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Technical Note

Rock Wedge Stability Analysis Using System Reliability Methods

By

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Summary

We present a system reliability approach to rock wedge stability analysis. Different failure modes are considered, and a disjoint cut-set formulation is employed – with each cut-set corresponding to a different failure mode – to explore the system aspects of the problem, so that the reliability of the system is assessed by computing the probability of failure of the slope under each failure mode. An example case is used to demonstrate different approaches to compute the reliability of the slope design. Our results show that an approximation to the "exact" probability of failure – given by Monte Carlo simulation results – may be obtained using a first order approximation to the failure domain, and that linear programming techniques may be used to obtain bounds of the probability of failure. Furthermore, we identify the most likely failure mode, and we explore the sensitivity of the computed probabilities to changes in the random variables considered. The results indicate that the reliability results are quite sensitive to the geometry of the wedge. Changes in water conditions are also found to have a significant impact on the computed probabilities, while changes in unit weight of the rock have a considerably smaller effect on the reliability.

Keywords: Wedge stability, reliability, FORM, linear programming, risk, Monte Carlo.

1. Introduction

The characterization of rock masses for engineering applications is subject to uncertainties due to the limited data that are typically available during site characterization, and due to inherent variability of properties within the rock mass. Accordingly, the problem of decision-making under uncertainty has become a topic of increasing interest for the rock engineering community. Within that context, hazard assessment and the quantification of the probability of undesirable events – i.e., failure probability – is a significant aspect of the decision-making process, and it has been widely discussed in the literature (Tamimi et al., 1989; Duzgun et al., 2003; Low, 1997).

Wedge failures are probably the most common and general case (Hoek and Bray, 1981) within the wide variety of mechanisms leading to failure of rock slopes (see e.g., Goodman and Kieffer, 2000). Hence, the problem has been extensively treated in the literature (Hoek and Bray, 1981; Warburton, 1981; Goodman, 1989; Wittke, 1990; Nathanail, 1996; Low, 1997; Wang and Yin, 2002). Here we explore the system aspects (Hudson, 1992; Jimenez-Rodriguez et al., 2006) of the problem of analysis of stability of rock wedges using limit equilibrium methods, and we develop a probabilistic approach in which state-of-the-art reliability methods (e.g., Ditlevsen and Madsen, 1996; Ambartzumian et al., 1998; Song and Der Kiureghian, 2003) are employed to compute the probability of failure of rock wedges in a systematic and quantitative way.

2. Wedge Stability Model

Unstable wedges may be formed in rock slopes cut by at least two sets of discontinuities upon which sliding can occur (Hoek and Bray, 1981). In this paper we address the problem of stability of individual wedges in rock slopes. We use the closed-form equations presented by Low (1997) for stability of tetrahedral wedges in slopes with an inclined upper ground surface that dips in the same direction as the slope face (see Fig. 1). Four different failure modes may be defined for a wedge (Goodman, 1989; Low, 1997): Sliding along the line of intersection of both planes forming the block (failure mode 1), sliding along plane 1 only (failure mode 2), sliding along plane 2 only (failure mode 3), and a “floating” type of failure (failure mode 4). “Floating” failure could be induced by high water pressures or in-situ stresses, by directly applied forces (e.g., the pull of a cable anchored within the wedge), or by a combination of both. It needs to be kept in mind that such failure modes represent a limited set of possibilities leading to failure of rock slopes (Goodman and Kieffer, 2000), and that the addition of more joints to the system could define additional blocks that may need

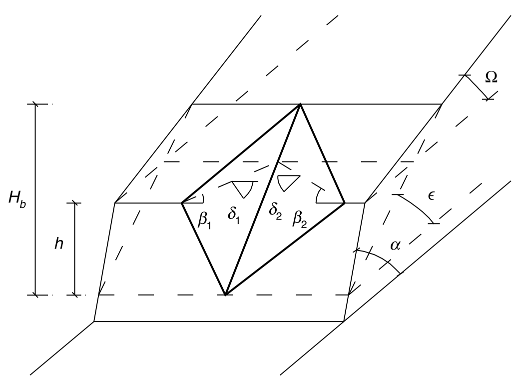

Fig. 1. Tetrahedral wedge model

to be considered. (Naturally, the same approach could be extended to other cases; e.g., see Jimenez-Rodriguez et al. (2006) for an application to the analysis of plane failure in rock slopes.)

