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Does Σ–Σ–α Form Quasi-Bound States?

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Abstract For the Σ–Σ–α system we theoretically look into the possible existence of a quasi-bound state in the framework of Faddeev calculations. We are particularly interested in the state of total iso-spin T=2, because there is no strong conversion between Ξ–N–α and Λ–Λ–α. An analytic continuation using the point method is applied to search the eigenvalue in the complex energy plane. In our results the Σ–Σ–α three-body system has two quasi-bound states ($J^\pi = 0^+$) where, depending on the potential parameters in the Nijmegen NSC97 model potential, the energy ranges between −1.4 and −2.4 MeV and the level width is about 0.4 MeV for the ground state. In addition, we obtained the excited state at −0.15 MeV (width 4 MeV).

1 Introduction

Strangeness S = −2 hypernuclei provide information on baryon–baryon forces in the state of S = −2. Only three nuclei have been identified so far, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{6}_{\Lambda\Lambda}$He, ${}^{13}_{\Lambda\Lambda}$B, etc [1–4]. The challenge is to understand their binding energies and decay properties. These nuclei are especially interesting since the S = −2 two-baryon system is rich in structure due to the conversions between $\Lambda\Lambda$, ΞN and $\Sigma\Sigma$. Baryon–baryon forces for S = 0,−1 and −2 are being investigated in the meson exchange picture [5–8] or using quark models [9]. While there is a wealth of data for S = 0, which allows to fix force parameters, the situation is still much open in the S = −1 and −2 sectors.

In this study we would like to focus on the system $\Sigma-\Sigma-\alpha$ in the state of total iso-spin T=2. If the α-particle would be inert, that system could not convert to $\Xi-N-\alpha$ or $\Lambda-\Lambda-\alpha$. Therefore in case the forces would be strong enough, there might exist a low lying state with a small width. We investigate that system $\Sigma-\Sigma-\alpha$ under effective simplifying assumptions. The $\Sigma-\alpha$ interaction is modeled via an optical potential based on the Nijmegen model D and the $\Sigma-\Sigma$ interaction in the state of total iso-spin T=2 is taken either

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Table 1 The complex energy eigen values of the ground state in MeV from the $\Sigma-\Sigma-\alpha$ breakup threshold using the simulated Gaussian and the original Nijmegen $\Sigma-\Sigma$ potentials together with the complex $\Sigma-\alpha$ potentials

	NSC97a	NSC97c	NSC97e
Nijmegen	−1.418 − i0.202	−2.34 − i0.014	−2.376 − i0.191
Gaussian	−1.492 − i0.218	−2.323 − i0.017	−2.354 − i0.211

directly as a meson theoretical Nijmegen potential of the type NSC97 [6,7] or a simulated version thereof of the Gaussian type [10]. Using these potentials we solve precisely in the Faddeev scheme [11].

We solved the Faddeev equation by S-wave approximation [11]. Total spin and parity of $\Sigma-\Sigma-\alpha$ system are chosen $J^{\pi}=0^{+}$ as the ground state. Here we recently find an excited 0^{+} state to serach along the energy trajectory in the complex energy plane for the ${}^{6}_{\Sigma\Sigma}$He system. In next section we would like to demonstrate the excited state as well as the ground state.

2 Results and Outlook

For the $\Sigma-\Sigma$ potential we use either the original Nijmegen potentials NSC97a,c,e [5] or the simulated Gaussian forms thereof [10]. The $\Sigma-\alpha$ potential is chosen to be complex to provide for absorptive processes, like the ones mentioned in the introduction. We use the form [10]

$$V_{\Sigma\alpha}(r)=\sum_{i=1}^{2}V_i e^{-(r/\mu_i)^2}+i\sum_{i=1}^{2}U_i e^{-(r/\mu_i)^2} \tag{1}$$

with the parameters $V_1=-21.3$ MeV, V_2[1] $=4.8$ MeV for the real part and $U_1=4.07$ MeV, $U_2=-11.73$ MeV for the imaginary part. Further, one has $\mu_1=1.3$ fm and $\mu_2=1.7$ fm. In the Brueckner theory frame the potential was derived from the original Nijmegen model D version [12], and transformed into this five Gaussian form. The imaginary part arises due to $\Lambda-N$ to $\Sigma-N$ conversion.

The Faddeev equations in the momentum representation given as a set of coupled integral equations that are discretized in the program code. We refer for numerical details to [13,14]. The energy eigenvalue E is determined as follows. The homogenous set of coupled equations is schematically written as

$$\eta(E)\psi=K(E)\psi \tag{2}$$

where $\eta(E)$ equals 1 at the energy eigenvalue E and $K(E)$ is the integral kernel of the Faddeev equation. The eigenvalue η is determined either by a simple power method or by a Lanczos type algorithm. The energy search in the complex energy plane was greatly simplified by using a method of analytical continuation in the form of the point method [15] which is recently applied to the Faddeev continuum equations [16,17] and to the Yakubovsky four-body continuum equations [18,19].

