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Light-Front Perturbation Without Spurious Singularities

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Abstract A new form of the light front Feynman propagators is proposed. It contains no energy denominators. Instead the dependence on the longitudinal subinterval $x_L^2 = 2x^+x^-$ is explicit and a new formalism for doing the perturbative calculations is invented. These novel propagators are implemented for the one-loop effective potential and various 1-loop 2-point functions for a massive scalar field. The consistency with results for the standard covariant Feynman diagrams is obtained and no spurious singularities are encountered at all. Some remarks on the calculations with fermion and gauge fields in QED and QCD are added.

1 Introduction

Wightman function for a free massive scalar field $\langle 0|\phi(x)\phi(0)|0\rangle = W_2(x)$ has its LF momentum representation

$$W_2(x^+, x^-, \boldsymbol{x}_\perp) = \int\limits_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} \int\limits_0^\infty \frac{dk^+}{4\pi k^+} e^{-ik^+x^-} e^{-i\frac{m^2+\boldsymbol{k}_\perp^2}{2k^+}x^+}. \tag{1}$$

The LF propagator $\Delta_{LF}(x)$ is defined by the chronological (in x^+) ordering

$$\begin{aligned}\Delta_{LF}(x) = \langle 0|T^+\phi(x)\phi(0)|0\rangle &:= \Theta(x^+)\langle 0|\phi(x)\phi(0)|0\rangle + \Theta(-x^+)\langle 0|\phi(0)\phi(x)|0\rangle \\ &= \Theta(x^+)W_2(x^+, x^-, \boldsymbol{x}_\perp) + \Theta(-x^+)W_2(-x^+, -x^-, -\boldsymbol{x}_\perp).\end{aligned} \tag{2}$$

Within the standard LF approach [1] (for review see [2]) one introduces the Fourier representation for the Heaviside step function

$$\Theta(x^+) = \int\limits_{\mathbb{R}} \frac{dk^-}{2\pi} e^{-ik^-x^+} \frac{i}{k^- + i0}, \tag{3}$$

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then changes the order of integrations and finally one shifts the integration variable which gives

$$\int_{\mathbb{R}} \frac{dk^-}{2\pi}\frac{i\,e^{-ik^-x^+}}{k^-+i0}\int_0^\infty \frac{dk^+}{k^+}e^{-ik^+x^-}e^{-ix^+(m^2+k_\perp^2)/(2k^+)} = \int_0^\infty \frac{dk^+}{k^+}e^{-ik^+x^-}\int_{\mathbb{R}} \frac{dk^-}{2\pi}e^{-ik^-x^+}\frac{i}{k^- - \frac{m^2+k_\perp^2}{2k^+}+i0}. \tag{4}$$

For $k^+ \to 0$, the pole in k^- moves to infinity, so the naive implementation of the residua theorem can lead to false results. This problem overlaps with the usual LF singularity due to $1/k^+$ pole. The very trick, presented in (4), is analogous to the equal-time propagators, where one makes the ordering in x^0 temporal variable, so one introduces

$$\Theta(x^0) = \int_{\mathbb{R}} \frac{dk_0}{2\pi}e^{-ik_0x^0}\frac{i}{k_0+i0}, \tag{5}$$

and then one proceeds as follows

$$\int_{\mathbb{R}} \frac{dk_0}{2\pi}e^{-ik_0x^0}\frac{i}{k_0+i0}\frac{1}{2\omega_k}e^{-ix^0\omega_k} = \int_{\mathbb{R}} \frac{dk_0}{2\pi}\frac{e^{-ik_0x^0}}{2\omega_k}\frac{i}{k_0-\omega_k+i0}. \tag{6}$$

In this calculation the shifting of k_0 is reliable since the limit $\omega_k \to \infty$ is suppressed by the inverse power of ω_k in the invariant measure factor, on contrary in the Eq. (4). Therefore it is not strange that the LF propagator with the pole structure as in (4) may lead to various artificial singularities of Feynman diagrams, which are absent in the analogous equal-time calculation.

