests

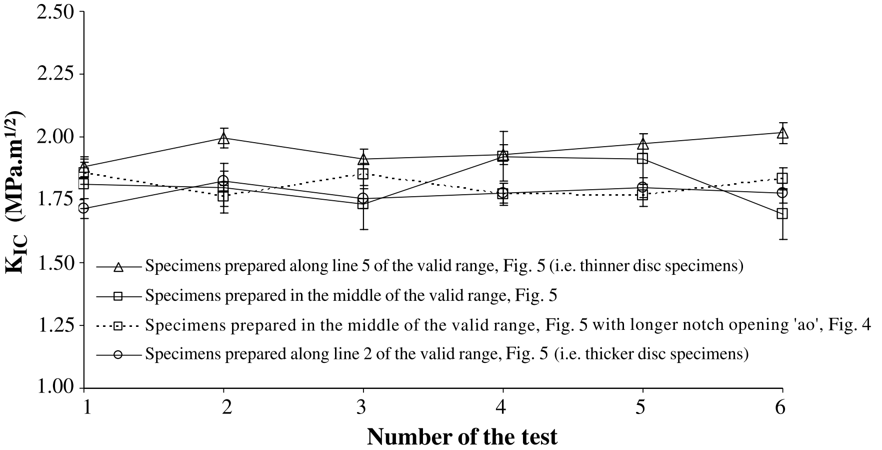


Fig. 13. Fracture toughness values of Laurentian granite obtained with CCNBD tests

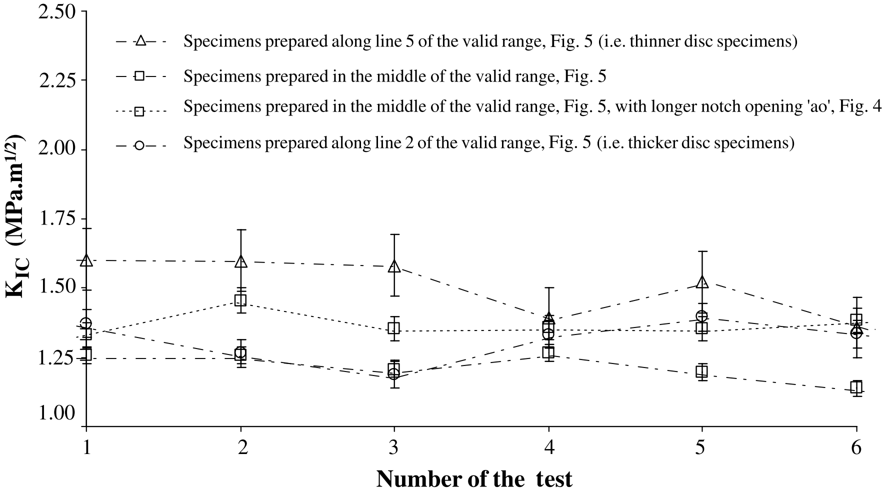


Fig. 14. Fracture toughness values of Stanstead granite obtained with CCNBD tests

Table 4. Summary of results of fracture toughness values obtained from the CB and CCNBD (using Eq. (2)) tests

Rock type	Results of CB tests			Results of CCNBD tests		
	z-axis notch (MPa.m$^{1/2}$)					
	32 mm	56 mm	76 mm	Line 5	In the middle	Line 2
Barre granite	$1.54_6 \pm 0.06$	$1.71_6 \pm 0.05$	$1.89_6 \pm 0.02$	$1.95_6 \pm 0.09$	$1.80_6 \pm 0.08$	$1.87_6 \pm 0.04$
Laurentian granite	$1.54_6 \pm 0.06$	$1.70_6 \pm 0.04$	$1.80_6 \pm 0.04$	$1.96_6 \pm 0.05$	$1.83_6 \pm 0.06$	$1.77_6 \pm 0.04$
Stanstead granite	$1.31_6 \pm 0.12$	$1.26_6 \pm 0.05$	$1.44_6 \pm 0.10$, $1.48_5 \pm 0.05$	$1.49_6 \pm 0.10$	$1.22_6 \pm 0.05$	$1.31_6 \pm 0.08$

All values in the table are average values of the number of tests shown in subscripts, along with the respective standard deviations.

in the values within each disc specimen thickness group, are shown in Figs. 12–14, respectively. The plots show that the fracture toughness values are generally comparable for all the four types of disc specimens i.e. the specimens prepared along line 5 (thinner discs), in the middle with short and long notch openings (Fig. 4) and along line 2 of the valid specimen geometrical range (Fig. 5). The plots also show that generally the fluctuation in the values of the disc specimens prepared along line 5 (i.e. thinner discs) of the valid range (Fig. 5) is higher than the values of the other three types of the disc specimens.

