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# Technical Note

# Effects of Stress on Cave Passage Shape in Karst Terranes

By

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## 1. Introduction

The shapes and orientations of caves in karst terranes are long-standing enigmas. Palmer (1991) recognizes four basic passage shapes-branchwork, network, ramiform and anastomosing, that he argues are due to different water chemistries, bedrock and caprock characteristics, and hydraulic controls including inundation histories. Jennings (1985) attributes the shape and orientation of caves to preexisting rock structure and the type of water flow inside the cavern. Veni (2005) argues that local lithologic, structural and hydrologic factors govern passage patterns, such as bedding, joints, and dip and strike. Tall caves have been attributed to formation along joints, whereas low, wide caves are commonly thought to result from preferential dissolution of particular strata. Although such explanations are correct or at least plausible in many instances, disagreement exists regarding many mechanisms. For example, joint orientations are clearly important in network caves, yet in many areas little correlation is found between the orientations of joints and cave passages (Orndorff et al., 2001). Thus, although special controls are clearly important in given instances, their significance is not universal.

We suggest that the stress fields surrounding caves exert important controls on passage shape that are ubiquitous and sometimes dominant. Compressional stresses may enhance dissolution of lateral cave walls below the water table, causing the characteristic oval sections of phreatic tubes. Tensional stresses probably cause the ceiling cracks that lie along cave passages, the arcuate fractures of breakdown rooms, the high vaulted shapes of old, inactive caves, and breakdown effects near entrances.

## 2. Stress Concentrations around Caves

The stress concentrations around a circular opening can be approximated with the biaxial equations (e.g., Obert and Duvall, 1967):

$$2\sigma_r = (\sigma_x + \sigma_z)\left(1 - \frac{a^2}{r^2}\right) + (\sigma_x - \sigma_z)\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos(2\theta) \quad (1a)$$

$$2\sigma_\theta = (\sigma_x + \sigma_z)\left(1 + \frac{a^2}{r^2}\right) - (\sigma_x - \sigma_z)\left(1 + \frac{3a^4}{r^4}\right)\cos(2\theta) \quad (1b)$$

$$2\tau_{r\theta} = (\sigma_z - \sigma_x)\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin(2\theta) \quad (1c)$$

These equations give the radial, tangential, and shear stresses ($\sigma_r$, $\sigma_\theta$, $\tau_{r\theta}$), acting on a stress element as a function of its distance from the cavity center ($r$), its angle from horizontal ($\theta$), the cavity radius ($a$) and the nominal horizontal and vertical stresses ($\sigma_x$, $\sigma_z$), as illustrated in Fig. 1a, b. The horizontal and vertical stresses are interrelated but their ratio depends on local conditions. Our calculations assume negligible

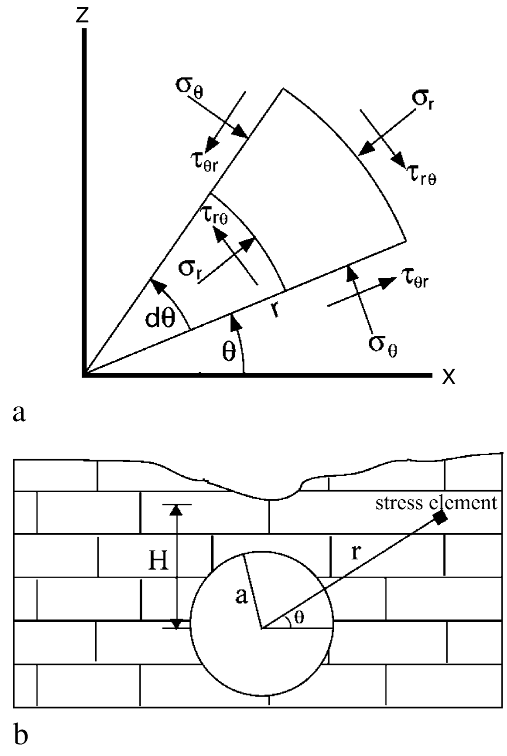


**Fig. 1. a** Orientation and direction of stresses for a stress element in a cylindrical coordinate system, after Obert and Duvall (1967). **b** Field relationship of stress element to a cylindrical cavity of radius "a" and depth *H*

