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Analysis of functional brain network based on electroencephalography and complex network

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Abstract

In this paper, electroencephalography data are used to establish a functional network connecting correlated human brain regions. Through analysis, it is found that the resulting network shows statistical characteristics of a complex network: its clustering coefficient is orders of magnitude larger than that of the equivalent random network, which is typical of a small-world network, and the distribution of degree is close to that of a scale-free network. All these characteristics reflect important functional information about brain states. For alcohol addicts, the characteristic indices of their brains are obviously different from those of the control group. The information entropy and standard information entropy of the brain neural network are also defined to measure the characteristics of the complex network. This gives a new criterion for clinical diagnosis and treatment of encephalopathy. Calculation results indicate that the brain neural network information entropy of alcohol addicts is quite distinct from that of the control group.

1 Introduction

The human brain is made up of huge numbers of neurons and sparse connections between them, which operate at multiple organizational levels. In addition, each level has its own temporal and spatial scales (Power et al. 2011). Therefore, neurons can be analyzed from multiple levels. Neuronal clusters include local circuits, special brain areas, large-scale tissues of the cortex and the entire brain. The early neuroscientific research focused on the function positioning of the brain regions, while the modern viewpoint tends to analyze the structural and dynamic behaviours of neural networks at different levels by using complex networks (Ingber and Nunez 1990; Ingber 2012, 2016). At present, the research of complex brain network based on neuroimaging has become a hot topic.

The brain neural network is a complex network that can extract and integrate all kinds of information perfectly from external and internal stimuli in real time. The brain is essentially a dynamic system, in which the connections between any two regions are closely related to the complex functional networks based on the dynamics theory. In fact, functional networks derive deviations from statistical independence between neural clusters with a certain distance in space (Shamshiri et al. 2017), including the measurement of their correlation, covariance, coherent spectrum, and phase synchronization of the brain network. It is time dependent and the measurement results are independent of each other. Actually, different methods of measuring brain activities usually result in different statistical values of functional network connections (Horwitz 2003).

Since Friston (2005) proposed brain functional networks based on positron emission tomography and functional magnetic resonance imaging, the complexity analysis of brain neural networks based on brain functional imaging data has become an important research direction. Subsequently, Friston et al. put forward the principal component analysis and independent component analysis of brain functional networks (Friston et al. 1999; Penny et al. 2004), and further studied their functional integration. MoIntosh (Mcintosh et al. 1996) used standard variable analysis and partial least squares to analyze brain functional networks. Dodel and Herrmann (2002) combined the FMRI signal time process and the graph theory to reveal the complex characteristics of brain functional networks. Further, Eguiluz et al. (2003) used FMRI to extract a large-scale

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human brain functional network, showing that its degree distribution and connection are scale-free, and have small-world network characteristics such as small path length and large clustering coefficient, which reflect important functional information about the brain state and provide a new beginning for the study on the dynamic behaviours of the brain.

At present, most of the researches on brain functional networks are based on FMRI imaging data. In comparison, although the electroencephalography (EEG) measurement does not have accurate positioning and the number of network nodes is relatively small, it has special advantages, such as high time resolution, real-time monitoring, low cost and easy access. In addition, due to special reasons, patients with some diseases (such as Alzheimer) cannot be examined or are not suitable for examination. Now EEG data have been widely used in research (Diykh and Li 2016; Balenzuela et al. 2014). Therefore, this paper attempts to use EEG time series to establish a brain functional network for analysis.

An important purpose of studying the brain is to help people give better diagnose, warning and treatment of encephalopathy, so it is necessary to link the research of the complex brain network with the clinical research. Alcohol addiction is a kind of alcoholism which causes mental or physical dependence on alcohol. It is one of the common diseases that afflict the mankind. In spite of the adverse consequences of alcohol consumption, alcohol addicts continue to drink in order to seek mental effects after drinking or to avoid withdrawal syndrome caused by abstinence. Prolonged excessive drinking can lead to obvious mental and physical impairments. The symptoms usually include varying degrees of memory loss and decline of computational power and judgement. In addition, there may be hallucinations, delusions, eyeball and limb tremors, ataxia, limb muscle atrophy and other symptoms. A survey shows that the average lifetime prevalence of alcohol addiction in the general population is 13.6% in the United States, and that the alcohol consumption and the prevalence of alcohol addiction have increased substantially in the last decade in China. Therefore, it is of great significance to study the brain functional networks of alcohol addicts based on the EEG data and compare their brain functional networks with those of the normal people.

