

Extraction of Astrophysical Cross Sections in the Trojan-Horse Method*

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Abstract. The Trojan-horse method has been proposed to extract S -matrix elements of a two-body reaction at astrophysical energies from a related reaction with three particles in the final state. This should be useful in cases where the direct measurement of the two-body reaction at the necessary low energies is experimentally difficult. The formalism of the Trojan-horse method for nuclear reactions is developed in detail from basic scattering theory including spin degrees of freedom of the nuclei and we specify the necessary approximations. The energy dependence of the three-body reaction is determined by characteristic functions that represent the theoretical ingredients for the method. In a plane-wave Born approximation of the T -matrix the differential cross section assumes a simple structure.

1 Introduction

The Trojan-horse method (THM) [1] has been suggested as an indirect method in order to determine cross sections of charged-particle reactions relevant to nuclear astrophysics. Ideally, reaction cross sections which serve as an input to various astrophysical models, as primordial nucleosynthesis or stellar evolution, should be measured directly in the laboratory. However, for the relevant low energies reaction rates become very small because of Coulomb repulsion of the interacting particles, and an experimental determination is very difficult or impossible. In order to circumvent this problem alternative methods have been proposed, where the considered reaction is not studied directly, but a closely related process is investigated which can be measured experimentally. An example is the Coulomb dissociation method for the determination of radiative capture cross sections which has been successfully used for several reactions in recent years [2–4]. In general, the relation to the astrophysical reaction is established with the help of nuclear reaction theories, however, approximations are necessary. These may depend on the reaction investigated giving the possibility to select the appropriate method. Because of

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these approximations one cannot expect to extract absolute cross sections for reactions of astrophysical interest, however, the Trojan-horse method will give reliable information on their energy dependence. This is important, because at higher energies the absolute cross section is usually well known from direct experiments and with the information from the Trojan-horse method it can be reliably extrapolated to the relevant energies. In particular, direct measurements can suffer from the effect of electron screening at very small energies, which is not understood sufficiently well theoretically. In contrast, electron screening does not affect cross sections extracted in the Trojan-horse method. The method is thus also of interest in understanding the electron screening effect.

Let us consider a nuclear reaction



with given Q -value. At low relative energies in the initial channel the Coulomb repulsion of the charged nuclei A and x will lead to a strong reduction of the cross section. In the Trojan-horse method the nuclear cluster x is hidden inside a projectile $a = b + x$ by attaching it to a nucleus b – therefore the name of the method – and the reaction



with three particles in the final channel is studied (Fig. 1). The relative energy in the $A + a$ channel is chosen above the Coulomb barrier, thus there is no Coulomb reduction of the three-body cross section. However, reactions between A and x can still be induced at small relative energies due to the Fermi motion of x inside a which can compensate, at least partially, the large relative momentum in the $A + a$ system. The nucleus b is thus assumed to be a mere spectator to the relevant process. The method, in principle, is very flexible, since one is not restricted to inelastic processes but also the elastic $A + x \rightarrow A + x$ scattering can be investigated. The method can also be employed where the particle c is a photon and C is a bound state of the $A + x$ system.

In general, various reaction mechanisms can lead to a final state with particles b , c , and C . Besides the process, where nucleus b is emitted and only x and A interact, one could also consider the formation of a compound nucleus D from $A + a$ and a subsequent decay with some flux into the $C + c + b$ channel. In case of elastic $A + x$ scattering the final $A + x + b$ state can be reached by a breakup of a into $x + b$ in the nuclear plus Coulomb field of A . These other mechanisms have to be distinguished experimentally from the process where the reaction $A + x \rightarrow C + c$ is not affected by the presence of particle b .

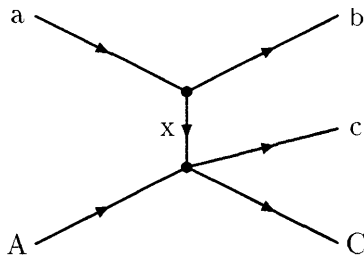


Fig. 1. Diagram for the three-body breakup reaction in the quasi-free mechanism

The THM has already been applied in some cases [5–8] in order to identify the quasi-free reaction mechanism and to test the method. The experimental results were analyzed essentially within the plane-wave-impulse approximation (PWIA), where nucleus b is assumed to be unaffected by the collision. The PWIA was originally developed for the description of α -particle knockout reactions in order to deduce momentum distributions of the α -cluster inside nuclei like ${}^6\text{Li}$ or ${}^7\text{Li}$ [9]. In PWIA the breakup cross section is written as [9]

$$\frac{d^3\sigma}{dE_{Cc} d\Omega_{Cc} d\Omega_{Bb}} = (\text{KF}) |\phi_a(k_{bB}^f)|^2 \frac{d\sigma^N}{d\Omega} \Big|_{Ax \rightarrow Cc}. \quad (1.3)$$

It is a product of three factors: (i) a kinematical factor KF, (ii) a momentum distribution of the nucleus a at the bB final momentum, and (iii) an off-shell two-body $A + x \rightarrow C + c$ cross section, which is replaced in the quasi-free approximation by an appropriate on-shell two-body cross section. In the THM one is interested in the latter quantity. Although this factorization is quite appealing some questions remain about the approximations involved. This concerns the replacement of the complete three-body T -operator by the two-body T -operator, i.e., the impulse approximation [9], and the relation between the off-shell and the on-shell two-body cross section. In the application of Eq. (1.3) $d\sigma^N/d\Omega$ has been interpreted as the nuclear cross section without Coulomb barrier effects and the astrophysically relevant two-body cross section $d\sigma/d\Omega$ has been written as

$$\frac{d\sigma}{d\Omega} = G \frac{d\sigma^N}{d\Omega}, \quad (1.4)$$

where G is supposed to account for the penetration factor for the dominant partial wave through the Coulomb barrier, which was taken in a simple semiclassical approximation. Such empirical procedures also have to be justified.

The relation between the cross section of two-body reaction (1.1) and the cross section of three-body reaction (1.2) can be clarified within the theory of direct nuclear reactions as will be shown in the following. In the original suggestion of the Trojan-horse method [1] the relation was established more or less qualitatively with emphasis on the energy dependence of the cross sections. The employed approximations were not specified in detail. Here, they will be stated more explicitly. They can be tested experimentally as well as theoretically. Besides the full formulation for the three-body cross section in distorted-wave Born approximation (DWBA) we deduce a plane-wave Born approximation (PWBA) which resembles in structure the result of the PWIA, but is more general. We treat both elastic and inelastic processes for the two-body reaction.

Our work is organized as follows: In Sect. 2 we will introduce the relevant quantities for the description of reaction (1.2) and express the cross section with three particles in the final state with the help of the T -matrix of the process. The following section is devoted to the approximations for the T -matrix in the post-form distorted wave description. In Sect. 4 the relation of the cross sections for reaction (1.1) and reaction (1.2) is deduced with the help of the asymptotic form of the two-body scattering wave function. In Sect. 5 the T -matrix for the three-body breakup is calculated in plane-wave Born approximation (PWBA) and compared to the PWIA. Finally, we close with a summary. For transparency we develop the

formalism-disregarding spin. The general expressions including spin are given in an Appendix.

2 The Three-Body Breakup Reaction $A + a \rightarrow C + c + b$

We will denote the mass, the spatial coordinate and the momentum of a nucleus i by m_i , \mathbf{r}_i , and $\mathbf{p}_i = \hbar\mathbf{k}_i = m_i\dot{\mathbf{r}}_i$, respectively. Furthermore we introduce relative coordinates

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j \quad (2.1)$$

between nuclei i and j , and the conjugated momenta

$$\mathbf{p}_{ij} = \hbar\mathbf{k}_{ij} = \mu_{ij}\dot{\mathbf{r}}_{ij} = \frac{m_j\mathbf{p}_i - m_i\mathbf{p}_j}{m_i + m_j}, \quad (2.2)$$

where $\mu_{ij} = (m_i m_j)/(m_i + m_j)$ is the corresponding reduced mass. For a three-body system with particles b, c , and C we have the total energy

$$E_{\text{tot}} = m_C + m_c + m_b + E_{Cc} + E_{(Cc)b} + \frac{\mathbf{P}^2}{2M} \quad (2.3)$$

with the total momentum $\mathbf{P} = \sum_i \mathbf{p}_i$ and total mass $M = \sum_i m_i$.

$$E_{ij} = \frac{\mathbf{p}_{ij}^2}{2\mu_{ij}} \quad (2.4)$$

is the kinetic energy of relative motion between particles i and j . The relative motion of the three particles is completely specified by the two Jacobi momenta \mathbf{p}_{Cc} and $\mathbf{p}_{(Cc)b}$. In case of the two-body system $A + a$ we have

$$E_{\text{tot}} = m_A + m_a + E_{Aa} + \frac{\mathbf{P}^2}{2M} \quad (2.5)$$

with only one relevant kinetic energy E_{Aa} . For simplicity we will assume $\mathbf{P} = 0$ in the following.

