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Manifestly Lorentz-Invariant Baryon Chiral Perturbation Theory

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Abstract The straightforward application of dimensional regularization and minimal subtraction leads to problems with the power counting in a manifestly Lorentz-invariant formulation of baryon chiral perturbation theory. These complications can be avoided by using alternative renormalization schemes, such as infrared regularization and the extended on-mass-shell scheme. Some recent applications of these formalisms are discussed, including the chiral expansion of the nucleon mass and nucleon form factors. The extension of these methods to include additional degrees of freedom is also addressed.

1 Introduction

Chiral perturbation theory (ChPT) [15,16,36] is the effective field theory (EFT) of the Standard Model, describing the interactions of hadrons at energies well below a scale of about 1 GeV (for an introduction also see Ref. [29]). Instead of the underlying quarks and gluons, ChPT uses the degrees of freedom relevant to the given energy regime, such as pions and nucleons. The connection to the Standard Model is made through symmetries, with the EFT having the same symmetries and pattern of symmetry breaking as the more fundamental theory. In particular, besides the discrete symmetries of charge conjugation, parity, and time reversal, this means that ChPT incorporates the chiral symmetry of QCD and its spontaneous breaking. The pions as the lightest hadrons are identified as the Goldstone bosons of spontaneous chiral symmetry breaking. Because of the explicit breaking of chiral symmetry by the nonzero light quark masses, the pions are not massless. Their interactions with each other and with nucleons and external fields are described by an effective Lagrangian which, as for any EFT, consists of an infinite number of terms, each accompanied by its own low-energy coupling (LEC). The Lagrangian can be arranged according to a power counting [36] in a series of terms with an increasing number of derivatives and powers of the light quark masses. This corresponds to a dual expansion of observables in powers of small momenta and quark masses. On the level of Feynman diagrams, the power counting assigns to each diagram a chiral order D. Diagrams with higher chiral orders D are suppressed by powers of a small quantity q, which generically denotes either a small external momentum or the pion mass. In the mesonic sector, the power counting establishes a connection between the chiral and the loop expansion, with higher-order loop diagrams having higher chiral order D and thus being suppressed.

ChPT was originally developed to describe the interactions of pions with each other and with external fields [15,16,36]. It has been applied to a large number of pion properties and reactions, see e.g., Refs. [5,10,23,25,27]. ChPT was subsequently extended to also include the interactions with nucleons [17], considering

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matrix elements with one-nucleon initial and final states. Baryon ChPT can be used to describe nucleon properties such as the nucleon mass and form factors, as well as scattering processes at low energies, such as pion-nucleon scattering. For reviews, see e.g., Refs. [2,4,28].

2 Power Counting and Renormalization in Baryon ChPT

In the mesonic sector of ChPT, the application of dimensional regularization combined with the modified minimal subtraction scheme of ChPT results in expressions for tree and loop diagrams that satisfy the power counting. However, in baryon ChPT this procedure leads to loop diagram contributions that violate the power counting [17]. As pointed out in Ref. [17], these power counting violating terms can be avoided by application of a more suitable renormalization scheme. Several solutions to the power counting problem have been proposed, including the infrared regularization of Ref. [1] and the extended on-mass-shell (EOMS) scheme of Ref. [12]. Both of these approaches, unlike the also commonly used heavy-baryon formulation [3,20], retain manifest Lorentz invariance at each stage of the calculation.

To illustrate the main features of infrared regularization and the EOMS scheme, consider the dimensionally regularized one-loop integral

$$H\left(M^2, m^2, q_1, p_1, \ldots; n\right) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{\left[(k+q_1)^2 - M^2\right] \ldots \left[(k+p_1)^2 - m^2\right] \ldots}, \tag{1}$$

where M is the lowest-order pion mass, m the nucleon mass in the chiral limit, and the q_i and p_i are external momenta. The pion and nucleon propagators can be combined using Feynman parametrization to give the expression

$$H\left(M^2, m^2, q_1, p_1, \ldots; n\right) = \int_0^1 dz\, f\left(z, M, q_1, p_1, \ldots\right), \tag{2}$$

where $f(z, M, q_1, p_1, \ldots)$ is a function of the masses and external momenta. In infrared regularization the integral is rewritten as

$$\begin{aligned} H\left(M^2, m^2, q_1, p_1, \ldots; n\right) &= \int_0^\infty dz\, f\left(z, M, q_1, p_1, \ldots\right) - \int_1^\infty dz\, f\left(z, M, q_1, p_1, \ldots\right) && (3)\\ &\equiv I + R. && (4) \end{aligned}$$

The terms I and R are the so-called infrared singular and infrared regular parts of the integral, respectively. Each term satisfies the Ward identities of the theory separately. Further, the infrared singular term satisfies the power counting, while the infrared regular one contains terms that violate it. As shown in Ref. [1], R only contains terms that are analytic in the small parameters of ChPT and can thus be absorbed by counter terms. The renormalized expression of H thus satisfies both the power counting and the Ward identities.

