TECHNICAL PAPER

Design and experimental study of broadband hybrid energy harvester with frequency-up conversion and nonlinear magnetic force

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Abstract

In this paper, a novel broadband hybrid piezoelectric and electromagnetic energy harvester using in the low frequency vibration environment is proposed, which combines nonlinear magnet force and frequency-up conversion mechanism simultaneously. Performances are studied by theoretical analysis and experimental test. Electromechanical governed equations of harvester are established, and analytical solutions of vibration response, output voltage and power are derived. Then, effects of nonlinear force, spacing between low frequency vibration beam and piezoelectric beam, load resistance and input excitation on harvester performances are investigated by experimental test. It can be concluded that the harvester can be used to work at the low-frequency environment efficiently, and the resonant frequency and harvesting bandwidth can be tuned by the nonlinear force between the magnets and the spacing between beams. Moreover, the larger the nonlinear magnetic force and the smaller the distance between two beams, the lower working frequency and the larger bandwidth. Compared with the corresponding linear apartment, output power and bandwidth of proposed harvester are improved 90% and 125% respectively.

1 Introduction

Vibration energy harvester, which converts the mechanical energy into electrical energy, makes possibly to harvest energy from the ambient environment and directly supply the low power devices to work without the chemical battery (Jeong et al. 2016; Li et al. 2018; Cao et al. 2019). In order to improve the performances in the practical environment, the wideband and high energy conversion efficiency at the low frequency are the two main difficulties needing to solve for the energy harvester (Deng et al. 2016; Chen et al. 2015; Bryn and Kean 2015; Mahmoudi et al. 2014). Some methods have been proposed in the literatures, among which the nonlinear energy harvesting method is one of the feasible solutions (Maryam and Stephen 2014; Al-Ashtari et al. 2012; Marzencki et al. 2009).

Especially, the vibration frequency of practical environment is mostly lower than the 100 Hz, which induces

the output power decreasing because the power is proportional to the frequency. Then, the frequency-up conversion method, which is one of nonlinear energy harvesting techniques, is proposed by researchers in order to improve the power (Abed 2016; Abed et al. 2016; Zhou et al. 2018). Seok-Min Jung proposed and demonstrated a wideband energy-harvesting device operating at low frequency range using bistability for mechanical frequency-up conversion. It shows that this phenomenon provides high acceleration to the attached cantilever beams and causes them to vibrate at their fundamental resonant frequency. One important advantage of the proposed device is that it eliminates the need to match the fundamental resonant frequency with the excitation frequency (Jung and Yun 2010). Halim designed a mechanical impact driven and frequency up-converted wide-bandwidth piezoelectric energy harvester, which allows the device to offer approximately 180% increased 3 dB bandwidth and more than 62% of the maximum power generation within the remaining operating frequency range as well (Halim and Park 2014). Wang proposes models and experiments of a wideband piezoelectric vibration energy harvester with a quadruple-well potential induced by the combined nonlinearity of cantilever-surface contact and magnetoelasticity. Moreover, dynamic responses of the present energy harvester with a quadruple-

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well potential are explored by numerical simulations and validated by experiments. The results show that the combined nonlinearity can not only improve the efficiency of electrical power transfer under low intensity excitation (Wang et al. 2017, 2018). Therefore, the frequency-up conversion can improve the harvesting efficiency efficiently, but the bandwidth needs to be further enhanced. Besides, the above references mainly focused only one working mechanisms.

Then, in this paper, a novel hybrid piezoelectric and electromagnetic energy harvester is designed, which combined nonlinear magnetic force and frequency up-converting simultaneously. For the proposed nonlinear hybrid energy harvester, the theoretical modeling is proposed, and the corresponding expressions of output voltage and power are derived. Moreover, performances are tested by the experiment, and effects of nonlinear force, spacing between high and low frequency beam, load and excitation on harvester performances are studied, and compared with its corresponding linear apartment.

2 Structure design

The designed energy harvester in Fig. 1 includes two mechanisms together, which are piezoelectric and electromagnetic harvesting respectively. The structure integrates frequency up-conversion mechanism and nonlinear magnetic force in order to improve the bandwidth and output power in the low frequency environment simultaneously, which consists of the low-frequency vibration element, high-frequency piezoelectric beam, coils and fixed magnet. A high frequency piezoelectric beam and plane coil is located on the upper and lower side of the low frequency vibration beam respectively. The dynamic magnet as the mass is opposite to the fixed magnet, and the magnetic

force is repulsive, which is used to broad the harvesting bandwidth.

Due to the magnetic force, the response of low-frequency beam is non-linear, which is the broadband vibration. During movable magnet vibrating, the coil outputs electric energy as the magnetic flux changing. On the other hand, the high-frequency piezoelectric beam, whose resonant frequency is much bigger than that of low frequency vibration beam, starts to resonant after touched by the movable magnet by the low vibration frequency, so as to output power at the high-frequency and increase the energy conversion efficiency of the device.

3 Theoretical modeling

The equivalent working model of designed broadband hybrid energy harvester is proposed and shown in Fig. 2. When the excitation is applied to the harvesting system, effective mass m_1 of low-frequency vibration element is bounded on a linear spring of effective stiffness k_1, a nonlinear spring of effective stiffness k_n, a damper of coefficient c_1 and an EM element. The nonlinear stiffness is induced by the force between movable magnet and fixed magnet. Moreover, m_1 can touch the effective mass m_2 of high-frequency piezoelectric beam, which bounded on a linear spring of effective stiffness k_2, a damper of coefficient c_2 and an PE element. Above parameters can be got by references (Spreemann and Manoli 2012; Erturk and Inman 2011). Besides, the initial spacing between two beams is s. Therefore, the vibration of low-frequency and high-frequency beams are both affected by the linear stiffness, damping, nonlinear magnetic force and coupling effect of energy harvesting elements.