2 Construction of the functional brain network

2.1 Data sources

The raw EEG data used in the study are from the neurodynamics laboratory at the New York State University Health Centre. These data were derived from a large-scale study on the EEG of alcohol addicts, measured at a frequency of 256 Hz and with 64 leads. The subjects were divided into two groups—the alcohol addict group and the normal people group. There were 122 subjects, including 78 patients and 44 normal persons, each undergoing 120 measurements under different stimuli. Meanwhile, the placement of electrodes complied with the standard points specified in the standardized EEG electrode position designation formulated in 1990. Stimuli were generally divided into three categories: single stimulus (S1), two matched stimuli (S2-M) and two mismatched stimuli (S2-N). The stimuli came from the standard picture groups that Sondgrass and Vanderwart proposed in 1980, which were drawn with black and white lines based on a set of specific rules, similar to a standard visual chart. The standard is based on four variables that are most related to the cognitive process: name consistency, image consistency, familiarity and visual complexity.

2.2 Construction of the functional brain network based on EEG data

The functional brain network is an abstract network. The measurement area of every lead defined by EEG is a node of the network, and its electrical activities are a number of time series. Through calculation of the correlation between the time series, a correlation matrix can be obtained, which is a symmetric matrix, where C_{ij} represents the correlation between the brain regions i and j. According to the definition of the correlation threshold r_c, when a relative value is greater than the threshold value, it is considered that two brain regions have a correlation and that the element of the functional brain network matrix is 1; conversely, the two brain regions have no correlation and the element of the functional brain network matrix is 0. Note that the definition of "correlation" or "no correlation" does not consider whether there is an anatomical connection between the two brain regions. Thus, a complete complex functional brain network based on EEG data is established.

The next step is to select the threshold r_c. In the FMRI-based functional network research conducted by Eguiluz et al. (2003), r_c was set at 0.5, 0.6 and 0.7, but the reasons for such selection were not clearly stated. The selection of threshold should adhere to the following rules: the present study has proved that the brain is a sparse network, which consists of about 1011 highly interconnected neurons, each of which has about 104 connections; however, the overall connectivity factor, which represents the ratio of the real number of connections to the equivalent global coupling network, is only about 10–6. Numerical simulation shows that the functional network is sparse when the value of r_c is 0.95. Only when the correlation between time series is

more than 0.95, it is considered that there is a functional connection between the two brain regions. Figure 1 briefly illustrates the process of establishing the functional brain network.

3 Analysis of the functional brain network

3.1 Degree distribution

Degree is a very important concept in the complex network theory (Sun and Bin 2018). Its calculation formula is very simple, which means if there are $k(i)$ nodes connected to the node i, its degree is $k(i)$ (Shamshiri et al. 2017). For the entire network, the degree distribution is

$$k = \frac{\sum_{i=1}^{n} k(i)}{n}. \tag{1}$$

The degree distribution of nodes in the network can be described by the function $p(k)$, which indicates the probability that a randomly selected node is k. In general, the degree distribution of a random network follows the Poisson distribution and the probability distribution is bell shaped, while the actual network has the scale-free characteristics and follows the power-law distribution:

$$p(k) \sim k^{-\gamma}. \tag{2}$$

As a result, when the number of nodes in the network is large, the power-law distribution is more obvious. Under the S1 condition, the calculation results the brain networks of the patients and the normal people and corresponding random network degrees are shown in Table 1, 2, 3, and 4, where k indicates the degree, $n(k)$ represents the node number of the corresponding degree, and n represents the total number of nodes in the network.

