Few-Body Syst (2017) 58:35
DOI 10.1007/s00601-016-1181-2

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The ^{4}He Trimer as an Efimov System: Latest Developments

Dedicated to the 30th anniversary of the "Few-Body Systems" journal

Received: 4 November 2016 / Accepted: 8 December 2016 / Published online: 12 January 2017

Abstract Kolganova et al. (Few-Body Syst 51:249, 2011) reviewed the results that demonstrate the Efimov nature of the ^{4}He three-atom system. The present note represents an extension of that survey to the time period which passed since its publication.

This short note may be viewed as a complement to our review paper on the same subject [1]. We decided to write it because there was a significant progress in studying Efimov systems since the time of publication of [1]. The progress is based mainly on the recent experimental studies of dilute ultracold gases of alkali atoms in magnetic traps [2–5] but there are interesting news also on the ^{4}He trimers [6].

We recall that the genuine Efimov effect is a remarkable phenomenon which may occur in a system of three particles with short-range pairwise interactions provided that none of the two-body subsystems has bound states. If at least two of the two-body subsystems are formed of distinguishable particles or identical bosons and have infinite s-wave scattering lengths then the three-particle system has infinitely many binding energies that exponentially converge to the three-body threshold. This is the essence of the Efimov effect in its "full-scale" form [7,8]. The corresponding bound states are called Efimov states. It should be emphasized that the asymptotic value of the ratio of the consecutive binding energies of the Efimov states is universal in the sense that it only depends on the ratios of particle masses (but not on the form of the pairwise interactions).

All known two-body systems (both nuclear and atomic/molecular) have finite scattering lengths. Thus, for an isolated three-body system, i.e., in the absence of external fields, it is rather impossible to observe the full-scale Efimov effect with an infinite set of binding energies. Nevertheless, three-body systems featuring at least some peculiarities of the Efimov effect are already of great interest. A qualitative analysis presented by Efimov himself [7] shows that the total number of bound states in the three-boson system is proportional to the logarithm of the ratio of the boson-boson scattering length and effective radius of the two-body forces, provided that this ratio is very large. In the case of the ^{4}He three-atom system the ratio of the atom-atom scattering length and the effective radius is quite large (about 25). However it is not "very large", so that the Efimov estimate implies the existence rather of a single excited state for the $^4He_3$ molecule (see discussion and references in [1]). That such a situation should take place in reality, is confirmed by the results of numerous

This article belongs to the Topical Collection "30th anniversary of Few-Body Systems".

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Table 1 ^{4}He dimer binding energy ϵ_d, ^{4}He dimer bond length $\langle R\rangle$, ^{4}He$-^4$He scattering length $\ell_{\rm sc}^{(1+1)}$, excited state energy E^* of the ^{4}He trimer, and the difference $|E^*-\varepsilon_d|$ for various He–He potentials, as compared to the experimental values from [6,29,44]

Potential	ε_d (mK)	$\ell_{\rm sc}^{(1+1)}$ (Å)	$\langle R\rangle$ (Å)	E^* (mK)	$\lvert E^*-\varepsilon_d\rvert$ (mK)
HFDHE2 [18]	-0.830	124.65	64.21[a]	1.67	0.84
LM2M2 [19]	-1.303	100.23	51.84[a]	2.27	0.97
TTY [20]	-1.309[d]	100.01	51.65[a]	2.28	0.97
CCSAPT07 [21]	-1.564[b]	91.82	47.78[a]	2.59[b]	1.02
PCKLJS [22]	-1.615[b]	90.42	47.09[a]	2.65[b]	1.03
HFD-B [23]	-1.685	88.50	46.46[a]	2.74	1.05
Jeziorska [24]	-1.728[c]	87.53		2.78[c]	1.06
SAPT96 [25]	-1.744[b]		45.45[a]	2.80[b]	1.06
Exp.	$1.1^{+0.3}_{-0.2}$ [29]	104^{+8}_{-18} [29]	52^{+4}_{-4} [29]		
	$1.76^{+0.15}_{-0.15}$ [44]				0.98 ± 0.2 [6]

[a] Results from [34]
[b] Results from [9,10]
[c] Results from [33]
[d] This result slightly differs from the one presented in [32] due to a different number of terms used in the dispersion series for the TTY potential [20]. Unmarked values in the second, third, and fifth columns were obtained by the authors (see, e.g., [1,35])

calculations of the ^{4}He trimer binding energies: For various realistic atom-atom potentials suggested in the last three decades, the ^{4}He trimer has exactly one excited state (see [1] and references therein; see also the recent papers [9–12]). The Efimov nature of this state was conjectured for the first time in [13]. Later on, this conjecture was strongly supported by several numerical calculations involving very small variation of the atom-atom potential strength but producing arbitrarily large change of the atom-atom scattering length [14–17]. For details and more references on this approach we again refer to [1].

