Few-Body Syst (2021) 62:88
https://doi.org/10.1007/s00601-021-01679-0

ORIGINAL PAPER

Yuriy Volkotrub · Jacek Golak · Roman Skibiński ·
Kacper Topolnicki · Henryk Witała

Correlation and Regression Analysis of 2N Scattering Observables

Received: 10 May 2021 / Accepted: 25 August 2021 / Published online: 8 September 2021

Abstract This contribution provides information about correlations among the neutron-proton (np) elastic scattering observables. The most basic prerequisite for this study is existence of models of the nucleon-nucleon (NN) force, for which a covariance matrix of potential parameters is available. Using the covariance matrices for the chiral SMS NN potential from the Bochum-Bonn group and the one for the OPE-Gaussian potential proposed by the Granada group, we performed a systematic analysis of the correlation coefficients between the differential cross section and the depolarization R, as well as between R and the polarization P.

1 Introduction

One of the main goals of theoretical low-energy nuclear physics is to establish the structure of the nuclear Hamiltonian. Currently, effective and phenomenological models of nuclear interactions are still of great importance. The examples of such models are: the semiphenomenological One-Pion-Exchange Gaussian (OPE-Gaussian) potential, proposed by the Granada group [1] and the chiral interaction derived up to the fifth-order N^{4}LO of the chiral expansion using the semilocal regularization in momentum space (SMS) by the Bochum-Bonn group [2]. This choice is dictated by the availability of the covariance matrix for the free parameters of these forces. The knowledge of the covariance matrix of the potential parameters helps to investigate correlations among various two- or three-nucleon (2N/3N) observables as well as between observables and specific potential parameters. The information about correlations among such observables is particularly interesting in the context of determining free strength parameters present in the 3N interaction. However, the study of the correlations in the 2N system is also interesting and can impact future procedures used to fix free parameters of the NN force. In this paper, we continue our previous studies [3], [4], [5] observables in the 2N system.

In the past a study of the correlation, at a statistically significant level, was not possible due to the lack of a sufficient number of NN potentials and data points. Recently, this situation has changed. Using the OPE-Gaussian or the chiral SMS forces allows us to prepare many sets of the potential parameters. This can be done in the following way. Having at our disposal the covariance matrix for the potential parameters, as well as the central values of the parameters, we sample 50 sets of the potential parameters from the multivariate normal distribution. This number of sets is sufficient to analyze the theoretical uncertainties and to draw sound conclusions, as was shown in Refs. [6], [7]. In addition, 50 sets of the potential parameters are sufficient to estimate correlation coefficients among scattering observable and qualitatively interpret their values. For both NN forces and each set of the potential parameters we computed the deuteron wave function by solving the Schrödinger equation and the t-matrix elements from the Lippmann-Schwinger equation. For more details, we refer to Ref. [8].

Y. Volkotrub (✉)· J. Golak · R. Skibiński · K. Topolnicki · H. Witała
Jagiellonian University: Uniwersytet Jagiellonski w Krakowie, Kraków, Poland
E-mail: yuvolkotrub@gmail.com