2 Novel LF Representation of Propagator and Convolutions of Propagators

We observe that we may make the following changes of integration variables: for $x^+ > 0$ we take $k^+ = 2\lambda x^+$

$$W_2(x) \simeq \int_0^\infty \frac{dk^+}{4\pi k^+}e^{-ik^+x^-}\,e^{-i\frac{m^2+\boldsymbol{k}_\perp^2}{2k^+}x^+} = \int_0^\infty \frac{d\lambda}{4\pi\lambda}e^{-i\lambda\boldsymbol{x}_L^2}\,e^{-i(m^2+\boldsymbol{k}_\perp^2)/(4\lambda)}, \tag{7}$$

while for $x^+ < 0$ we take $k^+ = -2\lambda x^+$

$$W_2(-x) \simeq \int_0^\infty \frac{dk^+}{4\pi k^+}e^{ik^+x^-}\,e^{i\frac{m^2+\boldsymbol{k}_\perp^2}{2k^+}x^+} = \int_0^\infty \frac{d\lambda}{4\pi\lambda}e^{-i\lambda\boldsymbol{x}_L^2}\,e^{-i(m^2+\boldsymbol{k}_\perp^2)/(4\lambda)} \tag{8}$$

This leads to the LF propagator in the form, which we call λ-representation,

$$\langle 0|T^+\phi(x)\phi(0)|0\rangle = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\int_0^\infty \frac{d\lambda}{4\pi\lambda}e^{-i\lambda\boldsymbol{x}_L^2}\,e^{-iM^2/(4\lambda)} = \Delta_{LF}(x), \tag{9}$$

where $\boldsymbol{x}_L^2 = 2x^+x^-$ and $M^2 = m^2 + \boldsymbol{k}_\perp^2$. If one wishes to compare this new representation with the covariant formula in the 4-momentum space, then one takes the Fourier transform in the 2-dimensional longitudinal subspace (31) and then integrates over λ, as defined in the sense of distributions in (30). This gives the equivalence between λ-representation and covariant Feynman propagators

$$\Delta_{LF}(x) = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_L}{(4\pi)^2}e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}e^{-i\boldsymbol{k}_L\cdot\boldsymbol{x}_L}\frac{4i}{k_L^2-k_\perp^2+m^2+i0} = \Delta_F(x). \tag{10}$$

Evidently λ-representation of $\Delta_{LF}(x)$ is singular at $x^+ = 0$, but one may check its consistency with the help of its Volterra equation, which is quite similar to the Volterra equation for the Wightman function of a free massive scalar field [3]. Next we consider the convolutions of LF propagators and we begin with the convolution of two LF propagators defined as follows

$$[\Delta_{LF} * \Delta_{LF}](x-z) = \Delta^2_{LF}(x-z) := \int_{\mathbb{R}^4} d^4y\, \Delta_{LF}(x-y)\Delta_{LF}(y-z). \tag{11}$$

By inserting λ-representation, with $(\lambda_1, \boldsymbol{p}_{1\perp})$ and $(\lambda_2, \boldsymbol{p}_{2\perp})$ for respective propagators, one may directly integrate over the transverse and longitudinal coordinates using (32a) and (32c)

$$\begin{aligned}\Delta^2_{LF}(x-z) = &\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_{1\perp}}{(2\pi)^2} \int_{\mathbb{R}^2} d^2\boldsymbol{p}_{1\perp}\, \delta^2(\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp}) e^{-i\boldsymbol{p}_{1\perp}\cdot(\boldsymbol{x}_\perp - \boldsymbol{z}_\perp)} \int_0^\infty \frac{d\lambda_1}{4\pi\lambda_1} \int_0^\infty \frac{d\lambda_2}{4\pi\lambda_2} \\ &\times \frac{\pi}{\lambda_1+\lambda_2} e^{-i\frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2}(x^+-z^+)(x^--z^-)}\, e^{-iM_1^2/(4\lambda_1)} e^{-iM_1^2/(4\lambda_2)}.\end{aligned} \tag{12}$$