As an example, we present the conditions that need to be fulfilled for occurrence of failure mode 1. The conditions for occurrence of the remaining failure modes can be derived in an equivalent way, and in the interest of brevity they are not listed herein. (See Low (1997) for details.)

2.1 Failure Mode 1

The closed-form expression for the factor of safety for a wedge under failure mode 1 is given by Low (1997) as:

$$FS = \left(a_1 - \frac{b_1 G_w}{S_\gamma}\right)\tan\phi_1 + \left(a_2 - \frac{b_2 G_w}{S_\gamma}\right)\tan\phi_2 + 3b_1\frac{c_1}{\gamma h} + 3b_2\frac{c_2}{\gamma h}. \tag{1}$$

Equation (1) is only valid under the condition that there is contact on both planes – i.e., the terms preceding $\tan\phi_1$ and $\tan\phi_2$ must be positive:

$$\left(a_1 - \frac{b_1 G_w}{S_\gamma}\right) > 0, \tag{2}$$

and

$$\left(a_2 - \frac{b_2 G_w}{S_\gamma}\right) > 0, \tag{3}$$

where a_1, a_2, b_1, b_2 are parameters that depend on the geometry of the slope (i.e., defined as a function of angles δ_1, β_1, δ_2, β_2, α, Ω, and ϵ, as shown in Fig. 1 and explained in Table 1; for details see Low (1997)); c_1 and c_2 represent the cohesion on planes 1 and 2; ϕ_1 and ϕ_2 represent the corresponding friction angles; G_w is a pore-pressure parameter ($G_w = H/2h$ for pyramidal water pressure distributions, as considered herein); and, finally, $S_\gamma = \gamma/\gamma_w$ is the specific density of the rock.

In addition, the following kinematical constraints have to be fulfilled:

$$\Omega < \epsilon < \alpha, \tag{4}$$

indicating the formation of removable wedges that can move toward the free face (Goodman and Shi, 1985).

Table 1. Description of angles that define the geometry of the wedge

Angle	Description
δ_1	Dip angle of plane 1
δ_2	Idem for plane 2
β_1	Horizontal angle (measured within the wedge) between the strike line of plane 1 and the (horizontal) line of intersection between the upper ground surface and the slope face
β_2	Idem for plane 2
α	Inclination (i.e., dip angle) of slope face
Ω	Inclination (i.e., dip angle) of upper ground surface
ϵ	Inclination (i.e., plunge angle) of the line of intersection between planes 1 and 2

3. System Reliability

The reliability analysis is performed by computing the probability of failure of removable wedges due to each of the four failure modes considered. In particular, we use a *disjoint cut-set formulation*, in which the performance of the system (i.e., the stability of the wedge) is modelled as a series assembly of disjoint parallel sub-systems. That is, failure of the system under each failure mode occurs when *all* the components in the corresponding parallel sub-system fail, and the total probability of failure of the slope may be obtained as the *sum of probabilities* of failure of the individual failure modes. In Fig. 2 we show the general system representation of the wedge stability model presented in Section 2, and in Table 2 we list the physical interpretation of the limit state functions, as well as the definitions of the limit state functions corresponding to failure mode 1 (see Section 2.1). (We assume that component i fails if the corresponding limit state function, g_i, is less than zero.)