In general, the cross section for reaction (1.2) is given by [10, 11]

$$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}^i} \frac{d^3 p_{Cc}^f}{(2\pi\hbar)^3} \frac{d^3 p_{Bb}^f}{(2\pi\hbar)^3} |T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i)|^2 \delta(E_{Cc}^f + E_{Bb}^f - E_{Aa}^i - Q), \quad (2.6)$$

where B stands for the system $C + c$. Quantities in the initial and final channels are given superscripts i and f , respectively. The δ -function with the Q -value

$$Q = m_A + m_a - m_C - m_c - m_b \quad (2.7)$$

of the reaction guarantees energy conservation in the scattering process. The T -matrix contains all information about the dynamics and its calculation is the principle problem for the description of the reaction. With $p_{Cc}^f dp_{Cc}^f = \mu_{Cc} dE_{Cc}^f$ we find the triple differential cross section

$$\frac{d^3 \sigma}{dE_{Cc}^f d\Omega_{Cc} d\Omega_{Bb}} = \frac{\mu_{Aa} \mu_{Bb} \mu_{Cc}}{(2\pi)^5 \hbar^6} \frac{k_{Bb}^f k_{Cc}^f}{k_{Aa}^i} |T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i)|^2, \quad (2.8)$$

which depends on the final relative energy E_{Cc}^f in system B and the directions of both relative momenta $\hat{\mathbf{p}}_{Cc}^f$ and $\hat{\mathbf{p}}_{Bb}^f$ for the given initial momentum \mathbf{p}_{Aa}^i . Here and in the following we have neglected possible spins of the participating nuclei in order to simplify the equations. The relevant formulas with inclusion of spin degrees of freedom will be given in the Appendix.

3 Approximations to the Three-Body T -Matrix

The exact T -matrix is given in terms of the interaction between the particles and the exact solution of the scattering problem for the total wave function. Of course, this is not known in the general case and we have to employ approximations which should retain the relevant physics of the reaction process. We assume that the total interaction in our system is given by a sum of two-body potentials $V_{\alpha\beta}$. Then the potential between two nuclei

$$V_{ij} = \sum_{\substack{\alpha \in i \\ \beta \in j}} V_{\alpha\beta} \quad (3.1)$$

depends not only on the relative coordinate \mathbf{r}_{ij} but also on the internal coordinates of the nucleons inside the nuclei i and j , respectively. On the other hand, we can write the kinetic energy operator for the relative motion as

$$T_{ij} = \frac{\check{\mathbf{p}}_{ij}^2}{2\mu_{ij}} \quad (3.2)$$

with the momentum operator of the relevant Jacobi momentum $\check{\mathbf{p}}_{ij}$. We define the internal Hamiltonian of a nucleus i to be h_i (including the rest mass) which depends only on internal coordinates with the solution ϕ_i of the corresponding Schrödinger equation. Considering the different partitions into nuclei in the initial and final channels we have for the total Hamiltonian the expressions in the initial state

$$H = h_A + h_a + T_{Aa} + V_{Aa}, \quad h_a = h_x + h_b + T_{xb} + V_{xb} \quad (3.3)$$

and in the final state

$$H = h_B + h_b + T_{Bb} + V_{Bb}, \quad h_B = h_C + h_c + T_{Cc} + V_{Cc}. \quad (3.4)$$

The exact T -matrix element is now given in the post formulation by [10, 11]

$$T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) = \langle \phi_0^f(Bb) \phi_B \phi_b | V_{Bb} | \Psi^{(+)}(Aa) \rangle, \quad (3.5)$$

where $\Psi^{(+)}(Aa)$ is the full solution of the scattering problem with a plane wave in the initial channel $A + a$ and outgoing spherical waves in all channels with or without rearrangement. The outgoing plane wave

$$\phi_0^f(Bb) = \exp(i\mathbf{k}_{Bb}^f \cdot \mathbf{r}_{Bb}) \quad (3.6)$$

appears on the left-hand side. The wave function ϕ_B for the system $C + c$ will be specified in Sect. 4, depending on whether one considers a bound state or a scattering state.

The T -matrix element is transformed with the help of the Gell-Mann–Goldberger relation into a sum of two contributions. For this we split the total

Hamiltonian H into two parts

$$H = H_i + (V_{Aa} - U_{Aa}), \quad H_i = h_A + h_a + T_{Aa} + U_{Aa}, \quad (3.7)$$

introducing an (optical) potential U_{Aa} depending only on the relative coordinate \mathbf{r}_{Aa} . The scattering problem for the Hamiltonian H_i can be solved exactly as the potential U_{Aa} only allows elastic Aa scattering without any excitation of internal degrees of freedom of the colliding nuclei. The relative motion is given by a distorted wave

$$(T_{Aa} + U_{Aa})\chi^{(+)}(Aa) = E_{Aa}^i \chi^{(+)}(Aa). \quad (3.8)$$

Similarly, in case of the final state we have the decomposition

$$H = H_f + (V_{Bb} - U_{Bb}), \quad H_f = h_B + h_b + T_{Bb} + U_{Bb} \quad (3.9)$$

and the distorted wave

$$(T_{Bb} + U_{Bb})\chi^{(+)}(Bb) = E_{Bb}^f \chi^{(+)}(Bb). \quad (3.10)$$

The Gell-Mann–Goldberger relation [10] now leads to the result

$$\begin{aligned} T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= \langle \chi^{(-)}(Bb) \phi_B \phi_b | V_{Aa} - (V_{Bb} - U_{Bb}) | \phi_0(Aa) \phi_A \phi_a \rangle \\ &\quad + \langle \chi^{(-)}(Bb) \phi_B \phi_b | V_{Bb} - U_{Bb} | \Psi^{(+)}(Aa) \rangle, \end{aligned} \quad (3.11)$$

which is still exact. Note that in the final channel we have to insert the wave function $\chi^{(-)}(Bb)$ which is the time-reversed of $\chi^{(+)}(Bb)$. The original Gell-Mann–Goldberger relation [12] was derived for identical partitions of the Hamiltonian in the initial and final channels whereas here we apply the more general result for different partitions [10, 11].

Next we observe that the first contribution to the T -matrix element vanishes exactly. By using Eqs. (3.3) and (3.4) we find

$$\begin{aligned} V_{Aa} - (V_{Bb} - U_{Bb}) &= (H - h_A - h_a - T_{Aa}) - (H - h_B - h_b - T_{Bb} - U_{Bb}) \\ &= h_B + h_b + T_{Bb} + U_{Bb} - (h_A + h_a + T_{Aa}). \end{aligned} \quad (3.12)$$

Since the wave functions in the matrix element are solutions of the Schrödinger equations

$$[h_B + h_b + T_{Bb} + U_{Bb}^\dagger] \chi^{(-)}(Bb) \phi_B \phi_b = E_{\text{tot}} \chi^{(-)}(Bb) \phi_B \phi_b \quad (3.13)$$

and

$$[h_A + h_a + T_{Aa}] \phi_0(Aa) \phi_A \phi_a = E_{\text{tot}} \phi_0(Aa) \phi_A \phi_a \quad (3.14)$$

with total energy E_{tot} the first matrix element vanishes. Then the T -matrix relevant for our calculation is still exactly given only by the second contribution

$$T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) = \langle \chi^{(-)}(Bb) \phi_B \phi_b | V_{Bb} - U_{Bb} | \Psi^{(+)}(Aa) \rangle. \quad (3.15)$$