where $M_1^2 = m^2 + \boldsymbol{p}_{1\perp}^2$ and $M_2^2 = m^2 + \boldsymbol{p}_{2\perp}^2$. While the integration over $\boldsymbol{p}_{2\perp}$ is immediate, then for evaluating the integrals over $\lambda_{1,2}$ one needs to parameterize them as $\lambda_1 = \lambda/\xi$, $\lambda_2 = \lambda/(1-\xi)$, with new parameters $\lambda \in \langle 0, \infty)$ and $\xi \in (0, 1)$. This allows to evaluate the integration over ξ explicitly, and finally we obtain λ-representation for the convolution of two LF propagators, (where we put $\boldsymbol{p}_{1\perp} = \boldsymbol{p}_\perp$)

$$\Delta^2_{LF}(x-z) = \frac{1}{4}\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} e^{-i\boldsymbol{p}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)} \int_0^\infty \frac{d\lambda}{4\pi\lambda^2} e^{-i\lambda(\boldsymbol{x}_L-\boldsymbol{z}_L)^2} e^{-i\frac{m^2+\boldsymbol{p}_\perp^2}{4\lambda}}. \tag{13}$$

By induction one finds the convolution of n propagators

$$\Delta^n_{LF}(x-z) = \frac{1}{4^{n-1}}\frac{1}{(n-1)!}\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} e^{-i\boldsymbol{p}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)} \int_0^\infty \frac{d\lambda}{4\pi\lambda^n} e^{-i\lambda(\boldsymbol{x}_L-\boldsymbol{z}_L)^2} e^{-i\frac{m^2+\boldsymbol{p}_\perp^2}{4\lambda}}. \tag{14}$$

The 1-loop effective potential (for the $g/(4!)\phi^4$ theory) is given by [4]

$$V^{(1)}_{eff}[\phi_c] = \frac{i}{2}\sum_{n=1} \frac{(-i)^n}{n}\left(\frac{g}{2}\phi_c^2\right)^n \Delta^n_{LF}(0), \tag{15}$$

thus we need to evaluate $\Delta^n_{LF}(0)$, where using the general formula (30) we find

$$\Delta^n_{LF}(0) = \frac{1}{4^{n-1}}\frac{1}{(n-1)!}\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} \int_0^\infty \frac{d\lambda}{4\pi\lambda^n} e^{-i\frac{m^2+\boldsymbol{p}_\perp^2}{4\lambda}} = \frac{i^{n-1}}{4\pi(n-1)}\int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\frac{1}{(m^2+\boldsymbol{p}_\perp^2)^{n-1}}. \tag{16}$$

This expression gives nontrivial contribution to the effective potential, which can be compared with the result obtained within the standard LF formulation [4]

$$\bar{\Delta}^n_{LF}(0) = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} \int_{\mathbb{R}} \frac{dp^+}{4\pi(p^+)^n} \int_{\mathbb{R}} dp^- \frac{1}{\left[p^- - \frac{m^2+\boldsymbol{p}_\perp^2}{2p^+} + i\,\mathrm{sgn}(p^+)0\right]^n}, \tag{17}$$

which for $n > 1$, vanishes by residua, unless the contribution from the arc is properly taken into account.

3 One Loop 2-Point Diagrams

Now we will consider one loop 2-point diagrams with a flow of non-zero external 4-momentum q^μ, starting with the simplest scalar self-energy diagram

$$\Sigma(q) = \int_{\mathbb{R}^4} d^4x e^{+iq\cdot x} \Delta_F(x)\Delta_F(x) = \int_{\mathbb{R}^4} \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i0} \frac{i}{(k+q)^2 - m^2 + i0}, \tag{18}$$

where the standard covariant Feynman propagators are inserted and further after the Wick rotation one may evaluate the Euclidean momentum integrals. Our aim is to calculate $\Sigma(q)$ with λ-representation for $\Delta_{LF}(x)$

$$\Sigma(q) = \int_{\mathbb{R}^4} d^4x\, e^{+iq\cdot x} \Delta_{LF}(x)\Delta_{LF}(x). \tag{19}$$