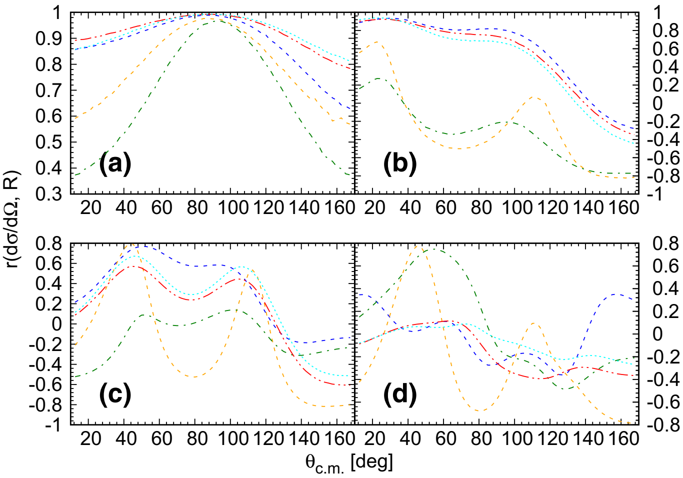

Fig. 1 The angular dependence of correlation coefficients r between $d\sigma/d\Omega$ and R in np scattering at the incoming neutron laboratory energy **a** $E_{\mathrm{lab}} = 10$ MeV, **b** $E_{\mathrm{lab}} = 30$ MeV, **c** $E_{\mathrm{lab}} = 65$ MeV, and **d** $E_{\mathrm{lab}} = 135$ MeV. The dashed orange, dash-dotted green, red double-dot-dashed, cyan dotted lines represent predictions of the chiral N^2LO, N^3LO, N^4LO, and N^4LO^+ SMS forces, respectively, and the blue dashed line shows predictions based on the OPE-Gaussian potential

2 Results

Elastic scattering of particles with spin 1/2 offers much more diverse measurements than only of the unpolarized cross section. Various spin observables, in particular the depolarization R and the polarization P, yield important information about the NN interaction. For each version of the chiral SMS NN potentials at N^2LO, N^3LO, N^4LO and N^4LO^+ with the cutoff parameter $\Lambda = 450$ MeV, and the OPE-Gaussian potential, we computed mentioned observables as well as their correlation coefficients at four incident neutron energies $E_{\mathrm{lab}} = 10$, 30, 65, and 135 MeV in the range of the scattering angle $\theta_{c.m.} \in [12.5°, 167.5°]$.

Figures 1 and 2 shows examples of the angular dependence of correlation coefficient for $(d\sigma/d\Omega, R)$ and (P, R) pairs. As seen from Fig. 1, the np differential cross section $d\sigma/d\Omega$ is, in general, strongly correlated with R over a wide range of scattering angle at the incoming neutron laboratory energies $E_{\mathrm{lab}} = 10$ MeV and 30 MeV. The magnitude of the correlation coefficient between the observables exceeds 0.8. For the higher energies ($E_{\mathrm{lab}} = 65$ MeV and 135 MeV) the correlation decreases, but still at some regions correlation is moderate. This is especially clearly seen for the chiral N^4LO and N^4LO^+ SMS, and the OPE-Gaussian predictions.

In the case of P and R, see Fig. 2, one gets a weak correlation for each of N^4LO, N^4LO^+ SMS and the OPE-Gaussian potentials for most of scattering angles and at all energies. The exceptional, maxima of correlation coefficients r are observed with $r \approx 0.65$ for N^4LO force ($r \approx 0.6$ for N^4LO^+) in $\theta_{c.m.} \in (60°, 80°)$ for $E_{\mathrm{lab}} = 65$ MeV. At $E_{\mathrm{lab}} = 135$ MeV the correlation coefficient can reach 0.8 at very backward scattering angles for the chiral force at N^4LO^+ order.

In Figs. 3 and 4 we demonstrate results on dependences between 2N scattering observables based on the two models (the OPE-Gaussian and the chiral N^4LO^+ SMS potentials) using 50 sets of potential parameters by plotting the scatter plots for pairs of observables at a given scattering angle. The top panel in Fig. 3 visualizes a strong positive correlation between $d\sigma/d\Omega$ and R at $E_{\mathrm{lab}} = 10$ MeV and at three scattering angles $\theta_{c.m.} = 30°, 90°, 150°$ with the corresponding magnitudes of the correlation coefficients $r(\theta_{c.m.} = 30°) = 0.89$, $r(\theta_{c.m.} = 90°) = 0.99$, $r(\theta_{c.m.} = 150°) = 0.86$ for the chiral N^4LO^+ SMS force ($r(\theta_{c.m.} = 30°) = 0.88$, $r(\theta_{c.m.} = 90°) = 0.99$, $r(\theta_{c.m.} = 150°) = 0.72$ for the OPE-Gaussian potential). For $E_{\mathrm{lab}} = 135$ MeV and the three scattering angles, the scatter plots in the bottom row of Fig. 3, indicate a weak correlation between the pair $(d\sigma/d\Omega, R)$ for both potentials.