This expression still contains the full solution of the scattering problem on the right side of the T -matrix. As a first approximation, we apply the distorted-wave Born approximation by replacing the full solution $\Psi^{(+)}(Aa)$ by the distorted wave $\chi^{(+)}(Aa) \phi_A \phi_a$

$$T_{fi}^{\text{DWBA}}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) = \langle \chi^{(-)}(Bb) \phi_B \phi_b | V_{Bb} - U_{Bb} | \chi^{(+)}(Aa) \phi_A \phi_a \rangle. \quad (3.16)$$

The next approximation is introduced as usual by replacing the potential

$$V_{Bb} - U_{Bb} = V_{Cb} + V_{cb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}, \quad (3.17)$$

where we assume that the transferred nucleus x is small and that the optical potential U_{Bb} fits elastic scattering of b by nucleus B . Neglecting the difference $V_{Ab} - U_{Bb}$ is expected to be of the same order as the uncertainties of DWBA. This approximation for the interaction potential has been extensively discussed, e.g., in ref. [13] where it was applied to deuteron stripping. In a plane-wave approximation of the T -matrix the potential U_{Bb} is not introduced and the above approximation means to neglect the interaction between nucleus A and the spectator b . This just corresponds to the impulse approximation for the three-body reaction. Finally the relevant T -matrix of the three-body breakup reaction assumes the form

$$T_{fi}^{\text{DWBA}}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) = \langle \chi^{(-)}(Bb) \phi_B \phi_b | V_{xb} | \chi^{(+)}(Aa) \phi_A \phi_a \rangle. \quad (3.18)$$

4 Relation Between the Two-Body and the Three-Body Cross Sections

We now formulate the relation between the cross section for the two-body reaction (1.1) and the three-body breakup reaction (1.2). The T -matrix element (3.18) for the three-body reaction is calculated with the wave function ϕ_B for the system B in the final state. In our case, this is the full scattering wave function of the unbound $B = C + c$ system, and thus the three-body T -matrix element also contains the information on the $C + c \rightarrow A + x$ reaction, therefore giving the connection between the two-body and three-body cross sections. In the following formulae we will neglect effects of antisymmetrization.

4.1 The Two-Body Scattering Cross Section

The two-body differential cross section is given as

$$\frac{d\sigma}{d\Omega_{Cc}}(Ax \rightarrow Cc) = \frac{v_{Cc}^f}{v_{Ax}^i} |f(Ax \rightarrow Cc)|^2 \quad (4.1)$$

with the appropriate scattering amplitude f . The velocities are determined from the corresponding momenta $p_{Cc}^f = \mu_{Cc} v_{Cc}^f$ and $p_{Ax}^i = \mu_{Ax} v_{Ax}^i$. The scattering amplitude is derived from the full scattering wave function $\Psi^{(+)}(Ax, \mathbf{k}_{Ax}^i)$ with the system $A + x$ in the initial channel. Its asymptotic form is given by (we only consider two-body final states)

$$\Psi^{(+)}(Ax, \mathbf{k}_{Ax}) \rightarrow \frac{4\pi}{k_{Ax}^i} \sum_{\alpha} \sum_{lm} \frac{\xi_l^{(+)}(\alpha; k_{\alpha} r_{\alpha})}{r_{\alpha}} i^l Y_{lm}(\hat{\mathbf{r}}_{\alpha}) \phi_{\alpha} Y_{lm}^*(\hat{\mathbf{k}}_{Ax}^i). \quad (4.2)$$

The sum runs over all possible final channels α , where ϕ_{α} is the corresponding wave function which depends on all internal variables of the two particles in the final state. The radial wave function

$$\begin{aligned} \xi_l^{(+)}(\alpha; k_{\alpha} r_{\alpha}) &= \frac{\exp[i\sigma_l(\eta_{Ax}^i)]}{2i} \sqrt{\frac{v_{Ax}^i}{v_{\alpha}^f}} \\ &\times [S_l(Ax \rightarrow \alpha) u_l^{(+)}(\eta_{\alpha}; k_{\alpha} r_{\alpha}) - \delta_{\alpha Ax} u_l^{(-)}(\eta_{\alpha}; k_{\alpha} r_{\alpha})] \end{aligned} \quad (4.3)$$

can be expressed by the (nuclear) S -matrix element $S_l(Ax \rightarrow \alpha)$ and linear combinations of the regular and irregular Coulomb wave functions (F_l and G_l), where we have introduced the notation

$$u_l^{(\pm)}(\eta_\alpha; k_\alpha r_\alpha) = G_l(\eta_\alpha; k_\alpha r_\alpha) \pm iF_l(\eta_\alpha; k_\alpha r_\alpha) \\ \rightarrow \exp \left[\pm i \left(k_\alpha r_\alpha - \eta_\alpha \ln(2k_\alpha r_\alpha) - \frac{l\pi}{2} + \sigma_l(\eta_\alpha) \right) \right]. \quad (4.4)$$

The Coulomb phase shift $\sigma_l(\eta_\alpha)$ for the partial wave l depends on the Sommerfeld parameter

$$\eta_\alpha = \frac{Z_{\alpha_1} Z_{\alpha_2} e^2}{\hbar v_\alpha} \quad (4.5)$$

with the charge numbers Z_{α_i} of the two nuclei i in channel α and their relative velocity v_α . The asymptotic form of the radial wave function (4.3) will be of importance for establishing the relation between the cross sections for the two- and three-body reactions. Assuming that the initial momentum \mathbf{p}_{Ax}^i is directed along the z -axis we have

$$Y_{lm}(\hat{\mathbf{k}}_{Ax}^i) = \sqrt{\frac{2l+1}{4\pi}} \delta_{m0} \quad (4.6)$$

and for the total scattering amplitude (including Coulomb scattering)

$$f(Ax \rightarrow Cc) = -i \frac{\sqrt{\pi}}{k_{Ax}^i} \sqrt{\frac{v_{Ax}^i}{v_{Cc}^f}} \sum_l \sqrt{2l+1} Y_{l0}(\hat{\mathbf{r}}_{Cc}) \\ \times [\exp\{i[\sigma_l(\eta_{Ax}^i) + \sigma_l(\eta_{Cc}^f)]\} S_l(Ax \rightarrow Cc) - \delta_{CcAx}]. \quad (4.7)$$

Finally, the total reaction cross section for an inelastic reaction ($Cc \neq Ax$)

$$\sigma(Ax \rightarrow Cc) = \frac{\pi}{(k_{Ax}^i)^2} \sum_l (2l+1) |S_l(Ax \rightarrow Cc)|^2 \quad (4.8)$$

is obtained by integrating Eq. (4.1) over all possible directions $\hat{\mathbf{r}}_{Cc}$. It is obvious that the S -matrix contains all the essential information of the scattering process. At very small energies E_{Ax}^i in the initial state the energy dependence of the S -matrix is dominated by the Coulomb barrier and decreases like $\exp(-\pi\eta_{Ax}^i)$. In astrophysical applications one therefore defines the S -factor

$$S(E_{Ax}^i) = \sigma(Ax \rightarrow Cc) E_{Ax}^i \exp(2\pi\eta_{Ax}^i) \quad (4.9)$$

which shows a much weaker variation with energy as compared to the cross section.

4.2 The Three-Body Scattering Cross Section

In case of the three-body breakup the triple differential cross section is given by Eq. (2.8).

We now use the approximation (3.18) for the T -matrix. Assuming that the optical potentials U_{Aa} and U_{Bb} depend only on r_{Aa} and r_{Bb} , respectively, we write

$$T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) \\ = \langle \chi^{(-)}(Bb, \mathbf{k}_{Bb}^f) \Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f) \phi_b | V_{xb} | \chi^{(+)}(Aa, \mathbf{k}_{Aa}^i) \phi_a \phi_a \rangle. \quad (4.10)$$

We have replaced the wave function ϕ_B in Eq. (3.18) by the exact two-body scattering wave function $\Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f)$ which asymptotically behaves as a plane wave in the Cc channel and ingoing spherical waves in all other channels. It can be obtained from Eq. (4.2) by applying the time-inversion operator and replacing Ax by Cc . Then, its asymptotic form is given by

$$\Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f) \rightarrow \frac{4\pi}{k_{Cc}^f} \sum_{\alpha} \sum_l \frac{\xi_l^{(-)}(\alpha; q_{\alpha} r_{\alpha})}{r_{\alpha}} i^l Y_{lm}(\hat{\mathbf{r}}_{\alpha}) \phi_{\alpha} Y_{lm}^*(\hat{\mathbf{k}}_{Cc}^f) \quad (4.11)$$

with radial wave functions

$$\begin{aligned} \xi_l^{(-)}(\alpha; k_{\alpha} r_{\alpha}) &= i \frac{\exp[-i\sigma_l(\eta_{Cc}^f)]}{2} \sqrt{\frac{v_{Cc}^f}{v_{\alpha}}} \\ &\times [S_l^*(Cc \rightarrow \alpha) u_l^{(-)}(\eta_{\alpha}; k_{\alpha} r_{\alpha}) - \delta_{\alpha Cc} u_l^{(+)}(\eta_{\alpha}; k_{\alpha} r_{\alpha})] \end{aligned} \quad (4.12)$$

for all possible partitions α .