We denote $(\lambda_1, \boldsymbol{p}_{1\perp})$ and $(\lambda_2, \boldsymbol{p}_{2\perp})$ for $\Delta_{LF}(x)$ respectively and start with the integration over the space-time coordinates, using (32b),

$$\Sigma(q) = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_{1\perp}}{(2\pi)^2} \int_{\mathbb{R}^2} d^2\boldsymbol{p}_{2\perp} \delta^2(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} - \boldsymbol{q}_\perp) \int_0^\infty \frac{d\lambda_1}{4\pi\lambda_1} \int_0^\infty \frac{d\lambda_2}{4\pi\lambda_2} e^{-i(\lambda_1+\lambda_2)(\boldsymbol{x}_L)^2}\, e^{-iM_1^2/(4\lambda_1)}\, e^{i\frac{\boldsymbol{q}_L^2}{4(\lambda_1+\lambda_2)}}, \tag{20}$$

where $M_1^2 = m^2 + \boldsymbol{p}_{1\perp}^2$ and $M_2^2 = m^2 + \boldsymbol{p}_{2\perp}^2$. Then we parameterize $\lambda_1 = \lambda\xi$, $\lambda_2 = \lambda(1-\xi)$ with the Jacobian $\mathscr{J} = \lambda$ and the transverse momenta as $\boldsymbol{p}_{1\perp} = \boldsymbol{p}_\perp\xi + \boldsymbol{k}_\perp$, $\boldsymbol{p}_{2\perp} = \boldsymbol{p}_\perp(1-\xi) - \boldsymbol{k}_\perp$ with the Jacobian $\mathscr{J} = 1$, which leads to

$$\Sigma(q) = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} \int_{\mathbb{R}^2} d^2\boldsymbol{k}_\perp \delta^2(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp) \int_0^\infty \frac{d\lambda}{16\pi\lambda^2} \int_0^1 \frac{d\xi}{\xi(1-\xi)} e^{i\frac{\boldsymbol{q}_L^2}{4\lambda}}\, e^{-iM_1^2/(4\xi\lambda)}\, e^{-iM_2^2/(4(1-\xi)\lambda)}. \tag{21}$$

The integration over $\boldsymbol{p}_\perp$ is simple and due to the property (33), one finds

$$\Sigma(q) = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \int_0^\infty \frac{d\lambda}{16\pi\lambda^2} \int_0^1 \frac{d\xi}{\xi(1-\xi)} e^{i(\boldsymbol{q}_L^2 - \boldsymbol{q}_\perp^2)/(4\lambda)}\, e^{-i(m^2+\boldsymbol{k}_\perp^2)/(4\xi(1-\xi)\lambda)}. \tag{22}$$

The integral over λ can be performed explicitly according to (30) for $n = 1$, so

$$\Sigma(q) = \frac{i}{4\pi} \int_0^1 d\xi \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \frac{1}{q^2\xi(1-\xi) - m^2 - \boldsymbol{k}_\perp^2 + i0}, \tag{23}$$

which coincides with the result of calculation with the covariant Feynman propagators. Then we wish to consider another 2-point function

$$\Sigma_\mu(q) = i\int_{\mathbb{R}^4} d^4x e^{iq\cdot x} \partial_\mu \Delta_F(x)\Delta_F(x) = \int_{\mathbb{R}^4} \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{k^2 - m^2 + i0} \frac{i^2}{(k+q)^2 - m^2 + i0}. \tag{24}$$

We now insert the propagators in λ-representation and take steps analogous to those in the calculation of $\Sigma(q)$. There is a slight difference,because $i\partial_\mu\Delta_{LF}(x)$ appears instead of $\Delta_{LF}(x)$. First, for the longitudinal partial derivatives (32d) one obtains the extra factor $\lambda_1 q_\pm/(\lambda_1 + \lambda_2)$, which after the re-parametrization of $\lambda_{1,2}$ boils down to the extra factor $q_\mu\xi$. Second, for the transverse partial derivatives one obtains another extra factor

$\boldsymbol{p}_{1\perp} = \boldsymbol{p}_{\perp}\xi + \boldsymbol{k}_{\perp}$. Third, the term linear in $k_{\perp}$, due to its antisymmetry, will vanish during the integration over $\boldsymbol{k}_{\perp}$. Ultimately one obtains the covariant expression

$$\Sigma_{\mu}(q) = \frac{i\,q_{\mu}}{4\pi}\int_0^1 d\xi \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_{\perp}}{(2\pi)^2}\frac{\xi}{q^2\xi(1-\xi) - m^2 - \boldsymbol{k}_{\perp}^2 + i0}. \tag{25}$$