The probability of failure of each component is computed by solving the following integral:

$$P_f = P(g(\mathbf{x}) \leqslant 0) = \int_{g(\mathbf{x}) \leqslant 0} f(\mathbf{x}) d\mathbf{x}, \tag{5}$$

where $f(\mathbf{x})$ is the probability density function (PDF) of the input variables of the stability model.

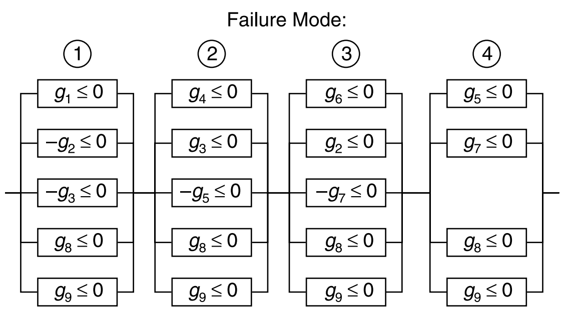

Fig. 2. Disjoint cut-set system formulation of rock wedge stability

Table 2. Interpretation of limit state functions in the system modeling failure of the wedge

LSF	Interpretation
$g_1 \equiv FS - 1$	Wedge unstable with contact on both planes
$g_2 \equiv a_1 - b_1 G_w / S_\gamma$	Contact on plane 1
$g_3 \equiv a_2 - b_2 G_w / S_\gamma$	Contact on plane 2
g_4	Wedge unstable with contact on plane 1 only
g_5	"Floating" conditions on plane 2
g_6	Wedge unstable with contact on plane 2 only
g_7	"Floating" conditions on plane 1
$g_8 \equiv \Omega - \epsilon$	Kinematic admissibility
$g_9 \equiv \epsilon - \alpha$	Kinematic admissibility

A number of methods have been developed to obtain the probability content of the failure domain in Eq. (5). Monte Carlo (MC) simulation methods have been used in the literature (Tamimi et al., 1989), together with approximations such as the first order reliability method (FORM) (Bjerager, 1990). Based on the FORM analysis of the components in each failure mode (i.e., parallel sub-system), a first order approximation to their probability of failure may be obtained as (Ditlevsen and Bjerager, 1989):

$$P_f \approx P\left(\bigcap_{i \in C_k} \beta_i \leqslant v_i \right) = \Phi(-\beta_{C_k}, \mathbf{R}), \tag{6}$$

where C_k is the cut-set corresponding to the failure mode of interest, β_{C_k} is the vector of reliability indices of the components, and $\mathbf{R}$ is the correlation matrix between the components. The probability in Eq. (6) may be efficiently computed using the SCIS algorithm (Ambartzumian et al., 1998).

Finally, bounds of the probability of failure of the system may be obtained using, for instance, the linear programming (LP) approach proposed by Song and Der Kiureghian (2003). This approach is based on the solution of the optimization problem of a linear objective function subjected to linear constraints. The strength of the LP bounds is that they have been shown to be the narrowest possible bounds for any given level of information on component probabilities.

4. Example Analysis

We consider an example case in which the input variables of the wedge stability model have the statistical distributions presented in Table 3, where μ indicates the mean of the distribution, σ is the standard deviation, and a and b indicate lower and upper bounds. Cohesion values are chosen to be lognormally distributed because the lognormal distribution is often used to model cohesion (Duzgun et al., 2003). Beta distribution is used to model friction angles because it is flexible and versatile; it is also

Table 3. Statistical distributions of input parameters in the stability model

Variable	Type	Parameters			
		p_1	p_2	p_3	p_4
c_1 [kPa]	Lognormal[a]	22	4		
ϕ_1 [deg]	Beta[b]	30	5	22	38
c_2 [kPa]	Lognormal[a]	25	4		
ϕ_2 [deg]	Beta[b]	32	5	24	36
G_w	Normal[a]	0.5	0.12		
δ_1 [deg]	Uniform[c]	47	53		
β_1 [deg]	Uniform[c]	58	66		
δ_2 [deg]	Uniform[c]	45	51		
β_2 [deg]	Uniform[c]	16	24		
γ_r [kN/m^3]	Normal[a]	26	2		

[a] $p_1 = \mu$, $p_2 = \sigma$.
[b] $p_1 = \mu$, $p_2 = \sigma$, $p_3 = a$, $p_4 = b$.
[c] $p_1 = a$, $p_2 = b$.