The expression (4.10) can be calculated in DWBA by replacing the exact scattering wave $\Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f)$ by a distorted wave. But then we do not obtain an explicit relation to the two-body cross section. We therefore introduce the essential, so-called surface approximation [14]. From the form of the T -matrix element (4.10) we can deduce which channel α gives the most important contribution. The potential V_{xb} only acts on the bound-state wave function ϕ_a of the $b + x$ system and does not depend on r_{Aa} . The distorted wave $\chi^{(+)}(Aa, \mathbf{k}_{Aa}^i)$ will become very small for small r_{Aa} since the optical potential U_{Aa} is strongly absorptive there. For small momenta \mathbf{p}_{Aa}^i the Coulomb barrier in the initial channel will additionally reduce the amplitude of the wave function at small r_{Aa} . On the other hand, asymptotically, the channels $A + a$ and $C + c$ become orthogonal. Therefore, there will be a substantial contribution to the T -matrix element only in the surface region of nucleus A , i.e., for r_{Aa} close to the radius of A . Thus we can expect that the product $V_{xb} \chi^{(+)}(Aa, \mathbf{k}_{Aa}^i) \phi_A \phi_a$ projects onto contributions mainly from the $A + (b + x)$ structure and only little onto the $C + (b + c)$ and other partitions. We conclude that in the wave function $\Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f)$ the channel α with the $A + x$ fragmentation is selected in the matrix element and further, that we can use the asymptotic form (4.11) of the wave function, since the substantial contributions to the matrix element arise from not too small r_{Ax} . Using this surface approximation [14] we replace the full wave function by its asymptotic form (in the $A - x$ channel)

$$\begin{aligned} &\Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f \rightarrow Ax) \\ &= \frac{4\pi}{k_{Cc}^f} \sum_{lm} \exp[-i\sigma_l(\eta_{Cc}^f)] \sqrt{\frac{v_{Cc}^f}{v_{Ax}^f}} i^l Y_{lm}(\hat{\mathbf{r}}_{Ax}) \phi_A \phi_x Y_{lm}^*(\hat{\mathbf{k}}_{Cc}^f) \\ &\times \left[S_l^*(Cc \rightarrow Ax) \frac{\zeta_l^{(-)}(Ax; k_{Ax}^f r_{Ax})}{r_{Ax}} - \delta_{Ax Cc} \frac{\zeta_l^{(+)}(Ax; k_{Ax}^f r_{Ax})}{r_{Ax}} \right] \end{aligned} \quad (4.13)$$

with the radial wave functions

$$\zeta_l^{(\pm)}(Ax; k_{Ax}^f r_{Ax}) = \Theta(r_{Ax} - R_{Ax}) \frac{i}{2} u_l^{(\pm)}(\eta_{Ax}^f; k_{Ax}^f r_{Ax}), \quad (4.14)$$

where we introduced a cut-off at a suitable radius R_{Ax} in the radial wave functions $\zeta_l^{(\pm)}$ to eliminate contributions from the interior including the divergence from the irregular Coulomb function at small r_{Ax} . The momentum in the Ax channel and the corresponding velocity v_{Ax}^f are determined by

$$m_A + m_x + \frac{(\mathbf{p}_{Ax}^f)^2}{2\mu_{Ax}} = m_C + m_c + \frac{(\mathbf{p}_{Cc}^f)^2}{2\mu_{Cc}}, \quad (4.15)$$

i.e., the $Cc \rightarrow Ax$ S -matrix is taken on-shell. The surface approximation can be tested in an actual calculation, e.g., by a comparison of the results obtained with a distorted wave approximation for the radial wave function without the radial cut-off and with the asymptotic wave function with proper radial cut-off. In case of inclusive deuterium breakup reactions the results of a full calculation and of the surface approximation were compared and found to be in satisfactory agreement [15].

For an inelastic process with $Cc \neq Ax$ the T -matrix finally assumes the form

$$\begin{aligned} T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= \sqrt{\frac{\mu_{Ax} k_{Ax}^f}{\mu_{Cc} k_{Cc}^f}} \\ &\times \sum_{lm} \exp[i\sigma_l(\eta_{Cc}^f)] S_l(Cc \rightarrow Ax) Y_{lm}^*(\hat{\mathbf{k}}_{Cc}^f) t_{fi}^{(-)lm}(k_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) \end{aligned} \quad (4.16)$$

with the reduced T -matrix element

$$\begin{aligned} t_{fi}^{(\pm)lm}(k_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= 4\pi i^l \langle \chi^{(-)}(Bb, \mathbf{k}_{Bb}^f) \frac{\zeta_l^{(\pm)}(Ax; k_{Ax}^f r_{Ax})}{k_{Ax}^f r_{Ax}} \\ &\times Y_{lm}(\hat{\mathbf{r}}_{Ax}) \phi_A \phi_x \phi_b | V_{xb} | \chi^{(+)}(Aa, \mathbf{k}_{Aa}^i) \phi_A \phi_a \rangle. \end{aligned} \quad (4.17)$$

In case of the elastic process with $Cc = Ax$ we find

$$\begin{aligned} T_{fi}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= \sum_{lm} \exp[i\sigma_l(\eta_{Ax}^f)] Y_{lm}^*(\hat{\mathbf{k}}_{Ax}^f) \\ &\times [S_l(Ax \rightarrow Ax) t_{fi}^{(-)lm}(k_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) - t_{fi}^{(+)lm}(k_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i)]. \end{aligned} \quad (4.18)$$

The reduced T -matrix elements $t_{fi}^{(\pm)}$, Eq. (4.17), are the essential ingredients in the THM, since they are needed to extract the two-body S -matrix $S_l(Cc \rightarrow Ax)$ from the measured three-body cross section. They have the form of a DWBA T -matrix for the transfer reaction $A + (bx)_a \rightarrow (Ax)_{\text{unbound}} + b$, i.e., for the transfer of x to an unbound state in $A + x$ with relative momentum p_{Ax}^f , Eq. (4.15), mediated by the interaction V_{xb} . Here p_{Ax}^f is the momentum for the astrophysical energy E_{Ax}^f , at which the $A + x \rightarrow C + c$ cross section is to be investigated. Thus the reduced T -matrix is the amplitude for preparing the (Ax) -system at the corresponding energy. Its structure will become clearer in the PWBA in the next section.

As seen in Eqs. (4.16) and (4.18) there is no simple relationship between the two-body and three-body cross sections in general. The S -matrix elements in Eq. (4.16) are for the inverse process $C + c \rightarrow A + x$, but both are related by unitarity.

In general, one has interference between different partial waves in the triple differential cross section (2.8), and one has to perform angular correlations between the different fragments to extract the partial-wave S -matrix elements. The reduced T -matrix elements, depending on three momenta, have to be supplied from theoretical calculations to the THM. In principle they can be evaluated in DWBA, even though they involve six-dimensional integration, i.e., are of the exact finite-range type.

The reduced T -matrix elements $t_{fi}^{(\pm)}$ are not model-independent. They depend on assumptions on the distorted waves, the radial cut-offs, the bound-state wave function ϕ_a , and the V_{bx} residual interaction. Thus the THM will not allow to determine the two-body S -matrix absolutely. However, it will give reliably the energy dependence, which is just the important information in the extrapolation to astrophysical energies.