The chiral expansion of the infrared regular piece R contains an infinite number of terms, including those terms that violate the power counting. The central idea of the EOMS scheme [12] is to only subtract the power counting violating terms instead of the closed-form expression of R. Because these terms are analytic in small quantities, they can be absorbed by counter terms, resulting in a renormalized expression for loop integrals that satisfies the power counting. To find the subtraction terms H_{subtr}, one expands the original integrand in small quantities and then interchanges summation and integration. The terms violating the power counting are now easily identified and can be subtracted, leaving the renormalized expression

$$H_R = H - H_{\mathrm{subtr}}, \tag{5}$$

which satisfies both the power counting and the Ward identities of ChPT. The EOMS scheme can also be extended to multi-loop diagrams [31] and diagrams containing additional heavy degrees of freedom, such as vector mesons [13] and the $\Delta(1232)$ resonance [18].

In its original formulation, the infrared regularization of Ref. [1] is applicable to one-loop integrals containing only pion and nucleon propagators. As shown in Ref. [32], infrared regularization can be reformulated analogously to the EOMS scheme. It can then also be applied to multi-loop diagrams and diagrams containing additional degrees of freedom [31]. For a different formulation of infrared regularization to include vector mesons see Ref. [6].

3 Nucleon Mass

The nucleon mass is one of the simplest quantities for the application of baryon ChPT since it does not depend on momentum transfers. The only relevant small quantity is the pion mass, and the chiral expansion of the nucleon mass therefore corresponds to a pion mass expansion. It has been calculated in a number of renormalization schemes, including heavy baryon ChPT [3,24], infrared renormalization [1], and the EOMS scheme [12]. Most calculations are restricted to the one-loop level, which corresponds to $\mathcal{O}(M^4)$ in the chiral counting, while Ref. [24] also considered terms at $\mathcal{O}(M^5)$. A complete calculation to $\mathcal{O}(M^6)$, including two-loop diagrams, was performed in Refs. [30,34]. The chiral expansion of the nucleon mass takes the form

$$\begin{aligned} m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \frac{M}{\mu} + k_4 M^4 \\ + k_5 M^5 \ln \frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2 \frac{M}{\mu} + k_8 M^6 \ln \frac{M}{\mu} + k_9 M^6, \end{aligned} \tag{6}$$

where the k_i are combinations of LECs and μ is a renormalization scale. The full expressions for the k_i can be found in Refs. [30,34]. Since a number of the LECs entering the coefficients k_i have not been determined, it is not possible to provide reliable numerical values for all terms in Eq. (6). However, the coefficients of the leading nonanalytic terms, such as $M^5 \ln \frac{M}{\mu}$ and $M^6 \ln^2 \frac{M}{\mu}$, only depend on known LECs and are renormalization-scheme independent. They can be used to study the pion-mass dependence of the nucleon mass, as shown in Fig. 1. The left panel shows the expected suppression of the higher-order term $k_7 M^6 \ln^2 \frac{M}{\mu}$ compared to $k_3 M^4 \ln \frac{M}{\mu}$ over a pion mass range up to approximately 400 MeV. However, the right panel shows that the higher-order term $k_5 M^5 \ln \frac{M}{\mu}$ becomes as large as the nominally lower-order term $k_2 M^3$ around pion masses of approximately 360 MeV. This does not constitute a complete analysis of the convergence behavior of Eq. (6) since not all terms at a given order are considered. However, the pion mass estimate at which the power counting is no longer reliable agrees with the analyses of Refs. [8,26].

4 Form Factors and Additional Degrees of Freedom

The electromagnetic form factors of the nucleons present an important test for any theoretical description of hadrons at low energies. Baryon ChPT at order $\mathcal{O}(q^4)$ fails to accurately describe the form factors beyond momentum transfers of $Q^2 \approx 0.1\,\mathrm{GeV}^2$ [14,21]. It was shown in Ref. [21] that the inclusion of vector mesons as dynamical degrees of freedom can improve the description of the form factors at higher Q^2 values. In standard baryon ChPT, vector mesons are integrated out and their contributions are taken into account through the LECs order by order. Keeping the vector mesons as dynamical degrees of freedom thus changes the values of the LECs, but at the same time also resums an infinite series of higher-order terms.