In order to search for the eigen energy E at $\eta(E)=1$, in the case of real number energy we simply scan the function $\eta(E)$ on the real axis of E. However, the eigen energy of the quasi-bound states and resonance states is complex number in general. Seaching the complex eigen energy we need an appropriate way which was already succeeded in the case of the ground state in the ${}^{6}_{\Sigma\Sigma}$He system [11]. If the strength is gradually varied back to the original potential we could find an energy trajectory in the complex energy plane (see Fig. [11]). Table 1 shows the eigen energies for the ground state. The behavior of the trajectory is experientially known that the imaginary part of the eigen energy is decreasing up to the multiplicative factor 0.4 by which the attractive real part and the overall imaginary parts are multiplied. (see Fig. 4 [11]). Beyond the factor 0.4 the imaginary part of eigen energy becomes small to be close to the real axis.

Now, we vary the only repulsive real part of the potential [the parameter V_2 in Eq. (1)]. The effect of increasing the strength for V_2 makes small the imaginary part of eigen value. Using the way we found a new trajectory in the complex energy plane not only to the ground state but also to the excited state. In Table 2 we demonstrate some eigen energies corresponding to the $\Sigma-\alpha$ potential which the real repulsive part V_2 were multiplied by 4, 3, 2 and 1. Figure 1 shows the new trajectories for the ground state and the excited one. Therefore, we obtained the excited state ($E=-0.151-$ i1.914 MeV) as a realistic quasi-bound state without the multiplicative factor.

[1] Later we vary this parameter V_2.

Table 2 The complex energy eigen values of the quasi-bound states in MeV from the $\Sigma-\Sigma-\alpha$ breakup threshold by changing the real repulsive part of the $\Sigma-\alpha$ potential. The potential parameter V_2 are varied by the multiplicative factors 4, 3, 2 and 1 from the left hand side. The $\Sigma-\Sigma$ potential is used by Nijmegen NSC97e version

	$V_2 = 19.2$ MeV	$V_2 = 14.4$ MeV	$V_2 = 9.6$ MeV	$V_2 = 4.8$ MeV
Ground state	−2.390 − i0.057	−2.407 − i0.084	−2.415 − i0.131	−2.376 − i0.191
Excited state	0.206 − i0.415	0.043 − i0.598	−0.133 − i0.952	−0.151 − i1.914

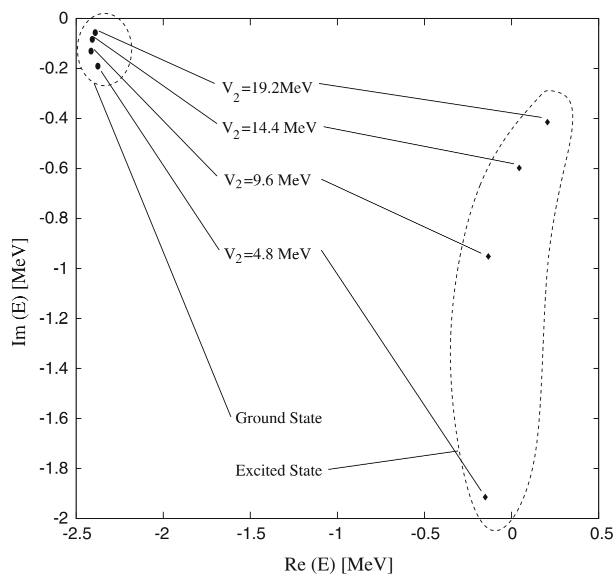

Fig. 1 Energy trajectories in the complex energy plane for ${}^{6}_{\Sigma\Sigma}$He system by changing the real repulsive part of the $\Sigma-\alpha$ potential. The $\Sigma-\Sigma$ potential is used by Nijmegen NSC97e version. The spots of bullet (*diamond*) mark belong to the ground (excited) states

Upcoming meson based $\Sigma-\Sigma$ potentials without a bound state should be used and in addition the effective $\Sigma-\alpha$ potential should be generated more consistently using realistic α particle wave functions in conjunction with Σ– nucleon forces related to the same theoretical model as for the $\Sigma-\Sigma$ interaction. Although the obtained excited state has a wide width these two low lying states (ground and excited) are for the $\Sigma-\Sigma-\alpha$ system with isospin $T = 2$ would provide interesting additional information on the dynamics in the strangeness $S = -2$ sector.

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