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Stability of Asymmetric Roof Wedge Under Non-Symmetric Loading

By

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Summary

An analytical method is presented for the calculation of the load carrying capacity of two-dimensional asymmetric rock wedges when the loading on the joint faces is non symmetric, such as the case of an asymmetric wedge formed in the roof of a circular tunnel in an inclined stress field. The pull out force that causes yield at one of the joint faces is evaluated from formulae based on the limiting equilibrium conditions assuming a purely frictional joint resistance. Next, the total pull out force required for the secondary face to yield is calculated. During this step, the wedge is further displaced and while the primary yielding face is plastically deformed, the other face is still in the elastic range until failure. Validation of the analytical procedure is obtained with the UDEC code, which provides an implementation of the Distinct Element Method in two dimensions. When the assumptions made in the analytical procedure are valid, the analytically calculated values for the pull out resistance of the wedge are computed to be close to the numerically obtained ones.

Keywords: Rock wedges, limit equilibrium, numerical simulation.

List of symbols

FS	Factor of safety
h	Height of the wedge
H_0, V_0	Horizontal and vertical force respectively, acting on the joint faces
k_{n1}, k_{n2}	Normal stiffness of joint 1 and 2 respectively
k_{s1}, k_{s2}	Shear stiffness of joint 1 and 2 respectively
N_{01}, N_{02}	Normal forces acting on the joint face 1 and 2 respectively
S_{01}, S_{02}	Shear forces acting on the joint face 1 and 2 respectively
P_0	Load carrying capacity of the wedge
q	Non-dimensional load carrying capacity of the wedge
R	Support force

W	Weight of the wedge
u	Displacement of the wedge as a rigid body
u_x, u_y	Components of u in x and y directions
α_1, α_2	Wedge semi-apical angles
σ, τ	Normal and shear stress respectively
ϕ_1, ϕ_2	Friction angles of joints 1 and 2 respectively

1. Background

The stability of a rock block formed in the roof or the walls of an underground opening depends on the orientation and length of the joints delineating the block, their strength and elastic properties, the deformability of the block and that of the surrounding rock mass and the stress field within the rock mass. When analyzing the stability of such a rock block it is necessary to take account of the stress field around the block, which in many cases produces confining stresses around the block that significantly increase its stability.

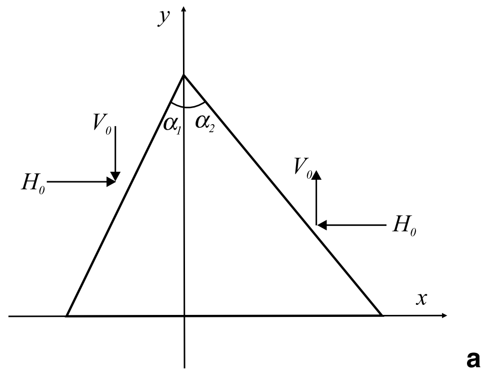
Bray (1977) provided an analytical solution for the stability analysis of rock blocks in the roof of an excavation confined by the lateral stress field by assuming a two-stage analytical procedure. In the first stage the rock mass around the excavation is considered as linearly elastic, homogeneous and isotropic, while the joints forming the wedge are assumed infinitely stiff. Thus, the forces acting on the joint surfaces of the wedge can be calculated by elastic analysis, where the usual assumption of a weightless medium is adopted. In the second stage the joints are deformable according to their natural characteristics while the rock mass is assumed rigid. During this stage, loading is due to the weight W of the wedge and any acting support force R. The pull out resistance P_0 of the wedge can be determined by using the limiting equilibrium condition on the joints of the wedge. For any wedge in the roof of an opening the safety factor of the wedge is then defined by

$$FS = \frac{R + P_0}{W} = \frac{R}{W} + q, \quad q = \frac{P_0}{W} \tag{1}$$

where, q is the normalized pull out resistance of the wedge.

