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Two-Dimensional Massless Light Front Fields and Solvable Models

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Abstract Quantum field theory formulated in terms of light front (LF) variables has a few attractive as well as some puzzling features. The latter hindered a wider acceptance of LF methods. In two space–time dimensions, it has been a long-standing puzzle how to correctly quantize massless fields, in particular fermions. Here we show that two-dimensional massless LF fields (scalar and fermion) can be recovered in a simple way as limits of the corresponding massive fields and thereby quantized without any loss of physical information. Bosonization of the fermion field then follows in a straightforward manner and the solvable models can be studied directly in the LF theory. We sketch the LF operator solution of the Thirring-Wess model and also point out the closeness of the massless LF fields to those of conformal field theory.

1 Introduction

Quantum field theory (QFT) formulated in terms of the Dirac's light-front (LF) space–time and field variables [1,2] differs in a few important aspects from the conventional (instant or space-like—SL) form of the theory. It has only three dynamical Poincaré generators and all three boost operators are kinematical—they do not contain interaction. The vacuum structure is drastically simplified (the Fock vacuum is very often an eigenstate of the full Hamiltonian) and permits one the probabilistic interpretation of the bound-state wave functions. However, it also raises serious questions concerning the description of the symmetry breaking and vacuum condensation in the LF theory. These and additional features (enhanced role of field zero modes, proliferation of constrained variables, etc.) seemed to many physicists so confusing that the LF field theory, despite its great potential [3–9], did not achieve so far a status comparable to the conventional theory.

A particularly disturbing feature was a lack of description of two-dimensional (2D) massless fields. The free massless scalar and fermion fields are the essential ingredients of the exact solutions of the 2D interacting models (Thirring, Thirring-Wess, Federbush, Schwinger…), that have played a major role in understanding the QFT non-perturbatively. If its LF version is to be considered as a more efficient and transparent formulation, it should provide a simple and consistent operator solution of this class of models. This however was not the case. A partial exception is the Schwinger model [10,11]. Unfortunately, even with an extension of the canonical

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LF framework (by introducing additional fermion degrees of freedom with negative LF momenta) the results were not comparable with the generally accepted physical picture of this prototype gauge theory.

At the previous LC meeting [12], we have shown how the 2D massless fermions can be consistently described in the LF theory. In the present contribution, we extend this solution to the LF massless scalar field (including the LF version of bosonization), demonstrating thereby that the problem with 2D massless LF fields is only fictitious. The key observation is that the two-point functions of the 2D massive scalar and fermion fields have non-vanishing massless limit, indicating the presence of corresponding massless fields. Their components can be obtained directly as limits of massive fields, or in some cases after a change of variables. Let us note that quantum fields should be rigorously treated as operator-valued distributions [13]. Such a formulation of the studied problem can be found in [14]. In the present paper, we shall be less formal.

The consistent quantum representation of the 2D massless LF fields enables one to address the solvable models in a genuine LF approach. We give some details of the LF operator solution of the Thirring-Wess model. A few remarks related to the LF version of conformal field theory are also included.

We use the following LF notation. The LF coordinate is $x^\mu = (x^+, x^-) = (x^0 + x^1, x^0 - x^1)$, while $k^\mu = (k^+, k^-)$ (or p^μ) denotes the momentum two-vector. Further symbols are $\partial_\pm = \frac{\partial}{\partial x^\pm}$, $\hat{k}^- = \frac{\mu^2}{k^+}$, $\hat{k}\cdot x = \frac{1}{2}k^+x^- + \frac{1}{2}\hat{k}^-x^+$. k^+ is the LF momentum and $\hat{k}^-$ the on-shell LF energy. It is evident that there is no sign ambiguity analogous to $E(k^1) = \pm\sqrt{(k^1)^2 + \mu^2}$ of the SL theory and both k^+, k^- can be taken positive.

2 Massless LF Scalar Field

The covariant Lagrangian density $\mathcal{L}$ and the corresponding field equation describing a free massive scalar field, $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2$, $\left(\partial_\mu\partial^\mu + \mu^2\right)\phi(x) = 0$, takes, in terms of the LF variables, the form

$$\mathcal{L} = 2\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2\phi^2, \quad \left(4\partial_+\partial_- + \mu^2\right)\phi(x) = 0. \tag{1}$$

The field time derivative and the conjugate momentum are $\theta(x) = 2\partial_+\phi(x) = -\frac{1}{2}\mu^2\partial_-^{-1}\phi(x)$, $\pi(x) = 2\partial_-\phi(x)$, where ∂_-^{-1} is the inverse derivative (unambiguously defined after prescribing boundary conditions).