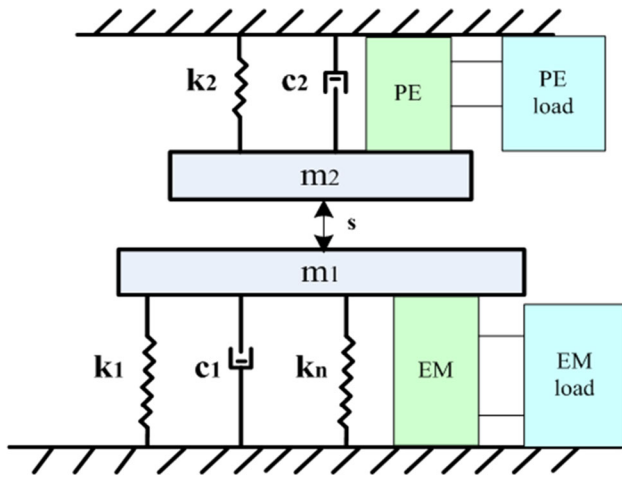Fig. 1 Structure model of designed harvester
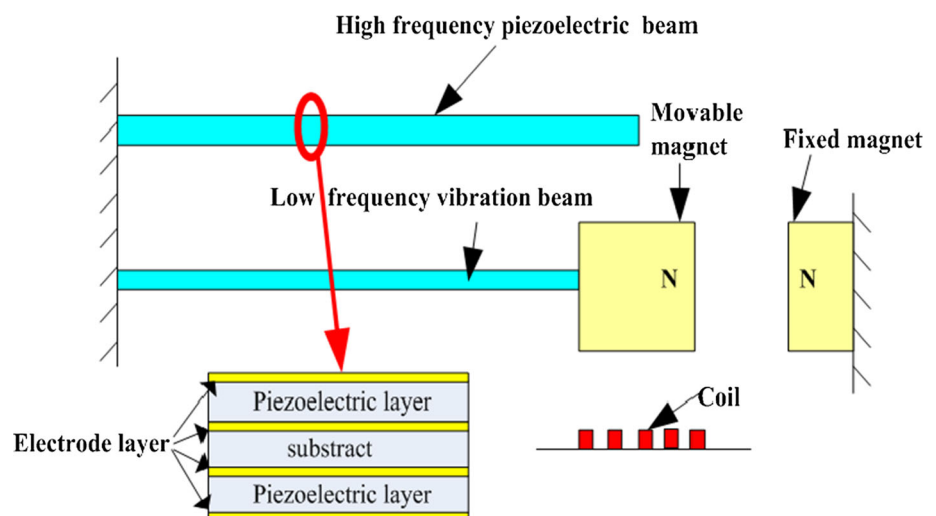
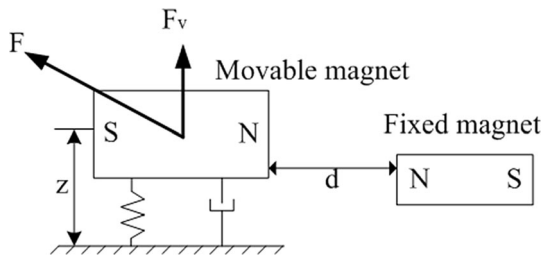**Fig. 2** Equivalent working model of the harvester

Fig. 3 Nonlinear repulsive force between two magnets

Besides, the repulsive magnet force is illustrated in Fig. 3. By reference (Ferrari et al. 2010), the nonlinear force can be expressed as Eq. (1):

$$F_{NL} = az - \beta z^3 \tag{1}$$

For a given spacing d between movable and fixed magnets, the repulsive force F varies with the mass moving in the direction by an angle θ, but it will be assumed to remain constant in magnitude. The horizontal component of the force F is balanced by the longitudinal stiffness of the cantilever which assumed sufficiently high (Ferrari et al. 2010). The vertical component F_v of F is given by:

$$F_v = F \sin \theta \tag{2}$$

The vertical tip displacement z and the angle are related by $z = d \tan \theta$. Therefore, the resulting relationship between the vertical component of the repulsive force and displacement is:

$$F_v = \frac{F}{d} \frac{z}{\sqrt{1 + (z^2/d^2)}} \tag{3}$$

The vertical force F_v, as written in Eq. (3), can be illustrated in Eq. (4) by Taylor expand:

$$F_v \cong \frac{F}{d} z - \frac{F}{d^3} z^3 \tag{4}$$

Suppose the amplitude of low-frequency vibration beam and high-frequency piezoelectric beam is z_1, z_2 respectively. When $z_1 < s$, the elements do not contact and work independently. Then, the free movement of the movable magnet and piezoelectric beam, which are given by the displacement z_1, z_2 respectively, is governed by the following differential equations:

$$m_1 \ddot{z}_1(t) + c_1 \dot{z}_1(t) + k_1 z_1(t) + g_{em} i_{em}(t) = -m_1 a(t) + F_V \tag{5}$$

$$m_2 \ddot{z}_2(t) + c_2 \dot{z}_2(t) + k_2 z_2(t) + \theta V_p(t) = -m_2 a(t) \tag{6}$$

