Few-Body Syst (2017) 58:110
DOI 10.1007/s00601-017-1264-8

M. Viviani · L. Girlanda · A. Kievsky · L. E. Marcucci

Three-Nucleon Force Effects in p-^{3}H and n-^{3}He Scattering

Received: 23 January 2017 / Accepted: 17 February 2017 / Published online: 11 March 2017

Abstract We present a preliminary study of the effect of a three-nucleon force (3NF) in p-^{3}H and n-^{3}He scattering at low energies. The used 3NF is derived from effective field theory at next-to-next-to-leading order. The four-nucleon scattering observables are calculated using the Kohn variational principle and the hyperspherical harmonics technique and the results are compared with available experimental data. We have found that the effect of introducing this type of 3NF is tiny, and sometimes worsens the agreement with the experimental data.

1 Introduction

Four nucleon systems have been the object of intense studies in recent years. In first place, there is a number of reactions involving four nucleons which are of extreme importance for astrophysics, energy production, and studies of fundamental symmetries. As an example, the reactions $d - d \rightarrow p$-^{3}H and $d - d \rightarrow n$-^{3}He play a key role in the theory of big-bang nucleosynthesis. Moreover, these systems are particularly interesting as a "theoretical laboratory" to test the validity of our present knowledge of the nucleon-nucleon (NN) and three nucleon forces (3NF).

The interest in ab initio calculations has been renewed after the advent of the theoretical framework of chiral effective field theory (χEFT), nowadays widely used to derive nuclear potentials and electroweak currents from the symmetries of QCD—the exact Lorentz, parity, and time-reversal symmetries, and the approximate chiral

This article belongs to the Topical Collection "The 23rd European Conference on Few-Body Problems in Physics.

M. Viviani (✉) · A. Kievsky · L. E. Marcucci
INFN- Pisa, Largo B. Pontecorvo 3, I-56127 Pisa, Italy
E-mail: michele.viviani@pi.infn.it

A. Kievsky
E-mail: alejandro.kievsky@pi.infn.it

L. Girlanda
Department of Mathematics and Physics, University of Salento, Lecce, Italy
E-mail: luca.girlanda@le.infn.it

L. Girlanda
INFN-Lecce, Via Arnesano, I-73100 Lecce, Italy

L. E. Marcucci
Department of Physics, University of Pisa, Pisa, Italy
E-mail: laura.elisa.marcucci@unipi.it

symmetry (see, for example, Refs. [1–4]). The test of these new potentials in few-nucleon scattering will give very stringent and critical information.

In a previous work, we have presented a study of the inclusion of the 3NF in low energy $n-{}^3$H and $p-{}^3$He elastic observables [5]. In the present paper, we extend the study to p-^{3}H and n-^{3}He scattering for energies where both channels are open but below the $d-{}-d$ threshold. These calculations present new challenges and are rather complex since the two reaction channels are coupled and involve both total isospin $T=0$ and $T=1$ states.

The NN potential used in this paper is the N3LO500 model by Entem and Machleidt [4], an interaction based on the χEFT approach and derived up to next-to-next-to-next-to-leading order of chiral perturbation theory (χPT). To this NN force, we have added the 3NF derived within the same approach at next-to-next-to-leading order (N2LO) [6,7], with parameters c_D and c_E fixed in order to reproduce the three-nucleon binding energy and the Gamow–Teller matrix element in the tritium β-decay [8–10]. The cutoff parameter for these interactions is fixed in this contribution to be $\Lambda=500$ MeV. The interaction obtained including this 3NF will be labeled hereafter as N3LO500/N2LO500. The study of the dependence of the results on the value of the cutoff parameter Λ and on the choice of c_D and c_E is still in progress and it will be presented elsewhere [11]. We are also planning the study of the contributions of the 3NF terms coming from successive orders of χPT, recently derived in Refs. [12–14].

The paper is organized as follows. In Section 2 we report the results of the theoretical calculations, and compare them with the available experimental data. Finally, in Section 3 we present our conclusions.

2 Results

The calculations presented in this contribution have been performed using the Kohn variational principle and expanding the wave functions in terms of the hyperspherical harmonic functions (for more details see, for example, Ref. [15]). Recently, a benchmark paper [16] has been completed where the results obtained with this method have been compared with the analogous results obtained in the framework of the Alt-Grassberger-Sandhas (AGS) equations [17–19], solved in momentum space, and of the Faddeev–Yakubovsky (FY) equations, solved in configuration space [20,21]. An overall good agreement among the results of the three methods has been found.