5 Plane-Wave Approximation

In order to see the structure of the above expressions and also for a comparison to the literature, we now use the plane-wave approximation. The calculation of the reduced T -matrix elements (4.17) simplifies considerably, when we replace the distorted waves $\chi^{(-)}(Bb, \mathbf{k}_{Bb}^f)$ and $\chi^{(+)}(Aa, \mathbf{k}_{Aa}^i)$ by plane waves $\exp(-i\mathbf{k}_{Bb}^f \cdot \mathbf{r}_{Bb})$ and $\exp(i\mathbf{k}_{Aa}^i \cdot \mathbf{r}_{Aa})$, respectively. Notice that in this approximation the wave function for the system B still contains the full effect of the Coulomb potential.

We Fourier transform the product of the potential V_{xb} and the bound-state wave function ϕ_a both depending on \mathbf{r}_{xb}

$$V_{xb}(\mathbf{r}_{xb})\phi_a(\mathbf{r}_{xb}) = \int \frac{d^3q}{(2\pi)^3} W(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}_{xb}) \phi_x \phi_b, \quad (5.1)$$

where we have introduced the momentum amplitude $W(\mathbf{q})$. A zero-range approximation for the product $V_{xb}\phi_a$ would correspond to a constant W . For the reduced T -matrix element we find

$$\begin{aligned} & t_{fi}^{(\pm)lm}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) \\ &= 4\pi \int \frac{d^3q}{(2\pi)^3} W(\mathbf{q}) \int d^3r_{Bb} \int d^3r_{Ax} \\ & \times \frac{\zeta_l^{(\pm)*}(Ax; k_{Ax}^f r_{Ax})}{k_{Ax}^f r_{Ax}} Y_{lm}^*(\hat{\mathbf{r}}_{Ax}) \exp(i[\mathbf{k}_{Aa}^i \cdot \mathbf{r}_{Aa} + \mathbf{q} \cdot \mathbf{r}_{xb} - \mathbf{k}_{Bb}^f \cdot \mathbf{r}_{Bb}]). \end{aligned} \quad (5.2)$$

Using

$$\mathbf{r}_{xb} = \mathbf{r}_{Bb} - \frac{m_A}{m_A + m_x} \mathbf{r}_{Ax}, \quad (5.3)$$

$$\mathbf{r}_{Aa} = \frac{m_b}{m_x + m_b} \mathbf{r}_{Bb} + \frac{m_x M}{(m_A + m_x)(m_x + m_b)} \mathbf{r}_{Ax}, \quad (5.4)$$

we can write for the exponential

$$\exp(i[\mathbf{k}_{Aa}^i \cdot \mathbf{r}_{Aa} + \mathbf{q} \cdot \mathbf{r}_{xb} - \mathbf{k}_{Bb}^f \cdot \mathbf{r}_{Bb}]) = \exp(i[\mathbf{q}_{Bb} \cdot \mathbf{r}_{Bb} + \mathbf{q}_{Ax} \cdot \mathbf{r}_{Ax}]) \quad (5.5)$$

with the momenta

$$\mathbf{q}_{Bb} = \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i + \mathbf{q} - \mathbf{k}_{Bb}^f, \quad (5.6)$$

$$\mathbf{q}_{Ax} = \frac{m_x M}{(m_A + m_x)(m_x + m_b)} \mathbf{k}_{Aa}^i - \frac{m_A}{m_A + m_x} \mathbf{q}. \quad (5.7)$$

Integrating over \mathbf{r}_{Bb} yields a δ -function and the \mathbf{q} integration becomes trivial. We obtain

$$\begin{aligned} & t_{fi}^{(\pm)lm}(k_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) \\ &= 4\pi W(\mathbf{Q}_{Bb}) \int d^3 r_{Ax} \frac{\zeta_l^{(\pm)*}(Ax; k_{Ax}^f r_{Ax})}{k_{Ax}^f r_{Ax}} Y_{lm}^*(\hat{\mathbf{r}}_{Ax}) \exp(i\mathbf{Q}_{Aa} \cdot \mathbf{r}_{Ax}) \end{aligned} \quad (5.8)$$

with

$$\mathbf{Q}_{Bb} = \mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i, \quad (5.9)$$

$$\mathbf{Q}_{Aa} = \mathbf{k}_{Aa}^i - \frac{m_A}{m_A + m_x} \mathbf{k}_{Bb}^f. \quad (5.10)$$

Here \mathbf{Q}_{Bb} is the difference between the momentum of b in the final channel and the fraction of the initial momentum in the projectile a . Thus it is the recoil of b . Similarly \mathbf{Q}_{Aa} is the recoil of A . In PWBA the reduced T -matrix is the product of the Fourier transforms of the bound (bx) wave function at \mathbf{Q}_{Bb} with that of the Ax -scattering state at \mathbf{Q}_{Aa} . Introducing a partial-wave expansion of the plane wave and performing the angular integration for \mathbf{r}_{Ax} we obtain

$$\begin{aligned} t_{fi}^{(\pm)lm}(k_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= (4\pi)^3 W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \\ &\times \sum_{l''m''} \sum_{l'm'} -i^{l''-l'} \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2l''+1)}} (l0l'0|l''0) (lml'm'|l''m'') \\ &\times R_{l''l'l}^{(\pm)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) Y_{l''m''}^*(\hat{\mathbf{k}}_{Aa}^i) Y_{l'm'}(\hat{\mathbf{k}}_{Bb}^f) \end{aligned} \quad (5.11)$$

with the radial integral

$$\begin{aligned} & R_{l''l'l}^{(\pm)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) \\ &= (k_{Ax}^f)^{-1} \int_0^\infty dr_{Ax} r_{Ax} j_{l''}(k_{Aa}^i r_{Ax}) j_{l'}(\lambda_{Ax} k_{Bb}^f r_{Ax}) \zeta_l^{(\pm)*}(Ax; k_{Ax}^f r_{Ax}) \end{aligned} \quad (5.12)$$

and the abbreviation

$$\lambda_{Ax} = \frac{m_A}{m_A + m_x}. \quad (5.13)$$

Due to the parity Clebsch-Gordan coefficient $(l0l'0|l''0)$ some combinations of l'' , l' , and l will give no contribution to the reduced T -matrix elements. The radial integral over continuum wave functions can conveniently be calculated with the methods given in ref. [16] by passing to the complex plane for the variable r_{Ax} with a suitable decomposition of the integrand. For small momenta p_{Ax}^f the contribution of the

irregular Coulomb wave function in $\zeta_l^{(\pm)}$ will dominate and we find with ref. [17] the very low energy behaviour of the radial integral

$$\begin{aligned} R_{l''l'}^{(\pm)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) \\ \propto -i \frac{\exp(\pi\eta_{Ax}^f)}{\sqrt{2\pi Q_{Ax} k_{Ax}^f}} \\ \times \int_{R_{Ax}}^{\infty} dr_{Ax} r_{Ax} j_{l''}(k_{Aa}^i r_{Ax}) j_{l'}(\lambda_{Ax} k_{Bb}^f r_{Ax}) \sqrt{2Q_{Ax} r_{Ax}} K_{2l+1}(2\sqrt{2Q_{Ax} r_{Ax}}) \end{aligned} \quad (5.14)$$

with the (constant) inverse Bohr length

$$Q_{Ax} = \eta_{Ax}^f k_{Ax}^f = \frac{Z_A Z_x e^2 \mu_{Ax}}{\hbar^2}. \quad (5.15)$$

In Eq. (5.14) only the factor in front of the integral depends on k_{Ax}^f . For low energies the exponential factor increases strongly cancelling the decrease of the two-body S -matrix elements in the three-body T -matrix element (4.16).