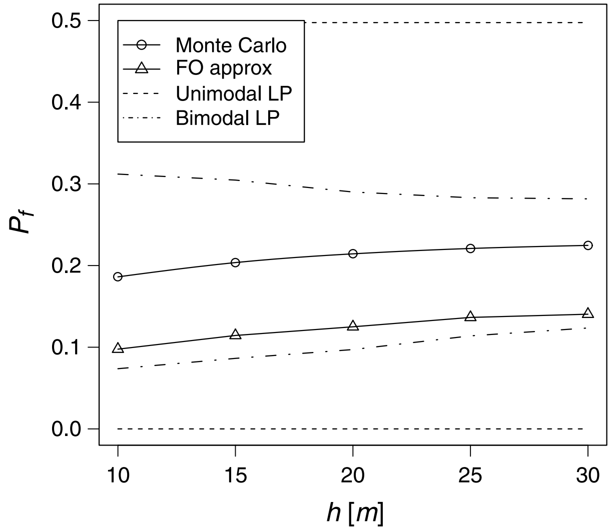

Fig. 3. Probability of failure of the slope for different wedge sizes

bounded, avoiding problems that may arise when using unbounded distributions to model friction angles. Since no previous information on the distribution of the angles defining wedge geometry is known, the uniform distribution is used to model δ_1, β_1, δ_2, and β_2. Finally, the normal distribution is used to model the distributions of water pressure and unit weight. Random variables in the model are considered to be independent of each other; c_1 and ϕ_1, as well as c_2 and ϕ_2 are, however, considered to be negatively correlated (with a correlation coefficient of $\rho = -0.3$), to model common shear test results in which the cohesion drops as the friction angle rises, and vice-versa (Hoek, 2000). In addition, the following deterministic parameters are considered in the analyses: $\alpha = 70$ [deg], $\Omega = 0$ [deg], and $\gamma_w = 9.8$ $[kN/\mathrm{m}^3]$.

Figure 3 shows the probability of failure of the slope as a function of wedge size. Monte Carlo simulation and FORM approximation results have been computed using program CALREL (Liu et al., 1989), whereas the first order approximation to the system's probability of failure (see Eq. (6)) has been computed using our own computer implementation of the SCIS algorithm. (Our program is built on top of the GNU Scientific library (Galassi et al., 2003), and it is available upon request.) Finally, we also show the LP bounds computed in this case. In general, the uni-modal bounds (i.e., bounds computed using probabilities of single components only) are too wide to be of any practical interest. Bi-modal bounds (i.e., bounds based on probabilities of intersections of two components) are, however, much narrower, providing significantly improved estimates. As expected, the "exact" solution (i.e., the Monte Carlo solution) is contained within the computed LP bounds.

Figure 4 shows the relative influence (using "exact" MC results) that each failure mode has on the overall failure probability of the slope. Useful information for slope design may be obtained from this analysis; in this case, for instance, failure mode 4 (i.e. corresponding to "floating" conditions) is the failure mode contributing most to