nominal horizontal stresses, and a vertical stress related linearly to depth (e.g. Obert and Duvall, 1967):

$$\sigma_z = \gamma H, \tag{2}$$

where $\sigma_z$ is the nominal vertical stress, $\gamma$ is the rock's unit weight, and $H$ is the depth (Fig. 1b). This equation when combined with Eq. (1a) indicates that the highest compressional stress occurs on the side walls of caves, while the largest tensional stresses are in the ceiling and floor (Fig. 2).

This useful, closed form solution of the stress field around a circular opening is symmetrical in both the horizontal and vertical directions, as it neglects asymmetries due to gravity. Certain inaccuracies arise in detail. The actual gravitational gradient increases tension in the ceiling but decreases it in the floor, so in real caves the roof and floor behave differently. These tensional stresses are also larger than would be expected under lithostatic or hydrostatic conditions where significant horizontal pressure exists. Additionally, Eqs. (1a), (1b), (1c) neglect certain stresses that can be significant

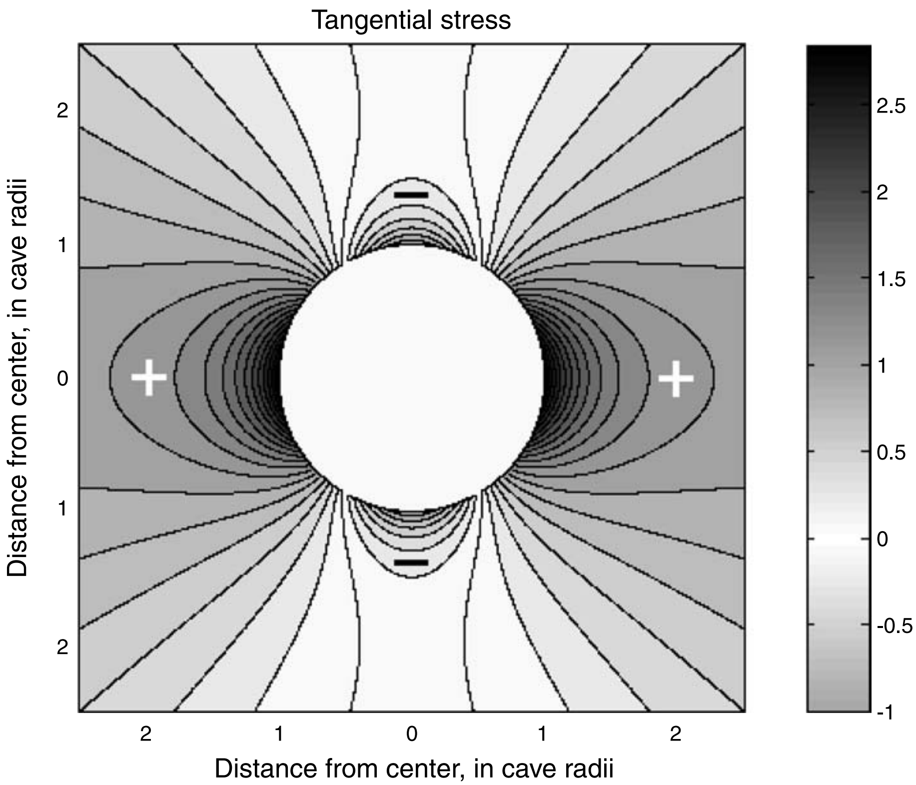


**Fig. 2.** Vertical section showing contours of the tangential stress concentration factor ($\kappa_\theta$) near a cylindrical cavity, defined as the ratio between tangential stress ($\sigma_\theta$), calculated in Eq. (1b), over the nominal vertical stress ($\sigma_z$). Positive values indicate compressive stresses. The nominal horizontal stress is assumed to be 0. Note the high compressive stress concentrations along the side walls of the cavity, with $\kappa_\theta$ approaching 3, and the large tensional stress near the ceiling and floor where $\kappa_\theta$ approaches $-1$

in caves. Pore pressures have a significant effect on the stress field. Sudden draining of cave passages can cause collapse, due to transient effects and loss of support.

