

Jerzy A. Przeszowski · Jan Żochowski

Scale and Lorentz Transformations at the Light-Front

Received: 2 November 2013 / Accepted: 19 November 2013 / Published online: 5 December 2013
© The Author(s) 2013. This article is published with open access at Springerlink.com

Abstract The light-front (LF) quantization is applied for the model of massive scalar field with self-interaction. We check some of the LF postulates by considering the Wightman function for this model. The scale symmetry imposed only on the LF quantization hypersurface and the Lorentz symmetry assumed for all points in Minkowski's space-time lead to a strong constraint for the Wightman functions, which is satisfied only by a free and massless scalar field. This result agrees with the recent Weinberg's result for a scale-symmetric theory. This means that one cannot expect the unitary equivalence of the Fock space for scalar fields with different masses at the LF hypersurface.

1 Introduction

In this paper we analyze the consistency and consequences of some postulates which are commonly accepted within the standard LF formulation. For this aim we consider the vacuum correlation function $\langle 0|\phi(x)\phi(y)|0\rangle$, which we will refer to as the 2-point Wightman function. This allows us to work mostly with the c-numbered (generalized) functions instead of the quantum field operators. We focus our attention on the following LF assumptions. First postulate—there are two generators of the Poincaré group, which have their spectra bounded from below, (our notation is explained in Appendix)

$$P_+ = P^- \geq 0, \quad P_- = P^+ \geq 0. \quad (1)$$

Commonly it is argued that P^+ is kinematical, thus it has no dependence on mass and interaction at the LF hypersurface. Then $P^- = H_{LF}$ is the LF Hamiltonian, which generates the temporal evolution. Second postulate—at the LF hypersurface $x^+ = 0$ the maximal number (7 out of 10) of the Poincaré generators are kinematical: P^+ , P^i , J_{+-} , J_{-i} , J_{ij} . Third postulate—the fields with different masses are unitarily equivalent at the LF hypersurface, thus after [1] *a null plane field theory is dilatation invariant in the null plane even if it has a mass*, where a null plane is a synonym of a light-front.

2 LF 2-Point Wightman Function

Let us consider the massive scalar hermitian field $\phi(x)$ with a self-interaction defined by the Lagrangian density in $D = 1 + 3$ dimensions

J. A. Przeszowski (✉) · J. Żochowski
Faculty of Physics, University of Białystok, ul. Lipowa 41, 15-424 Białystok, Poland
E-mail: jprzeszo@alpha.uwb.edu.pl

J. Żochowski
E-mail: jazo@alpha.uwb.edu.pl

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4. \quad (2)$$

Starting from this Lagrangian, the LF canonical quantization leads to the LF commutator for the quantum scalar field operator

$$[\phi(0, \bar{x}), \phi(0, \bar{y})] = -\frac{i}{4} \operatorname{sgn}(x^- - y^-) \delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp). \quad (3)$$

Since this commutator is a c-number, thus one easily finds its vacuum expectation value

$$\langle 0 | [\phi(0, \bar{x}), \phi(0, \bar{y})] | 0 \rangle = -\frac{i}{4} \operatorname{sgn}(x^- - y^-) \delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp), \quad (4)$$

which is the commutator function at the LF, or equivalently a relation for the 2-point Wightman function (2-WF)

$$\langle 0 | \phi(0, \bar{x}) \phi(0, \bar{y}) | 0 \rangle - \langle 0 | \phi(0, \bar{y}) \phi(0, \bar{x}) | 0 \rangle = -\frac{i}{4} \operatorname{sgn}(x^- - y^-) \delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp). \quad (5)$$

Since the hermiticity condition gives $\langle 0 | \phi(0, \bar{y}) \phi(0, \bar{x}) | 0 \rangle = \langle 0 | \phi(0, \bar{x}) \phi(0, \bar{y}) | 0 \rangle^*$, thus the canonical commutator fixes only the imaginary part of 2-WF

$$\Im \langle 0 | \phi(0, \bar{x}) \phi(0, \bar{y}) | 0 \rangle = -\frac{1}{8} \operatorname{sgn}(x^- - y^-) \delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp). \quad (6)$$