By now it is rather well established, both theoretically and experimentally, that the system of two ^{4}He atoms possesses a single bound state. The energy of this state is very small in molecular scale. In particular, the mostly used realistic potential models [18–25] predict the ^{4}He dimer energy between 0.8 and 1.8 mK, which results in a very large scattering length around 100 Å (see Table 1). In an experiment, ^{4}He dimers have been observed for the first time in 1993 by the Minnesota group [26], and in 1994 by Schöllkopf and Toennies [27]. Along with the dimers, the experimental work [27] has also proved the existence of ^{4}He trimers. A first experimental estimate for the size of the ^{4}He$_2$ molecule has been presented in [28]. According to [28], the root mean square distance between ^{4}He nuclei in the ^{4}He dimer is equal to 62 ± 10 Å. Several years later, the bond length for ^{4}He$_2$ was measured again by Grisenti et al. [29] who found for this length the value of 52 ± 4 Å. The estimates of [28,29] imply that the ^{4}He dimer is the most extended known diatomic molecular ground state. The measurements [29] also allowed to evaluate a ^{4}He–^{4}He scattering length $\ell_{\rm sc}^{(1+1)}$ of 104^{+8}_{-18} Å and a ^{4}He dimer energy ε_d of $1.1^{+0.3}_{-0.2}$ mK. The size of the ^{4}He$_3$ ground state has been estimated for the first time in the experiment [30]. According to [30] the He–He bond length in the ^{4}He$_3$ ground state is 11^{+4}_{-5} Å, in agreement with theoretical predictions.

Until 2015, there was no reliable experimental evidence for the existence of an excited state in the ^{4}He trimer. A good news [6] on the experimental observation of this long-predicted Efimov-type state came just in that year. The experiment [6] was based on a combination of the Coulomb explosion imaging technique [31] with cluster mass selection by matter wave diffraction [27]. The helium clusters were prepared in a molecular beam by expanding helium gas at a temperature of 8 K through a 5-μm nozzle. Helium trimers were extracted from the molecular beam by means of matter wave diffraction. Every ^{4}He atom of a cluster was then singly ionized by a strong ultrashort laser field, which led to the subsequent Coulomb explosion of the cluster. Momenta of the ions acquired during the explosion were measured by cold target recoil-ion momentum spectroscopy. These momenta were then used to reconstruct the initial pair-distance distribution and extract the cluster energy. As a result, the difference $|E^*-\varepsilon_d|$ between the binding energy E^* of the exited state of the trimer and the ground state energy ε_d of the dimer was found to be equal to 0.98 $\pm$0.2 mK [6]. This result is close to the theoretical predictions for $|E^*-\varepsilon_d|$ corresponding to various potentials (see column 6 of Table 1), although these potentials give quite different binding energies for the dimer and trimer relative to the breakup threshold (see columns 2 and 5 of Table 1, respectively). Moreover, the theoretical values for $|E^*-\varepsilon_d|$ lie inside the experimental error bar. It is also worth recalling that, since the ^{4}He$_3$ system is almost Efimov, the dependence of the excited state energy on the dimer energy lies on a universal curve (see, e.g. [36–38]) and the difference between these energies varies much slower than the energies themselves. Furthermore, the energy values given in Table 1 represent only a very small piece of this curve.

The experimental technique used in [6] and earlier in [39] also gave information about geometrical structure of helium trimers. From the results of [39] it follows that there is no exceptional mutual position of helium atoms (like, say, equilateral triangle or a linear chain) in the $^4He_3$ ground state. This state is described rather as a structureless random cloud. In [6] it was shown that the most probable geometry of the $^4He_3$ exited state is completely different: two atoms in this state are close to each other and the third atom is far away. The conclusions of [6,39] agree with theoretical predictions of the trimer shapes made in [40,41]. The experimental images of pair-distance distribution initiated additional theoretical calculations of geometrical properties of trimer bound states [11,42,43]. The results of these calculations are rather in good agreement with [6,39].

Another recent news, already of 2016, concerns binding energy of the ^{4}He dimer. This energy was evaluated in the experiment [44], just for the second time and in about 15 years after the previous experimental evaluation in [29]. The investigation [44] was based on a technique very similar to the one used in [6]. The dimer energy of $1.76^{+0.15}_{-0.15}$ mK obtained in [44] differs significantly from the experimental value of $1.1^{+0.3}_{-0.2}$ mK established in [29]. Such an uncertainty in the experimental results does not allow one to make a choice in favor of a particular potential model. Clearly, further experiments dedicated to determining binding energies of helium dimer and trimer are very necessary in order to verify He–He potential models and to choose the most appropriate one.

Acknowledgements This work was supported in part by the Heisenberg-Landau Program and the Russian Foundation for Basic Research.

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