The components of the energy-momentum tensor corresponding to the Lagrangian (1) are

$$T^{++}(x) = 4\partial_-\phi(x)\partial_-\phi(x), \quad T^{--}(x) = 4\partial_+\phi(x)\partial_+\phi(x), \quad T^{+-}(x) = \mu^2\phi^2(x) = T^{-+}(x). \tag{2}$$

The quantum solution of the field equation (1) takes the form [15]

$$\phi(x) = \int_0^{+\infty} \frac{dk^+}{\sqrt{4\pi k^+}}\left[a(k^+)e^{-\frac{i}{2}\hat{k}\cdot x} + a^\dagger(k^+)e^{\frac{i}{2}\hat{k}\cdot x}\right] f(k^+, \frac{\mu^2}{k^+}), \; \left[a(k^+), a^\dagger(l^+)\right] = \delta(k^+ - l^+), \tag{3}$$

with a test function $f(k^+, \frac{\mu^2}{k^+}) \in \mathcal{S}$, the space of rapidly decreasing functions [13]. Here we adopt the most simple form $\exp\left[-(k^+\delta^- + \frac{\mu^2}{k^+}\delta^+)\right]$ for f. Even if not displayed (like e.g. in Eq. (5) below), it is implicitly present to give mathematically well defined expressions. The field commutation relation at equal LF time is

$$\left[\phi(x^+, x^-), \phi(x^+, y^-)\right] = \frac{1}{4i}\epsilon(z^-) = \int_0^{\infty} \frac{dk^+}{4\pi}\left[e^{-\frac{i}{2}k^+(z^- - i\delta^-)} - e^{\frac{i}{2}k^+(z^- + i\delta^-)}\right], \; z^- = x^- - y^-, \tag{4}$$

where $\epsilon(z^-)$ is the sign function. From (3) we directly find

$$\theta(x) = \int_0^{+\infty} \frac{dk^+}{i\sqrt{4\pi k^+}}\frac{\mu^2}{k^+}\left[a(k^+)e^{-\frac{i}{2}\hat{k}\cdot x} - a^\dagger(k^+)e^{\frac{i}{2}\hat{k}\cdot x}\right], \; \pi(x) = \int_0^{+\infty} \frac{dk^+}{i\sqrt{4\pi k^+}}k^+\left[a(k^+)e^{-\frac{i}{2}\hat{k}\cdot x} - H.c.\right]. \tag{5}$$

The LF Hamiltonian and the momentum operator are $P^\mu = \frac{1}{2}\int_{-\infty}^{+\infty} dx^- T^{+\mu}(x) = \int_0^{+\infty} dk^+ \hat{k}^\mu a^\dagger(k^+)a(k^+)$.

One can form a few two-point correlation functions from the three field operators above:

$$D_0^{(+)}(z) = \langle 0|\phi(x)\phi(y)|0\rangle, \quad D_1^{(+)}(z) = \langle 0|\phi(x)\pi(y)|0\rangle, \quad D_2^{(+)}(z) = \langle 0|\phi(x)\theta(y)|0\rangle, \tag{6}$$

$$D_i^{(+)}(z) = \int_0^\infty \frac{dk^+}{4\pi} g_i(k^+) e^{-\frac{i}{2}k^+(z^- - i\delta^-) - \frac{i}{2}\frac{\mu^2}{k^+}(z^+ - i\delta^+)}, \quad g_0 = \frac{1}{k^+}, \quad g_1 = 1, \; g_2 = \frac{\mu^2}{(k^+)^2}. \tag{7}$$