Temperature variations, particularly freeze thaw, are similarly not addressed in the above equations, but will also promote tensional failure. Temperature variations affect various sections of the cave differently, particularly near entrances where breakdown is accordingly common. In contrast, cave interiors have uniform temperatures and lack such boundary effects.

## 3. Discussion

### *3.1 Compressional Stresses and their Effects*

Compressional stresses can influence cave shape during dissolution due to their thermodynamic effect on carbonate solubility. Both dissolution and precipitation of calcium carbonate occur in karst regions, as a sensitive response to environmental conditions that affect the dominant equilibrium relationship between calcite, dissolved carbon dioxide, and calcium and bicarbonate ions:

$$\mathrm{CaCO_3} + \mathrm{H_2O} + \mathrm{CO_2} = \mathrm{Ca^{++}} + 2\,\mathrm{HCO_3}^{-}. \tag{3}$$

A well-known thermodynamic equation (e.g., Nordstrom and Munoz, 1986) relates the pressure derivative of the equilibrium constant ($K$) of a reaction to the net volume change ($\Delta V_r$), the gas constant $R$ and temperature $T$.

$$\left(\frac{\partial(\ln K)}{\partial P}\right)_T = -\frac{\Delta V_r^\circ}{RT}. \tag{4}$$

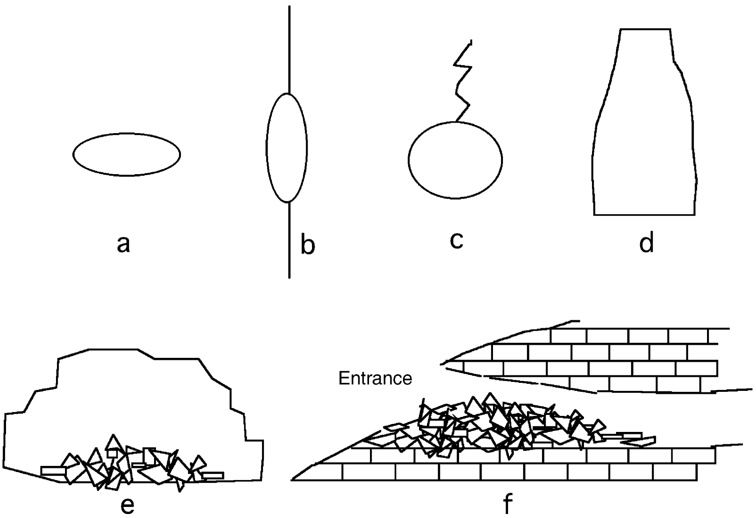


**Fig. 3.** Cross sectional sketches (**a**–**e**) and longitudinal section (**f**) of cave passage shapes illustrating different controls. **a** Oval shape of phreatic cave tube with elongated horizontal axis due to pressure dissolution. **b** Tall cave passage formed along a preexisting vertical joint. **c** Oval cave passage with subsequent tensional ceiling crack. **d** Tall, vault-shaped passage formed by upward stoping and collapse of ceiling. **e** Breakout dome. **f** Longitudinal section of cave entrance on a hillside, showing large pile of breakdown so that entry is near cave ceiling and far above bedrock floor

The common geologic occurrence of pressure solution proves that $\Delta V_r^\circ$ for calcite dissolution is negative. That increased pressure drives calcite into solution is empirically shown through dissolution on grain to grain contacts (e.g., de Boer, 1977) and can be observed in limestone at depths as shallow as 20–30 m (Railsback, 1993).

