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$\Lambda - \alpha$ Potential by Folding Λ-Nucleon Interaction with Realistic α Wave Function

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Abstract We construct a folding potential between the α and Λ particles based on underlying nucleon-nucleon and hyperon-nucleon interactions. Starting from a phenomenological Λ-N potential and a Gaussian form of the α-particle wave function we obtain the α-Λ potential and with this potential the binding energy of ${}^{5}_{\Lambda}$He is (3.10 MeV), which is consistent with recent experimental data 3.12 ± 0.02 MeV. When in turn an exact solution of the four-body Faddeev-Yakubovsky equation for the α-particle calculated with the CDBonn, Nijmegen or Argonne V18 realistic nucleon-nucleon potential is used and the phenomenological Gaussian Λ-N potential is replaced by the realistic (e.g. Nijmegen NSC97f) potential approximated by a rank-1 separable form, then ${}^{5}_{\Lambda}$He is overbound. In particular, its binding energy given by the folding potential generated with the α particle wave function based on the CDBonn potential is 7.47 MeV. Although the rank-1 separable Λ-N potential reproduces the exact scattering length and the effective range of the original Λ-N potential, the Λ-α folding potential results from these Λ-N potential give a large binding energy of ${}^{5}_{\Lambda}$He .

1 Introduction

The interaction between the hyperon (Λ, Σ and Ξ)and the α particle are important ingredients to consider the possible existence for various kinds of light hypernuclei. If reliable effective hyperon-nucleon interactions are known, one could easily calculate the hyperon-α potentials. Unfortunately, hyperon-nucleon interaction itself is not well known because of the technical difficulties of strangeness experiments. In the case of Λ-N or Σ-N (strangeness S=− 1 sector) the set of scattering data is very small [1–4] and is not sufficient to determine well the properties of those forces. In this connection, light hypernuclei such as e.g. the hypertriton[5], which are accessible for few-body methods, offer a unique opportunity to learn about the YN interaction. Certain heavier hypernuclei can also be viewed as few-body systems assuming their cluster structure in terms of the α -particles. Phenomenological α -Λ potentials of a simple Gaussian type were used e.g. to study double-Λ [6,7]. Recently, we predicted the existence of quasi-bound state of Σ -Σ -α system using a phenomenological Σ-α potential [8,9].

The common drawback of all phenomenological potentials is that they have some parameters to be fit to experimental data. In order to obtain the potential without any parameters, one need to solve the A-body problem using baryon-baryon interactions which is, however, very difficult beyond the four-body system. In

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this project we propose to derive a folding potential between the α and Λ particles without any additional parameters. In order to achieve this goal, we use the α-particle wave function based on realistic NN forces, e.g., a meson theoretical CD-Bonn [10], Nijmegen [11] and Argonne [12] potentials and we describe the Λ-N interaction by a phenomenological Gaussian form [13] and meson theoretical models, e.g., Chiral [14], Jülich [15], Nijmegen [16,17], Ehime[18] and Kyoto-Niigata[19].

2 Formulation of Λ-α Folding Potential

The folding potential V_{fold} is defined by evaluating matrix elements of the inner realistic potential V_{inner} between products of two-cluster wave functions $\psi_\alpha \psi_\Lambda$:

$$V_{fold} = \langle \psi_\alpha \psi_\Lambda | V_{inner} | \psi_\alpha \psi_\Lambda \rangle. \tag{1}$$

The schematic diagram of the $\Lambda - \alpha$ potential is shown in Fig. 1. The natural Jacobi momenta for the ((123)4)Λ partition [20,21] are

$$\mathbf{u}_1 = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2), \quad \mathbf{u}_2 = \frac{2}{3}\left\{\mathbf{k}_3 - \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)\right\},$$

$$\mathbf{u}_3 = \frac{3}{4}\left\{\mathbf{k}_4 - \frac{1}{3}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\right\}, \tag{2}$$

$$\mathbf{u}_\Lambda = \frac{4m_N \mathbf{k}_\Lambda - m_\Lambda(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)}{4m_N + m_\Lambda} \tag{3}$$

where $\mathbf{k}_i$, $i = 1, \ldots, 4$ are the individual nucleon momenta, $\mathbf{k}_\Lambda$ is the momentum of the Λ hyperon; m_N and m_Λ are the masses of the nucleon and the Λ particle.