When $z_1 > s$, the proof mass impacts the piezoelectric beam, and the low frequency beam vibrates together with the piezoelectric beams for a short period. Moreover, V_p is output voltage of PE layer; I_{em} is output current of coil; θ and g_{em} are PE and EM transfer factors respectively. Then, two elements separate and vibrate separately (Qian et al. 2018). So, the nonlinear system can be modeled a classical mass–spring-damper with an additive nonlinear term, and vibration response of mass is:

$$(m_1 + m_2)\ddot{z}_1(t) + (c_1 + c_2)\dot{z}_1(t) + (k_1 + k_2)z_1(t)$$
$$+ g_{em} i_{em}(t) + \theta V_p(t) - k_2 s \tag{7}$$
$$= -(m_1 + m_2)a(t) + F_v$$

Furthermore, the magnitude of piezoelectric beam is:

$$z_2 = z_1 - s \tag{8}$$

Therefore, the vibration of the low-frequency beam increases the harvesting frequency because of the collision. The amplitude of the mass and piezoelectric beam against excitation frequency can be obtained numerically based on Eqs. (7)–(8).

For piezoelectric beam, the output voltage is (Miah et al. 2014):

$$V_{op} = \frac{-d_{31} \times Y \times t_e}{\varepsilon_{33} \times l_b} \int_0^{l_b} \xi_1(x) dx \tag{9}$$

where $\xi_1(x)$ is strain distribution; d_{31} and ε_{33} are the piezoelectric charge constant and dielectric constant of the PZT layer, respectively; t_e and l_b are the thickness and length of the PZT layer respectively; Y is the elastic modulus. The strain distribution of the piezoelectric layer is:

$$\xi_{1T}(x) = \frac{3t_b(l_b - x)}{2l_b^3} z_2 \tag{10}$$

where t_b is the thickness of the subtract layer. So the output voltage of high frequency piezoelectric beam is:

$$V_{op} = \frac{-d_{31} \times Y \times t_e}{\varepsilon_{33} \times l_b} \frac{3t_b z_2}{4l_b^2} \tag{11}$$

And the power is:

$$P_p = \frac{V_{op}^2}{R_p} \tag{12}$$

where R_p is the resistance of piezoelectric element.

Similarly, based on Faraday law, the output voltage of coil is illustrated in Eq. (13) as the magnetic flux through the coil changes (Yang et al. 2014):

$$V_{oe} = -N\frac{\partial \Phi}{\partial t} = -N\frac{\partial \Phi}{\partial z_1}\frac{dz_1(t)}{dt} \tag{13}$$

where Φ is the magnetic flux and N is coil turn. Then the power of coil element is:

$$P_{em} = \frac{V_{0e}^2 R_e}{(R_{coil} + R_e)^2} \tag{14}$$

where R_{coil} and R_e is resistance of coil and load respectively.

In this way, output performances of the nonlinear energy harvester can be calculated by the MATLAB through above equations. The nonlinear bistable case can provide a wider bandwidth compared to the resonant behaviors of the linear case. At the same time, the frequency up-conversion by the impact of low frequency vibration beam improves the harvesting efficiency, which is affected by the spacing between beams, nonlinear magnetic force and structure parameters.

4 Experimental test and discussions

In this section, performances of frequency-up converting hybrid energy harvester are tested and compared with the linear apartment. Besides, effect of nonlinear force, distance between high and low frequency beams, load resistance and input excitation on harvester performances is also studied. Table 1 shows model parameters of designed energy harvester. Experimental setup is shown in Fig. 4. The whole setup of the device is mounted on the vibrating shaker which is connected to a signal generator through a power amplifier. The signal generator is used to provide the source vibration frequency and amplitude excitation. In addition, an accelerometer is used to record vibration acceleration, and dynamic signal analyzer is used to record output voltage of piezoelectric and electromagnetic energy harvesting element.

In the experiment, the initial distance between movable magnet and coil is 1.8 mm. At this case, the optimal loads of piezoelectric layer and coil are 19 kΩ and 47 Ω respectively. Moreover, when the spacing between magnets is big enough, the nonlinear magnetic force is negligible, and the resonant frequency of low frequency vibration beam and piezoelectric beam is 51 Hz and 215 Hz respectively. Figure 5 shows the acceleration signals and corresponding output voltage of piezoelectric layer.

