Few-Body Syst (2017) 58:47
DOI 10.1007/s00601-017-1217-2

A. Deltuva

Nucleon Transfer Reactions in Few-Body Nuclear Systems

Received: 4 November 2016 / Accepted: 4 January 2017 / Published online: 19 January 2017
© Springer-Verlag Wien 2017

Abstract Three- and four-body scattering is described solving Faddeev-Yakubovsky or equivalent Alt-Grassberger-Sandhas integral equations for transition operators in momentum-space. Several realistic nuclear interaction models are used; the Coulomb force between charged particles is taken into account via the screening and renormalization method. Differential cross sections and spin observables for various nucleon transfer reactions are calculated and compared with experimental data.

1 Introduction

Nucleon transfer reactions constitute a powerful tool for extracting the nuclear structure information as well as testing nuclear interaction models through comparison of theoretical predictions with experimental data. In the simplest nuclear three-body reaction, nucleon-deuteron scattering, the nucleon transfer and elastic scattering cannot be separated due to the identity of nucleons. Thus, a system of four or more nucleons is needed for the study of transfer reactions. Nevertheless, very often effective three-body models are used for reactions in systems consisting of a proton (p), a neutron (n), and a heavier nuclear core (A). There is a number of approximate methods for the description of three-body reactions, but the rigorous Faddeev-type scattering theory [1–3] has been practically applied to this problem only in the last decade [4,5]. It is more complicated than traditional approximate methods, but has an advantage that, once numerically well-converged results are obtained, the discrepancies with the experimental data can be attributed to the shortcomings of the used potentials or to the inadequacy of the three-body model. Recent progress in the application of rigorous Faddeev-type three-body theory to nucleon transfer reactions is reviewed in Sect. 2, while Sect. 3 discusses the extension to a four-body system, where practical results so far are limited to four-nucleon reactions.

2 Three-Body System

Both Faddeev [1,2] and Alt-Grassberger-Sandhas (AGS) equations [3] are equivalent to the Schrödinger equation but are more suitable for the numerical solution due to the connectedness of the kernel. I use Faddeev equations in the integral AGS form for transition operators

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha}\, G_0^{-1} + \sum_{\gamma=1}^{3} \bar{\delta}_{\beta\gamma}\, T_\gamma\, G_0 U_{\gamma\alpha}. \tag{1}$$

This article belongs to the Topical Collection “The 23rd European Conference on Few-Body Problems in Physics”.

A. Deltuva (✉)
Institute of Theoretical Physics and Astronomy, Vilnius University, Saulėtekio Al. 3, 10222 Vilnius, Lithuania
E-mail: arnoldas.deltuva@tfai.vu.lt

In Eq. (1) $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ and $G_0 = (E + i0 - H_0)^{-1}$ is the free resolvent, E being the available three-particle energy in the center of mass (c.m.) system and H_0 the free Hamiltonian. The two-particle transition matrix obeys the Lippmann-Schwinger equation

$$T_\gamma = v_\gamma + v_\gamma G_0 T_\gamma \tag{2}$$

where v_γ is the potential for the pair γ in the odd-man-out notation. On-shell matrix elements of $U_{\beta\alpha}$, i.e., $\langle\psi_\beta|U_{\beta\alpha}|\psi_\alpha\rangle$, are transition amplitudes leading directly to the scattering observables. Elastic scattering is determined by the matrix elements with $\beta = \alpha$, breakup observables are given by $\beta = 0$ while $0 \neq \beta \neq \alpha$ correspond to transfer reactions. The channel states $|\psi_\gamma\rangle$ for $\gamma = 1, 2, 3$ are the eigenstates of the corresponding channel Hamiltonian $H_\gamma = H_0 + v_\gamma$ with the energy eigenvalue E; thus, $|\psi_\gamma\rangle$ is a product of the bound state wave function for pair γ and a plane wave with fixed on-shell momentum corresponding to the relative motion of particle γ and pair γ in the initial or final state. The channel states $|\psi_0\rangle$ are the eigenstates of H_0 with the same eigenvalue E and describe the free motion of three particles. The equations are solved in the momentum-space partial-wave representation.