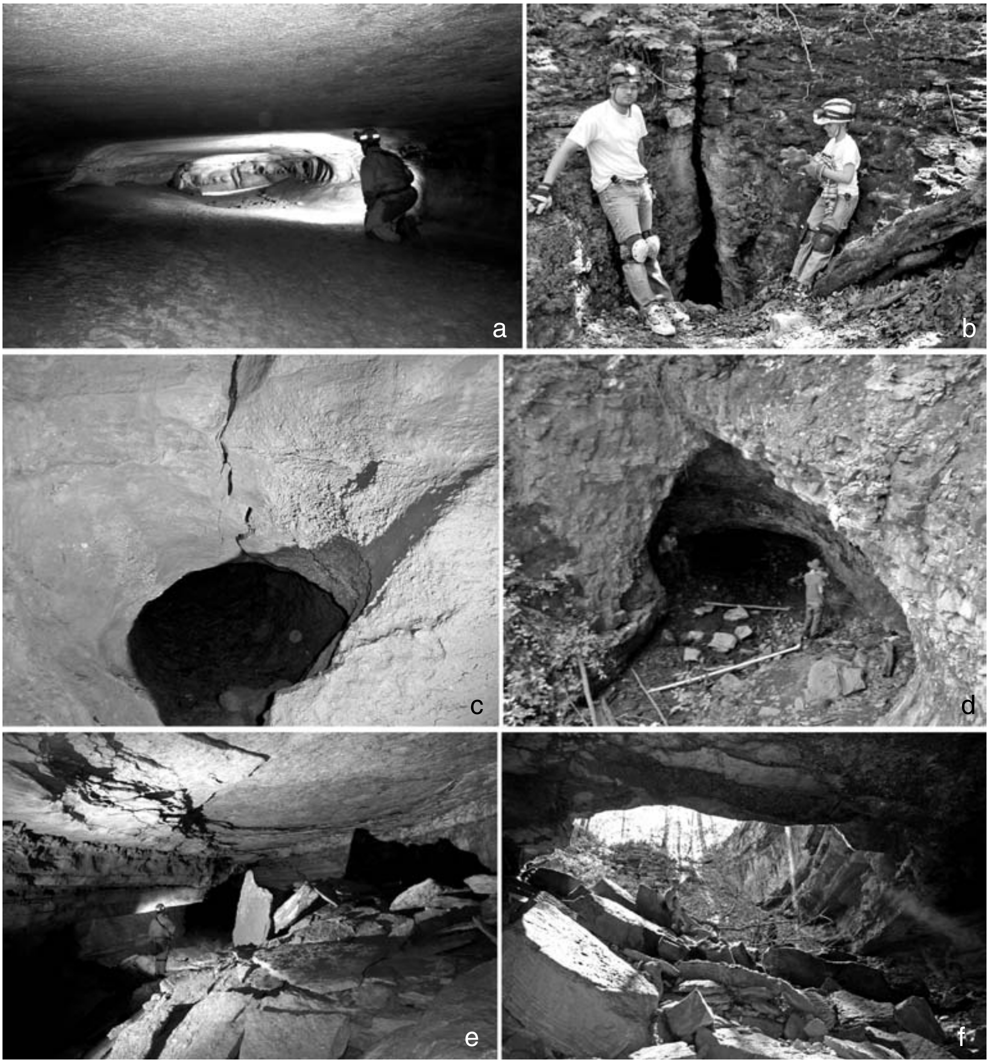

**Fig. 4.** Photos of cave passage shapes (cf. Figs. 3a–f). **a** Oval phreatic tube, Pohl Avenue, Mammoth Cave KY. **b** Structurally-controlled cave formed along preexisting vertical joint, Catacomb Cave, MO; the cave continues for 100 m, mostly as a 1 m-wide, 5–10 m-tall slot that trends due east–west. **c** Intersection of 1 m-high, 1.5 m-wide oval tube with main cave passage, showing jagged tensional ceiling crack formed subsequent to passage, Rankin Cave, MO. **d** Enlarged, vault-shaped entrance of Milburn Cave, MO; entrance breakdown has been efficiently removed by outflowing cave stream. **e** Breakout dome where stoping along arcuate ceiling fractures has modified the oval tube of Pohl Avenue (cf. Fig. 4a). **f** Entrance of Mertz Cave, MO, showing large, sloping breakdown pile; the cave stream is inflowing so breakdown is not efficiently removed, in contrast to Fig. 4d

Stylolites provide another example of pressure dissolution and have been described as a type of failure, an "anticrack" in the rock (Fletcher and Pollard, 1981). In areas of high compressive stress (Fig. 2), the additional pressure will enhance already occurring dissolution. Although this effect may be small, it will be significant near equilibrium. Were the ceiling of a phreatic cave at equilibrium with the ambient ground water, the side walls could continue to dissolve, as they are under higher pressure. Furthermore, as the cave cross section becomes elongated horizontally, the stress on the lateral walls will increase even more, accentuating the effects of pressure dissolution. The oval shape of phreatic tubes (Figs. 3a, 4a) could be due to pressure effects, rather than to the influence of bedding planes, the preferential dissolution of particular rock strata, or partial filling of cave passages with water.

### *3.2 Tensional Stresses and their Effects*

Tensional stresses have great relevance to cave behavior. Even if hosted by rock with great compressional strength, such as limestone, caves and tunnels can collapse due to tension. Ceiling cracks provide an obvious example; commonly, one dominant crack will run for great distances down the very center of a passage ceiling. Such cracks are usually considered to be joints, and sometimes even referred to as the "lifeline of a cave", the assumption being that the cave formed along this preexisting "master" joint (Fig. 3b). Whereas such examples exist (Fig. 4b), caves more commonly