We take the vacuum state $|0\rangle$ as a unique state of energy and momentum zero with the translational invariance $e^{-iP \cdot x} |0\rangle = |0\rangle$, where the energy-momentum operators P_μ are the generators of translations $\phi(x) = e^{iP \cdot x} \phi(0) e^{-iP \cdot x}$. Thus, the 2-point Wightman function $\langle 0 | \phi(x) \phi(y) | 0 \rangle$ is translationally invariant

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle = \langle 0 | \phi(0) e^{-iP \cdot (x-y)} \phi(0) | 0 \rangle = \langle 0 | \phi(x-y) \phi(0) | 0 \rangle, \quad (7)$$

and one may consider $\langle 0 | \phi(x) \phi(0) | 0 \rangle = W_{(2)}(x)$ as a generic case. Further, the LF generators $P_- = P^+$ and $P_+ = P^-$ have non-negative spectra, which means that the 2-point LF Wightman function $W_{(2)}(x^+, \bar{x}) = \langle 0 | \phi(x) \phi(0) | 0 \rangle$ is a boundary value of an analytic function of x^+ and x^- from a lower half plane. Thus after acting with ∂_- and then implementing the analyticity in x^- coordinate we obtain

$$\Im \partial_- W_{(2)}(0, \bar{x}) = -\frac{1}{4} \delta(x^-) \delta^2(\mathbf{x}_\perp) \Rightarrow \partial_- W_{(2)}(0, \bar{x}) = -\frac{\delta^2(\mathbf{x}_\perp)}{4\pi} \frac{1}{x^- - i0}, \quad (8)$$

where we denote the distribution $\mathcal{S}'(\mathbb{R})$ in x^- as $1/(x^- - i0) = \lim_{\epsilon \rightarrow 0^+} 1/(x^- - i\epsilon)$. We stress that this expression for $\partial_- W_{(2)}(0, \bar{x})$ is a non-perturbative exact result. It has no dependence on mass and interaction, thus in some sense it may suggest an agreement with the standard LF formulation, where at the LF one introduces the mass independent Fock representation for $\phi(0, \bar{x})$. However we stress that our result is fundamentally different since here we do not implement any explicit representation for $\phi(0, \bar{x})$.

3 Lorentz Symmetry

The proper Lorentz transformation $x'^\mu = \Lambda^\mu_{\nu} x^\nu$ is imposed in the quantum field theory by the unitary operator $U_L(\Lambda)$. Thus a scalar field operator transforms as

$$U_L(\Lambda) \phi(x) U_L^{-1}(\Lambda) = \phi(x') \approx \phi(x) + \frac{1}{2} \omega^{\mu\nu} (x_\nu \partial_\mu - x_\mu \partial_\nu) \phi(x) \quad (9)$$

where for an infinitesimal transformation we take $\Lambda_{\mu\nu} = g_{\mu\nu} + \omega_{\mu\nu}$ with an arbitrary parameter $\omega_{\mu\nu} = -\omega_{\nu\mu}$. The vacuum state is Lorentz invariant $|0\rangle = U_L(\Lambda)|0\rangle$, which leads to the Lorentz transformation law for the Wightman function $\langle 0 | \phi(x') \phi(0) | 0 \rangle = \langle 0 | \phi(x) \phi(0) | 0 \rangle$ for arbitrary Λ^μ_{ν} . Then for an infinitesimal proper Lorentz transformation this leads to the differential equations for 2-WF

$$(x_\nu \partial_\mu - x_\mu \partial_\nu) \langle 0 | \phi(x) \phi(0) | 0 \rangle = 0. \quad (10)$$