A summary of the results of fracture toughness values obtained with the CB and CCNBD tests is given in Table 4.

4. Analysis of Test Results

The technique of the analysis of variance (ANOVA) has been used extensively in assessing the experimental results in this investigation. The main objective of the method is to determine the effect of various factors on some response variable '*y*' of interest. The ANOVA method essentially breaks down the total variance of '*y*' into its component parts, allowing one to assess the significance of the component factors (Guttman et al., 1982; Walpole et al., 1998). Specifically the following are tested:

- Are all means from more than two populations equal?
- Are all means from more than two treatments on one population equal (this is equivalent to asking whether the treatments have any overall effect)?

Some key assumptions of the ANOVA are as follows:

- Within each sample, values are independent and identically normally distributed i.e. same mean and variance.
- Samples are independent of each other.
- Different samples are assumed to come from populations with same variance.

The key statistic in the ANOVA is the *f*-test of difference of group means, testing if the means of the groups formed by values of the independent variable or combinations of values for multiple independent variables are different enough not to have occurred by chance. If the effects are found to be non-significant then the differences between the means are not great enough to say that they are different. In this case, no further interpretation is attempted. When the effects are significant, the means must then be examined in order to determine the nature of the effects. There are many procedures, such as the Scheffè post-hoc comparison tests, to do this task (Guttman et al., 1982; Walpole et al., 1998).

The ANOVA analysis was carried out with the present test results to determine the confidence levels for the comparisons of various variables. The confidence level is the probability value associated with a confidence interval and it is often expressed as a percentage. The analyses were conducted to determine the effects of specimen size, testing methods and grain size at the 5% significance level, on the measured values of fracture toughness for the selected rock types. Scheffè post-hoc comparison tests were also conducted for each analysis case in order to see which group of the values actually differ from the other groups. The percentage variation of the average values of fracture toughness between all the analysed data sets was also calculated. The results of the statistical analysis are summarized in the following.

4.1 Effects of Specimen Size

The analyses were carried out on the values of fracture toughness obtained from the three groups of core specimens, 32, 56 and 76 mm diameter, for the CB test; and on the values obtained from the three groups of disc specimens, prepared along line 5 (thin discs), in the middle and along line 2 (thick discs) of the valid geometrical range (Fig. 5), for the CCNBD test for each of the selected rock types.

The analysis of the CB test results showed that at the 5% significance level, the size (diameter) of core specimens did have an effect on the values of the fracture toughness for all rock types. In the case of Barre and Laurentian granites, the Scheffè post-hoc comparison tests showed that the values of all the three groups of core specimens differ from each other. However, in the case of Stanstead granite, the comparison test showed that only the values of the 56 and 76 mm diameter core specimens differ from each other. The percentage variation of the average values of the 32 mm diameter cores from the values of the 56 and 76 mm diameter cores was approximately 10 and 18%, respectively for the Barre granite, 10 and 14% for the Laurentian granite, and 4 and 9% for the Stanstead granite. Whereas the percentage variation of the average values of the 56 mm diameter cores from the values of the 76 mm diameter cores was approximately 10% for the Barre granite, 6% for the Laurentian granite, and 12% for the Stanstead granite.

The analysis of the CCNBD test results showed that at the 5% significance level, the size (thickness) of disc specimens did have an effect on the values of fracture toughness for all the rock types. The Scheffè post-hoc comparison tests showed that, generally, the values of the group of disc specimens prepared along line 5 (thin discs) of the valid geometrical range (Fig. 5) differ from the values of the groups of disc specimens prepared in the middle and along line 2 (thick discs) of the valid range for all the rock types. The percentage variation of the average values of the disc specimens prepared along line 5 of the valid range (Fig. 5) from the values of the disc specimens prepared in the middle and along line 2 (Fig. 5) was approximately 7.5 and 4%, respectively for the Barre granite, 6.5 and 9.5% for the Laurentian granite, and 9.5 and 12% for the Stanstead granite. Whereas the percentage variation of the values of the disc specimens prepared in the middle of the valid range (Fig. 5) from the values of the disc specimens prepared along line 2 (Fig. 5) was approximately 3–3.5% for all the rock types.