do not follow regional joint sets, yet ceiling cracks are almost universal, forcing the conclusion that the so called "lifelines" of caves are evidence of their tensional failure rather than a cause of their formation (Figs. 3c, 4c). This explanation better explains the discontinuous, jagged, irregular, and even arcuate character of most ceiling cracks and their contrast with joints which are very straight. Such cracks generally occur in the very center of the ceiling where tensile stresses are greatest.

Generally crack formation is but an initial step in cave collapse; caves tend to fail incrementally through progressive crack formation, culminating in ceiling failure due to the unsupported weight of roof rock. Presence of bedding planes will often result in a characteristic "stair step" (Figs. 3e, 4e) as the roof incrementally collapses. Engineers have long observed that the collapse of unsupported materials will eventually form a vault (Lauchli, 1915). This shape can be observed, in different stages of completion, in various caves (Figs. 3d, 4d). This shape is also attained in three dimensions, through the production of "beehive" domes, as observed in the collapse of large rooms (Figs. 3e, 4e).

Breakdown is common near cave entrances, particularly in temperate zones where freeze-thaw magnifies transient stresses due to temperature variations. In many caves large passages are almost completely choked where they intersect Earth's surface, and entry is achieved by crawling beneath the ceiling (Figs. 3f, 4f). However, if breakdown removal is efficient, such processes can greatly enlarge the cave entrance relative to the size of the interior passage (Fig. 4d).

## 4. Conclusion

Stress phenomena inside caves are universal, but their importance is variable. Certain cave shapes clearly reflect influencing or dominant stresses. The oval sections of phreatic tubes may indicate preferential dissolution of the lateral walls where compressive stress is maximized. Ceiling cracks attest to the destructive power of roof tensional stresses. Both large cave entrances and entrances choked with breakdown indicate the influence of transient stresses due to temperature variations. Finally, arch-shaped cross sections, especially when coupled with breakdown, illustrate the structural response of ovoid cavities to rectify their dissolutionally efficient, yet structurally poor design by forming a vaulted shape.

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