The Lorentz transformations generated by J_{-i} , J_{+i} , J_{+-} and J_{ij} lead to the differential conditions on 2-WF in the LC coordinates,

$$x^+ \partial_i W_{(2)}(x^+, \bar{x}) + x^i \partial_- W_{(2)}(x^+, \bar{x}) = 0, \quad (11a)$$

$$x^- \partial_i W_{(2)}(x^+, \bar{x}) + x^i \partial_+ W_{(2)}(x^+, \bar{x}) = 0, \quad (11b)$$

$$x^+ \partial_+ W_{(2)}(x^+, \bar{x}) - x^- \partial_- W_{(2)}(x^+, \bar{x}) = 0, \quad (11c)$$

$$\epsilon^{ij} x^i \partial_j W_{(2)}(x^+, \bar{x}) = 0, \quad (11d)$$

respectively. We have already found $\partial_- W_{(2)}(0, \bar{x})$ in (8) using other arguments, thus it is worthy to insert it into above equations. Then (11c) leads to the nontrivial LF limits

$$\lim_{x^+ \rightarrow 0} x^+ \partial_+ W_{(2)}(x^+, \bar{x}) = \lim_{x^+ \rightarrow 0} x^- \partial_- W_{(2)}(x^+, \bar{x}) = x^- \partial_- W_{(2)}(0, \bar{x}) = -\frac{1}{4\pi} \delta^2(\mathbf{x}_\perp) \neq 0. \quad (12)$$

This means that for $x^+ \sim 0$, the Wightman function $W_{(2)}(x^+, \bar{x})$ has a logarithmic dependence on x^+ and x^- along a light-like direction. As a consequence the generator J_{+-} , which is defined canonically by

$$J_{+-} = x^+ P^- - \int d^3 \bar{x} T^{++} x^-, \quad (13)$$

is not a kinematic operator at the LF limit

$$\lim_{x^+ \rightarrow 0} J_{+-} \neq - \int d^3 \bar{x} T^{++} x^-. \quad (14)$$

This means that the LF limit taken as a strong condition for an operator may be misleading, thus rather one should consider limit in a weak sense—when J_{+-} appears inside the vacuum expectation value.

4 Scale Transformation

The scale transformation (dilatation) $x' = lx$ for a classical scalar field is $\phi(x) \rightarrow \phi'(x') = l^{-d} \phi(x)$, where the scaling dimensionality is $d = D/2 - 1$. In the $D = 1 + 3$ space-time dimension $d = 1$ and the unitary operator $U_S(l)$ generates the scale (dilatation) transformation for a quantum scalar field

$$U_S(l) \phi(x) U_S^{-1}(l) = l^d \phi(lx) \approx \phi(x) + \epsilon(1 + x^\mu \partial_\mu) \phi(x) \quad (15)$$

where for an infinitesimal transformation $l = 1 + \epsilon$. Then the scale invariant vacuum state $|0\rangle = U_S^{-1}(l)|0\rangle$ leads to the scale transformation of 2-WF $\langle 0|\phi(x)\phi(0)|0\rangle = l^2 \langle 0|\phi(lx)\phi(0)|0\rangle$. Thus for an infinitesimal scale transformation one obtains

$$(2 + x^\mu \partial_\mu) \langle 0|\phi(x)\phi(0)|0\rangle = 0, \quad (16)$$

which in the LC coordinates looks as

$$(2 + x^- \partial_- + x^+ \partial_+ + x^i \partial_i) \langle 0|\phi(x)\phi(0)|0\rangle = 0. \quad (17)$$

The combination of partial derivatives, which appears in this equation $x^- \partial_- + x^+ \partial_+$ is quite similar to that which appears in the Lorentz transformation generated by J_{+-}

$$(x^- \partial_- - x^+ \partial_+) \langle 0|\phi(x)\phi(0)|0\rangle = 0. \quad (18)$$

