TECHNICAL PAPER



Significance of Arrhenius activation energy in flow and heat transfer of tangent hyperbolic fluid with zero mass flux condition

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Received: 3 January 2020 / Accepted: 22 February 2020 / Published online: 28 February 2020 © Springer-Verlag GmbH Germany, part of Springer Nature 2020

Abstract

The current endeavor scrutinizes the flow of tangent hyperbolic fluid over a moving stretched surface. The characteristics of heat transfer are conferred by utilizing nonlinear radiation. Further features of mass transfer are characterized with activation energy. The problem is modeled in terms boundary layer equations by implementing the relative laws. The independent variables in the governing equations through suitable transformations are reduced which are further tackled numerically via RKF-45 technique. Several physical parameters are varied in order to evaluate the behaviors of velocity, temperature and concentration distributions. It is established that higher values of We parameter increases the velocity profile. Further it is obtained that rate of heat transfer enhances as N_r parameter increases.

List of symbols

- b Stretching rate
- C_f Skin friction
- c_p Specific heat
- *C* Nanoparticle volume fraction
- D_B Brownian diffusion coefficient
- D_T Thermophoresis diffusion coefficient
- *E* Activation energy
- *k* Thermal conductivity (W/mK)
- k^* Mean absorption coefficient (W/mK)
- Nb Brownian motion parameter
- *Nt* Thermophoresis parameter
- Nu Local Nusselt number

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- *Nr* Radiation parameter
- Pr Prandtl number
- Sc Schmidt number
- Sh_x Local Sherwood number
- *T* Temperature of the fluid (K)
- T_w Temperature at the wall (K)
- T_{∞} Ambient fluid temperature (K)
- *u*,*v* Velocity components along *x* and *y* Directions (ms^{-1})
- u_w Stretching sheet velocity (W/mK)
- *x* Coordinate along the stretching Sheet (m)
- y Distance normal to the stretching sheet (m)
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Greek symbols

- v Kinematic viscosity
- ϕ Rescaled nanoparticle volume fraction
- ρ Density of the base fluid (kg/m³)
- σ Reaction rate
- δ Temperature difference
- κ Boltzmann constant
- θ Dimensionless temperature
- η Similarity variable
- α Thermal diffusivity

1 Introduction

The motion of heat for a viscous fluid has numerous industrialized, biomedical and engineering demands for instance capacity generator, petroleum productions, plasma research, cancer therapy, the laminar boundary laver dominance in aerodynamics and numerous others. But this concept takes a long journey to shape up for this position. In 1904, Prandtl suggested the laminar boundary layer theory which humor that the viscous impact would be limited to thin shear surfaces adjacent borders in the case of the flow of fluids with very small viscosity (Schlichting et al. 1960). Sakiadis (1961a b) has studied the boundary layer Blasius motion due to surface supplying with constant speed from a slit into a liquid at relaxes. Crane (1970) has investigated the flow over a stretching sheet. Numerous researchers for instance Dutta et al. (1985), Char (1988), Gupta and Gupta (1977) expanded the research of Crane (1970). Varied features of such problems have been studied by different researchers for instance Hayat et al. (2008). Hayat et al. (2006), Xu and Liao (2005) and Cortell (2005, 2006). Motion past a cylinder is assumed to be 2-D boundary layer flow across a stretching cylinder was invoked by Datta et al. (2006). Elbarbary was analyzed by heat transfer effects on micropolar across a stretching cylinder (Elbarbary and Elgazery 2005). Wang (1988) has investigated the steady two dimensional flow of a fluid across a stretching cylinder. Ishak et al. (2008) have studied the MHD motion and transfer of heat past a stretching cylinder. Ishak et al. (2008) have numerical investigation on the effect of suction/blowing on the motion across a stretching cylinder. Butt et al. (2016) have explored Heat transfer effects on hydromagnetic entropy generation flow over a horizontal stretching cylinder. They are obtaining solutions by using byp4c technique in MATLAB.