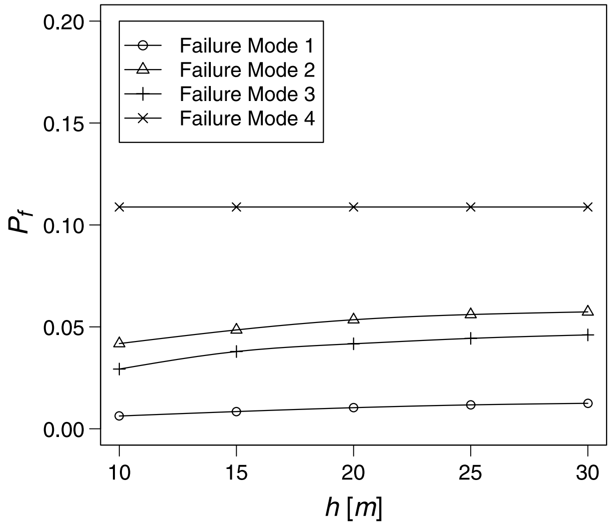

Fig. 4. Relative significance of failure modes

Table 4. Results of FORM analysis on components in failure mode 1

Variable	$g_1(\mathbf{x}) \leqslant 0$		$g_2(\mathbf{x}) \leqslant 0$		$g_3(\mathbf{x}) \leqslant 0$	
	$\mathbf{x}^*$	γ	$\mathbf{x}^*$	γ	$\mathbf{x}^*$	γ
c_1 [kPa]	2.2E+01	−0.02	2.2E+01	0.00	2.2E+01	0.00
ϕ_1 [deg]	3.9E+01	−0.05	3.0E+01	0.00	3.0E+01	0.00
c_2 [kPa]	2.5E+01	−0.06	2.5E+01	0.00	2.5E+01	0.00
ϕ_2 [deg]	3.5E+01	−0.01	3.5E+01	0.00	3.5E+01	0.00
G_w	5.0E−01	0.08	5.0E−01	0.01	5.0E−01	0.04
δ_1 [deg]	4.8E+01	0.46	5.6E+01	0.65	4.5E+01	−0.35
β_1 [deg]	5.6E+01	−0.50	4.8E+01	−0.58	5.7E+01	−0.17
δ_2 [deg]	4.7E+01	0.70	3.9E+01	−0.46	5.0E+01	0.90
β_2 [deg]	1.6E+01	−0.17	1.5E+01	−0.11	1.5E+01	−0.19
γ_r [kN/m^3]	2.6E+01	0.01	2.6E+01	−0.00	2.6E+01	−0.01

the probability of failure, indicating that assuring an adequate drainage should be a priority during design and construction of the slope.

Table 4 shows the sensitivity of the reliability results to changes in the random variables. $\mathbf{x}^*$ is the *design point*, corresponding to the transformation of the most likely failure point in the standard normal space back to the original space; and γ represents the sensitivities of the computed reliability results to changes in the random variables $\mathbf{x}$ (the larger the absolute value of the component of γ corresponding to random variable x_i, the higher the sensitivity with respect to changes in random variable x_i). These results show that, for instance, the limit state functions in the failure mode are mainly sensitive to changes in the geometry of the wedge. (Note the high values in the sensitivity vector, γ, corresponding to variables δ_1, β_1, δ_2, and β_2). That is, changes in the geometry of the wedge are shown to have a significant influence on the stability of the wedge and, accordingly, a good structural characterization of discontinuities is of paramount importance for an adequate assessment of wedge stability. Among the remaining random variables, the reliability results are most sensitive to changes in the water pressure parameter, G_w, while changes in the unit weight of the rock appear to be the least significant factor.

5. Conclusions

We present a methodology for the assessment of stability of wedges in rock slopes under uncertain information. System aspects of the problem are considered, and we model the stability of the wedge using a disjoint cut-set formulation, in which disjoint parallel sub-systems are used to represent the different failure modes of the slope.

An example case consisting of a tetrahedral wedge is used to demonstrate different approaches to compute the reliability of a slope design. The "exact" reliability results can be computed using Monte Carlo simulation methods; an approximation to the probability of failure may also be computed using a first order approximation to the failure domain, in which FORM information is employed. In addition, our results show that adequate bounds on the failure probability of the system may be obtained using linear programming techniques, as long as sufficient information on component reliabilities is considered.