Some of the preliminary results obtained for p-^{3}H elastic scattering and the p-^{3}H $\rightarrow$ n-^{3}He charge exchange reaction are reported in Figs. 1, 2 and 3, together with the available experimental data. We see that the contribution of the 3NF is small, and the only clearly visible effect is for p-^{3}H differential cross section at very low energies, see panels (a) and (b) of Fig. 1. For these energies, the n-^{3}He channel is closed, and the

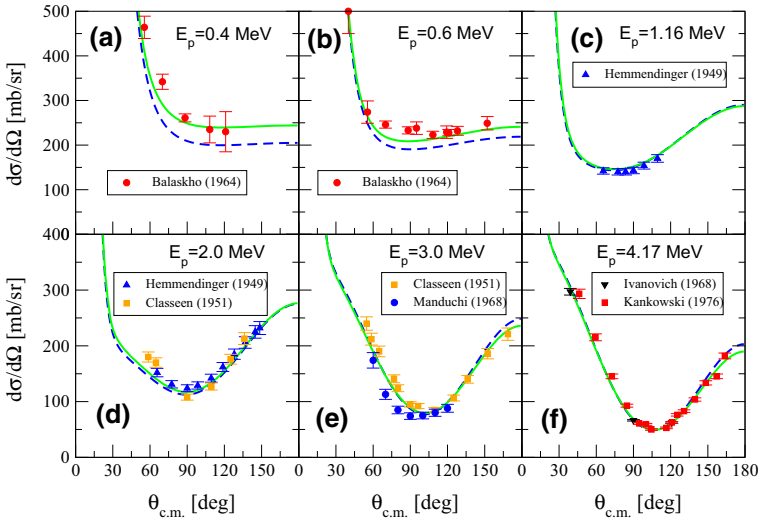

Fig. 1 p-^{3}H differential cross section, at several values of the proton laboratory beam energy E_p calculated with the N3LO500 (*blue dashed curve*) and with the N3LO500/N2LO500 (*solid green curve*) interactions. The experimental data are from Refs. [22–27]

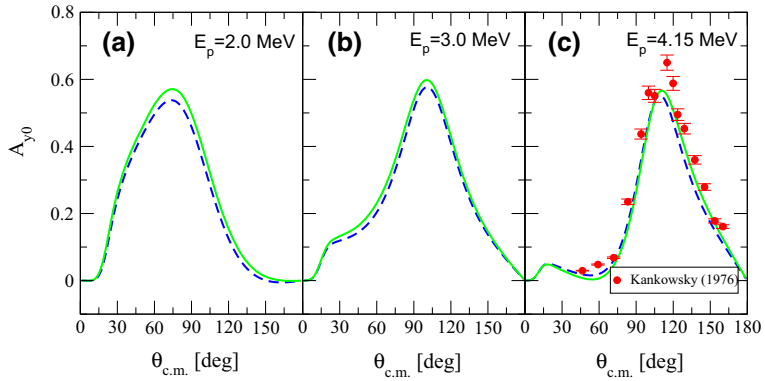

Fig. 2 p-^{3}H proton analyzing power at three values of the proton laboratory beam energy E_p calculated with the N3LO500 (*blue dashed curve*) and with the N3LO500/N2LO500 (*solid green curve*) interactions. The experimental data are from Ref. [27]