The relation between P and R constitutes a case of uncorrelated observables (Fig. 4). Here, at $E_{\mathrm{lab}} = 10$ MeV the correlation coefficients are $r(\theta_{c.m.} = 30°) = -0.4$, $r(\theta_{c.m.} = 90°) = -0.19$, $r(\theta_{c.m.} = 150°) = 0.03$ for the chiral N^4LO^+ SMS force ($r(\theta_{c.m.} = 30°) = -0.02$, $r(\theta_{c.m.} = 90°) = 0.11$, $r(\theta_{c.m.} = 150°) = 0.19$ for the OPE-Gaussian potential). But at $E_{\mathrm{lab}} = 135$ MeV we observe a significant increase of the magnitude of r, i.e. $r(\theta_{c.m.} = 30°) = 0.50$, $r(\theta_{c.m.} = 90°) = 0.56$, $r(\theta_{c.m.} = 150°) = -0.86$ for the chiral N^4LO^+ SMS force ($r(\theta_{c.m.} = 30°) = 0.25$, $r(\theta_{c.m.} = 90°) = 0.54$, $r(\theta_{c.m.} = 150°) = -0.83$ for the

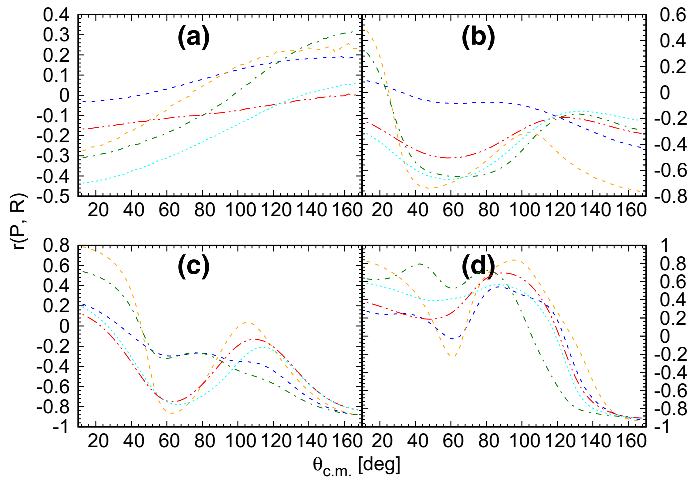

Fig. 2 Same as in Fig. 1, but for the (P, R) pair

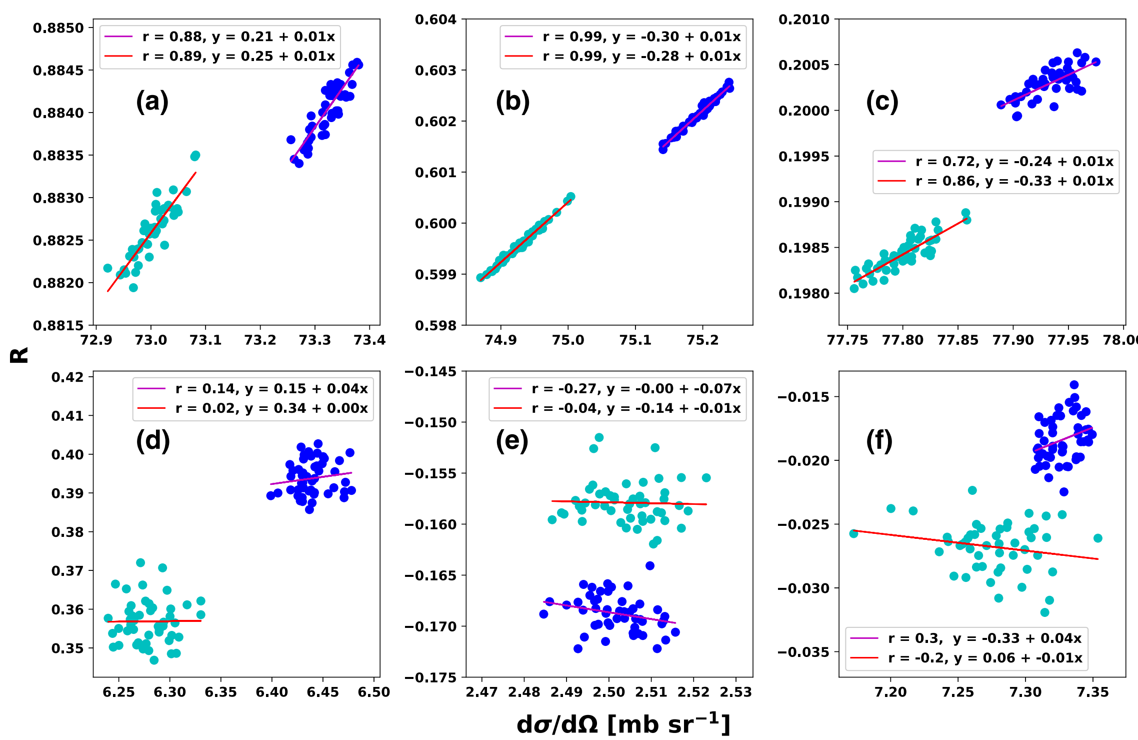

Fig. 3 The scatter plots for the np differential cross section $d\sigma/d\Omega$ and the depolarization R at the $\theta_{c.m.} = 30°$ (left), $\theta_{c.m.} = 90°$ (middle) and $\theta_{c.m.} = 150°$ (right) scattering angle and at the incident neutron laboratory energy $E_{\rm lab} = 10$ MeV (top) and $E_{\rm lab} = 135$ MeV (bottom). The cyan and blue circles represent 50 sets of potential parameters of the chiral N4LO$^+$ ($\Lambda = 450$ MeV) SMS force and the OPE-Gaussian potential, respectively. The red and magenta lines of best fit correspond to the chiral N4LO$^+$ SMS and the OPE-Gaussian models, respectively

OPE-Gaussian potential). However, for the remaining scattering angles, the relation between P and R looks moderate/weak correlation for both potentials, depending on angle.

3 Conclusions

We analyzed correlations among the chosen 2N observables. Summarizing all above given observations for 2N scattering observables we conclude that the angular dependence of the correlation coefficients has a complex