The above integrals are easily evaluated in terms of the (modified) Bessel functions $J_\nu(z), N_\nu(z), K_\nu(z), \nu = 0, 1$. As in the SL theory, the first one logarithmically diverges for vanishing mass μ. The second is given by

$$D_1^{(+)}(z) = -\theta\left(-z^2\right) \mathrm{sgn}(z^+) \frac{\mu}{4\pi} \sqrt{-\frac{z^+}{z^-}} K_1\left(\mu\sqrt{-z^2}\right) - \theta\left(z^2\right) \frac{i\mu}{4} \sqrt{\frac{z^+}{z^-}} \left[J_1\left(\mu\sqrt{z^2}\right) - i\ \mathrm{sgn}(z^+) N_1\left(\mu\sqrt{z^2}\right)\right]. \tag{8}$$

The important property is that both $D_1^{(+)}$ and $D_2^{(+)} = D_1^{(+)}(z^+ \leftrightarrow z^-)$ have a non-vanishing massless limit,

$$D_1^{(+)}(x - y; \mu^2 = 0) = \frac{1}{2\pi} \frac{1}{(x^- - y^- - i\delta^-)}, \quad D_2^{(+)}(x - y; \mu^2 = 0) = \frac{1}{2\pi} \frac{1}{(x^+ - y^+ - i\delta^+)}. \tag{9}$$

Technically, this is due to the behaviour of the Bessel function $K_1(z) \sim \frac{1}{z}$ for small value of z (the same is true for $N_1(z)$ in the timelike region), so that $\mu\sqrt{-\frac{z^\mp}{z^\pm}} K_1\left(-\mu\sqrt{z^+ z^-}\right)$ has the finite massless limit [16]. The results (9) suggest that there must exist massless analogs of the fields $\phi(x), \pi(x), \theta(x)$, with correlation functions reproducing the above massless limits of the massive correlators. The massless Klein–Gordon equation $\partial_+\partial_-\tilde{\phi}(x) = 0$ tells us that one should expect a general solution of the form

$$\tilde{\phi}(x) = \tilde{\phi}(x^+) + \tilde{\phi}(x^-). \tag{10}$$

The massless limit of the massive solution (3) yields $\tilde{\phi}(x^-)$ in a straightforward way:

$$\tilde{\phi}(x^-) = \int_0^\infty \frac{dk^+}{\sqrt{4\pi k^+}} \left[a(k^+) e^{-\frac{i}{2}k^+x^-} + a^\dagger(k^+) e^{\frac{i}{2}k^+x^-}\right], \; [a(k^+), a^\dagger(l^+)] = \delta(k^+ - l^+)]. \tag{11}$$

Remarkably, the second piece $\tilde{\phi}(x^+)$ is also contained in (3). A simple change of variables $k^+ \mapsto \frac{\mu^2}{k^-}$ yields $\phi(x) = \int_0^{+\infty} \frac{dk^-}{\sqrt{4\pi k^-}} \left[\frac{\mu}{k^-} a\left(\frac{\mu^2}{k^-}\right) e^{-\frac{i}{2}\frac{\mu^2}{k^-}x^- - \frac{i}{2}k^-x^+} + H.c.\right]$. The Fock commutator changes to

$$\left[\frac{\mu}{k^-} a\left(\frac{\mu^2}{k^-}\right), \frac{\mu}{l^-} a^\dagger\left(\frac{\mu^2}{l^-}\right)\right] = \frac{\mu^2}{k^- l^-} \delta\left(\frac{\mu^2}{k^-} - \frac{\mu^2}{l^-}\right) = \delta(k^- - l^-). \tag{12}$$

The rhs is mass independ and hence survives the massless limit. This should then be true for the lhs as well:

$$\lim_{\mu \to 0} \frac{\mu}{k^-} a\left(\frac{\mu^2}{k^-}\right) \equiv \tilde{a}(k^-) \neq 0, \; \left[\tilde{a}(k^-), \tilde{a}^\dagger(l^-)\right] = \delta(k^- - l^-), \; \left[a(k^+), \tilde{a}^\dagger(l^-)\right] = 0. \tag{13}$$

Performing the massless limit in the expression for $\phi(x)$ above Eq. (12) and in $\pi(x), \theta(x)$ (5), we find[1]

$$\tilde{\phi}(x^+) = \int_0^{+\infty} \frac{dk^-}{\sqrt{4\pi k^-}} \left[\tilde{a}(k^-) e^{-\frac{i}{2}k^-x^+} + \tilde{a}^\dagger(k^-) e^{\frac{i}{2}k^-x^+}\right], \tag{14}$$

$$\tilde{\theta}(x^+) = \int_0^{+\infty} \frac{dk^- k^-}{i\sqrt{4\pi k^-}} \left[\tilde{a}(k^-) e^{-\frac{i}{2}k^-x^+} - H.c.\right], \quad \tilde{\pi}(x^-) = \int_0^{+\infty} \frac{dk^+ k^+}{i\sqrt{4\pi k^+}} \left[a(k^+) e^{-\frac{i}{2}k^+x^-} - H.c.\right]. \tag{15}$$