Finally we may consider the 2-point function, which has been used by Melikhov and Simula in [5] for their discussion of spurious end-point singularities

$$\Sigma_{\mu}^{ms}(q) = i\int_{\mathbb{R}^4} d^4x e^{iq\cdot x}\partial_{\mu}\Delta_F(x)\Delta_F^2(x) = \int_{\mathbb{R}^4}\frac{d^4k}{(2\pi)^4}\frac{k_{\mu}}{k^2 - m^2 + i0}\frac{i^3}{[(k+q)^2 - m^2 + i0]^2}. \tag{26}$$

From λ-representations for $\Delta_F(x)$ in (9) and the convolution $\Delta_F^2(x)$ in (13), we see that $\Sigma_{\mu}^{ms}(q)$ contains extra term $1/(4\lambda_2) = [4\lambda(1-\xi)]^{-1}$ in comparison with $\Sigma_{\mu}(q)$. Thus following all steps preceding (25) we find

$$\Sigma_{\mu}^{ms}(q) = q_{\mu}\int_{\mathbb{R}^2}\frac{d^2\boldsymbol{k}_{\perp}}{(2\pi)^2}\int_0^{\infty}\frac{d\lambda}{16\pi\lambda^3}\int_0^1\frac{d\xi}{\xi(1-\xi)}\frac{\xi}{1-\xi}e^{i(\boldsymbol{q}_L^2 - \boldsymbol{q}_{\perp}^2)/(4\lambda)}\,e^{-i(m^2+\boldsymbol{k}_{\perp}^2)/(4\xi(1-\xi)\lambda)}. \tag{27}$$

The integral over λ can be performed explicitly according to (30) for $n = 1$, which leads to the desired form

$$\Sigma_{\mu}^{ms}(q) = \frac{iq_{\mu}}{4\pi}\int_0^1 d\xi\int_{\mathbb{R}^2}\frac{d^2\boldsymbol{k}_{\perp}}{(2\pi)^2}\frac{\xi^2}{[q^2\xi(1-\xi) - m^2 - \boldsymbol{k}_{\perp}^2 + i0]^2}, \tag{28}$$

with no sign of any end-point singularities.

4 Conclusions and Prospects

We conclude that the novel λ-representation for the scalar field LF propagator, which appears naturally within the LF quantization, is a very useful tool for evaluation 1-loop diagrams. For 2-point functions, we have derived the desired form of covariant expression with no spurious end-point singularities. For the effective potential we arrived at the nontrivial integrals over the transverse momenta, which are consistent with the equal-time results. Evidently, the next step should be to evaluate 3-point functions and to compare our method with the existing LF alternative approach proposed by Heinzl [6].

Also we would like to mention what happens for fields with non-zero spin. For the gauge field in the LC gauge $A^+ = 0$ one obtains for the chronological ordering in x^+

$$\langle 0|T^+A_+(x)A_+(0)|0\rangle = 2\partial_+\int_0^{x^-} d\tau\, D_{LF}(x^+, \tau, \boldsymbol{x}_{\perp}) - i\delta(x^+)\delta^2(\boldsymbol{x}_{\perp})|x^-|, \tag{29}$$

with the LF massless Feynman propagator function $D_{LF}(x) = \lim_{m\to 0}\Delta_{LF}$. For the first term, which has the form proposed by Bassetto [7], one may introduce λ-representation consistently, but the second term, which is local in time x^+ needs to be subtracted. This agrees with the standard LF quantization, where the LF propagator also has this additional local term. Analogous situation occurs for the fermion field propagator, where one obtains an additional local term for the "bad components" ψ_-, $\psi_-^{\dagger}$ of the fermion field. However in the LF Hamiltonian there are local interaction terms which exactly cancel the contribution from local terms in the LF propagators, thus effectively one obtains Feynman diagrams for the perturbative QED and QCD without local term contributions.

We hope that our novel representation for propagators may shed a new light on the equivalence problem between the LF and equal-time perturbative calculations [8]. At last, it will be quite interesting to apply λ-regularization for the LF Bethe-Salpeter equations as in [9].