Sofianos (1986) used the relaxation procedure proposed by Bray to calculate the pull out resistance of two-dimensional wedges in the roof of an excavation including that of an asymmetric rigid wedge under symmetric external loading. Elsworth (1986) provided a solution for the calculation of the forces acting on the joints of a symmetric wedge formed in the roof of a circular opening for the case of a hydrostatic stress field. For non-hydrostatic natural stress field, the calculation of the confining forces acting on a symmetric roof wedge is given by Sofianos et al. (1999) by virtue of the symmetry of geometry and loading conditions. Nomikos et al. (2002) provided a solution for the calculation of the confining forces as well as for the stability of a symmetric wedge, when the loading is non-symmetric, such as in the case of an inclined non-hydrostatic natural stress field.

In many cases, roof wedges formed in tunnel roofs in blocky rock masses, are asymmetric, while external loading conditions on their edges are often non-symmetric (Fig. 1a). Such is the case of the asymmetric rock wedge formed in the roof of a circular tunnel in an inclined biaxial stress field (Fig. 1b). Confining forces acting on the wedge may be calculated by analytical procedures proposed by Nomikos et al.



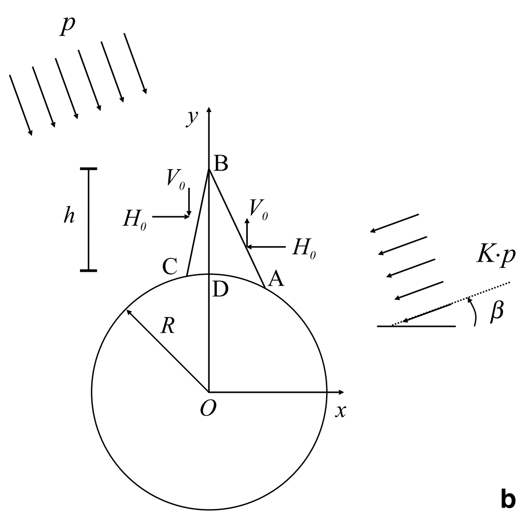


Fig. 1. Asymmetric rigid roof wedge with non-symmetric external loading: **a** in horizontal roof, **b** in the roof of a circular tunnel in an inclined stress field

(2002). However, in this case existing analytic formulae provided by Sofianos (1986) for the calculation of the pull out resistance of an asymmetric wedge need to be modified to include the asymmetric loading conditions on its edges.

2. Pull-out Resistance of the Wedge

Let us consider the case of Fig. 2, where a two-dimensional asymmetric rigid wedge is formed in the roof of an excavation. For the calculation of the wedge movement and of the load carrying capacity, the two-stage relaxation procedure is applied.

In Fig. 2 the free body diagram of the wedge body is shown with the forces acting on it. These are the normal and shear forces along the joint faces of the wedge, its own weight (W) and any artificial support force (R). In Fig. 3 the body of the wedge is shown with the forces W and R replaced by their resultant P. The force P_0 that brings

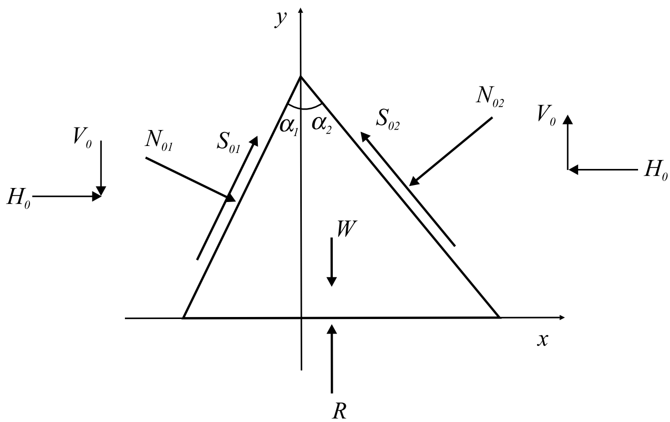


Fig. 2. Free body diagram of an asymmetric roof wedge subject to surface forces exerted by the surrounding rock, its weight (W) and support force (R)

