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Higher Fock Sectors in the Wick-Cutkosky Model*

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Abstract. In the Wick-Cutkosky model we analyze nonperturbatively, in light-front dynamics, the contributions of two-body and higher Fock sectors to the total norm and electromagnetic form factor. It turns out that two- and three-body sectors always dominate. For a maximal value of the coupling constant $\alpha = 2\pi$, corresponding to the zero bound-state mass $M = 0$, they contribute 90% to the norm. With decrease of α the two-body contribution increases up to 100%. The form factor asymptotic is always determined by a two-body sector.

1 Introduction

In field theory, the state vector $|p\rangle$ is described by an infinite set of the Fock components, corresponding to different numbers of particles. In light-front dynamics [1, 2] the state vector is defined on the light-front plane $\omega \cdot x = 0$, where ω is the null four-vector ($\omega^2 = 0$). The wave functions are expressed in terms of the variables $\boldsymbol{k}_\perp, x$: $\psi = \psi(\boldsymbol{k}_{1\perp}, x_1; \boldsymbol{k}_{2\perp}, x_2; \ldots; \boldsymbol{k}_{n\perp}, x_n)$. The total norm (equaled to 1) is given by the sum over all the sectors, $\sum_n N_n = 1$, where the n-body contribution N_n reads

$$N_n = (2\pi)^3 \int |\psi(\boldsymbol{k}_{1\perp}, x_1; \boldsymbol{k}_{2\perp}, x_2; \ldots; \boldsymbol{k}_{n\perp}, x_n)|^2 \times \delta^{(2)}\left(\sum_{i=1}^{n} \boldsymbol{k}_{\perp i}\right) \delta\left(\sum_{i=1}^{n} x_i - 1\right) 2 \prod_{i=1}^{n} \frac{d^2k_{\perp i}\, dx_i}{(2\pi)^3 2x_i}. \quad (1)$$

In applications, the infinite set of the Fock components is usually truncated to a few components only. The belief that a given Fock sector dominates (with two or

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three quarks, for instance) is often based on intuitive expectations and on "experimental evidences" rather than on field-theoretical analysis.

In the Wick-Cutkosky model two massive scalar particles interact by the ladder exchange of massless scalar particles. The two-body sector contains two massive particles. Higher sectors contain two massive and $1, 2, \ldots$ massless constituents.

In the present paper, based on the work [3], we present the results of our study in the Wick-Cutkosky model of contributions of the two- and three-body sectors to the total norm. Subtracting them from 1, we get a total contribution of all the sectors with $n \geq 4$. Besides, we also calculate their contributions to the electromagnetic form factor. Calculations are carried out nonperturbatively in full range of binding energy $0 \leq B \leq 2m$.

2 Bethe-Salpeter Amplitude in the Wick-Cutkosky Model

We use the Bethe-Salpeter (BS) amplitude known explicitly in the Wick-Cutkosky model [4]. For the ground state with zero angular momentum it reads

$$\Phi(k,p) = -\frac{i}{\sqrt{4\pi}} \int_{-1}^{+1} \frac{g_M(z)\, dz}{(m^2 - M^2/4 - k^2 - zp\cdot k - i\epsilon)^3}, \tag{2}$$

where k and p are relative and total four-momenta, m is the massive constituent mass, and M is the mass of the composite system. The representation (2) is valid and exact for the zero-mass exchange. The function $g_M(z)$ is determined by the integral equation

$$g_M(z) = \frac{\alpha}{2\pi} \int_{-1}^{1} K(z,z') g_M(z')\, dz' \tag{3}$$

with the kernel

$$K(z,z') = \frac{m^2}{m^2 - \frac{1}{4}(1-z'^2)M^2} \left[\frac{(1-z)}{(1-z')} \theta(z-z') + \frac{(1+z)}{(1+z')} \theta(z'-z) \right].$$

Here $\alpha = g^2/(16\pi m^2)$ and g is the coupling constant in the interaction Hamiltonian $H^{\rm int} = -g\varphi^2(x)\chi(x)$. In the nonrelativistic limit the interaction is reduced to the Coulomb potential $V(r) = -\alpha/r$.