Evidently if one imposes both the scale symmetry and the Lorentz symmetry, then one effectively arrives at the equation which does not contain the LF temporal derivative ∂_+

$$(2 + 2x^- \partial_- + x^i \partial_i) \langle 0|\phi(x)\phi(0)|0\rangle = 0 \quad (19)$$

or after a simple rearrangement

$$2x^- \partial_- \langle 0|\phi(x)\phi(0)|0\rangle + \partial_i \left[x^i \langle 0|\phi(x)\phi(0)|0\rangle \right] = 0. \quad (20)$$

This is a truly kinematical relation, which must hold at every fixed value of x^+ irrespective of the equation of motion for the scalar field. Thus it forms a nonperturbative constraint for 2-WF. We observe that $x^i \langle 0 | \phi(x) \phi(0) | 0 \rangle$ has a regular LF limit, since it has no contribution from points lying on a light-like line, thus at the LF $x^+ = 0$ we write

$$2x^- \partial_- W_{(2)}(0, \bar{x}) + \partial_i \left[x^i W_{(2)}(0, \bar{x}) \right] = 0. \quad (21)$$

Further, using (12) we arrive at the inhomogeneous partial differential equation for $x^i W_{(2)}(0, \bar{x})$

$$\partial_i \left[x^i W_{(2)}(0, \bar{x}) \right] = \frac{1}{2\pi} \delta^2(\mathbf{x}_\perp). \quad (22)$$

Moreover for the Lorentz transformation conditions we may impose the LF limits which produce further equations for $x^i W_{(2)}(0, \bar{x})$

$$0 = \lim_{x^+ \rightarrow 0} x^+ \partial_i W_{(2)}(x^+, \bar{x}) = - \lim_{x^+ \rightarrow 0} \partial_- \left[x^i W_{(2)}(x^+, \bar{x}) \right] = -\partial_- \left[x^i W_{(2)}(0, \bar{x}) \right] = 0 \quad (23a)$$

$$0 = \lim_{x^+ \rightarrow 0} \epsilon^{ij} x^i \partial_j W_{(2)}(x^+, \bar{x}) = \epsilon^{ij} \partial_j \lim_{x^+ \rightarrow 0} \left[x^i W_{(2)}(x^+, \bar{x}) \right] = \epsilon^{ij} \partial_j \left[x^i W_{(2)}(0, \bar{x}) \right] = 0, \quad (23b)$$

where we take $\lim_{x^+ \rightarrow 0} x^+ W_{(2)}(x^+, \bar{x}) = 0$, because for $x^+ \sim 0$ the singular part of 2-WF behaves as $\ln |x^+|$. These three equations $x^i W_{(2)}(0, \bar{x})$ can be solved as follows. From (23a) we see that there is no x^- -dependence of $x^i W_{(2)}(0, \bar{x})$, while (22) and (23b) are the divergence and rotation in \mathbb{R}^2 -space of \mathbf{x}_\perp . The solution is unique in the sense of distributions $\mathcal{S}'(\mathbb{R}^2)$

$$x^i W_{(2)}(0, \bar{x}) = \frac{1}{4\pi^2} \frac{x^i}{x_\perp^2}. \quad (24)$$

Thus if one imposes the scale symmetry at the LF hypersurface for the Lorentz-invariant scalar field model, then one finds both parts of 2-WF, which have regular LF limits, $x^i W_{(2)}(0, \bar{x})$ and $x^- \partial_- W_{(2)}(0, \bar{x})$. Moreover these expressions satisfy the partial differential equation

$$\Delta_\perp [x^i W_{(2)}(0, \bar{x})] + 2x^- \partial_- \partial_i W_{(2)}(0, \bar{x}) = 0, \quad (25)$$

which will be useful in our further analysis. Evidently $x^i W_{(2)}(0, \bar{x})$ and $x^- \partial_- W_{(2)}(0, \bar{x})$ are mass-independent, but this property does not necessarily mean that at the LF hypersurface one has the equivalence between fields with different masses. Here we stress that the scale symmetry at the LF hypersurface leads to a very strong constraint on the LF Wightman function.