With the development of the EOMS scheme and the reformulation of infrared regularization, a consistent power counting for diagrams including internal vector meson propagators can be implemented. In Ref. [33]

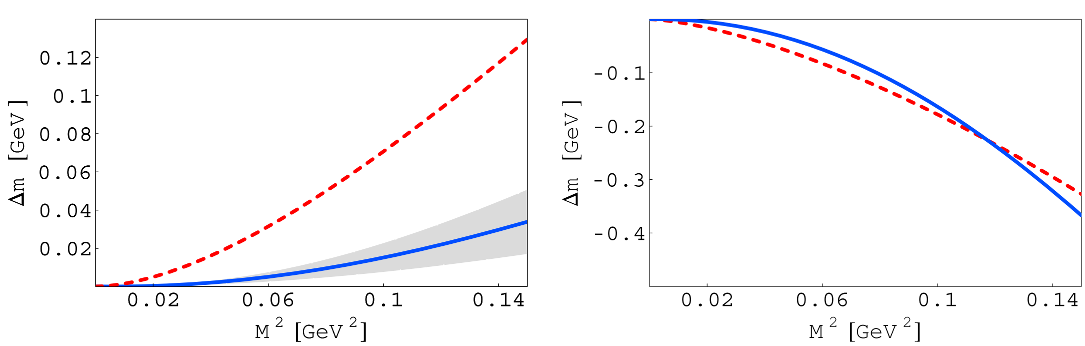

Fig. 1 Pion-mass dependence of terms in chiral expansion of nucleon mass. *Left panel*: *Solid blue line* represents $k_7 M^6 \ln^2 \frac{M}{\mu}$, *red dashed line* represents $k_3 M^4 \ln \frac{M}{\mu}$, the *grey band* denotes an error estimate (see Ref. [34]). *Right panel*: *Solid blue line* represents $k_5 M^5 \ln \frac{M}{\mu}$, *red dashed line* represents $k_2 M^3$ (color figure online)

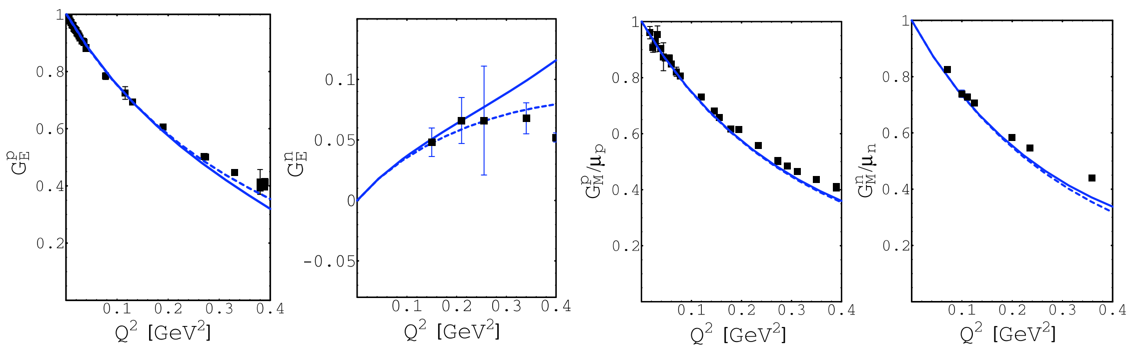

Fig. 2 Sachs form factors of the nucleon in manifestly Lorentz invariant baryon ChPT at $\mathcal{O}(q^4)$ including vector mesons. *Solid lines* EOMS scheme; *dashed lines* infrared regularization. The data are taken from Ref. [11]

the proton and neutron electromagnetic form factors were calculated in manifestly Lorentz invariant baryon ChPT to $\mathcal{O}(q^4)$ including vector mesons. To this order, loop diagrams containing vector meson propagators do not simultaneously contain pion propagators, and the contributions from these diagrams are completely absorbed in counter terms in infrared regularization. In this formalism, the only explicit contributions from vector mesons therefore come from tree-level diagrams. In the EOMS scheme, the loop contributions are small and the dominant vector meson contributions are again from tree-level diagrams. The results for the form factors in the EOMS scheme and in infrared regularization are shown in Fig. 2. The inclusion of vector mesons improves the description of the form factors considerably up to momentum transfers of $Q^2 = 0.4\,\mathrm{GeV}^2$.

5 Conclusions

There is no power counting problem in manifestly Lorentz invariant baryon ChPT provided a suitable renormalization scheme is employed. A number of such renormalization schemes have been proposed, including the infrared regularization and the EOMS scheme discussed here. The chiral expansion of the nucleon mass and the electromagnetic form factors only represent a small sample of the applications of these schemes. Further applications include, e.g., pion production [19], nucleon polarizabilities [22], the moments of GPDs [35], and pion momentum distributions in the nucleon [7]. A further interesting development is the application of the complex-mass scheme to extend the energy domain in which the EFT is applicable, see e.g., Ref. [9].

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