4.2 Effects of Test Methods

An analysis was carried out on the values of fracture toughness from the 76 mm diameter core specimens for the CB method and the values of three groups of disc specimens (specimens prepared along line 5 (thin discs), in the middle and along line 2 (thick discs) of valid geometrical range (Fig. 5)) for the CCNBD method. This was done for each of the selected rock types.

The analysis showed that at the 5% significance level, the testing methods did not have an effect on the values of the fracture toughness for all the rock types. However, the Scheffè post-hoc comparison tests showed that, in the case of Laurentian and Stanstead granites the values for the CB test differ from the values of one group of specimens for the CCNBD test. In the case of Laurentian granite only the values of the group of disc specimens prepared along line 5 (thin discs) of the valid range (Fig. 5) differ from the values of the CB test. Whereas in the case of Stanstead granite only the values of the group of disc specimens prepared in the middle of the valid range (Fig. 5) differ from the values of the CB test. The percentage variation of the average values for the CB test from the values of the three groups of disc specimens for the CCNBD test (i.e. the disc specimens prepared along line 5 (thin discs), in the middle and along line 2 (thick discs) of the valid range (Fig. 5)) was approximately 3, 4.5 and 1%, respectively for the Barre granite, 9, 1.5 and 1.5% for the Laurentian granite, and 3.5, 15 and 9% for the Stanstead granite.

4.3 Effects of Grain Size

The tested rock types were not from the same region and were not of the same type but their mineralogical composition showed that it would be fairly reasonable to conduct an analysis of effects of grain size among them (Nasseri et al., 2002). The analyses, based on grain sizes (coarse, medium-fine and fine grained), were carried out on all the values of fracture toughness of the three selected rock types for the CB and CCNBD tests.

The analysis of the CB test results showed that at the 5% significance level, the grain size did have an effect on the values of fracture toughness. The Scheffè post-hoc comparison tests showed that the values of the coarse grained rock (Stanstead granite) differ from the values of the medium-fine and fine grained rock types (Barre and Laurentian granites) for all three groups of core specimens (32, 56 and 76 mm). However, the values of the medium-fine and fine grained rock types were statistically the same for all the groups of core specimens. The percentage variation of the average values of fracture toughness of the coarse grained rock from the values of the medium-fine and fine grained rock types was approximately 15–30% for all the groups of core specimens.

The analysis of the CCNBD test results showed that at the 5% significance level, the grain size did have an effect on the values of fracture toughness. The Scheffè post-hoc comparison tests showed that the values of the coarse grained rock differ from the values of the medium-fine and fine grained rock types for all three groups of disc specimens (specimens prepared along line 5 (thin discs), in the middle and along line 2 (thick discs) of the valid range (Fig. 5)). However, the values of the medium-fine and fine grained rock types were statistically the same for all the groups of disc specimens. The percentage variation of the average values of fracture toughness of the coarse grained rock from the values of the medium-fine and fine grained rock types was approximately 25–30% for all the groups of disc specimens.