Radiation plays very important role in engineering, industry, and humankind also. In human kinds radiation is used for medicines, excavation, radiology lab, MRI scans etc. Akinshilo (2019) was scrutinized the flow of hydromagnetic fluid flow across vertical micro channel with radiation effects, in this paper they are obtaining solution by implementing Runge–Kutta scheme. A steady 3D nonlinear radiation on flow of a carreau fluid across a bidirectional stretching sheet was studied by Khan et al. (2018). They concluded that by escalating values of Hartmann number the velocity profile is declines and opposite behaviour is seen in temperature profile. Khan et al. (2018) was deliberated by impact of linear radiation on third grade fluid for coating analysis. In this paper they are obtaining solution by using HAM. Hashim et al. (Hamid et al. 2018) studied by heat and mass transfer effects on non-Newtonian fluid over a circular cylinder.

Magnetohydrodynamic plays a key role in industry, engineering, heat transfers etc. Hydromagnetics is the revise of the magnetic premises of electrically conducting fluid. There are several applications in magneto hydrodynamics they are cosmology, seismology, metal detectors etc.Heat transfer effects on hydromagnetic nano fluid by using Buongiorno's model were analyzed by Ajam et al. (2018).they are obtaining solution by using OHAM. Durga Prasad et al. (2018) was scrutinized by analytical approximation on MHD nano fluid across a semi-infinite plate. Khan et al. (2018) was invoked by homogenious -hetrogenious reactions on hydromagnetic non-newtonian fluid across a cylinder. Siti et al. (2018) was investigated by MHD flow past a stretching disk. Heat and fluid flow along a stretching cylinder was studied by Vikas et al. (2018). In this paper they are presenting graphical presentations along with stability analysis. Ismail et al. (2019) analyzed the study of stability analysis on an unsteady hydromagnetic flow across a shrinking sheet. Gholinia et al. (2019) was analyzed by heat and mass transfer effects on hydromagnetic nano fluid across a stretching sheet. In this paper they are presenting graphical representations by using Runge-Kutta method. Heat transfer effects on hydromagnetic natural convection in Darcy-Forchhimer were scrutinized by Sajjadi et al. (2019).

In modern days the abnormal trend of nanofluid is due to its unexpected thermal properties and have dynamic role in heat transfer enhancement of material processing and industrial thermal managements. Free convection flows permanently faced in engineering tools and nuclear reactor technology. Prasannakumara et al. (Prasannakumara et al. 2017) deliberate effort on radiative heat transfer of nanofluids using magnetic field over a plate. They perceive that the Nusselt number and Sherwood number are enhanced for nonlinear stretching sheet. Kumar et al. (2018) represented the Marangoni effects on nanofluid in the presence of heat. Recently various researchers studied heat transfer of nanofluids (Kumar et al. 2018; Kumar and Chamkha 2019; Kumar 2019; Usman et al. 2018; Anwar and Rasheed 2017; Mebarek-Oudina 2019; Mebarek-Oudina 2017; Raza et al. 2019; Gourari et al. 2019; Verma and Sinha 2015; Verma et al. 2017; Ganesh et al. 2020; Nadeem et al. 2020; Rashid et al. 2020; Abbas et al. 2020; Kahshan et al. 2019; Uddin et al. 2019; Akermi et al. 2019; Ganesh Kumar et al. 2019).

The above literature analysis discloses that no work occurs on incompressible tangent hyperbolic nanoliquid flow past a permeable surface. The combined effect of magnetic field and nanofluid across a permeable surface are accounted. By using similarity transformations PDE'S can be transformed into ODE'S and explained numerically. Outcomes are represented through graphs for the parameters of concern.

2 Mathematical formulation

Here firstly assume that flow is steady and the incompressible tangent hyperbolic nanoliquid flow past a permeable surface is considered. The flow occupies to y > 0, velocity of the sheet is $u_w(x) = bx, b > 0$ and further *x*-axis taken along the sheet (Fig. 1).