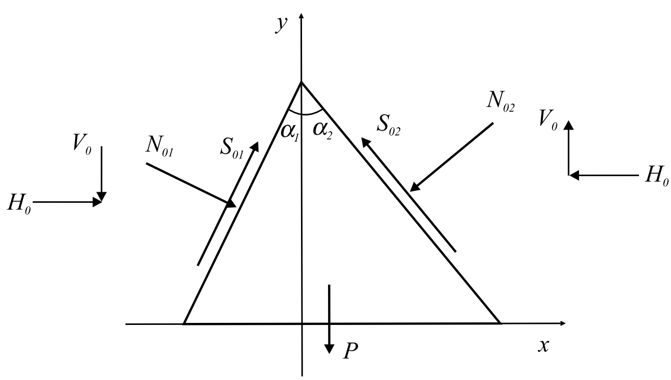


Fig. 3. Replacement of wedge weight (W) and support force (R) with the force P

the body of the wedge to the limit equilibrium state is the load carrying capacity of the wedge.

If the wedge is in the roof of a circular tunnel in an arbitrary inclined natural stress field (Fig. 1), the forces acting on the joint faces of the wedge after excavation and before the wedge begins to deform or be displaced can be obtained for any lateral stress coefficient and angle β by using the procedure developed by Nomikos et al. (2002). In the general case forces N_{01}, N_{02}, S_{01} and S_{02} are not equal and thus, as proposed by Sofianos (1986), the two joint faces will not simultaneously reach a state of limit equilibrium during the movement of the wedge.

For the calculation of the load carrying capacity of the wedge we consider an arbitrary displacement u of the wedge as a rigid body, with components u_x and u_y along the x and y directions respectively, as shown in Fig. 4. Let us suppose that during the movement of the wedge joint face 2 yields first, while face 1 remains elastic.

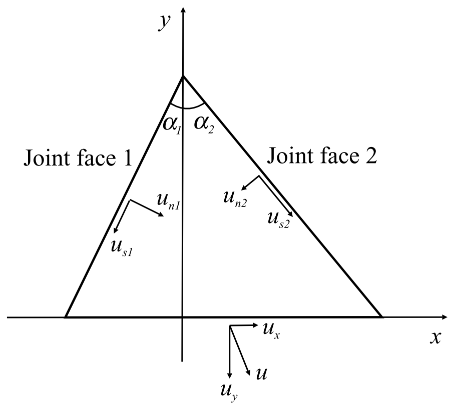


Fig. 4. Free body movement of the wedge as a rigid body

Solving the equations of elasticity, equilibrium, kinematic compatibility and yield, the values for u_x, u_y and P_0 for first yield at face 2 are obtained by the following equations

$$\begin{bmatrix} u_x \\ u_y \\ P_0^y \end{bmatrix} = \frac{r^y}{a^y \cdot d^y - b^y \cdot c^y} \cdot \begin{bmatrix} -b^y \\ a^y \\ a^y \cdot f^y - b^y \cdot e^y \end{bmatrix}, \tag{2}$$

where

$$a^y = -k_{s1} \cdot \sin^2\alpha_1 - k_{n1} \cdot \cos^2\alpha_1 - k_{s2} \cdot \sin^2\alpha_2 - k_{n2} \cdot \cos^2\alpha_2 \tag{3a}$$

$$b^y = -(k_{n1} - k_{s1}) \cdot \sin\alpha_1 \cdot \cos\alpha_1 + (k_{n2} - k_{s2}) \cdot \sin\alpha_2 \cdot \cos\alpha_2 \tag{3b}$$

$$c^y = k_{s2} \cdot \sin\alpha_2 - k_{n2} \cdot \cos\alpha_2 \cdot \tan\phi_2 \tag{3c}$$

$$d^y = k_{s2} \cdot \cos\alpha_2 + k_{n2} \cdot \sin\alpha_2 \cdot \tan\phi_2 \tag{3d}$$

$$e^y = (k_{n1} - k_{s1}) \cdot \sin\alpha_1 \cdot \cos\alpha_1 - (k_{n2} - k_{s2}) \cdot \sin\alpha_2 \cdot \cos\alpha_2 \tag{3e}$$

$$f^y = k_{s1} \cdot \cos^2\alpha_1 + k_{n1} \cdot \sin^2\alpha_1 + k_{s2} \cdot \cos^2\alpha_2 + k_{n2} \cdot \sin^2\alpha_2 \tag{3f}$$