[1] Strictly speaking, this derivation should be done on a classical level with field amplitudes and then taken over to the quantum field operators. Also, the above change of variables has to be done for θ to make it compatible with the massless field equation.

The field commutators, following from the Fock algebra in (11) and from (13) are

$$\left[\tilde{\phi}(x^-),\tilde{\phi}(y^-)\right] = -\frac{i}{4}\epsilon(x^- - y^-), \quad \left[\tilde{\phi}(x^+),\tilde{\phi}(y^+)\right] = -\frac{i}{4}\epsilon(x^+ - y^+). \tag{16}$$

In this way, the second half of the solution of the field equation has been recovered from the massive solution. Note also that the variables k^+ and k^- in fact coincide. This is analogous to the SL case where $k^0 = |k^1|$.

As a consistency check, the two-point functions calculated from the massless fields should coincide with the massless limit of the massive functions. This is indeed the case for $D_1^{(+)}(z)$ and $D_2^{(+)}(z)$. The massive $D_0^{(+)}(z)$ can be evaluated for small mass μ, it diverges as $\ln\mu$. On the other hand, $D_0^{(+)}(z)$ calculated from the massless solution is ill defined with μ carelessly set to zero from the start of the calculation. Heuristically it can be seen that if we introduce the infrared cutoffs $\lambda^+ = \lambda^- \equiv \lambda$ in the corresponding integrals,

$$\tilde{D}_0^{(+)}(z) = \int_\lambda^\infty \frac{dk^-}{4\pi k^-} e^{-\frac{i}{2}k^-(z^+ - i\delta^+)} + \int_\lambda^\infty \frac{dk^+}{4\pi k^+} e^{-\frac{i}{2}k^+(z^- - i\delta^-)}, \tag{17}$$

We find the same (regularized) $\ln\lambda$ divergent behaviour (see below).

It is instructive to study the massless limit of the LF momentum and Hamiltonian. One expects that these operators survive the limit since for a consistent quantum theory we have to find the Heisenberg equations $2i\partial_+\phi(x) = -[P^-,\phi(x)],\ 2i\partial_-\phi(x) = -[P^+,\phi(x)]$. The massless limit of the momentum operator P^+ is straightforward, while the above change of variables is necessary for the Hamiltonian. The resultant operators

$$P^+ = \int_0^{+\infty} dk^+ k^+ a^\dagger(k^+)a(k^+), \quad P^- = \int_0^{+\infty} dk^+ k^- a^\dagger(k^-)a(k^-) \tag{18}$$

generate, by virtue of (13) and due to $[P^+,\phi(x^+)] = [P^-,\phi(x^-)] = 0$, the correct Heisenberg equations

$$2i\partial_+\phi(x^+) = -[P^-,\phi(x^+)], \quad 2i\partial_-\phi(x^-) = -[P^+,\phi(x^-)]. \tag{19}$$

However, the massless limit of the LF Hamiltonian density $T^{+-}(x)$ itself vanishes, in agreement with the conformal symmetry of the massless theory, which requires Tr $T^{\mu\nu} = T^\mu_\mu = T^{+-} = 0$. It is also interesting to note that the correlation functions of the scalar conformal field [17] (and of its energy-momentum tensor) are directly reproduced from Eq. (9) by going over to the Euclidean time $t \to -i\tau$ and by defining the complex variables $\zeta = \tau + ix^1$, $\bar{\zeta} = \tau - ix^1$. This link to CFT deserves a further study.

3 Massless Light Front Fermion Field and Bosonization

The construction of the massless LF fermion fields proceeds in a completely analogous manner. The derivation is in a sense simpler since there are no infrared divergencies in this case. The only difference is that one has to carefully treat the constraint for the non-dynamical fermion field component.