The constitutive equation of tangent hyperbolic fluid is,

$$\bar{\tau} = \left[\mu_0 + (\mu_0 + \mu_\infty) \tanh\left(\Gamma \bar{\dot{\gamma}}\right)^n\right] \bar{\dot{\gamma}},\tag{a}$$

in which, $\bar{\tau}$ is the extra stress tensor, μ_{∞} the infinite shear rate viscosity, μ_0 the zero shear rate viscosity, Γ is the time dependent material constant, *n* the power law index i.e. flow behavior index and $\bar{\gamma}$ is defined as,

$$\bar{\dot{\gamma}} = \frac{1}{2} \sqrt{\frac{1}{2} \sum_{i} \sum_{j} \bar{\dot{\gamma}}_{ij} \bar{\dot{\gamma}}_{ji}}} = \sqrt{\frac{1}{2} \Pi}.$$
 (b)

Consider the equation (a), for the case when $\mu_{\infty} = 0$ because it is not possible to discuss the problem for the infinite shear rate viscosity and since we are considering tangent hyperbolic fluid that describing shear thinning effects so $\Gamma \bar{\gamma} < 1$. Then equation (a) will take the form,

$$\bar{\tau} = \mu_0 \left[\left(\Gamma \bar{\gamma} \right)^n \right]
\bar{\tau} = \mu_0 \left[1 + n (\Gamma \bar{\gamma} - 1) \right] \bar{\gamma}.$$
(c)

Governing equations for tangent hyperbolic fluid model after applying the boundary layer approximations can be defined as Abdul Maleque (2013), Dhlaminia et al. (2019), Kumar et al. (2017),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v(1-n)\frac{\partial^2 u}{\partial y^2} + \sqrt{2}vn\Gamma\left(\frac{\partial u}{\partial y}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u,$$
(2)



Fig. 1 Physical model and coordinate system

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial y^2}\right) + \tau \left\{ D_B \left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_{\infty}}\right) \left[\left(\frac{\partial T}{\partial y}\right)^2 \right] \right\} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y},$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right) \left(\frac{\partial^2 T}{\partial y^2}\right) - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty}\right)^m \exp\left(\frac{-Ea}{\kappa T}\right)$$
(4)

The following imposed conditions of the model are;

$$u = u_w(x), v = v_w, T$$

= T_w , $D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0$ (passive control of ϕ) at y
= 0

$$u \to 0, T \to T_{\infty}, C \to C_{\infty} as \ y \to \infty$$
 (5)

Here $v_w > 0$ notes as injection, $v_w < 0$ noted as suction. The third term on RHS of Eq. (5) denotes modified Arrhenius formula in which k_r^2 for rate of reaction, *Ea* for activation energy, κ for Boltzmann constant and *m* dimensionless fitted rate constant which lies in the range -1 < m < 1.

Implementation Rosseland approximation q_r defined as,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = \frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial y},\tag{6}$$

use of (6) in (3), we get;

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[\left(\alpha + \frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y} \right) \frac{\partial T}{\partial y} \right] + \tau \left\{ D_B \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial y} \right)^2 \right] \right\},$$
(7)

To obtain the similarity result use the following variables

$$\eta = \left(\frac{u_w}{vx}\right)^{1/2} y, \ f(\eta) = \frac{\psi}{\left(xvu_w\right)^{1/2}}, \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(\eta)$$
$$= \frac{C - C_\infty}{C_\infty} (\text{passive control of } \phi)$$
(8)

where $T = T_{\infty}(1 + (\theta_w - 1)\theta(\eta))$ and $\theta_w = T_w/T_{\infty}, \theta_w > 1$ being temperature ration parameter.