Since the Lorentz symmetry holds for different LF times, we may differentiate the Lorentz transformation condition $(x^+ \partial_i + x^i \partial_-) W_{(2)}(x) = 0$, with respect to x^+ , which gives

$$\left(x^+ \partial_+ \partial_i + \partial_i + x^i \partial_- \partial_+ \right) W_{(2)}(x) = 0. \quad (26)$$

Then by rearranging terms, we arrive at the equation at arbitrary x^+

$$\Delta_\perp [x^i W_{(2)}(x)] + 2x^+ \partial_+ \partial_i W_{(2)}(x) + x^i [2\partial_- \partial_+ - \Delta_\perp] W_{(2)}(x) = 0. \quad (27)$$

Taking the restriction to the LF hypersurface $x^+ = 0$ we get

$$\Delta_\perp [x^i W_{(2)}(0, \bar{x})] + 2x^- \partial_- \partial_i W_{(2)}(0, \bar{x}) + x^i \lim_{x^+ \rightarrow 0} [2\partial_- \partial_+ - \Delta_\perp] W_{(2)}(x) = 0 \quad (28)$$

thus due to (25) we find

$$x^i \lim_{x^+ \rightarrow 0} \square W_{(2)}(x) = 0 \quad \square = 2\partial_- \partial_+ - \Delta_\perp. \quad (29)$$

Moreover we check the sequence of limits

$$\lim_{x^+ \rightarrow 0} x^+ \square W_{(2)}(x) = 2\partial_- \lim_{x^+ \rightarrow 0} x^+ \partial_+ W_{(2)}(x^+, \bar{x}) - \Delta_\perp \lim_{x^+ \rightarrow 0} x^+ W_{(2)}(x^+, \bar{x}) = -\frac{1}{2\pi} \partial_- \delta^2(\mathbf{x}_\perp) = 0. \quad (30)$$

Finally we may collect these equations into one relation

$$\lim_{x^+ \rightarrow 0} (2x^+ x^- - x_\perp^2) W_{(2)}(x^+, \bar{x}) = 0, \quad (31)$$

which is the LF restriction of Weinberg's equation [2] $x^2 \square W_{(2)}(x) = 0$.

5 General Scalar Field

Starting from assumptions for our scalar field model we may introduce the Källén–Lehmann [3,4] spectral representation for 2-WF

$$\langle 0|\phi(x)\phi(0)|0\rangle = \int_0^\infty d\sigma^2 \rho(\sigma^2)\Delta_+(x; \sigma), \quad (32)$$

where the spectral density, defined as

$$\rho(q) = (2\pi)^3 \sum_n \delta^4(p_n - q) |\langle 0|\phi(0)|n\rangle|^2, \quad (33)$$

is a non-negative real valued function, while $|n\rangle$ are the eigenstates of 4-momenta operators $P^\mu|n\rangle = p_n^\mu|n\rangle$. From the Lorentz symmetry, the spectral density $\rho(q)$ is non-vanishing only for momenta in a forward light-cone $q^2 \geq 0$

$$\rho(q) = \rho(q^2)\theta(q^0) \geq 0. \quad (34)$$

The Lorentz invariant singular function $\Delta_+(x; \sigma)$, defined as

$$\Delta_+(x; \sigma) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^4} d^4k e^{-ik \cdot x} \Theta(k^0) \delta(k^2 - \sigma^2), \quad (35)$$

satisfies the Lorentz covariance relations $(x_\nu \partial_\mu - x_\mu \partial_\nu)\Delta_+(x; \sigma) = 0$ and near $x^+ \sim 0$ it behaves like [5]

$$\begin{aligned} \Delta_+(x; \sigma) = & -\frac{1}{4\pi} \left[\ln(\sigma^2|x^+x^-|) + \gamma_E + i\frac{\pi}{2} (\text{sgn}(x^+) + \text{sgn}(x^-)) \right] \\ & \delta^2(\mathbf{x}_\perp) - \frac{1}{4\pi^2} D_i \left[\frac{x^i}{x_\perp^2} K_0(\sigma x_\perp) \right] + 0(x^+x^-), \end{aligned} \quad (36)$$