Table 1 Structural parameters and material properties

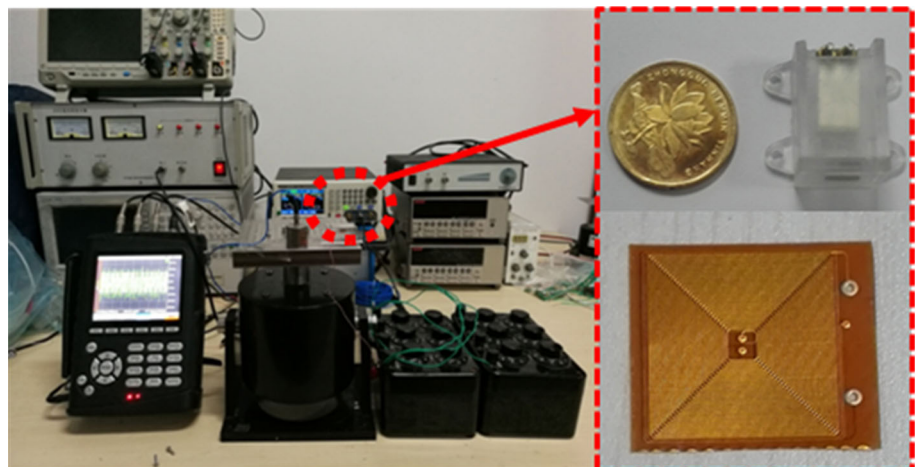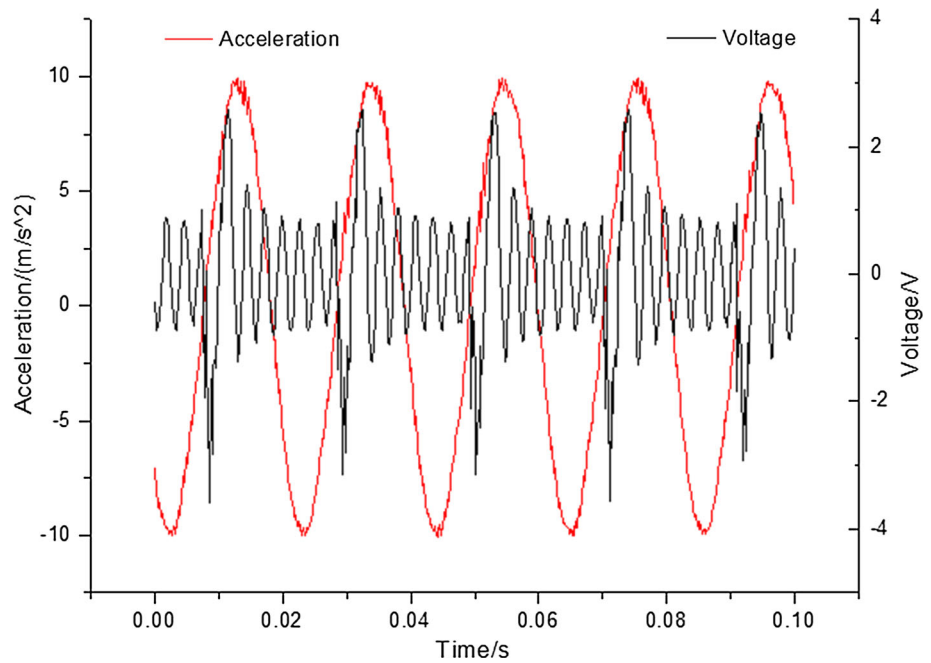|	Length/mm	Width/mm	Thickness/mm	
Low frequency beam	20	25	0.4	Stainless steel
Piezoelectric beam	30	25	0.4	Stainless steel
PZT layer	20	25	0.2	PZT-5H
Coil	15	15	1	Coppper
Movable magnet	8	8	8	NdFeB

Fig. 4 Experimental setup

Fig. 5 Input acceleration signals and corresponding output voltage of piezoelectric layer

4.1 Effect of nonlinear force on output characteristics

Output power and voltage of the harvester with or without a fixed magnet are compared experimentally. The nonlinear magnetic force is changed by adjusting the distance between magnets. Under different nonlinear forces, performances of the piezoelectric element and the electromagnetic unit of the harvester are tested, and the results show in Figs. 6 and 7. In the experiment, the excitation acceleration is 0.5 g.

The results show that, compared with the conventional linear harvester (without fixed magnet), the designed nonlinear composite structure can increase the output power and bandwidth under the action of nonlinear force. Moreover, for different nonlinear forces, the working frequency can be adjusted in different vibrations environment, and the smaller the distance between magnets, the smaller the resonance frequency of the harvester. That is to say, the working frequency of the harvester can be adjusted by changing the distance between the stationary magnet and the dynamic magnet, especially in the low-frequency environment.

When the horizontal distance between magnets is 15 mm, i.e. under the action of nonlinear magnetic force, the output voltage of piezoelectric beam and coil is increased from 1.14 V, 23 mV to 1.5 V and 33 mV respectively. The corresponding output power is increased from 0.13mW, 11 μW to 0.24mW and 24 μW, and the bandwidth is increased from 4 Hz to 9 Hz. In can be got that output power and bandwidth are improved 90% and 125% respectively under the nonlinear force. Therefore,
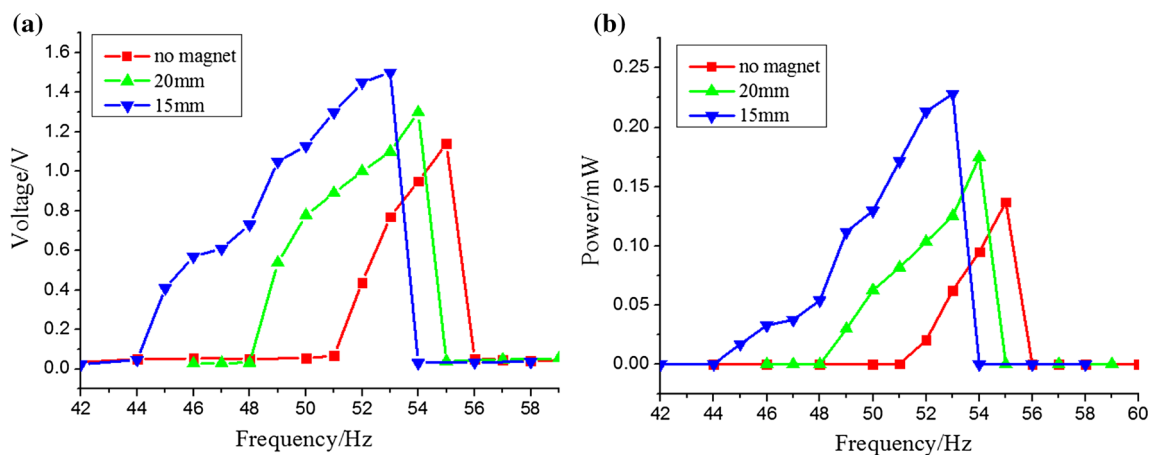