$$r^y = N_{02} \cdot \tan\phi_2 - S_{02}. \tag{3g}$$

Another value for P_0 is obtained for first yield occurring at joint face 1, where now the coefficients c^y, d^y and r^y become:

$$c^y = -k_{s1} \cdot \sin\alpha_1 + k_{n1} \cdot \cos\alpha_1 \cdot \tan\phi_1 \tag{3h}$$

$$d^y = k_{s1} \cdot \cos\alpha_1 + k_{n1} \cdot \sin\alpha_1 \cdot \tan\phi_1 \tag{3i}$$

$$r^y = N_{01} \cdot \tan\phi_1 - S_{01}. \tag{3j}$$

Evaluation of which joint is to yield first (primary yielding face) is required, in order to continue to the next step of calculating the pull out force. In the case of a symmetric wedge with joint faces of similar mechanical and deformational properties, this evaluation involves only the magnitude and direction of the confining forces. When the wedge is asymmetric though, and the mechanical and deformational properties of the faces differ, geometrical and geotechnical factors are introduced that need also be taken into consideration. Therefore, since explicit definition of the primary yielding face cannot be achieved, the lower value of P_0 and the corresponding primary yielding face must be chosen.

Without limiting the generality of the above, let us suppose that face 2 is the primary yielding face. We now need to calculate the additional pull out force required for the secondary face to yield. During this step, the wedge is further displaced and while the primary face deforms by yielding, the other face is still in the elastic area until failure.

Solving again the equations of elasticity, equilibrium, kinematic compatibility and yield, the values for u_x, u_y and P_0, for yield at both face 2 and face 1 and thus failure of the wedge, are obtained by the following equations:

$$\begin{bmatrix} u_x \\ u_y \\ P_0^f \end{bmatrix} = \frac{1}{a^f \cdot d^f - b^f \cdot c^f} \cdot \begin{bmatrix} d^f & -b^f & 0 \\ -c^f & a^f & 0 \\ e^f \cdot d^f - c^f \cdot f^f & a^f \cdot f^f - e^f \cdot b^f & a^f \cdot d^f - b^f \cdot c^f \end{bmatrix} \cdot \begin{bmatrix} r_1^f \\ r_2^f \\ r_3^f \end{bmatrix}, \tag{4}$$

where

$$a^f = -k_{s1} \cdot \sin^2\alpha_1 - k_{n1} \cdot \cos^2\alpha_1 - k_{n2} \cdot \cos\alpha_2 \cdot (\cos\alpha_2 + \tan\phi_2 \cdot \sin\alpha_2) \tag{5a}$$

$$b^f = -(k_{n1} - k_{s1}) \cdot \sin\alpha_1 \cdot \cos\alpha_1 + k_{n2} \cdot \sin\alpha_2 \cdot (\cos\alpha_2 + \tan\phi_2 \cdot \sin\alpha_2) \tag{5b}$$

$$c^f = -k_{s1} \cdot \sin\alpha_1 + k_{n1} \cdot \cos\alpha_1 \cdot \tan\varphi_1 \tag{5c}$$

$$d^f = k_{s1} \cdot \cos\alpha_1 + k_{n1} \cdot \sin\alpha_1 \cdot \tan\varphi_1 \tag{5d}$$

$$e^f = (k_{n1} - k_{s1}) \cdot \sin\alpha_1 \cdot \cos\alpha_1 - k_{n2} \cdot \cos\alpha_2 \cdot (\sin\alpha_2 - \tan\phi_2 \cdot \cos\alpha_2) \tag{5e}$$

$$f^f = k_{s1} \cdot \cos^2\alpha_1 + k_{n1} \cdot \sin^2\alpha_1 + k_{n2} \cdot \sin\alpha_2 \cdot (\sin\alpha_2 - \tan\phi_2 \cdot \cos\alpha_2) \tag{5f}$$

$$r_1^f = -N_{01} \cdot \cos\alpha_1 - S_{01} \cdot \sin\alpha_1 + N_{02} \cdot (\cos\alpha_2 + \tan\phi_2 \cdot \sin\alpha_2) \tag{5g}$$

$$r_2^f = N_{01} \cdot \tan\varphi_1 - S_{01} \tag{5h}$$

$$r_3^f = -N_{01} \cdot \sin\alpha_1 + S_{01} \cdot \cos\alpha_1 + N_{02} \cdot (-\sin\alpha_2 + \tan\phi_2 \cdot \cos\alpha_2). \tag{5i}$$