where γ_E is the Euler–Mascheroni constant, D_i is a distributional partial derivative and $K_0(z)$ is the modified Bessel function. This function has a logarithmic singularity at the LF hypersurface along lines with $\mathbf{x}_\perp = 0$, which connect points with a light-like separation. But its partial derivative ∂_- has a regular LF limit

$$\lim_{x^+ \rightarrow 0} \partial_- \Delta_+(x) = \partial_- \Delta_+(0, \bar{x}) = -\frac{1}{4\pi} \left[\mathcal{P} \frac{1}{x^-} + i\pi \delta(x^-) \right] \delta^2(\mathbf{x}_\perp) = -\frac{1}{4\pi} \frac{\delta^2(\mathbf{x}_\perp)}{x^- - i0}, \quad (37)$$

where \mathcal{P} denotes the Cauchy principal prescription. Thus $\partial_- \Delta_+(0, \bar{x})$ is mass-independent at x^+ and the spectral representation gives

$$\partial_- \langle 0|\phi(0, \bar{x})\phi(0)|0\rangle = \int_0^\infty d\sigma^2 \rho(\sigma^2) \partial_- \Delta_+(0, \bar{x}; \sigma) = -\frac{1}{4\pi} \frac{\delta^2(\mathbf{x}_\perp)}{x^- - i0} \int_0^\infty d\sigma^2 \rho(\sigma^2). \quad (38)$$

Thus from (8) we obtain the integral condition for the spectral amplitude

$$\int_0^\infty d\sigma^2 \rho(\sigma^2) = 1, \quad (39)$$

which agrees with the analysis in other formulations (like the equal-time approach). Further, also $x^i \Delta_+(x)$ has a finite LF limit, for $x_\perp > 0$

$$\lim_{x^+ \rightarrow 0} x^i \Delta_+(x) = -\frac{x^i}{4\pi^2} \frac{1}{x_\perp} \frac{d}{dx_\perp} K_0(\sigma x_\perp) = \frac{x^i}{4\pi^2} \frac{\sigma}{x_\perp} K_1(\sigma x_\perp), \quad (40)$$

so the spectral representation for $x_{\perp} > 0$ gives

$$x^i \langle 0 | \phi(0, \bar{x}) \phi(0) | 0 \rangle = \int_0^{\infty} d\sigma^2 \rho(\sigma^2) x^i \Delta_+(0, \bar{x}; \sigma) = \int_0^{\infty} d\sigma^2 \rho(\sigma^2) \frac{x^i}{4\pi^2 x_{\perp}} \sigma K_1(\sigma x_{\perp}). \quad (41)$$

Thus the scaling symmetry at the LF hypersurface leads to another integral condition for the spectral density

$$\frac{1}{x_{\perp}} = \int_0^{\infty} d\sigma^2 \rho(\sigma^2) \sigma K_1(\sigma x_{\perp}), \quad (42)$$

which must be satisfied for arbitrary $x_{\perp} > 0$. One may take the Bessel transform of this relation by integrating its both sides with $x_{\perp} J_1(\alpha x_{\perp})$, keeping $\alpha > 0$,

$$\int_0^{\infty} dx_{\perp} J_1(\alpha x_{\perp}) = \int_0^{\infty} d\sigma^2 \rho(\sigma^2) \sigma \int_0^{\infty} dx_{\perp} x_{\perp} J_1(\alpha x_{\perp}) K_1(\sigma x_{\perp}). \quad (43)$$