Applying (8) in (1, 2, 4 and 7), one can have;

$$(1 - n)f'''(\eta) - [f'(\eta)]^{2} + f''(\eta)f(\eta) + nW_{e}f''^{2}(\eta)f'''(\eta) - Mf'(\eta) = 0, (9) \left([(1 + Nr(\theta_{w} - 1))\theta]^{3}\theta' \right)' + \Pr(f\theta' + Nb\theta'\phi' + Nt\theta'^{2}) = 0, (10)$$

$$\phi'' + Scf \phi' + \frac{Nt}{Nb} \theta'' - Sc\sigma (1 + \delta\theta)^m \phi \, \exp\left(\frac{-E}{1 + \delta\theta}\right) = 0,$$
(11)

COresponding boundary conditions are;

$$f(0) = S, f'(0) = 1, \theta(0) = 1, Nb\phi'(0) + Nt\theta'(0)$$

= 0(passive control of ϕ), $f'(\infty) \to 0, \theta(\infty)$
 $\to 0, \phi(\infty) \to 0.$ (12)

The dimentionless numbers in Eqs. (9–12) are $\lambda = \lambda_2^2 a^2$ for material parameter, $Nr\left(=\frac{16\sigma^*T_{\alpha}^3}{3kk^*}\right)$ for radiation parameter, $Nb\left(=\frac{\tau D_B C_{\infty}}{v}\right)$ for Brownian movement. $Nt\left(=\frac{\tau D_T(T_f-T_{\infty})}{vT_{\infty}}\right)$ for thermophoretic, $\Pr\left(=\frac{v}{\alpha}\right)$ for Prandtl number, $Sc\left(=\frac{v}{D_B}\right)$ for Schmidt number, $\sigma\left(=\frac{k_f^2}{b}\right)$ for reaction rate, $\delta\left(=\frac{T_f-T_{\infty}}{T_{\infty}}\right)$ for temperature difference, $E\left(=\frac{Ea}{\kappa T_{\infty}}\right)$ for activation energy, $W_e = \frac{\sqrt{2b}\Gamma u_w}{\sqrt{v}}$ is the Weissenberg number and *n* is the power law index parameter and $S = -\frac{v_w}{\sqrt{bv}}$ for mass transfer parameter.

The physical quantities of interest like skin friction coefficient (C_f) and local Nusselt number (Nu_x) , and Sherwood number are defined as,

$$c_f = \frac{\tau_w}{\rho u_w^2}, Nu_x = \frac{u_w q_w}{bk(T_\infty - T_w)} \quad \text{and} \\ Sh_x = \frac{u_w q_m}{aD_b(C_w - C_\infty)},$$
(13)

where τ_w is known as shear stress along the wall, q_w is known as heat flux, q_m is nano particle mass flux,

$$\begin{aligned}
& \mathcal{L}_{w} = \mu_{0} \left((1-n) \frac{\partial u}{\partial y} + \frac{n\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial y} \right)^{2} \right), \quad q_{w} = -k \frac{\partial T}{\partial y}|_{y=0} \\
& \text{and} \quad q_{m} = -D \frac{\partial C}{\partial y}|_{y=0}.
\end{aligned}$$
(14)

Finally the expression of drag friction (C_f) , local Nusselt number (Nu) and Sherwood number (Sh) written as,

$$\sqrt{Re}C_{f} = \left[(1-n)f''(\eta) + \frac{n}{2}W_{e}(f''(\eta))^{2} \right]_{\eta=0}, Nu\operatorname{Re}_{x}^{-1/2}$$

$$= -\left(1 + Nr\theta_{w}^{3}\right)\theta'(0) \text{ and}$$

$$Sh\operatorname{Re}_{x}^{-1/2} = \frac{Nt}{Nb}\theta'(0) \text{ (passive control of }\phi)$$
(15)

where $\operatorname{Re}_{x} = \frac{u_{w}x}{v}$ for local Reynolds number.