The corresponding relative orbital angular momenta will be denoted by l_i, $i = 1, 2, 3, \Lambda$ and the total spin, total angular momenta and total isospins in the various subsystems by s_i, j_i and t_i, respectively (see Fig. 2). The 4N-Λ basis states are introduced via

$$|u_1 u_2 u_3 u_\Lambda a\rangle$$
$$:= |u_1 u_2 u_3 u_\Lambda \left[l_2((l_1 s_1) j_1 \frac{1}{2}) s_2\right] j_2, (j_2 \frac{1}{2}) j_3, (l_3 j_3) j_\alpha, (l_\Lambda \frac{1}{2}) j_\Lambda, (j_\alpha j_\Lambda) J, (t_1 \frac{1}{2}) t_2 (t_2 \frac{1}{2}) T\rangle, \tag{4}$$

where the brackets indicate self-explanatory consecutive couplings to the total five-baryon angular momentum J and total isospin T with the corresponding magnetic quantum numbers (not shown for brevity).

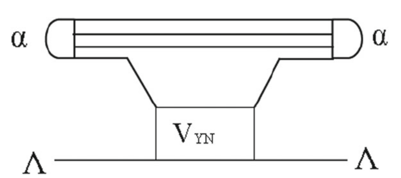

Fig. 1 A schematic representation of the α-Λ folding potential

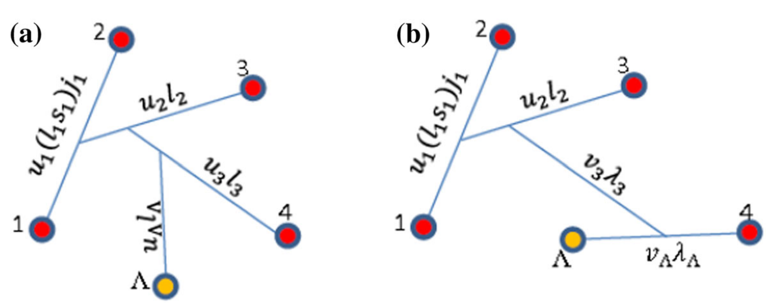

Fig. 2 Definition of continuous and discrete quantum numbers for the ((123)4)Λ partition (left panel) and for the (123)(4Λ) partition (right panel)

The natural Jacobi momenta for the fragmentation (123)(4Λ) are

$$\mathbf{v}_\Lambda = \frac{1}{m_N + m_\Lambda}(m_\Lambda \mathbf{k}_4 - m_N \mathbf{k}_\Lambda),$$
$$\mathbf{v}_3 = \frac{1}{4m_N + m_\Lambda}\{3m_N(\mathbf{k}_4 + \mathbf{k}_\Lambda) - (m_\Lambda + m_N)(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\}. \tag{5}$$

The corresponding discrete quantum numbers will be denoted by Greek letters, see the right panel of Fig. 2. The basis states are

$$|u_1 u_2 v_3 v_\Lambda b\rangle$$
$$:= |u_1 u_2 v_3 v_\Lambda \left[l_2((l_1 s_1) j_1 \frac{1}{2}) s_2 \right] j_2, (\lambda_\Lambda \Sigma_\Lambda)\tau_\Lambda, (\lambda_3 j_2)\tau_3, (\tau_\Lambda \tau_3) J, (t_1 \frac{1}{2}) t_2 (t_2 \frac{1}{2}) T\rangle, \tag{6}$$

where the brackets indicate again the sequences of couplings of angular momenta and isospins. The Jacobi momenta in these two sets are related via

$$\mathbf{u}_3 = \mathbf{v}_3 + \frac{3}{4}\mathbf{u}_\Lambda, \quad \mathbf{v}_\Lambda = -\mathbf{u}_\Lambda - \frac{m_\Lambda}{m_N + m_\Lambda}\mathbf{v}_3. \tag{7}$$

In order to calculate the folding potential of Eq. (1) we first prepare the α-particle wave function ψ_α and the YN interaction V_{YN} as

$$\psi_\alpha(u_1, u_2, u_3, a) = \langle u_1 u_2 u_3 a | \psi_\alpha \rangle,$$
$$V_{YN}(v_\Lambda, v'_\Lambda) = \langle v_\Lambda | V_{YN} | v'_\Lambda \rangle. \tag{8}$$