Formally, AGS equations are applicable only to systems with short-range interactions v_γ. The inclusion of the Coulomb force is however possible using the method of screening and renormalization [6,7] extended to few-body systems [8,9] with at most two charged clusters. Following Taylor's idea [6,7], in addition to a nuclear potential a screened Coulomb one with the screening radius R is included in the pair potential v_γ, standard form of AGS equations is solved, the resulting amplitudes are decomposed into short- and long-range parts, and renormalized in the $R \to \infty$ limit, leading to proper physical amplitudes. However, the screened Coulomb interaction, owing to its longer range, brings additional difficulties, i.e., quasisingular nature of the potential and slow convergence of the partial-wave expansion, that become more and more serious with increasing R. Thus, it is desirable to work with R as small as possible when solving AGS equations numerically. For the most complicated short-range part, the infinite R limit is reached with good accuracy at finite R, but one always needs to check the stability of results with R.

The rate of convergence with R depends on the reaction energy, involved charges, and, as found recently for (d, n) transfer reactions [10], also on the binding energy of the final (Ap) subsystem: it is slower for weakly bound nuclei such as ^{8}B or ^{17}F. The comparison of convergence rates for $^{16}\mathrm{O}(d, n)^{17}\mathrm{F}$ and $^{16}\mathrm{O}(d, p)^{17}\mathrm{O}$ is presented in Fig. 1; nucleon separation energies for $^{17}\mathrm{F}(5/2^+)$, $^{17}\mathrm{F}(1/2^+)$, and $^{17}\mathrm{O}(1/2^+)$ nuclei are 0.600, 0.105, and 3.272 MeV, respectively.

Differential cross section results for proton transfer reactions $^{12}\mathrm{C}(d, n)^{13}\mathrm{N}$ and $^{16}\mathrm{O}(d, n)^{17}\mathrm{F}$ are shown in Figs. 2 and 3. The predictions are obtained using realistic CD Bonn potential [11] for the np pair but several parametrizations of the nucleon-core optical potential, namely, the local Koning-Delaroche (KD) [12] potential and the nonlocal (NL) potential introduced by Giannini and Ricco [13] but with the parameters readjusted in Ref. [14] to the experimental data for the nucleon scattering from ^{12}C and ^{16}O. For $^{12}\mathrm{C}(d, n)^{13}\mathrm{N}$ the angular shape of the experimental data is not well reproduced: the data points from Refs. [15–18] are overestimated at forward angles $\Theta_{\mathrm{c.m.}} < 30^\circ$ and underestimated at large angles $\Theta_{\mathrm{c.m.}} > 90^\circ$. The agreement is better for $^{16}\mathrm{O}(d, n)^{17}\mathrm{F}$. For the transfer to the ^{17}F ground state $5/2^+$ the theoretical results follow the data [19] up to $\Theta_{\mathrm{c.m.}} = 30^\circ$ at $E_d = 7.7$ MeV, up to 60° at $E_d = 11$ MeV, and up to 90° at $E_d = 12$ MeV but underpredict at larger angles. This discrepancy is most sizable at $E_d = 7.7$ MeV, possibly being due to the compound-nucleus reaction mechanism. For the transfer to the ^{17}F excited state $1/2^+$ the agreement is even better, especially when using the nonlocal potential. This possibly indicates that a simple proton plus core model for the ^{17}F excited state $1/2^+$ is adequate, and supports the conjecture of Ref. [21] on this state being one-proton halo. In contrast, the disagreement for $^{12}\mathrm{C}(d, n)^{13}\mathrm{N}$ points out the inadequacy of the proton plus ^{12}C core model for ^{13}N nucleus.

The AGS equations can be formulated in an extended Hilbert space with several sectors corresponding to different states of the core nucleus, thereby including the core excitations [22,23]. As example the differential cross sections for neutron transfer reactions $^{11}\mathrm{Be}(p, d)^{10}\mathrm{Be}$ at 35.3 MeV/nucleon energy are shown in Fig. 4. They are quite sensitive to the choice of the underlying optical potential. Inclusion of the core excitation reduces the differential cross section for the transfer to ground state $^{10}\mathrm{Be}(0^+)$, and yields quite reasonable results for the transfer to excited state $^{10}\mathrm{Be}(2^+)$.