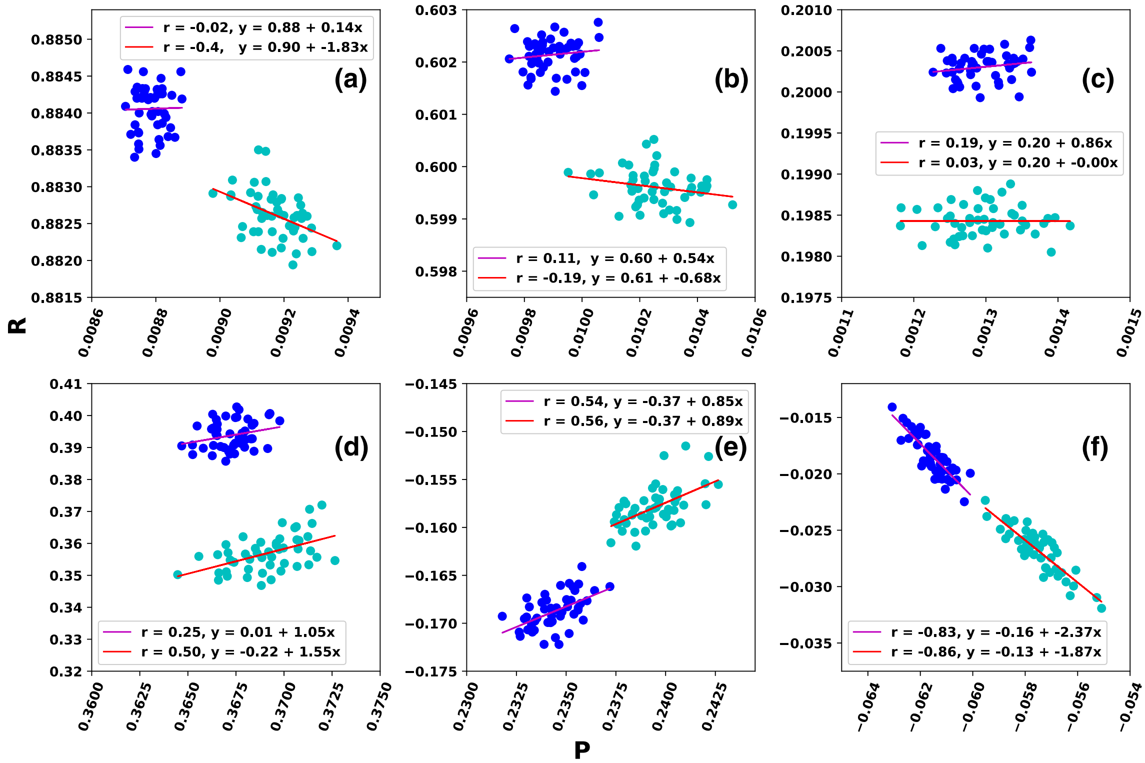

Fig. 4 Same as in Fig. 3, but for the (P, R) pair

behavior depending on the scattering angle and the scattering energy. The differential cross section $d\sigma/d\Omega$ is strongly correlated with the depolarization R at energies up to $E_{\text{lab}} = 30$ MeV, in specific intervals of $\theta_{c.m.}$. The polarisation P is characterized by a weak correlation with R, except selected scattering angles at $E_{\text{lab}} = 135$ MeV where we observe a moderate or even strong relation between them. In the case of the chiral SMS interaction the magnitude of r increases at some scattering angles at increasing orders of chiral expansion. The predictions of the OPE-Gaussian potential are in good agreement with the chiral N4LO$^+$ SMS ones. This is interesting that we have found pairs of 2N observables which are strongly correlated or remain uncorrelated independently of the model of the nuclear force and the reaction energy. We believe the reason for this is sensitivity of various observables to different partial waves (different potential parameters) contributing to the scattering amplitude. We hope that the results of our study will be useful for fitting procedures aiming at determining NN potential parameters, especially in the context of chiral interactions.

Acknowledgements This work is a part of the LENPIC project and was supported by the Polish National Science Center under Grants No. 2016/22/M/ST2/00173. We would like to thank P. Reinert for providing us with the potential subroutine, and Dr. E. Ruiz Arriola and Dr. R. Navarro Pérez for sending us sets of parameters for the OPE-Gaussian potential. The numerical calculations were partially performed on the supercomputer cluster of the JSC, Jülich, Germany.

References

1. R. Navarro Pérez et al., Statistical error analysis for phenomenological nucleon-nucleon potentials. Phys. Rev. C **89**, 064006 (2014)

2. P. Reinert et al., Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order. Eur. Phys. J. A **54**, 1–49 (2018)
3. Y. Volkotrub et al., Correlations among observables in two- and three-nucleon systems. Acta Phys. Pol. B **51**, 273 (2020)
4. Y. Volkotrub et al., Correlations among 3N observables. Sci. Post Phys. Proc. **3**, 055 (2020)
5. Y. Volkotrub et al., Correlations among observables two-nucleon scattering observables. Acta Phys. Pol. B Proc. Suppl. **13**, 971 (2020)
6. R. Skibiński et al., Theoretical uncertainties of the elastic nucleon-deuteron scattering observables. Phys. Rev. C **98**, 014001 (2018)
7. Y. Volkotrub et al., Uncertainty of three-nucleon continuum observables arising from uncertainties of two-nucleon potential parameters. J. Phys. G Nucl. Part. Phys. **47**, 014001 (2020)
8. W. Glöckle, *The Quantum Mechanical Few-Body Problem* (Springer-Verlag, Berlin Heidelberg, 1983), p. 207