3 Numerical method

1

The dimensionless arrangement of Eqs. (9)-(11) with the conditions (12) are profoundly coupled differential conditions. One needs to turn towards numerical strategies to acquire the arrangement of such conditions. In this investigation, we have utilized the method Runge–Kutta–Fehlberg fourth–fifth order with shooting system. The calculations have been done utilizing the representative programming Maple.

The algorithm of Runge–Kutta–Fehlberg –forth- fifth order method is given by;

$$\begin{split} k_0 &= F(\bar{x}_m, \bar{y}_m), \\ k_1 &= F\left(\bar{x}_m + \frac{h}{4}, \bar{y}_m + \frac{hk_0}{4}\right), \\ k_2 &= F\left(\bar{x}_m + \frac{3}{8}h, \bar{y}_m + \left(\frac{3}{32}k_0 + \frac{9}{32}k_1\right)h\right), \\ k_3 &= F\left(\bar{x}_m + \frac{12}{13}h, \bar{y}_m + \left(\frac{1932}{2197}k_0 - \frac{7200}{2197}k_1 + \frac{7296}{2197}k_2\right)h\right), \\ k_4 &= F\left(\bar{x}_m + h, \bar{y}_m + \left(\frac{439}{216}k_0 - 8k_1 + \frac{3860}{513}k_2 - \frac{845}{4104}k_3\right)h\right), \\ k_5 &= F\left(\bar{x}_m + \frac{h}{2}, \bar{y}_m + \left(-\frac{8}{27}k_0 + 2k_1 - \frac{3544}{2565}k_2 + \frac{1859}{4104}k_3 - \frac{11}{40}k_4\right)h\right), \\ \bar{y}_{m+1} &= \bar{y}_m + h\left(\frac{25}{216}k_0 + \frac{1408}{2565}k_2 + \frac{2197}{4109}k_3 - \frac{1}{5}k_4\right), \\ \bar{y}_{m+1} &= \bar{y}_m + h\left(\frac{16}{135}k_0 + \frac{6656}{12825}k_2 + \frac{28561}{56430}k_3 - \frac{9}{50}k_4 + \frac{2}{55}k_5\right), \end{split}$$

4 Results and discussion

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The flawless perception of the physical problem is discussed in this section. The prominent characteristics of the impact of a combination of nanofluids on velocity, temperature and concentration augmentation are evaluated through graphical representation of numerical data and tables. Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 represent the fluid flow, temperature and concentration profiles. For this computation we choose the parameter values Nb = Nt = 0.3, Nr = 3, Pr = 3, n = $0.5, W_e = 0.8, \theta_w = 1.2, \sigma = \delta = E = 1$ and Sc = 0.5.

Figure 2 captured the impact of M on velocity profiles against similarity variable η respectively. We perceived from Fig. 2 that by enhancing magnetic number M the fluid flow diminished progressively. Because the Lorentz force is an opposite force for strengthen magnetic number which resists the liquid motion. The influence of Weissenberg parameter W_e can be seen in Fig. 3. From these figures show that the velocity profile enhances with boost up values of W_e . Furthermore, the thickness of hydrodynamic boundary layer is high for rising values of W_e .

The behaviour of Radiation parameter Nr is illustrated in Fig. 4 An increase in Radiation parameter Nr shows the significant enhancement in the fluid temperature function. Physically speaking, by strengthening Nr yields more heat into the liquid, as a consequence the thickness of associated the energy boundary layer enhanced. Thus, Nr effects plays a key role in magnify the rate of heat transfer.

The influence of temperature ratio parameter θ_w on $\theta(\eta)$ is illustrated in Fig. 5. The temperature and species profiles rises by improving values of θ_w thickness of boundary layer increased. From Figs. 6, 7 illustrates that for escalating values of δ and *E* concentration profile declines for δ and opposite behaviour is seen in *E*.