3 Four-Nucleon Reactions

The theoretical description of four-nucleon reactions is technically and computationally highly challenging. Exact four-particle scattering equations have been formulated 50 years ago by Faddeev and Yakubovsky (FY)

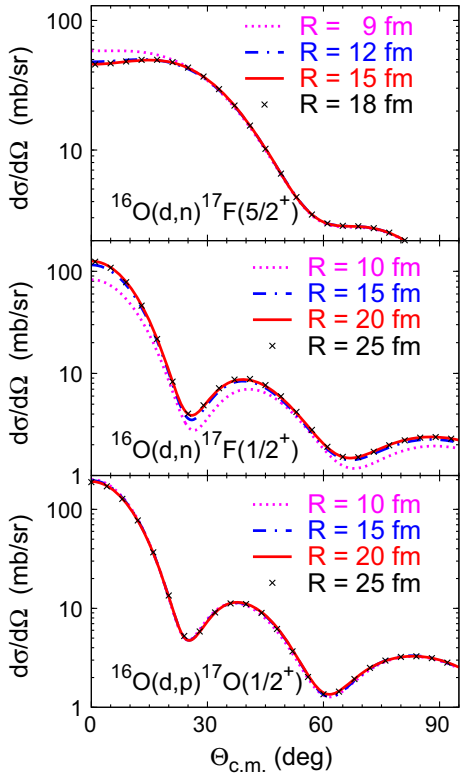

Fig. 1 (Color online) Differential cross section for ^{16}O$(d, n)^{17}$F and ^{16}O$(d, p)^{17}$O transfer reactions at $E_d = 12$ MeV. Results obtained with different values of the Coulomb screening radius R are compared. See Ref. [10] for more details

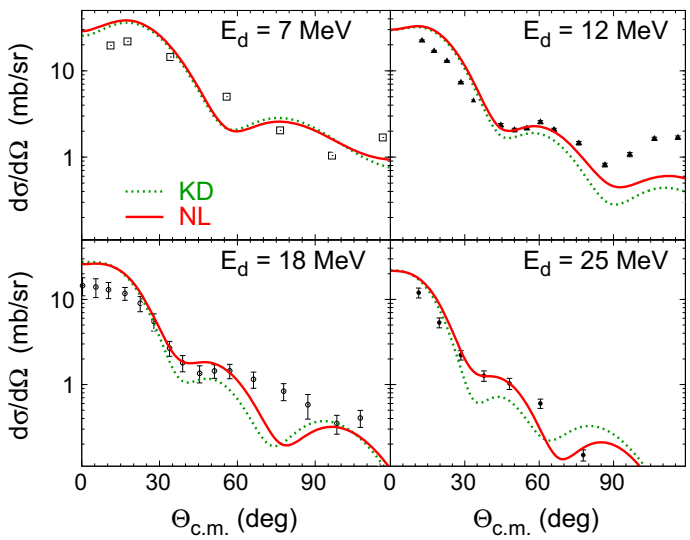

Fig. 2 (Color online) Differential cross section for the ^{12}C$(d, n)^{13}$N reaction at 7, 12, 18, and 25 MeV deuteron energy. Predictions obtained using the Koning-Delaroche (*dotted curves*) and nonlocal (*solid curves*) optical potentials are compared with the experimental data from Refs. [15] (7 MeV), [16] (12 MeV), [17] (18 MeV), and [18] (25 MeV)

[25,26] and by Alt, Grassberger, and Sandhas [27,28]. However, they were solved numerically with realistic force models many years later. In the last decade accurate numerical calculations for nucleon-trinucleon scattering below breakup threshold have been performed using several rigorous coordinate- or momentum-space methods, namely, the hyperspherical harmonics (HH) expansion method [29,30], the FY equations for the wave function components [31,32], and the AGS equations for transition operators [33,34].