The normalization condition for $g_M(z)$ is found from the requirement that the full electromagnetic form factor $F_{\rm full}(Q^2)$ (calculated with full state vector $|p\rangle$ and, hence, incorporating all the Fock components) equals to 1 at $Q = 0$. The form factor is expressed in terms of the BS amplitude:

$$(p+p')^\mu F_{\rm full}(Q^2) = -i \int \frac{d^4k}{(2\pi)^4} (p+p'-2k)^\mu\, (m^2-k^2)\, \Phi\left(\frac{1}{2}p - k, p\right) \Phi\left(\frac{1}{2}p' - k, p'\right). \tag{4}$$

We substitute here the BS amplitude (2) and find the normalization of $g_M(z)$ from the equality $F_{\rm full}(0) = 1$. The details of the calculations are given in ref. [3].

The function $g_M(z)$ is found from Eq. (3) analytically in the limiting cases of a small binding energy $B = 2m - M$ ($\alpha \to 0$, $B \to 0$, $M \to 2m$) and of an extremely large binding energy ($\alpha = 2\pi$, $B = 2m$, $M = 0$). In the case $M \to 2m$ it reads

$$g_M(z) = 8\sqrt{2}\pi\alpha^{5/2}m^3\left(1+\frac{5\alpha}{\pi}\log\alpha\right)\left[1-|z|+\frac{\alpha}{2\pi}(1+|z|)\log(z^2+\alpha^2/4)\right]. \tag{5}$$

In contrast to the solution found in ref. [4], Eq. (5) is calculated to the next α order, keeping, however, the leading $\log\alpha$ term (i.e., neglecting const relative to $\log\alpha$).

In the opposite case $M = 0$ $g_{M=0}(z)$ has the form

$$g_{M=0}(z) = 6\sqrt{30}\pi^{3/2}m^3(1-z^2). \tag{6}$$

For arbitrary M the function $g_M(z)$ is found from Eq. (3) numerically.

3 Two- and Three-Body Contributions

Knowing the BS amplitude, we extract from it the two-body wave function [1]

$$\psi(\boldsymbol{k}_\perp, x) = \frac{(\omega\cdot k_1)(\omega\cdot k_2)}{\pi(\omega\cdot p)}\int_{-\infty}^{+\infty}\Phi(k+\beta\omega, p)\,d\beta. \tag{7}$$

This relation is independent of any model. In the Wick-Cutkosky model, substituting Eq. (2) into Eq. (7), we find

$$\psi(\boldsymbol{k}_\perp, x) = \frac{x(1-x)g_M(1-2x)}{2\sqrt{\pi}(\boldsymbol{k}_\perp^2+m^2-x(1-x)M^2)^2}. \tag{8}$$

Substituting Eq. (8) into Eq. (1), we obtain the two-body contribution to the full normalization,

$$N_2 = \frac{1}{192\pi^3}\int_0^1 \frac{x(1-x)g_M^2(2x-1)\,dx}{(m^2-x(1-x)M^2)^3}. \tag{9}$$

The three-body contribution N_3 is found in ref. [3] by calculating the amplitudes of Fig. 1 (at $Q = 0$), where the two-body vertices are determined by the wave function (8).

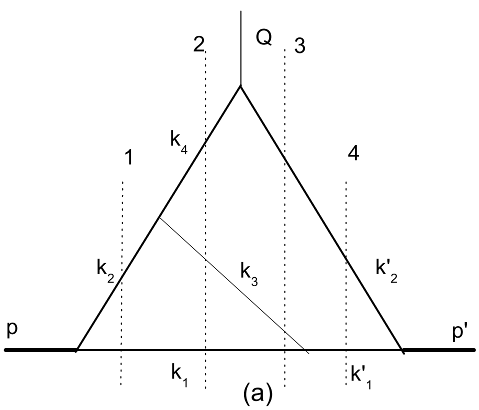

Fig. 1. Three-body contributions to the form factor

4 Results

For small α, with Eq. (5) for $g_M(z)$, the contributions N_2 and N_3 to the total norm are found analytically (up to order $\alpha \log \alpha$),

$$N_2 = 1 - \frac{2\alpha}{\pi} \log \frac{1}{\alpha}, \qquad N_3 = \frac{2\alpha}{\pi} \log \frac{1}{\alpha}, \qquad N_{n\geq 4} = \mathcal{O}(\alpha^2). \tag{10}$$