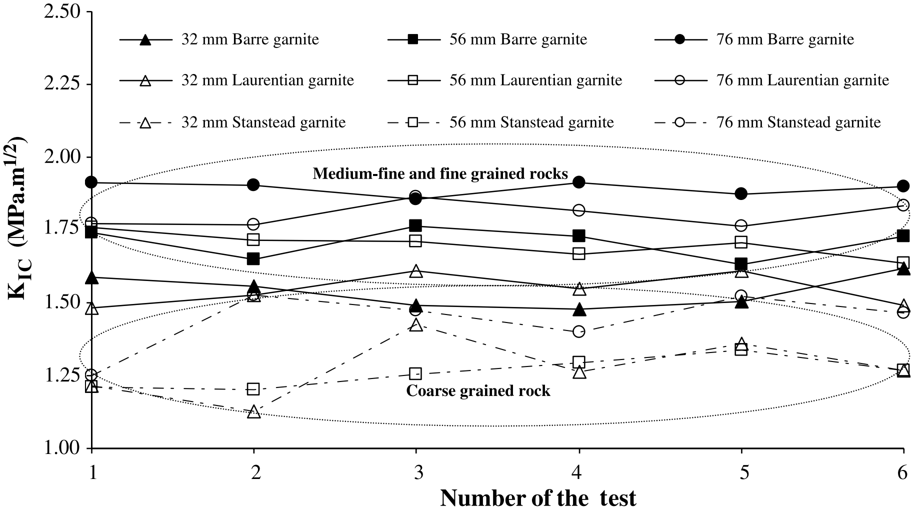


Fig. 15. Fracture toughness values of all selected rock types for CB tests

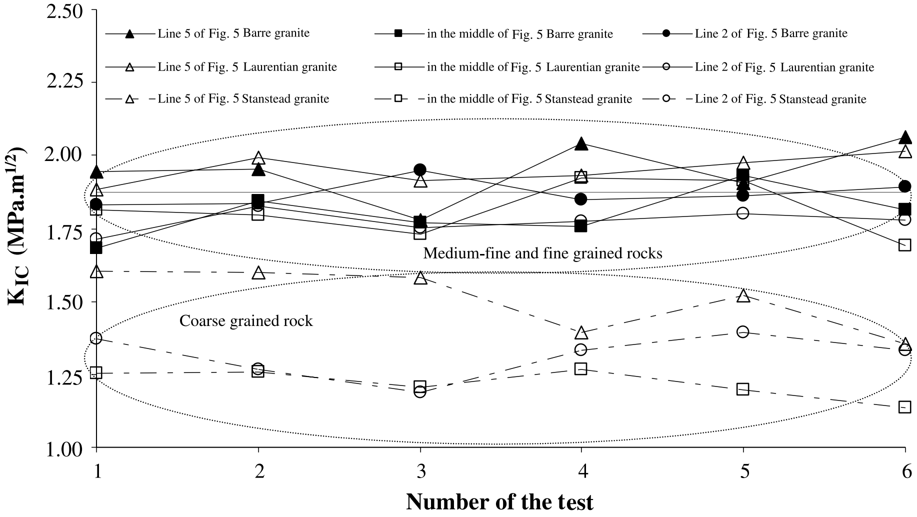


Fig. 16. Fracture toughness values of all selected rock types for the CCNBD tests

The combined plots of all the values of fracture toughness of the selected rock types for the CB and CCNBD tests are shown in Figs. 15 and 16, respectively. The plots also show that the values of the medium-fine and fine grained rock types (Barre and Laurentian granites) are higher than the values of the coarse grained rock (Stanstead granite). This indicates that the values of fracture toughness increase with decrease in the grain size for the selected rock types.

5. Discussion

5.1 Size of Specimen

The analysis of the CB test results showed that the size of specimen did have an effect on values of the fracture toughness. It was found that in general, the values increased with increase in diameter of the core specimens. It was also found that the standard deviation in the values, within each specimen size (diameter) group, relatively decreased with increase in diameter as shown in Fig. 17. It should however be noted that in the case of Stanstead granite the increase in the values of fracture toughness with increase in the core diameters applied only to size larger than 56 mm (Fig. 17). In this case, the values for the 32 mm diameter core specimens are more scattered than the other two diameters. This greater scatter in the values may be due to the fact that the 32 mm diameter core size did not fulfill the requirement of minimum diameter required for core specimen, for this rock type. The required minimum diameter of core for this rock type, as discussed above, was 56 mm. On the other hand, the larger standard deviation in the values of the core specimens of 76 mm diameter, in this case, was due to one lower value from the first test as shown in Fig. 11. If we ignore the value of the first test, the standard deviation in the rest of the values for the 76 mm diameter samples decrease considerably as shown in Fig. 17.