Finally, additional information of interest in the design process may be obtained using the approach presented in this paper. For example, in the case presented here, the most likely failure mode corresponded to "floating" of the wedge, indicating the importance of an adequate drainage in the design of this particular slope. In addition, the reliability results were found to be highly sensitive to variations in the geometry of the wedge (indicating the importance of an adequate structural characterization of the rock mass) and to variations in water level conditions, whereas variations in the unit weight of the slope were found to have a significantly smaller influence on the probability of failure.

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References

Ambartzumian, R., Der Kiureghian, A., Ohanian, V., Sukiasian, H. (1998): Multinormal probability by sequential conditioned importance sampling: theory and application. Probabil. Engng. Mech. 13(4), 299–308.

Bjerager, P. (1990): On computation methods for structural reliability analysis. Struct. Saf. 9, 79–96.

Ditlevsen, O., Bjerager, P. (1989): Plastic reliability analysis by directional simulation. J. Engng. Mech. 115 (6), 1347–1362.

Ditlevsen, O., Madsen, H. (1996): Structural reliability methods. Wiley, Chichester.

Duzgun, H. S. B., Yucemen, M. S., Karpuz, C. (2003): A methodology for reliability-based design of rock slopes. Rock Mech. Rock Engng. 36 (2), 95–120.

Galassi, M., Davies, J., Theiler, J., Gough, B., Jungman, G., Booth, M., Rossi, F. (2003): GNU Scientific Library Reference Manual, 2nd edn. Network Theory Ltd.

Goodman, R. E. (1989): Introduction to rock mechanics, 2nd edn. Wiley, New York.

Goodman, R. E., Kieffer, D. S. (2000): Behavior of rock in slopes. J. Geotech. Geoenviron. Engng. ASCE 126 (8), 675–684.

Goodman, R. E., Shi, G. (1985): Block theory and its application to rock engineering. Prentice-Hall international series in civil engineering and engineering mechanics. Prentice-Hall, Englewood Cliffs, N.J.

Hoek, E. (2000): Practical rock engineering. http://www.rocscience.com/hoek/PracticalRockEngineering.asp, World Wide Web edn.

Hoek, E., Bray, J. (1981): Rock slope engineering, rev. 3rd edn. Institution of Mining and Metallurgy, London.

Hudson, J. A. (1992): Rock engineering systems: Theory and practice. Series in Civil Engineering, Geotechnics. Ellis Horwood, New York.

Jimenez-Rodriguez, R., Sitar, N., Chacón, J. (2006): System reliability approach to rock slope stability. Int. J. Rock Mech. Min. Sci. 43 (6), 847–859.

Liu, P.-L., Lin, H.-Z., Der Kiureghian, A. (1989): CALREL user manual. Structural engineering, mechanics and materials. Department of Civil Engineering. University of California at Berkeley.

Low, B. K. (1997): Reliability analysis of rock wedges. J. Geotech. Geoenviron. Engng. 123 (6), 498–505.

Nathanail, C. (1996): Kinematic analysis of active/passive wedge failure using stereographic projection. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 33 (4), 405–407.

Song, J., Der Kiureghian, A. (2003): Bounds on system reliability by linear programming. J. Engng. Mech. 129 (6), 627–636.

Tamimi, S., Amadei, B., Frangopol, D. M. (1989): Monte Carlo simulation of rock slope stability. Comput. Struct. 33 (6), 1495–1505.

Wang, Y. J., Yin, J. H. (2002): Wedge stability analysis considering dilatancy of discontinuities. Rock Mech. Rock Engng. 35 (2), 127–137.

Warburton, P. M. (1981): Vector stability analysis of an arbitrary polyhedral rock block with any number of free faces. Int. J. Rock Mech. Min. Sci. 18 (5), 415–427.

Wittke, W. (1990): Rock mechanics: theory and applications with case histories. Springer, Berlin Heidelberg New York Tokyo.

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