From the integrals

$$\int_0^{\infty} dx_{\perp} J_1(\alpha x_{\perp}) = \frac{1}{\alpha}, \quad \int_0^{\infty} dx_{\perp} x_{\perp} J_1(\alpha x_{\perp}) K_1(\sigma x_{\perp}) = \frac{\alpha}{\sigma(\alpha^2 + \sigma^2)}, \quad (44)$$

the Bessel transformation leads to a simple form of the integral condition

$$\frac{1}{\alpha} = \int_0^{\infty} d\sigma^2 \rho(\sigma^2) \frac{\alpha}{\alpha^2 + \sigma^2}, \quad \Rightarrow \quad \int_0^{\infty} d\sigma^2 \rho(\sigma^2) \frac{\sigma^2}{\alpha^2 + \sigma^2} = 0, \quad (45)$$

where we have used (39). Thus the spectral density is uniquely determined as $\rho(\sigma^2) = 2\delta(\sigma^2)$, which means that there is only a free massless mode.

6 Conclusions and Prospects

Our analysis agrees with Weinberg's result, that the scaling and Lorentz symmetries allow only for a free and massless field. While Weinberg discusses fields with arbitrary spin, then we consider here only a scalar field case. Another difference is that we assume the scaling symmetry only at the LF hypersurface, while Weinberg imposes it for all points in Minkowski's space-time. Our analysis is intended for the LF canonical formulation and it falsifies one of the commonly accepted postulate of the unitary equivalence of scalar fields with different masses at the LF hypersurface. We point out that the scaling symmetry is necessary for the mass-independence, thus our result means that one cannot build a Fock space which is mass-independent at the fixed LF hypersurface. We also find that the LF restriction for a quantum operator may lead to an inconsistency, as for J_{+-} at the LF hypersurface $x^+ = 0$.

We plan to extend our analysis for fields with higher spins, specially for fermions. This would allow to study the Yukawa model at the LF hypersurface. Much more interesting cases of QED and QCD are eventually our next aim, but we are aware that the Lorentz and scaling transformations would be non-trivial for gauge fields.

Acknowledgments We would like to thank the organizers of the LC2013 Workshop *Venturing off the lightcone—local versus global features* for all their efforts and hospitality.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

Appendix: Notation

We define the light-cone (LC) coordinates as $x^+ = (x^0 + x^3)/\sqrt{2}$, $x^- = (x^0 - x^3)/\sqrt{2}$, $\mathbf{x}_\perp^i = (x^1, x^2)$, $i \in \{1, 2\}$, $x_\perp = |\mathbf{x}_\perp|$. The non-vanishing metric components are $g^{+-} = g^{-+} = 1$, $g^{ij} = -\delta_{ij}$ and the partial derivatives for LC coordinates are $\partial_+ = \partial^- = \partial/\partial x^+$, $\partial_- = \partial^+ = \partial/\partial x^-$, $\partial_i = -\partial^i = \partial/\partial x^i$. The 2-dimensional tensor ϵ^{ij} is normalized as $\epsilon^{12} = 1$. The LF hypersurface is described by a fixed x^+ coordinate and we consider the canonical quantization hypersurface with the LC coordinates $x^+ = 0$ and $\bar{x} = (x^-, \mathbf{x}_\perp)$. The LF evolution develops with x^+ being the LF temporal parameter.

References

1. Rohrlich, F.: Null plane field theory. Acta Physica Austriaca, Suppl. VIII, 227–322, lecture given at the X Internationale Univeritätswochen für Kernphysik, Schladming (1971)
2. Weinberg, S.: Minimal fields of canonical dimensionality are free. Phys. Rev. D **86**, 105015 (2012)
3. Källén, C.: On the definition of the renormalization constants in quantum electrodynamics. Helv. Phys. Acta **25**, 417 (1952)
4. Lehmann, H.: Über Eigenschaften von Ausbreitungsfunktionen und Renormierung-konstanten quantisierter Felder. Nuovo Cim. **11**, 342 (1954)
5. Przeszowski, J.A.: Lorentz symmetry for the light-front Wightman functions. Acta Phys. Polon. Proc. Suppl. B **6**, 327–333 (2012)