3. Numerical Simulation

The two-stage procedure adopted in the analytical solution has been simulated by use of the UDEC code (Itasca Co., 1998), which provides an implementation of the

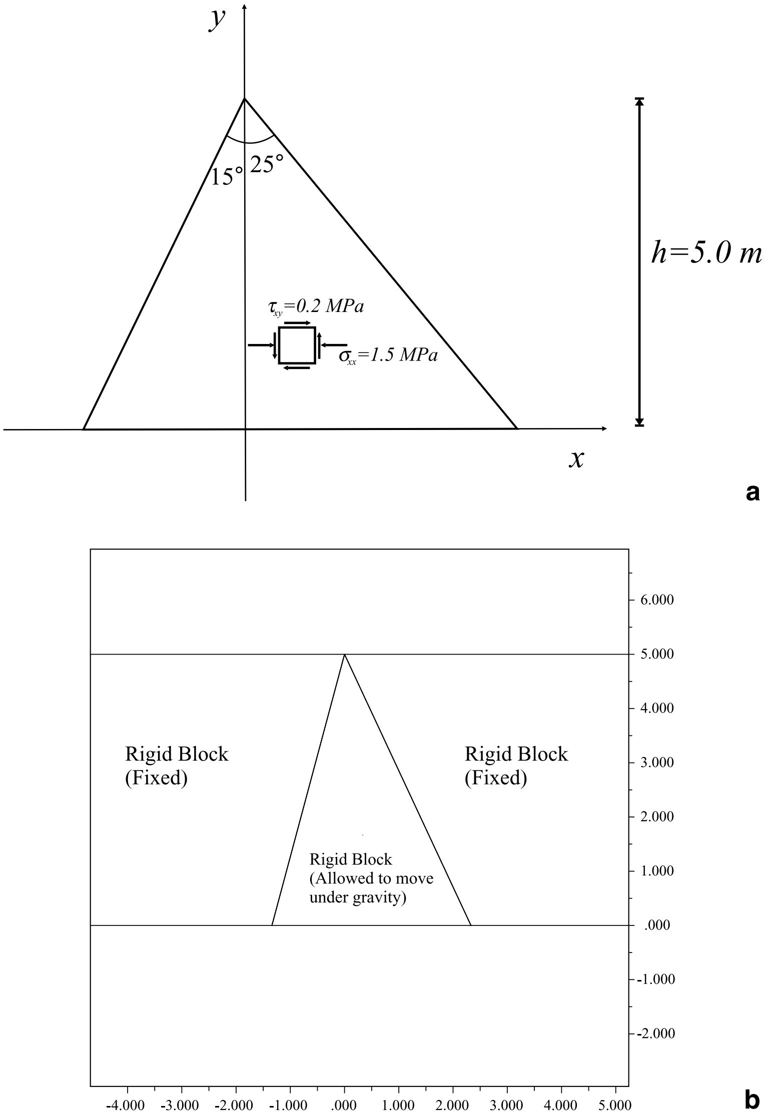


Fig. 5. **a** Geometry and loading conditions of the asymmetric wedge used in the numerical models, **b** UDEC model

Distinct Element Method in two dimensions. The model used is depicted in Fig. 5. In Fig. 5a, a roof wedge is formed by two joints with semi-apical angles $a_1 = 15°$ and $a_2 = 25°$ respectively. The height of the wedge is $h = 5.0$ m. The in-situ state of stress has a uniform horizontal stress of $\sigma_{xx} = 1.5$ MPa and a shear stress of $\tau_{xy} = 0.2$ MPa. Shear strength on the joint faces of the wedge is assumed to be purely frictional, without dilatation and with friction angles $\phi_1 = \phi_2 = 30°$, 35° and 40°. The UDEC model shown in Fig. 5b consists of three rigid blocks. The two side-blocks represent the surrounding rock mass and they are fixed. The central block represents the isolated asymmetric wedge and is allowed to move under gravity.