Then Eq.(1) turns into

$$\begin{aligned} V_{fold}(u_\Lambda, u'_\Lambda) &= 4 \sum_{a,a'} \int d\mathbf{u}_1 d\mathbf{u}_2 d\mathbf{u}_3 d\mathbf{u}_\Lambda \int d\mathbf{u}'_1 d\mathbf{u}'_2 d\mathbf{u}'_3 d\mathbf{u}'_\Lambda \psi_\alpha(u_1 u_2 u_3 a) \psi_\Lambda(u_\Lambda) \\ &\quad \times \langle u_1 u_2 u_3 u_\Lambda a | V_{YN} | u_1 u'_2 u'_3 u'_\Lambda a' \rangle \psi_\alpha(u'_1 u'_2 u'_3 a') \psi_\Lambda(u'_\Lambda) \\ &= 4 \sum_{aa'bb'} \int_0^\infty v_3^2 dv_3 \int_{-1}^{1} dx \int_{-1}^{1} dx' \frac{K_\alpha(u_3 u'_3 a a')}{u_3^{l_3} u'^{l'_3}_3} G_{a,b}(u_\Lambda, v_3, x) \\ &\quad \times \frac{V_{YN}(v_\Lambda, v'_\Lambda)}{v_\Lambda^{\lambda_\Lambda} v'^{\lambda'_\Lambda}_\Lambda} G_{b',a'}(v_3, u'_\Lambda, x'), \end{aligned} \tag{9}$$

where

$$K_\alpha(u_3, u'_3, a, a') = \int d\mathbf{u}_1 d\mathbf{u}_2 \psi_\alpha(u_1 u_2 u_3 a) \psi_\alpha(u_1 u_2 u'_3 a') \tag{10}$$

and ψ_Λ is taken is as a plane wave. The geometrical functions $G_{a,b}(u_\Lambda, v_3, x)$ and $G_{b',a'}(v_3, u'_\Lambda, x')$ have been introduced in Refs. [22,23]

3 Numerical Results

First let us consider the results for $\Lambda - \alpha$ potential obtained with the spin-dependent phenomenological Λ-N interaction is parameterized by Hiyama et al.,[13] as

$$V_{\Lambda N}(r) = V^0_{\Lambda N}(1 + \eta\, \sigma_\Lambda \cdot \sigma_N) e^{-(r/\beta)^2}, \tag{11}$$

with $V^0_{\Lambda N} = -38.19\,\text{MeV}$, $\beta = 1.034$ fm and $\eta = -0.1$. The Λ-N potential for the spin triplet is shown in Fig. 3 . The integral kernel K of Eq. (10) originating from the Gaussian α particle S-wave function is given as

$$K(u_3, u'_3) = 4\pi (\frac{2}{3\Omega\pi})^{\frac{3}{2}} \exp\{-\frac{(u_3^2 + u'^2_3)}{3\Omega}\}, \tag{12}$$

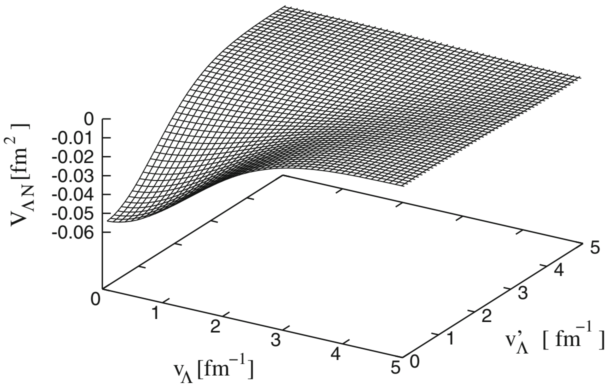

Fig. 3 Spin triplet Λ-nucleon interaction in momentum space

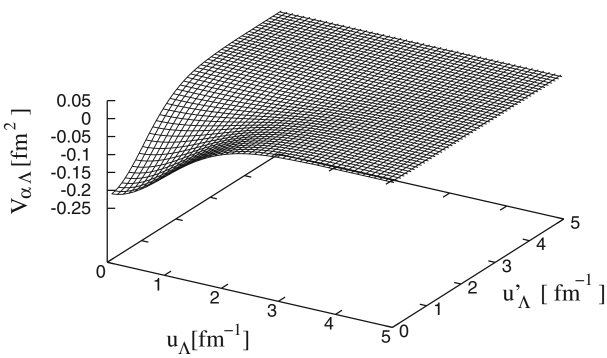

Fig. 4 α-Λ folding potential for S-wave

where the width parameter Ω is a common shell model mode [6,24] taken to be 0.275 fm^{-2}.