Fig. 6 Performances of piezoelectric element at different magnetic spacing(no magnet, 15 mm and 20 mm): **a** output voltage; **b** Output power
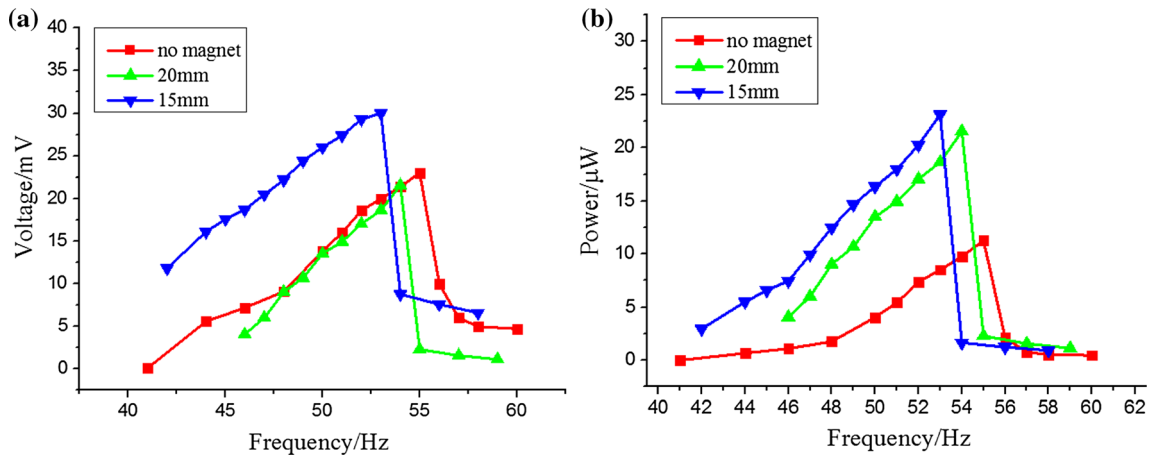
Fig. 7 Performances of coil at different at different magnetic spacing (no magnet, 15 mm and 20 mm): **a** output voltage; b output power
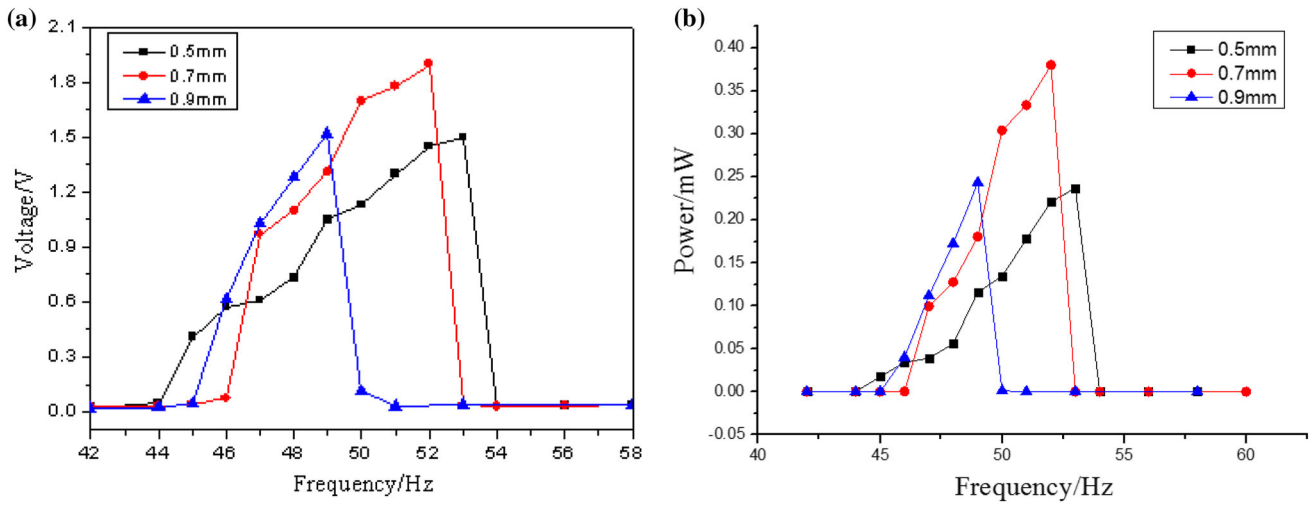