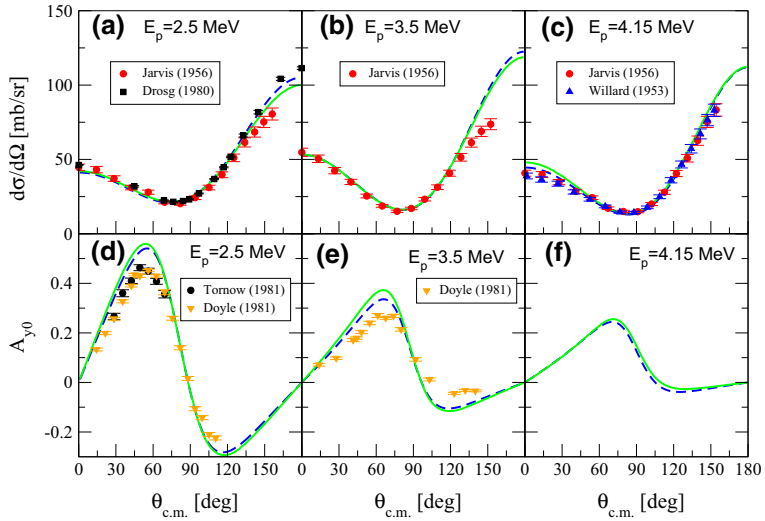

Fig. 3 p-^{3}H $\rightarrow$ n-^{3}He differential cross section and proton analyzing power at three values of the proton laboratory beam energy E_p calculated with the N3LO500 (*blue dashed curve*) and with the N3LO500/N2LO500 (*solid green curve*) interactions. The experimental data are from Ref. [28–32]

theoretical results are extremely dependent on the correct position of the trinucleon binding energies. The p-^{3}H analyzing power is found to be slightly increased when the 3NF is included, but this does not significantly help in reducing the disagreement observed with the experimental data at $E_p = 4.15$ MeV, as can be seen in panel (c) of Fig. 2.

Regarding the charge exchange reaction shown in Fig. 3, we note that the inclusion of the 3NF gives negligible effects for the differential cross section, while slightly increases A_{y0} in the forward peak. In this case, the disagreement with the experimental data, found when only the NN force is considered, is worsened by the inclusion of the adopted 3NF.

3 Conclusions

In this work, we have studied some low energy n-^{3}He and p-^{3}H elastic and charge-exchange observables by using 1) an interaction including an NN force only and 2) an interaction including also 3NF. Inclusion of 3NF for these reactions was considered earlier only in Ref. [21], in the framework of the FY equations, and in Ref. [33], in the framework of the AGS equations.

We have observed that the effects of the inclusion of the 3NF are, for most of the cases, tiny. Only for energies below the opening of the $n - {}^3$He channel some sizable effects are observed. In that case, the inclusion of 3NF

is essential for a better reproduction of the experimental data. Above the opening of the $n - {}^3$He channel, the inclusion of 3NF in the p-^{3}H elastic scattering differential cross section is almost negligible, while it improves slightly the agreement with the experimental analyzing power.

On the other hand, for the charge exchange reaction p-^{3}H $\rightarrow$ n-^{3}He, the inclusion of the 3NF increases slightly the differences between the experimental and theoretical analyzing power. This observable, however, is known to depend sizeably on the adopted NN force [18]. From the present study, it seems that this observable does not depend significantly on the 3NF. Therefore, it could be an important testground for investigating our knowledge of the NN force. Consequently, it would be rather important to have new and more accurate measurements of this quantity, as the available experimental data are rather old and of limited angular range. More studies of these reactions with other interactions are in progress [11].