We obtained a Λ-α folding potential (shown in Fig.4) for which the ${}^{5}_{\Lambda}$He binding energy is -3.10 MeV. The calculated binding energy compares well with the data (-3.12 ± 0.02 MeV) [25]. This consistence is to be expected, since the YN potential of Eq. (11) is adjusted to the experimental ${}^{5}_{\Lambda}$He binding energy when the RGM technique is employed[7].

Next we replace the simple Gaussian wave function of the α particle by the wave function based on the realistic NN forces: the CD-Bonn[10], Nijmegen [11], and Argonne V18 [12] potentials. Now we consider, the Λ-N potentials are given in the separable form of rank 1 :

$$V_{\Lambda \mathrm{N}}(v_\Lambda, v'_\Lambda) = -\lambda g(v_\Lambda) g(v'_\Lambda), \tag{13}$$

where λ and $g(p)$ are the coupling constant and the form factor, respectively. This rank 1 potential can easily and analytically relate to well known low energy scattering parameters (see Eq. (16)). In order to check the accuracy of the separable approximation we prepare two kinds of the separable potentials, e.g, the Yamaguchi type (Y) and the Gaussian type (G). The form factors of these potentials are given as

$$\begin{aligned} g_{\mathrm{Y}}(v_\Lambda) &= \frac{1}{v_\Lambda^2 + \beta_{\mathrm{Y}}^2}, \\ g_{\mathrm{G}}(v_\Lambda) &= \exp\{-\beta_{\mathrm{G}}^2 v_\Lambda^2\}. \end{aligned} \tag{14}$$

The meson theoretical Λ-N potentials are constructed to describe the Λ-N scattering data. However, due to the sparsity of the data parameters of all potential models are not well determined. Therefore, we use the fact that the Λ-N scattering amplitude in the low energy limit can be determined from the well-known effective range expansion which has the form

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 + \cdots \tag{15}$$

where k denotes the scattering momentum in the center-of-mass system and the parameters a [fm] and r [fm] are often called the scattering length and the effective range, respectively. The phase shift δ of each partial wave is linked to the scattering amplitude.

These effective range expansion parameters are directly connected to the quantities $\beta_{\rm Y}$, $\lambda_{\rm Y}$, $\beta_{\rm G}$ and $\lambda_{\rm G}$ from Eq. (14) by the following relations.

$$\beta_{\rm Y} = \frac{3+\sqrt{9-16\frac{r}{a}}}{2r}, \lambda_{\rm Y} = \frac{4\beta_{\rm Y}^3}{\pi\mu(r\beta_{\rm Y}-1)},$$
$$\beta_{\rm G} = \frac{\sqrt{2}a+\sqrt{a(2a-\pi r)}}{2\sqrt{\pi}}, \lambda_{\rm G} = \frac{ar}{\sqrt{2}\mu\sqrt{a(2a-\pi r)}-(2a-\pi r)\mu}, \quad (16)$$

where μ is the reduced mass of the Λ-N system. In the case of the Yamaguchi type form factors these relations are proved in Ref. [26]. Tables 1 and 2 collect the scattering lengths and the effective ranges from several Λ-N potentials.

In Tab. 3 we demonstrate the calculated binding energies of the $^5_\Lambda$He hypernucleus. Each row of the table is prepared for one Λ-N potential. The row containing results based on the full potential from Hiyama *et al.*[13]) is separated from the other ones by a line to indicate that the predictions in all other rows are obtained with separable approximations employing the Yamaguchi type (Y) or the Gaussian type (G) form factors. Columns tell which realistic NN potential is used to calculate the α particle wave function, necessary to construct the integral kernel K_α in Eq.(10).

From the comparison of the binding energies for the Gaussian wave function with the full Hiyama Gaussian ΛN potential (-3.10 MeV), the approximate Hiyama potential (type Y) (-2.46 MeV) and the approximate Hiyama potential (type G) (3.07 MeV), we can estimate the accuracy of the separable approximation. The accuracy is not better than approximately 0.7 MeV. Using the realistic α particle wave functions we obtain clear underbinding. It is most evident for the AV18 potential and predictions based on this NN force differ from the others by up to 1.2 MeV.

Surprisingly, the most realistic input for calculations, namely the realistic Λ-N potential and the α particle wave functions generated by the realistic NN interactions, leads to rather strong overbinding of the $^5_\Lambda$He hypernucleus and moves the predictions away from the data. These numbers are listed below the second horizontal line in Tab. 3.