Finally, in case of the inelastic two-body reaction the result for the total three-body T -matrix becomes

$$\begin{aligned} T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) &= \sqrt{\frac{\mu_{Ax} k_{Ax}^f}{\mu_{Cc} k_{Cc}^f}} W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \\ &\times \sum_l (2l+1) \exp[i\sigma_l(\eta_{Cc}^f)] S_l(Cc \rightarrow Ax) \\ &\times \sum_{l''l'} R_{l''l'}^{(-)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) X_l^{l''l'}(\hat{\mathbf{k}}_{Cc}^f, \hat{\mathbf{k}}_{Bb}^f; \hat{\mathbf{k}}_{Aa}^i) \end{aligned} \quad (5.16)$$

with the angular distribution function

$$\begin{aligned} X_l^{l''l'}(\hat{\mathbf{k}}_{Cc}^f, \hat{\mathbf{k}}_{Bb}^f; \hat{\mathbf{k}}_{Aa}^i) &= (4\pi)^{5/2} i^{l''-l'} (l0l'0|l''0) \sqrt{\frac{2l'+1}{(2l''+1)(2l+1)}} \\ &\times \sum_{\substack{m''m' \\ m}} (lm'l'm''|l''m'') Y_{l''m''}^*(\hat{\mathbf{k}}_{Aa}^i) Y_{l'm'}(\hat{\mathbf{k}}_{Bb}^f) Y_{lm}^*(\hat{\mathbf{k}}_{Cc}^f). \end{aligned} \quad (5.17)$$

In the elastic case we find

$$\begin{aligned} T_{fi}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) \\ = W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \sum_l (2l+1) \exp[i\sigma_l(\eta_{Ax}^f)] \sum_{l''l'} X_l^{l''l'}(\hat{\mathbf{k}}_{Ax}^f, \hat{\mathbf{k}}_{Bb}^f; \hat{\mathbf{k}}_{Aa}^i) \\ \times [S_l(Ax \rightarrow Ax) R_{l''l'}^{(-)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) - R_{l''l'}^{(+)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f)]. \end{aligned} \quad (5.18)$$

The essential feature of the THM is the occurrence of the factor $\exp(\pi\eta_{Ax}^f)$ in the low-energy approximation (5.14) of the radial integrals $R_{l''l'}^{(\pm)}$ which compensates the low-energy suppression of the S -matrix element $S_l(Cc \rightarrow Ax)$ from the Coulomb barrier in the Ax channel. Therefore we have no suppression of the cross section in the three-body reaction [1]. Of course, the energy dependence derived here in the

PWBA also applies to the more general case with distorted waves in the T -matrix element.

Rewriting the plane-wave approximation for the T -matrix

$$T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i) = W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \tilde{T}_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i), \quad (5.19)$$

we obtain for the three-body cross section

$$\begin{aligned} & \frac{d^3\sigma}{dE_{Cc}^f d\Omega_{Cc} d\Omega_{Bb}} \\ &= \frac{\mu_{Aa}\mu_{Bb}\mu_{Cc} k_{Bb}^f k_{Cc}^f}{(2\pi)^5 \hbar^6} \frac{k_{Aa}^i}{k_{Aa}^i} \left| W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \right|^2 |\tilde{T}_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f; \mathbf{k}_{Aa}^i)|^2, \quad (5.20) \end{aligned}$$

a result which resembles the PWIA (1.3) in structure. We also have a product of three factors: (i) a kinematical factor, (ii) a momentum distribution, and (iii) an expression like a cross section. But there are clear differences as compared to PWIA. The momentum distribution $|W|^2$ is not simply the Fourier transform of the ground-state wave function of nucleus a but the Fourier transform of the product of this wave function with the interaction V_{xb} . In case of a nucleus a with an $l = 0$ ground-state wave function for the xb relative motion, e.g., ${}^6\text{Li}$, $|W|^2$ will peak at zero momentum. This means

$$\mathbf{k}_{Bb}^f \approx \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i,$$

i.e., the final momentum of the spectator b is just the corresponding mass fraction of the initial momentum of projectile a . Therefore it remains constant during the reaction as assumed in the impulse approximation. In the PWIA the last factor is directly the two-body cross section of the $A + x \rightarrow C + c$ reaction [9]. Here it is a function, where the S -matrix elements of this reaction enter. This function contains the off-shell effects for the two-body cross section and goes beyond the quasi-free approximation. In particular it contains an explicit expression for the barrier penetration factors through the radial integrals $R^{(\pm)}$. Although we have used the plane-wave approximation for the Aa and Bb relative motion the Coulomb effects in the $C + c$ scattering are still fully included.

6 Summary

We specified approximations involved in the Trojan-horse method in order to determine the cross section of a two-body reaction from a related process with three bodies in the final state. The full expression for the triple differential cross section of a three-body breakup reaction has been derived in a post-form distorted-wave Born description. The relation to the two-body cross section of interest can be established via the S -matrix elements if the surface approximation is applied. In a further plane-wave Born approximation of the T -matrix element the three-body cross section assumes a form similar to the plane-wave impulse approximation. In our formulation the full effect of Coulomb penetration in the relevant two-body process is included as well as off-shell effects in the breakup. The Trojan-horse method has

already been shown to be useful in the application with the simplified, semi-empirical PWIA expressions of Eq. (1.3) [6–8]. For an application of the Trojan-horse method to a specific reaction of astrophysical interest various combinations of spectator nucleus and projectile energy with different three-body final states can be selected. The actual choice depends strongly on the experimental possibilities for the detection of the particles in the final state. In particular, the range in relative energy of the two-body reaction which is accessible in the experiment is connected to projectile energy and the momentum of the spectator in the final state. It should be chosen to be close to the maximum of the momentum distribution so that the quasi-free reaction mechanism dominates. Thus, the application of the method depends strongly on the experimental realization. A Trojan-horse experiment under suitably chosen conditions should be performed and the more detailed formulation of the THM given here should be applied and discussed in that context. A comparison of the extracted cross section for a two-body reaction with data from a direct measurement would be useful to assess the applicability of the Trojan-horse method for nuclear astrophysics.

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Appendix

Considering particles with spin leads to more complex expressions for the cross sections, wave functions, and matrix elements. In the following we give the most important formulae, where we denote the spin of a nucleus j by s_j and s_{jk} is the channel spin of the system $j + k$ with projection ν_{jk} . For cross sections we usually have to sum over all final states and over all initial states in a reaction. In case of a two-body reaction we calculate the differential cross section

$$\begin{aligned} \frac{d\sigma}{d\Omega_{Cc}}(Ax \rightarrow Cc) \\ = \frac{1}{(2s_A + 1)(2s_x + 1)} \sum_{s_{Ax}\nu_{Ax}} \sum_{s_{Cc}\nu_{Cc}} \frac{v_{Cc}^f}{v_{Ax}^i} |f(Ax, s_{Ax}, \nu_{Ax} \rightarrow Cc, s_{Cc}, \nu_{Cc})|^2 \end{aligned} \quad (\text{A.1})$$

from the scattering amplitude f which can be derived from the full scattering wave function $\Psi^{(+)}(Ax, \mathbf{k}_{Ax}^i, s_{Ax}, \nu_{Ax})$ with the system $A + x$ in the initial channel. Its asymptotic form reads

$$\begin{aligned} \Psi^{(+)}(Ax, \mathbf{k}_{Ax}, s_{Ax}, \nu_{Ax}) \\ \rightarrow \frac{4\pi}{k_{Ax}^i} \sum_{\alpha s_\alpha} \sum_{JM} \sum_{l_\alpha l_{Ax}} \frac{\xi_{l_\alpha s_\alpha l_{Ax} s_{Ax}}^{J(+)}(\alpha; k_\alpha r_\alpha)}{r_\alpha} \mathcal{Y}_{JM}^{l_\alpha s_\alpha}(\alpha, \hat{\mathbf{r}}_\alpha) \mathcal{Z}_{JM}^{l_{Ax} s_{Ax} \nu_{Ax} *}(\hat{\mathbf{k}}_{Ax}^i) \end{aligned} \quad (\text{A.2})$$

with the functions

$$\mathcal{Y}_{JM}^{ls}(\alpha, \hat{\mathbf{r}}) = \sum_{m\nu} (lms\nu | JM) i^l Y_{lm}(\hat{\mathbf{r}}) \phi_\alpha(s\nu) \quad (\text{A.3})$$

and

$$\mathcal{Z}_{JM}^{ls\nu}(\hat{\mathbf{k}}) = \sum_m (lms\nu | JM) Y_{lm}(\hat{\mathbf{k}}), \quad (\text{A.4})$$