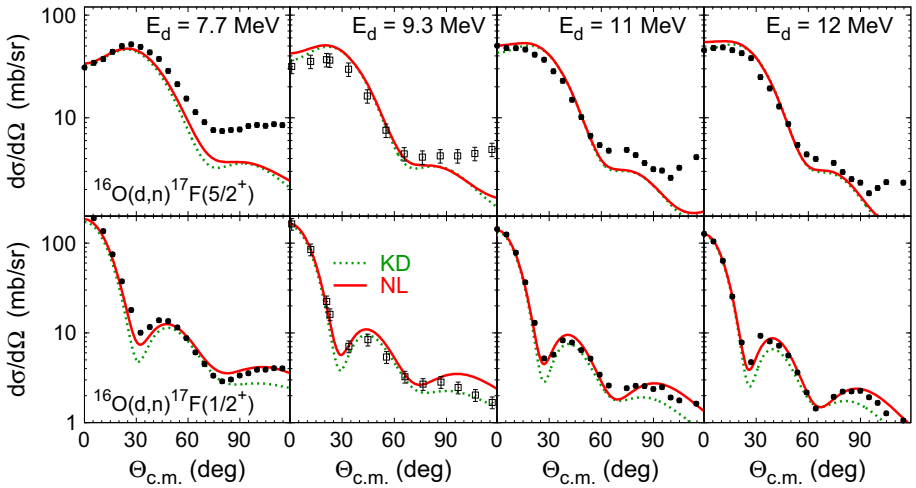

Fig. 3 (Color online) Differential cross section for $^{16}\mathrm{O}(d,n)^{17}\mathrm{F}$ reactions leading to the $^{17}\mathrm{F}$ ground state $5/2^+$ (*top*) and excited state $1/2^+$ (*bottom*) at the deuteron energy $E_d = 7.7$, 9.3, 11, and 12 MeV. Curves are as in Fig. 2, and the experimental data are from Refs. [20] (9.3 MeV) and [19] (other energies)

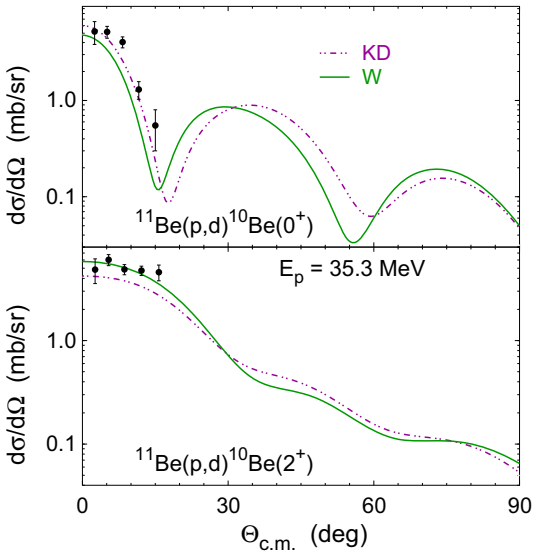

Fig. 4 (Color online) Differential cross section for $^{11}\mathrm{Be}(p,d)^{10}\mathrm{Be}$ transfer reactions at $E_p = 35.3$ MeV leading to the ground (0^+) and excited (2^+) states of $^{10}\mathrm{Be}$. Predictions including the $^{10}\mathrm{Be}$ core excitation within rotational model based on Koning-Delaroche (*double-dotted-dashed curves*) and Watson (*solid curves*) potentials are compared with the experimental data from Ref. [24]

First calculations above the breakup threshold emerged few years ago and mostly were limited to nucleon-trinucleon elastic scattering and charge-exchange reactions [35–37]. Currently the work on (d,p) and (d,n) transfer reactions in deuteron-deuteron collisions up to 25 MeV lab energy is in progress. Differential cross sections, deuteron analyzing powers, outgoing nucleon polarization, and deuteron-to-neutron polarization transfer coefficients are calculated. Due to the large spatial size of the deuteron, the calculation of transfer reactions is more demanding in terms of partial waves as compared to nucleon-trinucleon scattering. This manifests itself in a complicated angular dependence of observables, exhibiting a number of local minima and maxima. Nevertheless, well converged results are obtained and good overall agreement with the experimental data is found. As an example deuteron analyzing powers and outgoing nucleon polarization are shown in Fig. 5.