Table 1 Scattering lengths a and effective ranges r in fm for Λ-neutron potential

Model ΛN potential	a ($^1S_0$)	r ($^1S_0$)	a ($^3S_1$)	r ($^3S_1$)
Hiyama *et al.* [13]	– 1.28*	2.33*	– 0.67*	3.08*
Chiral (Λ = 600) [14,26]	– 2.91*	2.78*	– 1.54*	2.74*
Jülich04 [15,26]	– 2.56*	2.75*	– 1.66*	2.93*
Nimegen ESC16	– 1.96	3.65	– 1.84	3.33
Nijmegen NSC97e [17,26]	– 2.24	3.24	– 1.83	3.14
Nijmegen NSC97f [17,26]	– 2.68	3.07	– 1.67	3.34
Nijmegen NSC89 [27]	– 2.86	2.91	– 1.24	3.33
Nimegen HC-D model [16,26]	– 2.03	3.66	– 1.84	3.32
Ehime set 2 [18]	– 2.65*	3.24*	– 1.80*	3.71*
Ehime set A [18]	– 2.76*	3.19*	– 2.064*	3.46*
Ehime set B [18]	– 2.71*	3.21*	– 1.95*	3.56*

The Hiyama, Chiral and Jülich models do not differentiate between the Λ-neutron and the Λ-proton channel, which is indicated with a star(*)

Table 2 Scattering lengths a and effective ranges r in fm for Λ-proton potential

Model ΛN potential	a ($^1S_0$)	r ($^1S_0$)	a ($^3S_1$)	r ($^3S_1$)
Nimegen ESC16 [28]	– 1.88	3.58	– 1.86	3.37
Nijmegen NSC97e [17,26]	– 2.10	3.19	– 1.86	3.19
Nijmegen NSC97f [17,26]	– 2.51	3.03	– 1.75	3.32
Nijmegen NSC89 [27]	– 2.73	2.87	– 1.48	3.04
Nimegen HC-D model [16,26]	– 2.06	3.78	– 1.77	3.18

Table 3 The binding energies of $^5_\Lambda$He using the model $\alpha\Lambda$ potentials

NN potential for α particle						
ΛN potential	RGM [7]	CD-Bonn [10]	Nijm93 [11]	Nijm I [11]	Nijm II [11]	AV18 [12]
Hiyama *et al.* [13]	− 3.10	− 2.99	− 2.18	− 2.54	− 1.98	− 1.95
Hiyama (Y)	− 2.46	− 2.36	− 1.62	− 1.94	− 1.44	− 1.42
Hiyama (G)	− 3.07	− 2.89	− 2.12	− 2.47	− 1.93	− 1.90
Chiral (Λ = 600) [14,26] (Y)	− 8.44	− 8.26	− 6.63	− 7.38	− 6.23	− 6.15
Chiral (Λ = 600) (G)	− 9.25	− 8.82	− 7.41	− 8.07	− 7.03	− 6.98
Jülich04 [15,26] (Y)	− 8.46	− 8.25	− 6.71	− 7.42	− 6.29	− 6.20
Jülich04 (G)	− 9.07	− 8.60	− 7.28	− 7.91	− 6.92	− 6.89
Nimegen ESC16 (Y)	− 7.57	− 7.26	− 5.98	− 6.57	− 5.65	− 5.61
Nimegen ESC16 (G)	− 7.38	− 6.84	− 5.93	− 6.38	− 5.67	− 5.67
Nijmegen NSC97e [17,26] (Y)	− 8.21	− 7.94	− 6.51	− 7.16	− 6.14	− 6.09
Nijmegen NSC97e (G)	− 8.32	− 7.79	− 6.70	− 7.23	− 6.40	− 6.38
Nijmegen NSC97f [17,26] (Y)	− 7.98	− 7.69	− 6.31	− 6.94	− 5.95	− 5.91
Nijmegen NSC97f (G)	− 8.00	− 7.47	− 6.44	− 6.94	− 6.16	− 6.14
Nijmegen NSC89 [27] (Y)	− 7.11	− 6.88	− 5.54	− 6.15	− 5.19	− 5.15
Nijmegen NSC89 (G)	− 7.47	− 7.02	− 5.93	− 6.45	− 5.63	− 5.61
Nimegen HC-D model [16,26] (Y)	− 7.62	− 7.32	− 6.02	− 6.88	− 5.91	− 5.64
Nimegen HC-D model (G)	− 7.48	− 6.95	− 6.01	− 6.48	− 5.75	− 5.74
Ehime set 2 [18] (Y)	− 7.66	− 7.32	− 6.09	− 6.66	− 5.76	− 5.73
Ehime set A [18] (Y)	− 8.78	− 8.44	− 7.04	− 7.69	− 6.67	− 6.63
Ehime set B [18] (Y)	− 8.32	− 7.98	− 6.65	− 7.27	− 6.30	− 6.26