References

1. S. Weinberg, Nuclear forces from chiral lagrangians. Phys. Lett. B **251**, 288–292 (1990)
2. C. Ordonez, L. Ray, U. van Kolck, Two-nucleon potential from chiral Lagrangians. Phys. Rev. C **53**, 2086–2105 (1996)
3. E. Epelbaum, H.W. Hammer, U.G. Meissner, Modern theory of nuclear forces. Rev. Mod. Phys. **81**, 1773–1825 (2009)
4. R. Machleidt, D.R. Entem, Chiral effective field theory and nuclear forces. Phys. Rep. **503**, 1–75 (2011)
5. M. Viviani, L. Girlanda, A. Kievsky, L.E. Marcucci, Effect of three-nucleon interactions in $p - {}^3$He elastic scattering. Phys. Rev. Lett. **111**, 172302 (2013)
6. U. van Kolck, Few-nucleon forces from chiral Lagrangians. Phys. Rev. C **49**, 2932–2941 (1994)
7. E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, Meißner Ulf-G, H. Witała, Three-nucleon forces from chiral effective field theory. Phys. Rev. C **66**, 064001 (2002)
8. A. Gårdestig, D.R. Phillips, How low-energy weak reactions can constrain three-nucleon forces and the neutron–neutron scattering length. Phys. Rev. Lett. **96**, 232301 (2006)
9. D. Gazit, S. Quaglioni, P. Navrátil P, Three-nucleon low-energy constants from the consistency of interactions and currents in chiral effective field theory. Phys. Rev. Lett. **103**, 102502 (2009)
10. L.E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla, M. Viviani, Chiral effective field theory predictions for muon capture on deuteron and ^{3}He. Phys. Rev. Lett. **108**, 052502 (2012)
11. M. Viviani, L. Girlanda, A. Kievsky, L.E. Marcucci, in preparation (2017)
12. L. Girlanda, A. Kievsky, M. Viviani, Subleading contributions to the three-nucleon contact interaction. Phys. Rev. C **84**, 014001 (2011)
13. H. Krebs, A. Gasparyan, E. Epelbaum, Chiral three-nucleon force at N^{4}LO: longest-range contributions. Phys. Rev. C **85**, 054006 (2012)
14. H. Krebs, A. Gasparyan, E. Epelbaum, Chiral three-nucleon force at N^{4}LO. II. Intermediate-range contributions. Phys. Rev. C **87**, 054007 (2013)
15. A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, L. Girlanda, A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states. J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)
16. M. Viviani et al. Benchmark calculation of $p - {}^3$H and $n - {}^3$He scattering, arXiv:1610.09140 (2016)
17. A. Deltuva, A.C. Fonseca, Four-nucleon scattering: ab initio calculations in momentum space. Phys. Rev. C **75**, 014005 (2007)
18. A. Deltuva, A.C. Fonseca, Ab initio four-body calculation of $n -{}^3$ He, $p -{}^3$ H, and $d - d$ scattering. Phys. Rev. C **76**, 021001 (2007)
19. A. Deltuva, A.C. Fonseca, Calculation of neutron-^{3}He scattering up to 30 MeV. Phys. Rev. C **90**, 044002 (2014)
20. R. Lazauskas, J. Carbonell, Testing nonlocal nucleon–nucleon interactions in four-nucleon systems. Phys. Rev. C **70**, 044002 (2004)
21. R. Lazauskas, Elastic proton scattering on tritium below the $n - {}^3$He threshold. Phys. Rev. C **79**, 054007 (2009)
22. A. Hemmendinger, G.A. Jarvis, R.F. Taschek, Scattering of protons by tritons. Phys. Rev. **76**, 1137–1141 (1949)
23. R.S. Claassen, R.J.S. Brown, G.D. Freier, W.R. Stratton, The scattering of protons by tritons. Phys. Rev. **82**, 589–596 (1951)
24. Y.G. Balashko, I.Y. Barit, L.S. Dulkova, A.B. Kurepin, Elastic scattering protons on the tritium by energy below (p, n) reactions. Bull. Russ. Acad. Sci. Phys. **28**, 1028 (1965)
25. C. Manduchi, G. Moschini, G. Tornielli, G. Zannoni, Experimental study on low-energy levels of ^{4}He. Nuovo Cim. B **57**, 340 (1968)
26. M. Ivanovich, P.G. Young, G.G. Ohlsen, Elastic scattering of several hydrogen and helium isotops from tritium. Nucl. Phys. A **110**, 441 (1968)
27. R. Kankowsky, J.C. Fritz, K. Kilian, A. Neufert, D. Fick, Elastic scattering of polarized protons on tritons between 4 and 12 MeV. Nucl. Phys. A **263**, 29 (1976)
28. H.B. Willard, J.K. Bair, J.D. Kington, The reactions T(p, n)He^3 and T(p, γ)He^4. Phys. Rev. **90**, 865 (1953)
29. G.A. Jarvis, Charged particle cross sections, the reaction $T(p, n)^3$He, Los Alamos Scientific Report No. 2014, p. 35 (1956)
30. M. Drosg, The ^{3}H$(p, n)^3$He differential cross sections below 5 MeV and the $n - {}^3$He cross section (Los Alamos Scientific, Report No, 1980). 8215
31. M.A. Doyle Sr., H.W. Clark, J.L. Dries, J.L. Regner, T.R. Donoghue, G.M. Hale, Comparison of analyzing power and polarization in the reaction ^{3}H$(p, n)^3$He. (I). New analyzing power results below 4 MeV. Nucl. Phys. **A371**, 225 (1981)

32. W. Tornow, R.C. Byrd, P.W. Lisowski, R.L. Walter, T.R. Donoghue, Comparison of analyzing power and polarization in the reaction ^{3}H$(p, n)^3$He. (II). New analyzing power results below 4 MeV. Nucl. Phys. **A371**, 235 (1981)
33. J. Esterline, W. Tornow, A. Deltuva, A.C. Fonseca, Analyzing power $A_y(\theta)$ of $\vec{n}$ $-^3$He elastic scattering between 1.60 and 5.54 MeV. Phys. Rev. Lett. **110**, 152503 (2013)