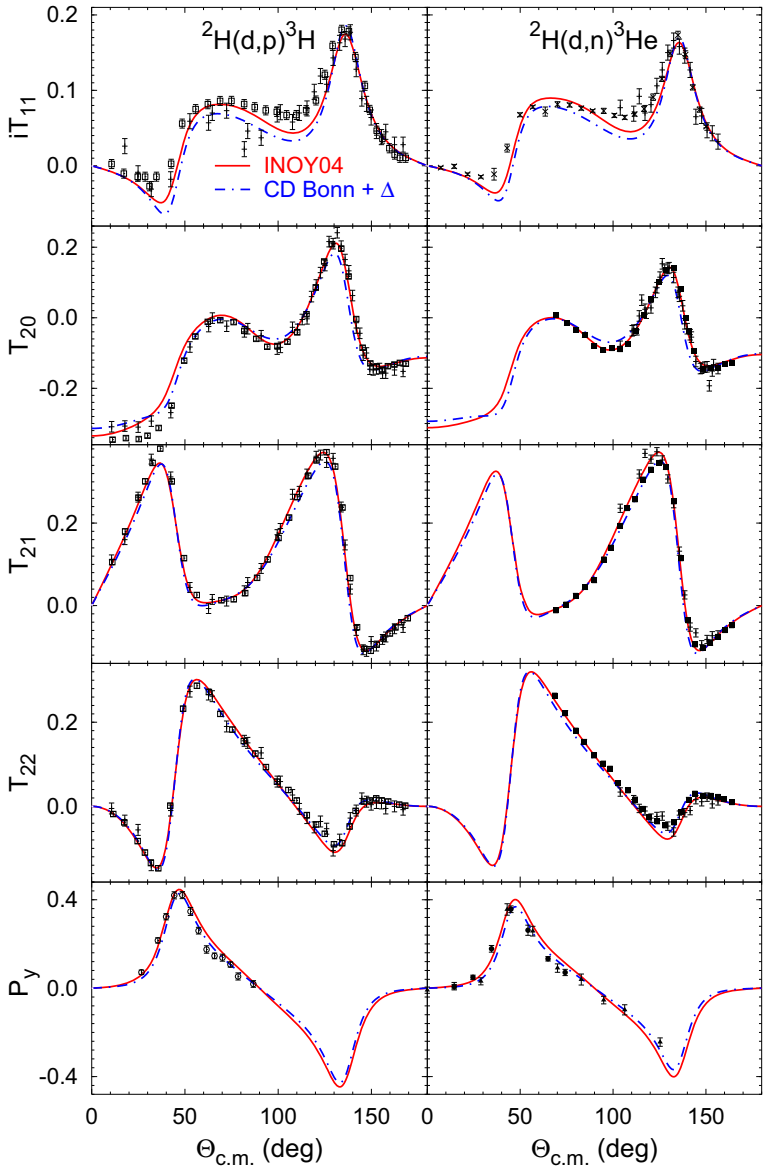

Fig. 5 Deuteron analyzing powers and outgoing nucleon polarization of ^{2}H(**d**, p)^{3}H (*left*) and ^{2}H(**d**, n)^{3}He (*right*) transfer reactions at 10 MeV deuteron energy. Results obtained with potentials INOY04 (solid curves) and CD Bonn + Δ (*dashed-dotted curves*) are compared with the data from Refs. [38] (+), [39] (□), [40] (×), [41] (■), [42] (○), [43] (●), and [44] (▲)

4 Summary

Accurate description of three- and four-body nuclear reactions (in the latter case presently limited to four-nucleon system) is achieved using integral AGS equations for transition operators and solving them in momentum-space partial-wave framework. Discrepancies with the experimental data observed in some cases point out inadequacy of employed interaction or structure models. The agreement between theoretical predictions and data for four-nucleon reactions ^{2}H(**d**, p)^{3}H and ^{2}H(**d**, n)^{3}He is remarkably good.

Acknowledgements This work was supported by Lietuvos Mokslo Taryba (Research Council of Lithuania) under Contract No. MIP-094/2015.