Appendix: Definitions and Useful Formulas

The LF longitudinal coordinates are defined as $x^{\pm} = (x^0 \pm x^3)/\sqrt{2}$ and the partial derivatives are denoted as $\partial_{\pm} = \partial/\partial x^{\pm}$. The Minkowski space-time metric tensor has non-vanishing components $g_{+-} = g_{-+} = 1$, $g_{ij} = -\delta_{ij}$.

We have the formula for $n \in \mathbb{N} - \{1\}$

$$\int_0^{\infty} \frac{d\lambda}{\lambda^{n+1}} e^{iA/\lambda} = \frac{i^n (n-1)!}{(A + i0)^n}. \tag{30}$$

The Fourier transform in the longitudinal coordinates

$$e^{-i\lambda \boldsymbol{x}_L^2} = e^{-i2x^+x^-\lambda} = \int_{\mathbb{R}^2} \frac{dk^+dk^-}{4\pi\lambda} e^{-i(k^+x^-+k^-x^-)} e^{ik^+k^-/(2\lambda)} = \int_{\mathbb{R}^2} \frac{d^2\boldsymbol{k}_L}{4\pi\lambda} e^{-i\boldsymbol{k}_L\cdot\boldsymbol{x}_L} e^{i\boldsymbol{k}_L^2/(4\lambda)}, \tag{31}$$

The integrations over the transverse and longitudinal coordinates give respectively

$$\int_{\mathbb{R}^2} d^2\boldsymbol{y}_\perp e^{-i\boldsymbol{p}_{1\perp}\cdot(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)} e^{-i\boldsymbol{p}_{2\perp}\cdot(\boldsymbol{y}_\perp - \boldsymbol{z}_\perp)} = (2\pi)^2 \delta^2(\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp}) e^{-i\boldsymbol{p}_{1\perp}\cdot\boldsymbol{x}_\perp} e^{i\boldsymbol{p}_{2\perp}\cdot\boldsymbol{z}_\perp}, \tag{32a}$$

$$\int_{\mathbb{R}^2} d^2\boldsymbol{x}_\perp e^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{p}_{1\perp}} e^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{p}_{2\perp}} e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp} = (2\pi)^2 \delta^2(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} - \boldsymbol{q}_\perp), \tag{32b}$$

$$\int_{\mathbb{R}^2} d^2\boldsymbol{y}_L e^{-i\lambda_1(\boldsymbol{x}_L - \boldsymbol{y}_L)^2} e^{-i2\lambda_2(\boldsymbol{y}_L - \boldsymbol{z}_L)^2} = \frac{\pi}{\lambda_1 + \lambda_2} \exp -i\left(\frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}(\boldsymbol{x}_L - \boldsymbol{z}_L)^2\right), \tag{32c}$$

$$\int_{\mathbb{R}^2} d^2\boldsymbol{x}_L \left[e^{-i\boldsymbol{x}_L^2\lambda_1}, \left(i\partial_{\pm} e^{-i\boldsymbol{x}_L^2\lambda_1}\right)\right] e^{-i\boldsymbol{x}_L^2\lambda_2} e^{i\boldsymbol{q}_L\cdot\boldsymbol{x}_L} = \frac{\pi}{\lambda_1 + \lambda_2}\left[1, \frac{\lambda_1 q_{\pm}}{\lambda_1 + \lambda_2}\right] \exp\frac{i\,\boldsymbol{q}_L^2}{4(\lambda_1 + \lambda_2)}. \tag{32d}$$

The parameterization of transverse momenta leads to

$$\frac{M_1^2}{\xi} + \frac{M_2^2}{1-\xi} = \frac{m^2 + (\boldsymbol{p}_\perp\xi + \boldsymbol{k}_\perp)^2}{\xi} + \frac{m^2 + (\boldsymbol{p}_\perp(1-\xi) - \boldsymbol{k}_\perp)^2}{1-\xi} = \frac{m^2 + \boldsymbol{k}_\perp^2}{\xi(1-\xi)} + \boldsymbol{p}_\perp^2. \tag{33}$$

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