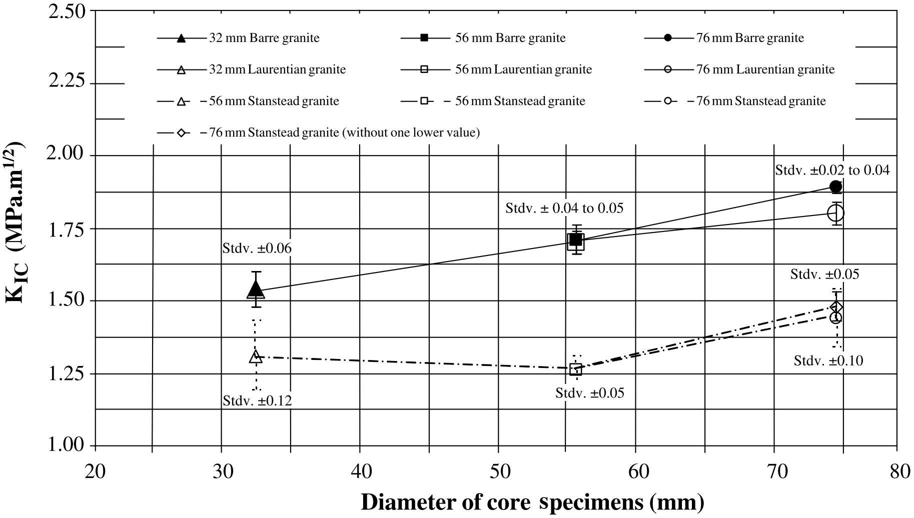


Fig. 17. Fracture toughness values vs. diameter of core specimens for CB test

The fracture toughness values should be size independent in order to represent a material property. The CB test results showed that the values increase with increase in the diameter of core specimens. The analysis of the test results led to selection of the values of 76 mm diameter core specimens as the representative fracture toughness values for all the selected rock types. This decision was based on lower standard deviation within the values of the 76 mm diameter samples and due to gradual but progressive decrease from 32 to 76 mm diameters. The other factor in selection of the 76 mm diameter core specimen values for the CB tests was the comparison with the corresponding values from the CCNBD tests. In order to have more confidence in the selected representative values of fracture toughness for the CB tests, the required minimum diameter of the core specimens were recalculated using Eq. (3) and actual K_{IC} values of the 76 mm diameter cores. This exercise resulted in the required minimum diameter of the core specimens to range between 65 and 80 mm for all the selected rock types. This showed that the values of fracture toughness obtained from the 76 mm diameter core specimens could be considered as representative K_{IC} values. This also showed that Eq. (3), used to calculate the required minimum diameter of core specimen, was reasonably valid provided a reasonable estimate of K_{IC} was known a priori. Equation (4), used to obtain an estimate of K_{IC}, did not work well in this case, requiring the following modification:

$$K_{IC}^{e} = 0.13(E)^{0.65} \tag{8}$$

Similarly, in order to have more confidence that results for the CCNBD test also represent the K_{IC} values of the selected rock types, the required minimum diameter of disc specimens was recalculated using Eq. (5) and actual K_{IC} values obtained with this method. This exercise resulted in the required minimum diameter of the disc specimens to range from 65 to 75 mm for all the selected rock types. This showed that the values of fracture toughness obtained from 76 mm diameter disc specimens could be considered as representative K_{IC} values. This also showed that Eq. (5), used to calculate the required minimum diameter of the disc specimen, worked reasonably well provided a reasonable estimate of K_{IC} was known.

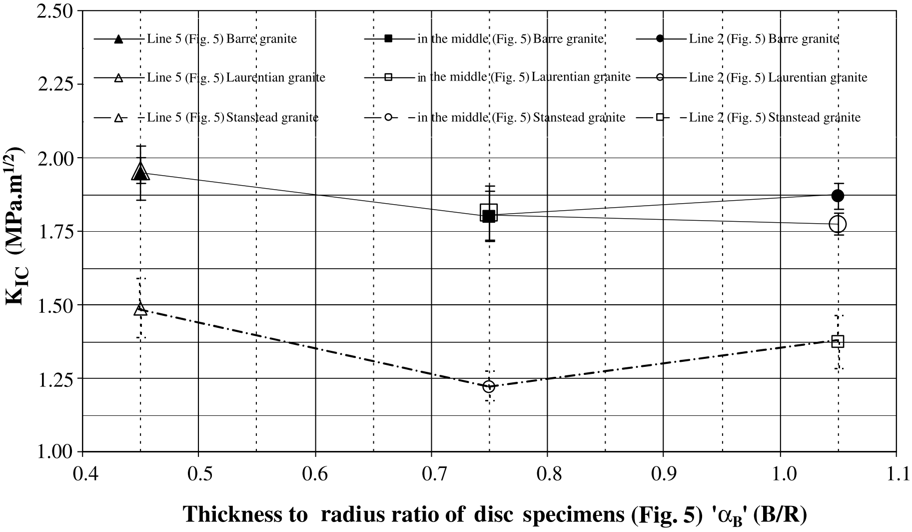


Fig. 18. Fracture toughness values vs. B/R ratio of disc specimen (α_B) for CCNBD test