References

1. L.D. Faddeev, Rigorous three-particle scattering problem. Zh. Eksp. Teor. Fiz. **39**, 1459 (1960)
2. L.D. Faddeev, Rigorous three-particle scattering problem. Sov. Phys. JETP **12**, 1014 (1961)
3. E.O. Alt, P. Grassberger, W. Sandhas, Reduction of the three-particle collision problem to multi-channel two-particle Lippmann-Schwinger equations. Nucl. Phys. B **2**, 167 (1967)
4. A. Deltuva, A.M. Moro, E. Cravo, F.M. Nunes, A.C. Fonseca, Three-body description of direct nuclear reactions: comparison with the continuum discretized coupled channels method. Phys. Rev. C **76**, 064602 (2007)
5. A. Deltuva, A.C. Fonseca, Three-body Faddeev-Alt-Grassberger-Sandhas approach to direct nuclear reactions. Phys. Rev. C **79**, 014606 (2009)
6. J.R. Taylor, A new rigorous approach to Coulomb scattering. Nuovo Cimento B **23**, 313 (1974)
7. M.D. Semon, J.R. Taylor, Scattering by potentials with Coulomb Tails. Nuovo Cimento A **26**, 48 (1975)
8. E.O. Alt, W. Sandhas, Scattering amplitudes and integral equations for the collision of two charged composite particles. Phys. Rev. C **21**, 1733 (1980)
9. A. Deltuva, A.C. Fonseca, P.U. Sauer, Momentum-space treatment of Coulomb interaction in three-nucleon reactions with two protons. Phys. Rev. C **71**, 054005 (2005)
10. A. Deltuva, Faddeev-type calculation of (d, n) transfer reactions in three-body nuclear systems. Phys. Rev. C **92**, 064613 (2015)
11. R. Machleidt, High-precision, charge-dependent Bonn nucleon-nucleon potential. Phys. Rev. C **63**, 024001 (2001)
12. A.J. Koning, J.P. Delaroche, Local and global nucleon optical models from 1 keV to 200 MeV. Nucl. Phys. A **713**, 231 (2003)
13. M.M. Giannini, G. Ricco, An energy-independent nonlocal potential model for bound and scattering states. Ann. Phys. (NY) **102**, 458 (1976)
14. A. Deltuva, Three-body direct nuclear reactions: nonlocal optical potential. Phys. Rev. C **79**, 021602(R) (2009)
15. H.R. Schelin, E. Farrelly Pessoa, W.R. Wylie, E.W. Cybulska, K. Nakayama, L.M. Fagundes, R.A. Douglas, The ^{12}C(d, n)^{13}N Reaction between E(d) = 7.0 and 13.0 MeV. Nucl. Phys. A **414**, 67 (1984)
16. G.S. Mutchler, D. Rendic, D.E. Velkley, W.E. Sweeney Jr., G.C. Phillips, The (d, n) reaction on 1p shell nuclei at E = 11.8 MeV. Nucl. Phys. A **172**, 469 (1971)
17. M.A. Kayumov, S.S. Kayumov, S.P. Krekoten, A.M. Mukhamedzhanov, K.D. Razikov, K. Khamidova, R. Yarmukhamedov, (d, n) and (p, n) reactions on light nuclei and their dispersion peripheral model analysis. Sov. J. Nucl. Phys. **48**, 403 (1988)
18. D.W. Lee, J. Powell, K. Perajarvi, F.Q. Guo, D.M. Moltz, J. Cerny, Study of the 11C(p, g) reaction via the indirect d(11C,12N)n transfer reaction. J. Phys. G **38**, 075201 (2011)
19. C.J. Oliver, P.D. Forsyth, J.L. Hutton, G. Kaye, J.R. Mines, Spectroscopic factors derived from 16O(d, n)17F measurements. Nucl. Phys. A **127**, 567 (1969)
20. S. Thornton, The 16O(d, n)17F reaction at Ed = 8.0 and 9.3 MeV. Nucl. Phys. A **137**, 531 (1969)
21. R. Lewis, A.C. Hayes, Deuteron stripping as a probe of the proton halo in ^{17}F. Phys. Rev. C **59**, 1211 (1999)
22. A. Deltuva, Faddeev-type calculation of three-body nuclear reactions including core excitation. Phys. Rev. C **88**, 011601(R) (2013)
23. A. Deltuva, Core excitation in three-body nuclear reactions: improved nucleon-core potential. Phys. Rev. C **91**, 024607 (2015)
24. J. Winfield, S. Fortier, W. Catford, S. Pita, N. Orr, J.V. de Wiele, Y. Blumenfeld, R. Chapman, S. Chappell, N. Clarke, N. Curtis, M. Freer, S. Gals, H. Langevin-Joliot, H. Laurent, I. Lhenry, J. Maison, P. Roussel-Chomaz, M. Shawcross, K. Spohr, T. Suomijrvi, A. de Vismes, Single-neutron transfer from 11Be(g.s.) via the (p, d) reaction with a radioactive beam. Nucl. Phys. **A683**, 48 (2001)
25. O.A. Yakubovsky, On the integral equations in the theory of N particle scattering. Yad. Fiz **5**, 1312 (1967)
26. O.A. Yakubovsky, On the integral equations in the theory of N particle scattering. Sov. J. Nucl. Phys. **5**, 937 (1967)
27. E.O. Alt, P. Grassberger, W. Sandhas, Systematical and practical treatment of the few-body problem. JINR Rep. **E4–6688**, 1 (1972)
28. P. Grassberger, W. Sandhas, Systematical treatment of the non-relativistic n-particle scattering problem. Nucl. Phys. B **2**, 181 (1967)
29. M. Viviani, A. Kievsky, S. Rosati, E.A. George, L.D. Knutson, The A_y problem for $p-^3He$ elastic scattering. Phys. Rev. Lett. **86**, 3739 (2001)
30. A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, L. Girlanda, A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states. J. Phys. G **35**, 063101 (2008)
31. R. Lazauskas, J. Carbonell, Testing nonlocal nucleon-nucleon interactions in four-nucleon systems. Phys. Rev. C **70**, 044002 (2004)
32. R. Lazauskas, Elastic proton scattering on tritium below the n-^{3}He threshold. Phys. Rev. C **79**, 054007 (2009)
33. A. Deltuva, A.C. Fonseca, Four-nucleon scattering: Ab initio calculations in momentum space. Phys. Rev. C **75**, 014005 (2007)
34. A. Deltuva, A.C. Fonseca, Ab initio four-body calculation of n-^{3}He, p-^{3}H, and d-d scattering. Phys. Rev. C **76**, 021001(R) (2007)
35. A. Deltuva, A.C. Fonseca, Neutron-^{3}H scattering above the four-nucleon breakup threshold. Phys. Rev. C **86**, 011001(R) (2012)
36. A. Deltuva, A.C. Fonseca, Calculation of multichannel reactions in the four-nucleon system above breakup threshold. Phys. Rev. Lett. **113**, 102502 (2014)
37. R. Lazauskas, Modern nuclear force predictions for $n-^3$ H scattering above the three- and four-nucleon breakup thresholds. Phys. Rev. C **91**, 041001 (2015)
38. W. Grüebler, V. König, P.A. Schmelzbach, R. Risler, R.E. White, P. Marmier, Investigation of excited states of 4He via the 2H(d, p)3H and 2H(d, n)3He reactions using a polarized deuteron beam. Nucl. Phys. A **193**, 129 (1972)