The massless LF Dirac equation requires that [12] $\psi_2(x) = \psi_2(x^-),\ \psi_1(x) = \psi_1(x^+)$. The massless limit of the solution of the massive Dirac equation then indeed yields

$$\tilde{\psi}_2(x^-) = \int_0^{+\infty} \frac{dp^+}{\sqrt{4\pi}}\left[b(p^+)e^{-\frac{i}{2}p^+x^-} + d^\dagger(p^+)e^{\frac{i}{2}p^+x^-}\right], \quad \{b(p^+),b^\dagger(q^+)\} = \{d(p^+),d^\dagger(q^+)\} = \delta(p^+ - q^+), \tag{20}$$

$$\tilde{\psi}_1(x^+) = \int_0^{+\infty} \frac{dp^-}{\sqrt{4\pi}}\left[\tilde{b}(p^-)e^{-\frac{i}{2}p^-x^+} - \tilde{d}^\dagger(p^-)e^{\frac{i}{2}p^-x^+}\right], \quad \{\tilde{b}(p^-),\tilde{b}^\dagger(q^-)\} = \{\tilde{d}(p^-),\tilde{d}^\dagger(q^-)\} = \delta(p^- - q^-), \tag{21}$$

with $\tilde{b}(p^-)$ and $\tilde{d}(p^-)$ given analogously to the scalar case (13). The field anticommutators acquire the form

$$\left\{\tilde{\psi}_1(x^+),\tilde{\psi}_1^{\dagger}(y^+)\right\} = \delta(x^+ - y^+), \quad \left\{\tilde{\psi}_2(x^-),\tilde{\psi}_2^{\dagger}(y^-)\right\} = \delta(x^- - y^-). \tag{22}$$

All this shows that the massless fields can be recovered from the original massive solution. No new fermionic variables have to be introduced artificially. This is confirmed by the fact that while the mixed two-point function $S_{12}^{(+)}$ vanishes for $m=0$, the two-point functions calculated from the massless representation (20, 21)

$$S_{22}^{(+)}(x-y;m=0)=\frac{1}{2i\pi}\frac{1}{(x^- - y^- - i\delta^-)},\quad S_{11}^{(+)}(x-y;m=0)=\frac{1}{2i\pi}\frac{1}{(x^+ - y^+ - i\delta^+)} \tag{23}$$

coincide with the massless limits of the massive two-point functions [12].

Since the solvable models are based on free Heisenberg fields, the above derivation of the 2D massless LF fermion fields opens the prospect for the genuine light-front solution of this class of models [12,14]. Following the consistent quantization of the massless LF scalar and fermion fields, it is natural to analyze the celebrated bosonization property [18] in a genuine LF form. Since the massless $\phi(x)$ and $\psi(x)$ fields decompose as $\phi(x)=\phi(x^+)+\phi(x^-)$, $\psi^T(x)=(\psi_1(x^+),\psi_2(x^-))$ (we omit tilde for the massless fields henceforth), the bosonization is straightforward. One simply observes [14] that the normal-ordered exponentials

$$\hat{\varphi}_1(x^-)=C:e^{i\alpha\phi(x^+)}:=Ce^{i\alpha\phi^{(-)}(x^+)}e^{i\alpha\phi^{(+)}(x^+)},\ \hat{\varphi}_2(x^-)=C:e^{i\alpha\phi(x^-)}:=Ce^{i\alpha\phi^{(-)}(x^-)}e^{i\alpha\phi^{(+)}(x^-)}, \tag{24}$$

with $C=\sqrt{\frac{\lambda e^{\gamma_E}}{4\pi}}$, $\alpha=2\sqrt{\pi}$ (γ_E is the Euler's constant), satisfy the fermionic properties

$$\hat{\varphi}_1(x^+)\hat{\varphi}_1(y^+)=-\hat{\varphi}_1(y^+)\hat{\varphi}_1(x^+),\quad \hat{\varphi}_2(x^-)\hat{\varphi}_1(y^-)=-\hat{\varphi}_2(y^-)\hat{\varphi}_1(x^-), \tag{25}$$

$$\left\{\hat{\varphi}_1(x^+),\hat{\varphi}_1^\dagger(y^+)\right\}=\delta(x^+-y^+),\quad \left\{\hat{\varphi}_2(x^-),\hat{\varphi}_2^\dagger(y^-)\right\}=\delta(x^--y^-). \tag{26}$$

The above construction allows one to study the bosonized LF Thirring model as well as the LF version of the sine-Gordon—massive Thirring model correspondence [19,20]. Here we demonstrate the simplicity of the LF treatment with the example of the exactly solvable Thirring-Wess model.