Table 1. Load carrying capacity of the asymmetric wedge model in horizontal roof with straight free wedge face

ϕ (°)	k_n (GPa/m)	k_s/k_n	q_{anal}	$q_{UDEC,stable}$	$q_{UDEC,unstable}$
30	10	0.01	3.3	3.2	3.3
30	10	0.10	5.5	5.3	5.4
30	10	0.50	8.6	8.4	8.5
35	10	0.01	4.3	4.2	4.3
35	10	0.10	7.6	7.3	7.4
35	10	0.50	12.8	12.3	12.4
40	10	0.01	5.1	4.9	5.0
40	10	0.10	9.3	9.0	9.1
40	10	0.50	16.9	16.1	16.2

Modelling the wedge stability problem with UDEC involves two stages in the same manner as in the analytical procedure. In the first stage the joints are assumed infinitely stiff and all the blocks surrounding the wedge are fixed. The only forces acting on the wedge are the horizontal and vertical forces on the joint faces, which are applied by assigning the in-situ stress state. At the end of this stage the forces acting on the joint faces are calculated from the UDEC model as $H_0 = 7.41$ MN and $V_0 = 0.88$ MN.

In the second stage the joints are assumed deformable with normal stiffness $k_n = 10$ GPa/m and ratio of shear to normal stiffness $k_s/k_n = 0.01$, 0.1 and 0.5. Loading of the wedge during the second stage is achieved through body forces increasing from a minimum value equal to the weight of the wedge W to the value P_0 where the wedge fails, at multiples of W increments.

The normalized pull out resistance of the wedge as calculated by the UDEC code is given in Table 1 in comparison with that calculated by the analytical solution using Eqs. (4) and (5a)–(5i). For each UDEC model two values of the pull out resistance are given. The first corresponds to the maximum applied value of q where the wedge is still stable and the second to the minimum applied value of q where the wedge is unstable. As observed from Table 1, there is a close agreement between the analytically calculated values of q and those obtained by the numerical code.

4. Conclusions – Discussion

An analytical method is presented for the calculation of the load carrying capacity of two-dimensional asymmetric rock wedges when the loading on joint faces is not symmetric, such as the case of an asymmetric wedge formed in the roof of a circular tunnel in an inclined stress field. Confining forces acting on the wedge joint faces may be evaluated analytically from the elastic stress distribution around the opening.

These forces along with the geometrical, mechanical and deformational properties of the joints determine the joint face that is to yield first. The pull out force, that causes yield at one of the joint faces, as well as the displacement of the wedge are evaluated from formulae based on the limiting equilibrium conditions and solving the equations of elasticity, equilibrium, kinematic compatibility and yield, assuming a purely frictional joint resistance.

In order to derive the latter formulae, a uniform shear to normal stress ratio along each joint face is assumed. By increasing the pull out force, the latter ratio reaches its maximum value equal to the friction coefficient, $\tan\phi$. Next the total pull out force required for the secondary face to yield and the corresponding displacement of the wedge are calculated. During this step, the wedge is further displaced and while the primary yielding face is deformed at yield, the other face is still in the elastic range until failure.

Validation of the analytical procedure is obtained by use of the UDEC code, which provides an implementation of the Distinct Element Method in two dimensions. When the assumptions made in the analytical procedure are valid, the analytically calculated values for the pull out resistance of the wedge are close to the numerically obtained ones.

The applicability of the analytical procedure is limited by the assumption of uniform ratio of shear to normal stress along the joint faces for each loading step. When this ratio is not uniform, as in the case of a wedge formed in the roof of an excavation with a curved boundary, progressive shear failure of joint faces is expected. This will reduce the final joint shear forces and consequently the load carrying capacity of the wedge. Thus, for many cases, the pull out resistance of the wedge will be overestimated by the analytical solution presented here. Despite these shortcomings of the model, which is a first-order approximation of the actual behaviour, the analysis is still useful for parametric evaluation of wedge stability, as it contains the basic features influencing the behaviour of the wedge.

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