39. W. Grüebler, V. König, P.A. Schmelzbach, B. Jenny, J. Vybiral, New highly excited 4He levels found by the 2H(d, p)3H reaction. Nucl. Phys. A **369**, 381 (1981)
40. P. Guss, K. Murphy, R. Byrd, C. Floyd, S. Wender, R. Walter, T. Clegg, W. Wylie, The analyzing power for the 2H(d, n)3He reaction from 5.5 to 11.5 MeV. Nucl. Phys. A **395**, 1 (1983)
41. V. König, W. Grüebler, R.A. Hardekopf, B. Jenny, R. Risler, H. Brgi, P. Schmelzbach, R. White, Investigation of charge symmetry violation in the mirror reactions 2H(d, p)3H and 2H(d, n)3He. Nucl. Phys. A **331**, 1 (1979)
42. R. Hardekopf, P. Lisowski, T. Rhea, R. Walter, T. Clegg, Four-nucleon studies: (III). Measurement of the 2H(d, p)3H polarization via the 3H(p, d)2H reaction and comparison to polarization in 2H(d, n)3He. Nucl. Phys. A **191**, 468 (1972)
43. G. Spalek, R. Hardekopf Jr., J.T. Stammbach, R. Walter, Four-nucleon studies: (I). A redetermination of the neutron polarization for the 2H(d, n)3He reaction from 6 to 14 MeV. Nucl. Phys. A **191**, 449 (1972)
44. G. Salzman, G.G. Ohlsen, J. Martin, J. Jarmer, T. Donoghue, Polarization and polarization transfer in the 2H(d, n)3He reaction at 10 MeV. Nucl. Phys. A **222**, 512 (1974)