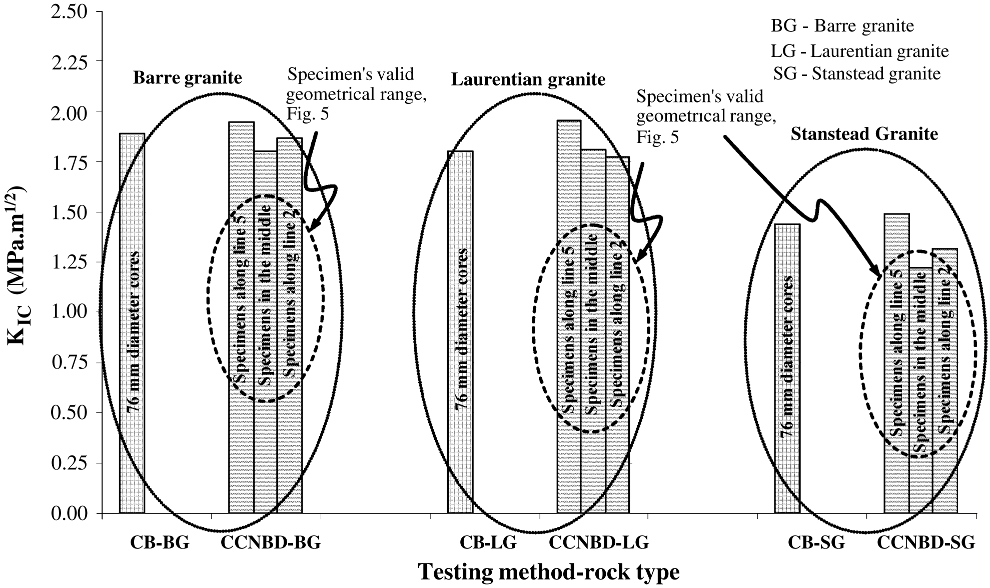


Fig. 19. Comparison of values of fracture toughness obtained with CB and CCNBD (using Eq. (2) i.e. the correct calculation equation) test methods

As far as the valid specimen geometrical range (Fig. 5) for the CCNBD method is concerned, the analysis showed that the fracture toughness values of the disc specimens prepared in the middle and along line 2 (thicker discs) of the valid range were generally comparable. However, the values of the disc specimens prepared along line 5 of the valid range (Fig. 5) were higher (approximately 5–10%) than the other two disc specimen groups as shown in Fig. 18. This shows that valid specimen geometrical range held reasonably well with the selected rock types.

5.2 Comparison of Test Methods

The analysis of effects of test methods showed that values of the fracture toughness obtained from 76 mm diameter core specimens for the CB tests were comparable with those obtained for the CCNBD tests as shown in Fig. 19.

Table 5. Summary of results of fracture toughness values obtained from the CB and CCNBD (using Eq. (1)) tests

Rock type	Results of CB tests			Results of CCNBD tests		
	z-axis notch (MPa.m$^{1/2}$)					
	32 mm	56 mm	75.5 mm	Line 5	In the middle	Line 2
Barre granite	$1.54_6 \pm 0.06$	$1.71_6 \pm 0.05$	$1.89_6 \pm 0.02$	$1.38_6 \pm 0.07$	$1.27_6 \pm 0.06$	$1.32_6 \pm 0.03$
Laurentian granite	$1.54_6 \pm 0.06$	$1.70_6 \pm 0.04$	$1.80_6 \pm 0.04$	$1.38_6 \pm 0.03$	$1.28_6 \pm 0.07$	$1.26_6 \pm 0.03$
Stanstead granite	$1.31_6 \pm 0.12$	$1.26_6 \pm 0.05$	$1.44_6 \pm 0.10$, $1.48_5 \pm 0.05$	$1.05_6 \pm 0.07$	$0.86_6 \pm 0.04$	$0.93_6 \pm 0.05$

All values in the table are average values of the number of tests shown in subscripts, along with the respective standard deviations.