Fig. 8 Piezoelectric output performances at different spacing between beams (0.5 mm, 0.7 mm and 0.9 mm): **a** output voltage; **b** output power
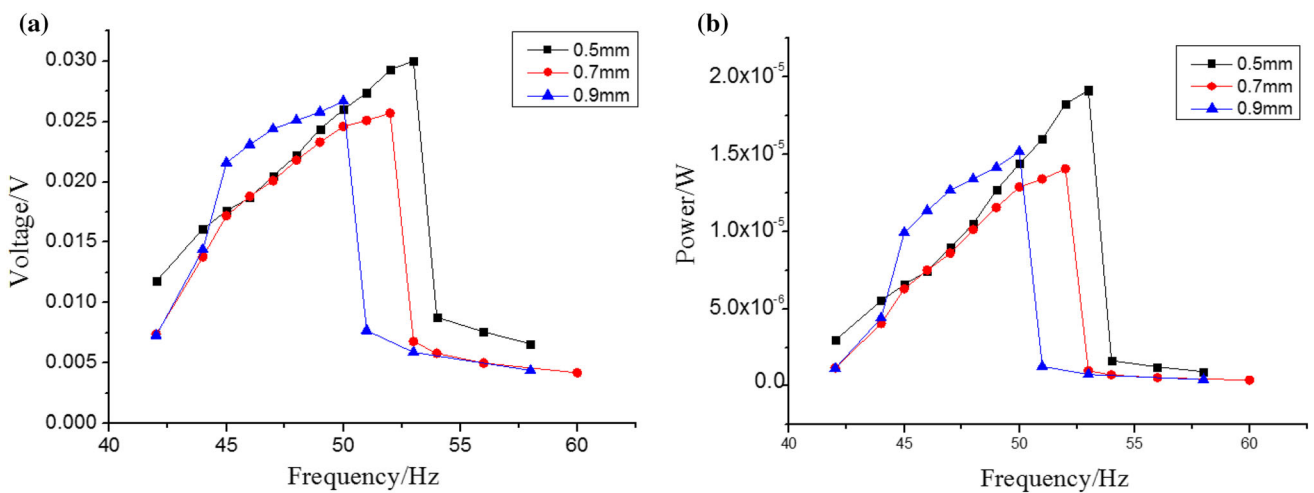

Fig. 9 Output performances of coils at different spacing between beams (0.5 mm, 0.7 mm and 0.9 mm): **a** output voltage; **b** Output power

the magnetic force increases the displacement of low frequency beam at the same excitation compared with the line apartment, which causes bigger force impacting on the piezoelectric beam and induces greater amplitude of piezoelectric beam. By Eqs. (12) and (13), the harvester can output better performances in this case.

In the experiment, the nonlinear force is adjusted by changing the magnet spacing, and the output voltage and power are optimized in this case. When the magnet spacing is 25 mm and 15 mm, the output power of the piezoelectric beam are 0.17 mW and 0.24 mW respectively, and the power of the coil is 22 μW and 24 μW respectively. Therefore, the larger nonlinear magnetic force, the lower resonance frequency and the larger bandwidth, and the output voltage and power of the coil and piezoelectric unit increase with the nonlinear magnetic force increasing.

4.2 Effect of spacing between low frequency vibration beam and piezoelectric beam on output characteristics

During frequency up-conversion mechanism design, the low-frequency beam strikes the piezoelectric beam under the excitation, whose resonant frequency is several times higher than that of low frequency beam. So it can improve the output electric energy. In addition, during the vibration of the low-frequency beam, the coil also outputs electric energy due to the change of magnetic flux of the dynamic magnet in the coil.

When the excitation acceleration is 0.5 g, the influence of the distance between two beams on output voltage, power and bandwidth is tested. In the study, the spacing is set to 0.5 mm, 0.7 mm and 0.9 mm respectively, and the distance between the movable magnet and the fixed magnet is 25 mm.

According to the experimental results in Figs. 8 and 9, there is an optimal spacing to maximize the output power of the piezoelectric beam and coil. As shown in Fig. 8, the maximum power is 0.38 mW at 0.7 m spacing for piezoelectric beam, but output power of the coil decreases with the spacing increasing. When the spacing is 0.5 mm, the maximum output power is 20 μW.

In addition, the larger distance between the piezoelectric beam and the low-frequency vibration beam, the lower resonance frequency of nonlinear hybrid energy harvester, which means that the working frequency can be also tuned by the beams spacing. When the spacing increases from 0.5 mm to 0.9 mm, the frequency decreases from 53 to 48.5 Hz. At the same time, the bandwidth of harvester decreases with the increase of spacing, but the bandwidth of high frequency piezoelectric beam is basically equal at 0.7 mm and 0.9 mm. Therefore, in order to maximize the output power and bandwidth, it is necessary to optimize the beams spacing when designing the frequency up-conversion nonlinear hybrid energy harvester.
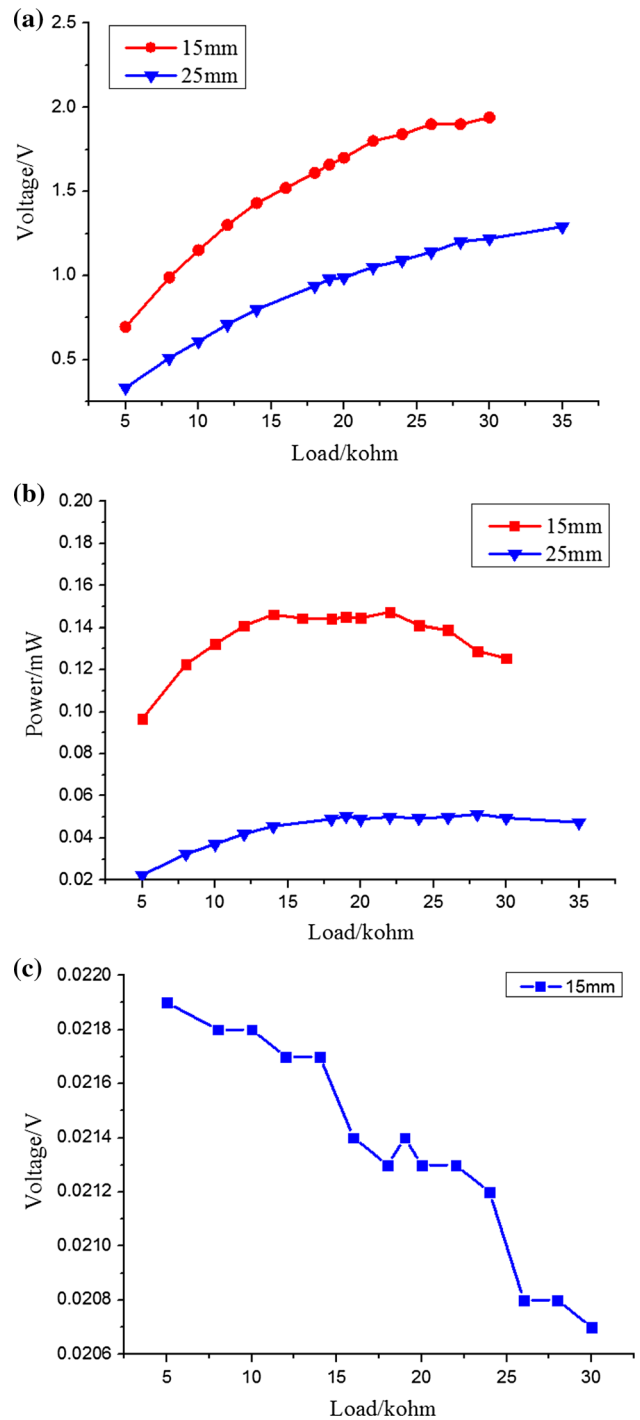