Figures 2, 3, 4 and 5 give the degree distribution of the brain network.

Because the number of nodes of the network is limited (64 channels of EEG, so there are only 64 nodes), the conclusion of the network scale-free and the power-law distribution of the corresponding network node degree cannot be obtained. However, from Tables 1 and 3, Figs. 2 and 4, it can be seen that a large number of nodes in the network have a small number of connections, and the proportion of nodes with more connections (9 or 10 connections) is very small. As shown in the figure, the proportion is about 2% for the normal people, while it is about 3% for the patients. By comparison, from Tables 2 and 4, Figs. 3 and 5, it is clear to see that the degree distribution of nodes among the corresponding random networks is much more uniform.

Essentially, an important feature of the scale-free network is that a few nodes in the network have a large number of connections, which are called "distributed nodes", and that a large number of nodes only have a small number of connections. The obtained results seem to be consistent with the theory, but it is also necessary to calculate much more nodes of brain networks to verify whether it has a scale-free feature. Besides, the results suggest that normal people and patients appear to be different in the distribution of brain networks, which indicates that the distribution of brain networks of normal people seems more likely to be scale-free, which is to be further studied.

3.2 Clustering coefficient

Thec clustering coefficient is an important statistical characteristic of a complex network. Assuming that node i is connected to other $k(i)$ nodes, and there is a maximum of $k(i)(k(i) - 1)/2$ edges between the $k(i)$ nodes, but actually, there exists $E(i)$ edges. So the clustering coefficient of the node i is expressed as
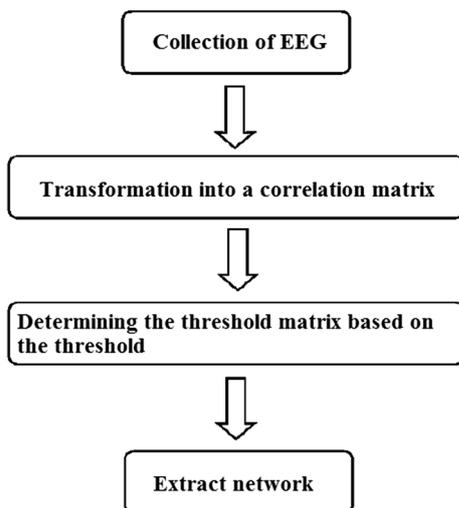
Fig. 1 Constructing a functional network method from EEG data

Table 1 Degree distribution of EEG network for patients

K	N(k)	$\frac{n(k)}{n}$ /%
1	9.00	33.82
2	3.33	12.87
3	2.00	8.96
4	3.33	12.91
5	2.67	11.86
6	1.33	5.86
7	0.67	3.00
8	0.33	1.28
9	0.67	9.44

Table 2 Corresponding random network degree distribution of EEG network for patients

K	N(k)	$\frac{n(k)}{n}$/%
1	22.67	53.95
2	14.33	34.13
≥ 3	5.00	11.92

Table 3 Degree distribution of EEG network for normal persons

K	N(k)	$\frac{n(k)}{n}$/%
1	6.33	20.66
2	7.67	19.02
3	7.67	19.02
4	6.33	14.06
5	5.67	10.74
6	4.33	5.78
7	3.00	3.30
8	2.33	5.78
9	0.67	0.82
10	0.33	0.82

Table 4 Corresponding random network degree distribution of EEG network for normal persons

K	N(k)	$\frac{n(k)}{n}$/%
1	17.67	30.62
2	16.00	29.50
≥ 3	22.00	39.88
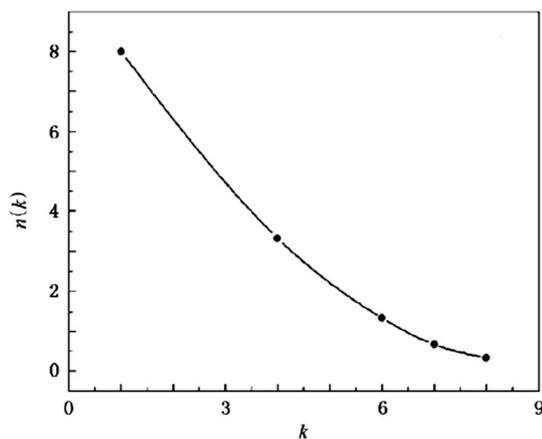
Fig. 2 Brain network degree distribution curve for patients