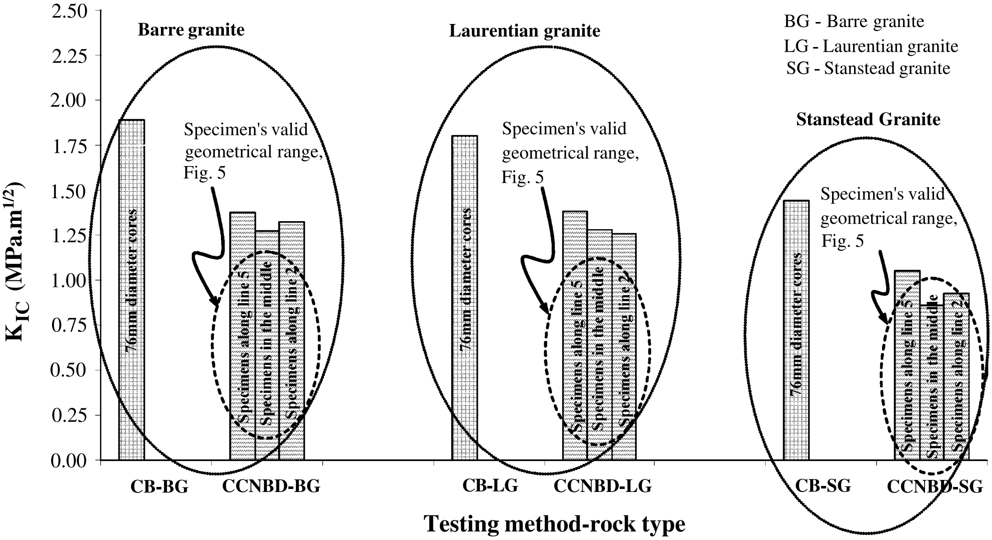


Fig. 20. Comparison of values of fracture toughness obtained with CB and CCNBD (using Eq. (1), i.e. the equation with error) test methods

A summary of the test results using the fracture toughness calculation formula in the ISRM standard document for the CCNBD method, ISRM (1995) (Eq. (1), i.e. formula with an error) is given in Table 5 along with the CB test results. It was found that the values for the CCNBD tests in this case were much lower than those for the CB tests as shown in Fig. 20. This result was similar to what was found in the literature (Dwivedi et al., 2000), where the difference between the two methods was approximately 30–40% for all the selected rock types. This exercise confirmed that lower values of fracture toughness for the CCNBD method, as quoted in literature, were most likely due to erroneous substitution of ‘radius’ by ‘diameter’ of the disc specimen in the equation given in the ISRM (1995) document, used to calculate values of fracture toughness.

As far as the general comparison between the two methods is concerned, the CCNBD method definitely had advantages in terms of less rock material required for testing and simplicity of the specimen installation fixture. The CCNBD method has an additional advantage over the CB method in terms of recording precision, as the former requires a much higher load for initiating failure.

6. Conclusions

The methods of determining fracture toughness in rock with both the Chevron-Bend and the Cracked Chevron Notch Brazilian Disc were found to be valid provided the following conditions were met: a) the correct equation for the CCNBD method (and not the one contained in the ISRM document, 1995) for calculating fracture toughness was used, and b) the issue of specimen size of the rock material are handled carefully. With the above provisions, there seems to be no real discrepancy in the fracture toughness values obtained by the two methods, in contrast to claims by previous investigators. The effect of anisotropy, if any, on the measured values of K_{IC} is minimized in this investigation as all cores in each rock type were obtained along the same axis referenced to a specific seismic wave velocity direction in the block sample. Without such a reference, it is realized that this could significantly affect the measured values of K_{IC}, and this constitutes part of a continuing study on relating micro-structure and grain size with fracture toughness anisotropy. The CCNBD method proved to be the best method in terms of accuracy, precision (compared to the CB method) and ease of conducting the experiments. The measured fracture toughness values were self consistent in each method, i.e. yielded consistent values of fracture toughness with very acceptable scatter in the measured values. The valid specimen geometrical range for the CCNBD method (Fig. 5) appears to be valid except that the fracture toughness values of the disc specimens prepared along line 5 of the valid range were generally more scattered and higher than the values of the specimens prepared in the middle and along line 2 of the range. The requirement of a minimum diameter of the core specimen, (i.e. 10× largest grain size) for the CB test appears to be a sound guideline but should be used with caution. It is not possible to arrive at a useable specimen diameter, by using the suggested empirical equation, for the CB test, without first having an accurate estimate of K_{IC} used in the calculation. The same conclusion applied to sample diameter specification for the CCNBD test.

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Authors' address: Muhammad Javid Iqbal, Department of Civil Engineering and Lassonde Institute, University of Toronto, GB 313b-35 St. George Street, Toronto, Ontario M5S 1A4, Canada; e-mail: iqbalm@ecf.utoronto.ca