From Fig. 8 shows that for boost up values of Schmidt number *Sc* concentration decreases for both the phases. Physically, enhancing values of Schmidt number *Sc*the mass diffusivity declines. Figures 9 and 10 are demonstrates the influence of *Nt* and *Nb* on $\theta(\eta)$. It shows that an



Fig. 2 Impact of M on $f'(\eta)$



Fig. 3 Impact of W_e on $f'(\eta)$



Fig. 4 Impact of N_r on $\theta(\eta)$



Fig. 5 Impact of θ_w on $\theta(\eta)$

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Fig. 6 Impact of δ on $\phi(\eta)$



Fig. 7 Impact of *E* on $\phi(\eta)$



Fig. 8 Impact of *Sc* on $\phi(\eta)$



Fig. 9 Impact of *Nt* on $\theta(\eta)$



Fig. 10 Impact of *Nb* on $\theta(\eta)$

increase in *Nt* and *Nb* enhances the $\theta(\eta)$ and hence, the associated thickness of the boundary layer also enhances.

Figure 11 elucidates the effect of Nb on ϕ field. A reduction in ϕ field is obtained for the larger values of Nb parameter. Correspondingly thickness of boundary layer exhibits the same behavior as that of nanoparticle concentration profile. This is because of the larger Brownian motion of the suspended particle, due to its increment in its corresponding parameter.

In the boundary layer region Nt plays a vital role on the ϕ field. This effect is captured in Fig. 12. From this figure it is clear that for gradually increasing values of Nt concentration profile is increased. Physically, the thermophoretic is a force that one particle exerts on the other particle. Due to which the particles in the hotter region are



Fig. 11 Impact of *Nt* on $\phi(\eta)$



Fig. 12 Impact of *Nb* on $\phi(\eta)$



Fig. 13 Influence of *Nb* and *Nt* on $-NuRe_x^{-\frac{1}{2}}$

moved to the colder region. As a result an acclivity in the concentration is noted. The effects of *Nb* and *Nt* on $-NuRe_x^{-\frac{1}{2}}$ is established in Fig. 13. It shows that the



Sc

Fig. 14 Influence of Sc and E on $-ShRe_x^{-\frac{1}{2}}$



Fig. 15 Stream lines for when M = 0.5

 $-NuRe_x^{-\frac{1}{2}}$ decreases for both injection and suction case for large values of *Nb* and *Nt*. Figure 14 IS prepared to establish the influence of *Sc* verses *E* on $-ShRe_x^{-\frac{1}{2}}$. It is clear that, $-ShRe_x^{-\frac{1}{2}}$ decays for enlarged values of *Sc* verses *E*.

Finally, to get a clear view of the flow field patterns, the stream line dynamics are reported in Figs. 15, 16, 17 and 18 for with peculiar values of Magnetic parameter M and Weissenberg number W_e For Magnetic parameter M and Weissenberg number W_e is strengthened, the fluid particle traces a definite curve along x-direction where the surface is present. But it is clear that the there is a retardation in the flow pattern.



Fig. 16 Stream lines for when M = 2.0



Fig. 17 Stream lines for when $W_e = 0.1$

5 Conclusion

Fully developed steady of flow and heat transfer of tangent hyperbolic nanofluid fluid over a stretching sheet is considered. In this paper the effect of zero mass flux, activation energy and impact of applied magnetic field is taken into consideration. The outcome of the present work is as follows:

• Velocity of the fluid increases as W_e parameter increases.



Fig. 18 Stream lines for when $W_e = 1.0$

- Both concentration and temperature profiles enhance for increasing values of *Nb* parameter.
- Temperature and velocity profiles shows reverse trend in rising values of *Nt*.
- Momentum boundary layer thickness decreases for rising values of *M*.
- The concentration profile and corresponding boundary layer thickness decreases for enhancing values of δ and *E*.
- Higher *Nr* plays a key role in magnify the rate of heat transfer. This is because, higher *Nr* enhances the temperature profile and also enhances the corresponding boundary layer thickness.

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