Fig. 10 Effect of load of piezoelectric element on harvester performances: **a** output voltage of piezoelectric element; **b** output power of piezoelectric element; **c** output voltage of coil

4.3 Effect of loads on output characteristics

In order to output the maximum power, it is necessary to optimize the load of the nonlinear hybrid energy harvester. Moreover, the nonlinear force, which changes with the distance between magnets, induces the different stiffness

and corresponding resonance frequency of the harvester. It can change the optimal load of harvester. Therefore, in the experiment, when the excitation acceleration is 0.5 g and the beam spacing is 0.5 mm, the influence of load on the output voltage, power and bandwidth of the piezoelectric beam and coil is tested at different magnet spacing, and the effect of nonlinear force on the optimal load is analyzed. The results are shown in Figs. 10 and 11.

According to the experimental results, for piezoelectric beam, when the spacing is 15 mm, the output voltage and power are both larger than those of 25 mm at the same load. In addition, the smaller spacing is, the smaller optimal load of high frequency piezoelectric beams is. When the spacing is 15 mm and 25 mm, the optimal load of piezoelectric beams is 13 kΩ and 21 kΩ respectively, which may be caused by that the smaller spacing is, the lower resonant frequency is.

For the electromagnetic coil, the influence of magnet spacing on output voltage and power is much less than that on piezoelectric beam. Similarly, the smaller spacing is, the greater output voltage and power of the coil are. However, at different spacing, the optimal load of the coil has hardly been changed.

4.4 Effect of excitations on output characteristics

Under different input excitations, the output voltage and power of piezoelectric and electromagnetic units are tested respectively, and the output characteristics under two different states of up-sweeping and down-sweeping are compared in the experiment. The test results shown in Figs. 12 and 13 illustrate that with the input excitation increasing, the output voltage, power and bandwidth of the harvester enhances; besides, performances during the up-sweeping are larger than those of the down-sweeping. It may be induced that the bigger excitation acceleration is, the bigger bending deformation of low frequency beam and
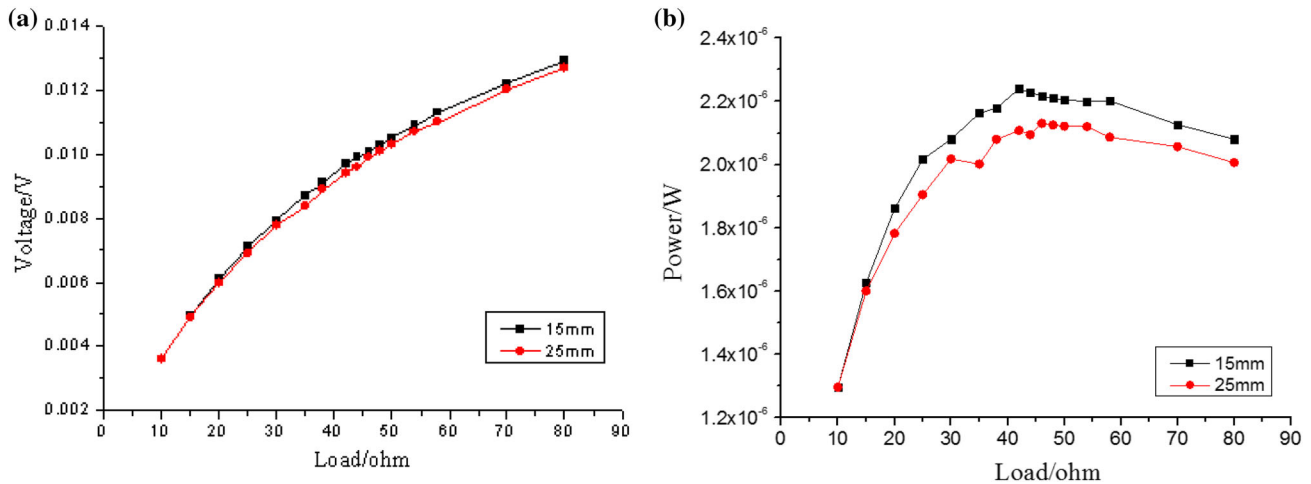