4 The LF Thirring-Wess Model

The dynamics of the model is characterized by the covariant-form Lagrangian

$$\mathcal{L}=\frac{i}{2}\overline{\Psi}\gamma^\mu\overset{\leftrightarrow}{\partial_\mu}\Psi-\frac{1}{4}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu}+\frac{1}{2}\mu_0^2\tilde{B}_\mu\tilde{B}^\mu-e\tilde{B}_\mu J^\mu,\quad \tilde{G}_{\mu\nu}=\partial_\mu\tilde{B}_\nu-\partial_\nu\tilde{B}_\mu, \tag{27}$$

where $\Psi(x)$ ($\tilde{B}^\mu(x)$) is the interacting massless fermion (massive vector) field. The solvability of the theory implies that one can find an operator solution of the coupled system of the Dirac and Proca equations:

$$i\gamma^\mu\partial_\mu\Psi=e\gamma_\mu\tilde{B}^\mu\Psi,\quad \partial_\mu\tilde{G}^{\mu\nu}+\mu_0^2\tilde{B}^\nu=eJ^\nu,\quad J^\mu(x)=\overline{\Psi}(x)\gamma^\mu\Psi(x). \tag{28}$$

For the conserved current $J^\mu(x)$ the vector field satisfies $\partial_\mu\tilde{B}^\mu(x)=0$. The LF form of the field equations is

$$2i\partial_+\Psi_2=e\tilde{B}^-\Psi_2,\quad 2i\partial_-\Psi_1=e\tilde{B}^+\Psi_1,\quad 4\partial_+\partial_-\tilde{B}^++\mu_0^2\tilde{B}^+=eJ^+,\quad 4\partial_+\partial_-\tilde{B}^-+\mu_0^2\tilde{B}^-=eJ^-. \tag{29}$$

The classical solution of the Dirac equation involves the free massless LF fermion field components:

$$\Psi_1(x)=\exp\left\{-\frac{ie}{2}\int_{-\infty}^{+\infty}\mathrm{d}y^-\frac{1}{2}\epsilon(x^--y^-)\tilde{B}^+(x^+,y^-)\right\}\psi_1(x^+),$$

$$\Psi_2(x)=\exp\left\{+\frac{ie}{2}\int_{-\infty}^{+\infty}\mathrm{d}y^-\frac{1}{2}\epsilon(x^--y^-)\tilde{B}^+(x^+,y^-)\right\}\psi_2(x^-). \tag{30}$$

The interacting current appearing in the Proca equation can be determined from the solution (30) (or rather its quantum version, see below), using the point-split regularized definition

$$J^+(x)=\lim_{\epsilon\to 0}\left[\Psi_2^\dagger\left(x+\frac{\epsilon}{2}\right)\Psi_2\left(x-\frac{\epsilon}{2}\right)+H.c.\right],\quad J^-(x)=\lim_{\epsilon\to 0}\left[\Psi_1^\dagger\left(x+\frac{\epsilon}{2}\right)\Psi_1\left(x-\frac{\epsilon}{2}\right)+H.c.\right]. \tag{31}$$

The interacting current is hence equal to the (normal-ordered) free current plus a quantum correction,

$$J^+(x) = 2 : \psi_2^\dagger(x^-)\psi_2(x^-) : -\frac{e}{2\pi}\tilde{B}^+(x^+, x^-), \;\; J^-(x) = 2 : \psi_1^\dagger(x^+)\psi_1(x^+) : -\frac{e}{2\pi}\tilde{B}^-(x^+, x^-). \tag{32}$$