For $\alpha = 2\pi$ ($B = 2m$, $M = 0$), with $g_M(z)$ given by Eq. (6), we get

$$N_2 = \frac{9}{14} \approx 64\%, \qquad N_3 \approx 26\%, \qquad N_{n\geq 4} \approx 10\%. \tag{11}$$

For α in the interval $2\pi \geq \alpha \geq 0$, corresponding to $0 \leq M \leq 2m$, the values of N_2, N_3, and $N_{n\geq 4}$ versus M are found numerically and they are shown in Fig. 2.

We find that the two-body sector always dominates. The sum $N_2 + N_3$ contributes 90% even in the extremely strong coupling case, as we see in Eq. (11). This result is non-trivial, since for $\alpha = 2\pi$ one might expect just the opposite relation of the $N_2 + N_3$ and $N_{n\geq 4}$ contributions. For any α, the asymptotic behavior of the form factor $F_{\text{full}}(Q^2)$ is determined by the two-body Fock sector [3].

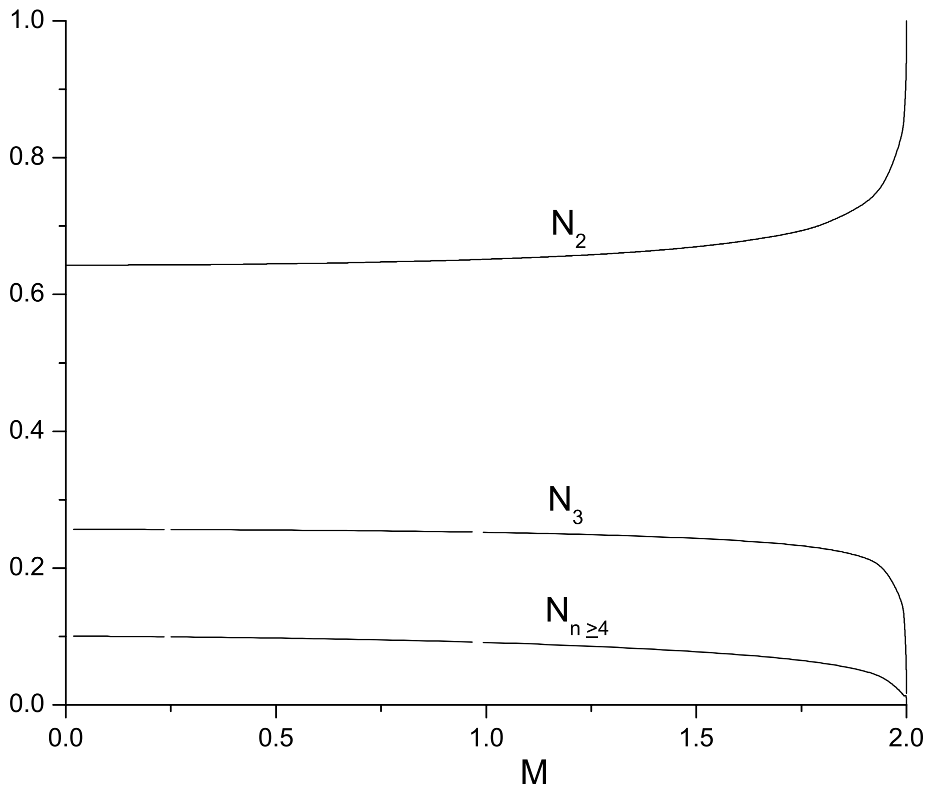

Fig. 2. Contributions to the total norm $N_{n=2} + N_{n=3} + N_{n\geq 4} = 1$ of the Fock sectors with the constituent numbers $n = 2$, $n = 3$, and $n \geq 4$ versus the bound-state mass M (in units of m)

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