which describe the angular dependence. The radial wave function

$$\begin{aligned} \xi_{l_\alpha s_\alpha l_{Ax} s_{Ax}}^{J(+)}(\alpha; k_\alpha r_\alpha) &= \frac{\exp[i\sigma_{l_{Ax}}(\eta_{Ax}^i)]}{2i} \sqrt{\frac{v_{Ax}^i}{v_\alpha^f}} \\ &\times [S_{l_\alpha s_\alpha l_{Ax} s_{Ax}}^J(Ax \rightarrow \alpha) u_{l_\alpha}^{(+)}(\eta_\alpha; k_\alpha r_\alpha) - \delta_{\alpha Ax} \delta_{l_\alpha l_{Ax}} \delta_{s_\alpha s_{Ax}} u_{l_\alpha}^{(-)}(\eta_\alpha; k_\alpha r_\alpha)] \end{aligned} \quad (\text{A.5})$$

contains the (nuclear) S -matrix element $S_{l_\alpha s_\alpha l_{Ax} s_{Ax}}^J(Ax \rightarrow \alpha)$ which depends on the total angular momentum J and on the orbital angular momenta and spins in the initial and final channels. The total scattering amplitude (including Coulomb scattering) is now given by

$$\begin{aligned} f(Ax, s_{Ax}, \nu_{Ax} \rightarrow Cc, s_{Cc}, \nu_{Cc}) &= -i \frac{\sqrt{\pi}}{k_{Ax}^i} \sqrt{\frac{v_{Ax}^i}{v_{Cc}^f}} \\ &\times \sum_{l_{Cc} m_{Cc}} \sum_{l_{Ax}} \sqrt{2l_{Ax} + 1} Y_{l_{Cc} m_{Cc}}(\hat{r}_{Cc}) \sum_{JM} (l_{Cc} m_{Cc} s_{Cc} \nu_{Cc} | JM) (l_{Ax} 0 s_{Ax} \nu_{Ax} | JM) \\ &\times [\exp\{i[\sigma_{l_{Ax}}(\eta_{Ax}^i) + \sigma_{l_{Cc}}(\eta_{Cc}^f)]\} S_{l_{Cc} s_{Cc} l_{Ax} s_{Ax}}^J(Ax \rightarrow Cc) - \delta_{Cc Ax} \delta_{l_{Cc} l_{Ax}} \delta_{s_{Cc} s_{Ax}}], \end{aligned} \quad (\text{A.6})$$

and the total reaction cross section for an inelastic reaction ($Cc \neq Ax$) reads

$$\sigma(Ax \rightarrow Cc) = \frac{1}{(2s_A + 1)(2s_x + 1)} \frac{\pi}{(k_{Ax}^i)^2} \sum_J (2J + 1) \sum_{s_{Ax} s_{Cc}} \sum_{l_{Ax} l_{Cc}} |S_{l_{Cc} s_{Cc} l_{Ax} s_{Ax}}^J(Ax \rightarrow Cc)|^2. \quad (\text{A.7})$$

In case of the three-body breakup the triple differential cross section including spin degrees of freedom is given by

$$\begin{aligned} \frac{d^3\sigma}{dE_{Cc}^f d\Omega_{Cc} d\Omega_{Bb}} &= \frac{\mu_{Aa} \mu_{Bb} \mu_{Cc}}{(2\pi)^5 \hbar^6} \frac{k_{Bb}^f k_{Cc}^f}{k_{Aa}^i} \\ &\times \frac{1}{(2s_A + 1)(2s_a + 1)} \sum_{s_{Aa} \nu_{Aa}} \sum_{s_{Bb} \nu_{Bb}} |T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa})|^2, \end{aligned} \quad (\text{A.8})$$

where the T -matrix depends on the spins and their projections

$$\begin{aligned} T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) &= \sum_{s_{Cc}} \sum_{\nu_{Cc} \nu_b} \sum_{\nu_{Aa}} (s_{Cc} \nu_{Cc} s_b \nu_b | s_{Bb} \nu_{Bb}) (s_A \nu_A s_a \nu_a | s_{Aa} \nu_{Aa}) \\ &\times \langle \chi^{(-)}(Bb, \mathbf{k}_{Bb}^f) \Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f, s_{Cc} \nu_{Cc}) \phi_b(s_b \nu_b) | V_{xb} | \chi^{(+)}(Aa, \mathbf{k}_{Aa}^i) \phi_A(s_A \nu_A) \phi_a(s_a \nu_a) \rangle. \end{aligned} \quad (\text{A.9})$$

The asymptotic scattering wave function in the $C + c$ channel can be written

$$\begin{aligned} \Psi^{(-)}(Cc, \mathbf{k}_{Cc}^f, s_{Cc}, \nu_{Cc} \rightarrow Ax) &= \frac{4\pi}{k_{Cc}^f} \sum_{s_{Ax}} \sum_{JM} \sum_{l_{Ax} l_{Cc}} \exp[-i\sigma_{l_{Cc}}(\eta_{Cc}^f)] \sqrt{\frac{v_{Cc}^f}{v_{Ax}^f}} \mathcal{Y}_{JM}^{l_{Ax} s_{Ax}}(Ax, \hat{r}_{Ax}) \mathcal{Z}_{JM}^{l_{Cc} s_{Cc} \nu_{Cc} *}(\hat{\mathbf{k}}_{Cc}^f) \\ &\times \left[S_{l_{Ax} s_{Ax} l_{Cc} s_{Cc}}^{J*}(Cc \rightarrow Ax) \frac{\zeta_{l_{Ax}}^{(-)}(Ax; k_{Ax}^f r_{Ax})}{r_{Ax}} - \delta_{Ax Cc} \delta_{l_{Ax} l_{Cc}} \delta_{s_{Ax} s_{Cc}} \frac{\zeta_{l_{Ax}}^{(+)}(Ax; k_{Ax}^f r_{Ax})}{r_{Ax}} \right] \end{aligned} \quad (\text{A.10})$$

with the radial wave functions

$$\zeta_{l_{Ax}}^{(\pm)}(Ax; k_{Ax}^f r_{Ax}) = \Theta(r_{Ax} - R_{Ax}) \frac{i}{2} u_{l_{Ax}}^{(\pm)}(\eta_{Ax}^f; k_{Ax}^f r_{Ax}). \quad (\text{A.11})$$

In an inelastic process with $Cc \neq Ax$ the T -matrix element assumes the form

$$\begin{aligned}
T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
= \sqrt{\frac{\mu_{Ax} k_{Ax}^f}{\mu_{Cc} k_{Cc}^f}} \sum_J \sum_{s_{Cc} s_{Ax}} \sum_{l_{Ax} l_{Cc}} \exp[i\sigma_{l_{Cc}}(\eta_{Cc}^f)] S_{l_{Ax} s_{Ax} l_{Cc} s_{Cc}}^J (Cc \rightarrow Ax) \sum_{\nu_{Cc} \nu_b} (s_{Cc} \nu_{Cc} s_b \nu_b | s_{Bb} \nu_{Bb}) \\
\times \sum_M \mathcal{Z}_{JM}^{l_{Cc} s_{Cc} \nu_{Cc}}(\hat{\mathbf{k}}_{Cc}^f) t_{fi}^{(-)JM}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, l_{Ax}, s_{Ax}, \nu_b; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa})
\end{aligned} \tag{A.12}$$

with the reduced T -matrix element

$$\begin{aligned}
t_{fi}^{(\pm)JM}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, l_{Ax}, s_{Ax}, \nu_b; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
= 4\pi \sum_{\nu_A \nu_a} (s_A \nu_A s_a \nu_a | s_{Aa} \nu_{Aa}) \\
\times \langle \chi^{(-)}(Bb, \mathbf{k}_{Bb}^f) \frac{S_{l_{Ax}}^{(\pm)}(\alpha; k_{Ax}^f r_{Ax})}{k_{Ax}^f r_{Ax}} \mathcal{Y}_{JM}^{l_{Ax} s_{Ax}}(Ax, \hat{\mathbf{r}}_{Ax}) \phi_b(s_b \nu_b) | V_{xb} | \chi^{(+)}(Aa, \mathbf{k}_{Aa}^i) \rangle \\
\times \phi_A(s_A \nu_A) \phi_a(s_a \nu_a).
\end{aligned} \tag{A.13}$$