Fig. 11 Effect of load of coil on harvester performances: **a** output voltage of coil element; **b** output power of coil element
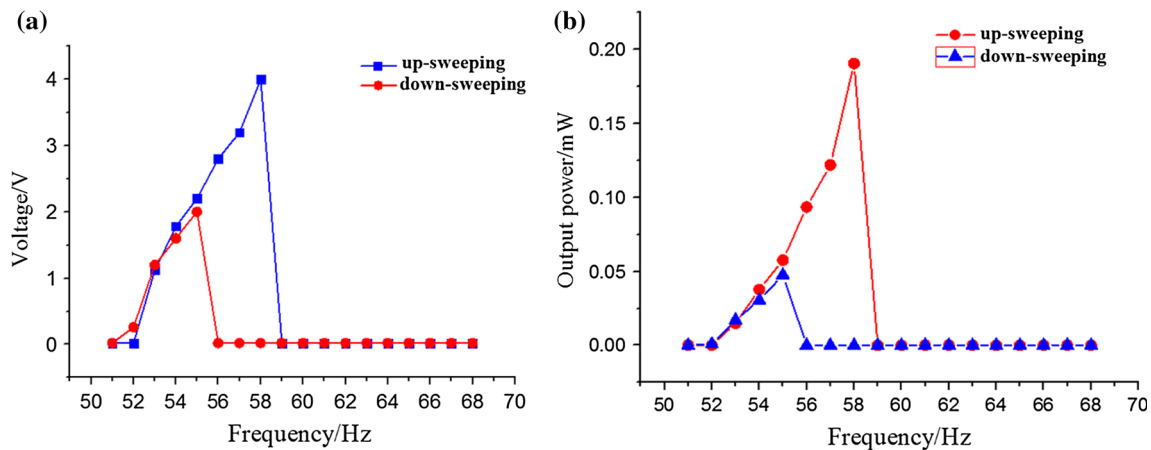

Fig. 12 Harvester performances at 0.5 g for up-sweeping and down-sweeping: **a** output voltage; **b** output power
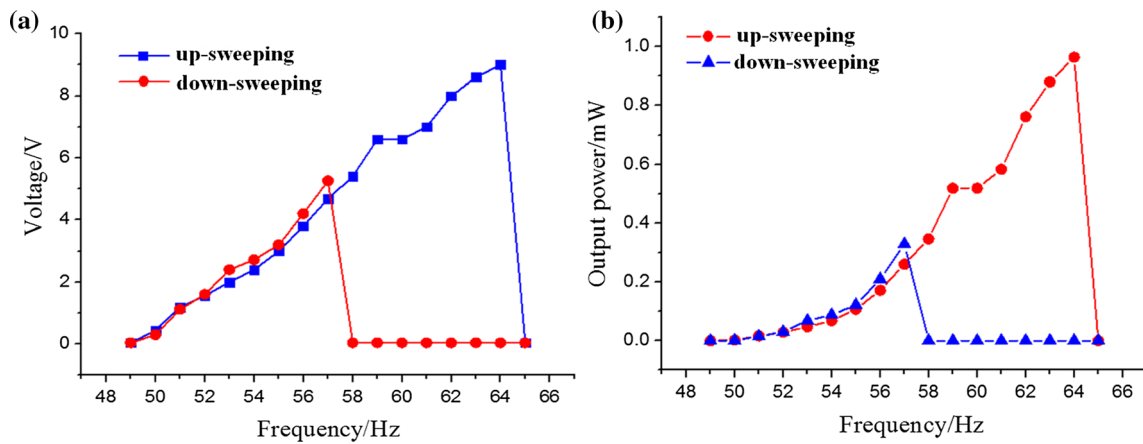
Fig. 13 Harvester performances at 1 g for up-sweeping and down-sweeping: **a** output voltage; **b** output power

piezoelectric beam under collision is. When the excitation acceleration is 1 g, the output voltage and power of the harvester are 5.5 V and 9.2 V respectively during the up-sweeping and down-sweeping, and the corresponding output power is 0.35 mW and 0.97 mW respectively.

5 Conclusions

In this paper, a novel nonlinear hybrid vibration energy harvester is designed, which combine the advantages of piezoelectric and electromagnetic harvesting mechanism simultaneously. At the same time, due to the frequency up-conversion and nonlinear magnetic force, the harvester can work at the low-frequency environment efficiently and broadband harvesting. By the governing electromechanical equations, expressions of amplitude, output power, voltage and current are derived. It is shown that the performances of nonlinear hybrid energy harvester mainly depend on the spacing between magnets, the distance between beams, input excitation, load and structural parameters.

Through experimental test, output voltage and power of piezoelectric beam and coil are studied at different cases. It can be concluded that compared with the conventional linear harvester, the designed nonlinear harvester can increase the output power and bandwidth under the action of nonlinear force and frequency up-conversion, and the working frequency of the harvester can be adjusted by changing the distance between magnets, especially in the low-frequency environment. Moreover, there is an optimal beams spacing to maximize the output power of piezoelectric beam and coils, whose optimal loads also varies with the distance between the dynamic magnet and the static magnet, and the smaller the spacing is, the smaller the optimal load of high frequency piezoelectric beams is. Meanwhile, with the input excitation increasing, performances of the harvester enhance, and output power during

the up-sweeping is larger than those of the down-sweeping. Therefore, the analysis results can provide certain criterion to improve the power and bandwidth for hybrid piezo-electric-electromagnetic energy harvester design.

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