The notations (Y) and (G) are corresponding to the Yamaguchi separable form and Gaussian one, respectively. Unit is in MeV

This overbinding problem could be due to the effect of Λ N - Σ N conversion. In our calculation we focus on Iso-spin zero and the α particle would be inert and Λ-N to Σ-N conversion effect has been ignored. There may be another reason that these realistic potentials generally have a repulsive core in short-range areas. It is possible that the separable potential of rank 1 does not fully incorporate the feature of its repulsive force. However, the separable potentials of Y and G type are not directly derived from the realistic potential, but simply derived from the scattering length and effective range of ΛN scattering, so it is thought that the effect of Σ particle is already included included implicitly.

In order to uniquely determine the parameters of the separated potential, it was decided to rank 1, but as a future plan, there are a lot of improvement possibilities that there could perform high rank calculations or use the direct potential.

However, considering that these folding potentials of the RGM method do not, in principle, diagonalize the Hamiltonian of the entire system, but only calculate the expected value in the sense of variation principle. Even in the NN potential based on the meson theory, a certain form factor is introduced into the nucleon and meson vertex to reduce the overall strength. By incorporating these effects, the phenomenological potential of $\alpha\Lambda$ also needs such a attenuation factor.

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References

1. G. Alexander et al.: Phys. Rev. **173**, 1452 (1968)
2. B. Sechi-Zorn et al.: Phys. Rev. **175**, 1735 (1968)
3. R. Engelmann et al.: Phys. Lett. **21**, 587 (1966)
4. J.A. Kadyk et al.: Nucl. Phys. B **27**, 13 (1971)
5. K. Miyagawa et al.: Phys. Rev. C **51**, 2905 (1995)
6. E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, Phys. Rev. C **97**, 881 (1997)
7. E. Hiyama et al.: Prog. Theor. Phys. **97**, 887 (1997)
8. H.H. Oo, K.S. Myint, H. Kamada, W. Glöckle, Prog. Theor. Phys. **113**, 809 (2005)
9. H.H. Oo, K.S. Myint, H. Kamada, W. Glöckle, Few Body Syst. **54**, 1283 (2013)
10. Machleidt, R., Phys. Rev. C **63**, (2001) 024001

11. Stoks, V.G.J., Klomp, R.A.M., Rentmeester M. C. M. and de Swart, J. J.: Phys. Rev. C **48**, 792 (1993)
12. R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C **51**, 38 (1995)
13. E. Hiyama et al., Phys. Rev. C **53**, 2075 (1996)
14. J. Haidenbauer et al., Nucl. Phys. A **915**, 24 (2013)
15. Haidenbauer, J., Meißner, Ulf-G,: Phys. Rev. C **72**(4), 044005 (2005)
16. T.A. Rijken, V.G.J. Stoks, Y. Yamamoto, Phys. Rev. C **59**, 21 (1999)
17. M.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D **15**, 2547 (1977)
18. K. Tominaga, T. Ueda, M. Yamaguchi, N. Kijima, D. Okamoto, K. Miyagawa, T. Yamada, Nucl. Phys. A **642**, 483 (1998)
19. Fujiwara, Y., et al.: Phys. Rev. C **65**, 014002 (2002)
20. W. Glöckle, H. Kamada, Phys. Rev. Lett. **71**, 971 (1993)
21. Kamada, H., W. Glöckle, Nucl. Phys. A **548**, 205 (1992)
22. W. Glöckle, H. Witała, D. Hüber, H. Kamada, J. Golak, Phys. Rep. **274**, 107 (1996)
23. W. Glöckle, *The quantum mechanical few-body problem* (Springer, Berlin, 1983)
24. H. Horiuchi, Prog. Theor. Phys. Suppl. **62**, 90 (1977)
25. A. Gal, Adv. Nucl. Phys. **8**, 1 (1975)
26. Afnan, I.R., Gibson, B.F,: Phys. Rev. C **92**, (2015) 054608
27. P.M.M. Maessen, Th.A. Rijken, J. de Swart, J. Phys. Rev. C **40**, 2226 (1989)
28. Nagels, M.M., Rijken, Th.A., Yamamoto, Y,: Phys. Rev. C **99**, 044003 (2019)