In case of the elastic process with $Cc = Ax$ we find

$$\begin{aligned}
T_{fi}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
= \sum_J \sum_{s_{Ax} s'_{Ax}} \sum_{l_{Ax} l'_{Ax}} \exp[i\sigma_{l_{Ax}}(\eta_{Ax}^f)] \sum_{\nu'_{Ax} \nu_b} (s'_{Ax} \nu'_{Ax} s_b \nu_b | s_{Bb} \nu_{Bb}) \sum_M \mathcal{Z}_{JM}^{l_{Ax} s'_{Ax} \nu'_{Ax}}(\hat{\mathbf{k}}_{Ax}^f) \\
\times [S_{l_{Ax} s_{Ax} l'_{Ax} s'_{Ax}}^J (Ax \rightarrow Ax) t_{fi}^{(-)JM}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, l_{Ax}, s_{Ax}, \nu_b; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
- t_{fi}^{(+)JM}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, l_{Ax}, s_{Ax}, \nu_b; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa})].
\end{aligned} \tag{A.14}$$

In the plane-wave approximation we introduce the Fourier transform of the product

$$\begin{aligned}
V_{xb}(\mathbf{r}_{xb}) \phi_a(s_a \nu_a, \mathbf{r}_{xb}) \\
= \int \frac{d^3 k}{(2\pi)^3} W(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{r}_{xb}) \sum_{\nu_x \nu'_b} (s_x \nu_x s_b \nu'_b | s_a \nu_a) \phi_x(s_x \nu_x) \phi_b(s_b \nu'_b)
\end{aligned} \tag{A.15}$$

and obtain for the reduced T -matrix

$$\begin{aligned}
t_{fi}^{(\pm)JM}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, l_{Ax}, s_{Ax}, l_{Cc}, s_b, \nu_b; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
= (4\pi)^3 (-1)^{s_x + s_A + s_{Aa} + s_b} \sqrt{2s_{Ax} + 1} \sqrt{2s_a + 1} \begin{Bmatrix} s_x & s_A & s_{Ax} \\ s_{Aa} & s_b & s_a \end{Bmatrix} \\
\times \sum_{m_{Ax} \nu_{Ax}} (l_{Ax} m_{Ax} s_{Ax} \nu_{Ax} | JM) (s_{Ax} \nu_{Ax} s_b \nu_b | s_{Aa} \nu_{Aa}) W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \\
\times \sum_{lm} \sum_{l'm'} i^{l-l'} \sqrt{\frac{(2l_{Ax} + 1)(2l' + 1)}{4\pi(2l + 1)}} (l_{Ax} 0 l' 0 | l 0) (l_{Ax} m_{Ax} l' m' | lm) \\
\times R_{ll' l_{Ax}}^{(\pm)}(Ax; k_{Aa}^i, k_{Bb}^f, k_{Ax}^f) Y_{lm}^*(\hat{\mathbf{k}}_{Aa}^i) Y_{l'm'}(\hat{\mathbf{k}}_{Bb}^f)
\end{aligned} \tag{A.16}$$

after performing the angular algebra. Finally, in case of the inelastic two-body reaction the result for the total three-body T -matrix becomes

$$\begin{aligned}
T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
= \sqrt{\frac{\mu_{Ax} k_{Ax}^f}{\mu_{Cc} k_{Cc}^f}} W\left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i\right) \\
\times \sum_J (2J + 1) \sum_{l_{Ax} l_{Cc}} \sum_{s_{Cc} s_{Ax}} \exp[i\sigma_{l_{Cc}}(\eta_{Cc}^f)] S_{l_{Ax} s_{Ax} l_{Cc} s_{Cc}}^J (Cc \rightarrow Ax) \\
\times \sum_{l''} R_{ll'' l_{Ax}}^{(-)}(Ax; q_{Aa}^i, q_{Bb}^f, q_{Ax}^f) X_{l_{Ax} s_{Ax} l_{Cc} s_{Cc}}^{l''}(\hat{\mathbf{k}}_{Cc}^f, \hat{\mathbf{k}}_{Bb}^f, s_{Bb}, \nu_{Bb}; \hat{\mathbf{k}}_{Aa}^i, s_{Aa}, \nu_{Aa})
\end{aligned} \tag{A.17}$$

with the angular distribution function

$$\begin{aligned}
& X_{l_{Ax} s_{Ax} l_{Cc} s_{Cc}}^{jll'}(\hat{\mathbf{k}}_{Cc}^f, \hat{\mathbf{k}}_{Bb}^f, s_{Bb}, \nu_{Bb}; \hat{\mathbf{k}}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
&= (4\pi)^{5/2} i^{l-l'} (l_{Ax} 0 l' 0 | l 0) (-1)^{s_x + s_A + s_{Aa} + s_b + l_{Ax} + s_{Ax} - l_{Cc} - s_{Cc}} \\
&\quad \times \sqrt{2s_{Ax} + 1} \sqrt{2s_a + 1} \sqrt{\frac{2l' + 1}{2l + 1}} \sqrt{\frac{(2s_{Aa} + 1)(2s_{Bb} + 1)}{2l_{Cc} + 1}} \\
&\quad \times \begin{Bmatrix} s_x & s_A & s_{Ax} \\ s_{Aa} & s_b & s_a \end{Bmatrix} \sum_K (2K + 1) \begin{Bmatrix} s_b & s_{Ax} & s_{Aa} \\ s_{Cc} & J & l_{Cc} \\ s_{Bb} & l_{Ax} & K \end{Bmatrix} \\
&\quad \times \sum_{m_{Ax} m_{Cc}} \sum_{\kappa} (s_{Bb} \nu_{Bb} K \kappa | l_{Ax} m_{Ax}) (s_{Aa} \nu_{Aa} K \kappa | l_{Cc} m_{Cc}) \\
&\quad \times \sum_{mm'} (l_{Ax} m_{Ax} l' m' | l m) Y_{lm}^*(\hat{\mathbf{k}}_{Aa}^i) Y_{l'm'}(\hat{\mathbf{k}}_{Bb}^f) Y_{l_{Cc} m_{Cc}}^*(\hat{\mathbf{k}}_{Cc}^f). \tag{A.18}
\end{aligned}$$

In the elastic case we have

$$\begin{aligned}
& T_{fi}(\mathbf{k}_{Ax}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
&= W \left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i \right) \sum_J (2J + 1) \sum_{l_{Ax} l'_{Ax}} \sum_{s_{Ax} s'_{Ax}} \exp[i\sigma_{l_{Ax}}^f(\eta_{Ax}^f)] \\
&\quad \times \sum_{ll'} X_{l_{Ax} s_{Ax} l'_{Ax} s'_{Ax}}^{jll'}(\hat{\mathbf{k}}_{Ax}^f, \hat{\mathbf{k}}_{Bb}^f, s_{Bb}, \nu_{Bb}; \hat{\mathbf{k}}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
&\quad \times [S_{l_{Ax} s_{Ax} l'_{Ax} s'_{Ax}}^j(Ax \rightarrow Ax) R_{ll' l_{Ax}}^{(-)}(Ax; q_{Aa}^i, q_{Bb}^f, q_{Ax}^f) - R_{ll' l_{Ax}}^{(+)}(Ax; q_{Aa}^i, q_{Bb}^f, q_{Ax}^f)]. \tag{A.19}
\end{aligned}$$

The plane-wave approximation for the full T -matrix can again be factorized

$$\begin{aligned}
& T_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \\
&= W \left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i \right) \tilde{T}_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa}) \tag{A.20}
\end{aligned}$$

with the result

$$\begin{aligned}
& \frac{d^3 \sigma}{dE_{Cc}^f d\Omega_{Cc} d\Omega_{Bb}} = \frac{\mu_{Aa} \mu_{Bb} \mu_{Cc}}{(2\pi)^5 \hbar^6} \frac{k_{Bb}^f k_{Cc}^f}{k_{Aa}^i} \left| W \left(\mathbf{k}_{Bb}^f - \frac{m_b}{m_x + m_b} \mathbf{k}_{Aa}^i \right) \right|^2 \\
&\quad \times \frac{1}{(2s_A + 1)(2s_a + 1)} \sum_{s_{Aa} \nu_{Aa}} \sum_{s_{Bb} \nu_{Bb}} |\tilde{T}_{fi}(\mathbf{k}_{Cc}^f, \mathbf{k}_{Bb}^f, s_{Bb}, \nu_{Bb}; \mathbf{k}_{Aa}^i, s_{Aa}, \nu_{Aa})|^2 \tag{A.21}
\end{aligned}$$

for the cross section.

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