Nevertheless, the interacting quantum current is conserved due to the condition $\partial_\mu \tilde{B}^\mu = 0$. On the other hand, the axial-vector current $J^\mu(x) = \overline{\Psi}(x)\gamma^\mu\gamma^5\Psi(x)$ is "anomalous", $\partial_\mu J_5^\mu = -\frac{e}{2\pi}\partial_+\tilde{B}^+ = \frac{e}{4\pi}\epsilon_{\mu\nu}\tilde{G}^{\mu\nu}$. When (32) is inserted to the Proca equation, the quantum correction renormalizes the bare mass μ_0 as $\mu_0^2 + \frac{e^2}{2\pi} \equiv \mu^2$:

$$4\partial_+\partial_-\tilde{B}^\nu + \mu^2\tilde{B}^\nu = ej^\nu \Rightarrow \tilde{B}^\nu = B^\nu + \frac{e}{\mu^2}j^\nu, \quad \text{where } (4\partial_+\partial_- + \mu^2)B^\nu = 0. \tag{33}$$

The interacting vector-meson field is thus given entirely in terms of the free fields. This permits us to regularize the solutions (30) on the quantum level by normal ordering:

$$\Psi_1(x) = e^{-\frac{ie}{2}F^{(-)}(x^+,x^-)}e^{-\frac{ie}{2}F^{(+)}(x^+,x^-)}\psi_1(x^+), \;\; \Psi_2(x) = e^{\frac{ie}{2}F^{(-)}(x^+,x^-)}e^{\frac{ie}{2}F^{(+)}(x^+,x^-)}\psi_2(x^-), \tag{34}$$

where the fields were decomposed into the positive and negative frequency parts,

$$F^{(\pm)}(x^+, x^-) = \int_{-\infty}^{+\infty} \mathrm{d}y^- \frac{1}{2}\epsilon(x^- - y^-)\left[B^{+(\pm)}(x^+, y^-) + \frac{e}{\mu^2}j^{+(\pm)}(y^-)\right]. \tag{35}$$

The free current $j^\mu(x)$ built from the components (20, 21) can be bosonized by a Fourier transformation:

$$j^+(x^-) = -\frac{i}{\sqrt{\pi}}\int_0^\infty \frac{dk^+\,k^+}{\sqrt{4\pi k^+}}\left[c(k^+)e^{-\frac{i}{2}k^+x^-} - H.c.\right] = j^{+(+)} + j^{+(-)}, \quad \left[c(k^+), c^\dagger(l^+)\right] = \delta(k^+ - l^+),$$

$$j^-(x^+) = -\frac{i}{\sqrt{\pi}}\int_0^\infty \frac{dk^-\,k^-}{\sqrt{4\pi k^-}}\left[c(k^-)e^{-\frac{i}{2}k^-x^+} - H.c.\right], \quad \left[c(k^-), c^\dagger(l^-)\right] = \delta(k^- - l^-). \tag{36}$$

The composite boson operators $c(k^+), c(k^-)$ are bilinear in the fermion Fock operators present in (20, 21) [12]. The free LF vector-meson field, quantized by $[a(k^+), a^\dagger(l^+)] = \delta(k^+ - l^+)$, is expanded as

$$B^+(x) = \int_0^{+\infty} \frac{dk^+}{\sqrt{4\pi k^+}}\frac{k^+}{\mu}\left[a(k^+)e^{-i\hat{k}\hat{x}} + a^\dagger(k^+)e^{i\hat{k}\hat{x}}\right], \quad B^-(x) = -\int_0^{+\infty} \frac{dk^+}{\sqrt{4\pi k^+}}\frac{\hat{k}^-}{\mu}\left[a(k^+)e^{-i\hat{k}\hat{x}} + H.c.\right],$$

$$\left[B^+(x^-), \Pi_{B^+}(y^-)\right] = i\delta(x^- - y^-), \quad \Pi_{B^+} = \partial_+B^+ - \partial_-B^- = 2\partial_+B^+. \tag{37}$$

The field expansions (37, 36) can be used to obtain the Fock form of the corresponding LF Hamiltonian

$$P^- = \frac{1}{2}\int_{-\infty}^{+\infty} dx^-\left[-\mu_0^2B^+B^- - \mu_0^2\frac{e}{\mu^2}\left(B^+j^- + B^-j^+\right) - \mu_0^2\frac{e^2}{\mu^4}j^+j^-\right]. \tag{38}$$

This, along with the nonperturbative computation of the correlation functions of the model, will be published separately [14]. Finally, as indicated at the end of Sect. 2, there is a direct link [14] between the quantum theory of two-dimensional massless LF fields, developed in the present work, and the conformal field theory [17].

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