Fig. 3 Corresponding random network node distribution ratio of the brain networks of patients

Fig. 4 Brain network degree distribution curve for normal persons
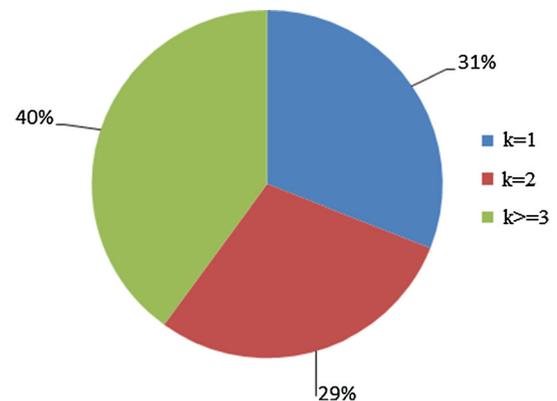
Fig. 5 Corresponding random network node distribution ratio of the brain networks of normal persons

$$C(i) = \frac{2E(i)}{k(i)(k(i) - 1)}. \tag{3}$$

The clustering coefficient of the network is

$$C = \frac{\sum\limits_{i=1}^{n} C(i)}{n}. \tag{4}$$

Obviously, for a fully connected regular network, the value of C is 1, while for a completely isolated "network" (without any side connection), the value is 0. It is found that the clustering coefficient of a completely random network with i nodes follows $C \sim O\left(\frac{1}{n}\right)$, while the real world network has a small world characteristic, where $O\left(\frac{1}{n}\right) < C < 1$.

The calculated results of the EEG functional networks of alcohol addicts and normal people and the clustering coefficients of their corresponding random networks are shown in Tables 5, 6, Figs. 6 and 7. C_{S1}, C_{S2-M} and C_{S2-N} represent the clustering coefficients under three different conditions, and C_h represents the clustering coefficient of the brain EEG network (including normal people and patients). More specifically, C_{hn} indicates the clustering coefficient of EEG network of the normal people, C_{hu} indicates the clustering coefficient of the EEG network of the patients, and C_{hrand} indicates the corresponding random network clustering coefficient.

It can be seen from Tables 5, 6, to Figs. 6 and 7 that, for EEG networks of patients and normal people, $C_{S1} < C_{S2-M} < C_{S2-N}$, and $C_h \gg C_{hrand}$. According to the rule of random network clustering coefficient $C \sim O\left(\frac{1}{n}\right)$, for a random network with 64 nodes, $C \sim O\left(\frac{1}{64}\right)$, which means that $C \sim O(0.01)$. In fact, although there are some differences in the clustering coefficients of random networks under various conditions, they all conform to the rule $C \sim O(0.01)$. Accordingly, the clustering coefficients of the functional networks of the patients and the normal people both follow $C \sim O(0.1)$, which is consistent with the conclusion that $O\left(\frac{1}{n}\right) < C < 1$ in the real world network, and also conforms to the criteria of the clustering coefficient of a small world network.

In the previous studies on various biological neural networks, such as the nematode's neural network, the ape's visual cortex network, and the cat's visual cortex network, a conclusion has been obtained that the clustering coefficient is far greater than that of the corresponding random network (Table 7). The conclusion is drawn from the EEG functional network of human brain. Even if the number of nodes is limited (only 64 nodes), the clustering coefficient of human brain EEG network is higher than that of the corresponding random network by an order of magnitude. If there is a large number of network nodes, the result will

be even better, which verifies the "small-world" characteristics shared by the human brain network and other biological neural networks.

Besides, the results also suggest that the clustering coefficient may be a parameter indicating the degree of attention concentration in neurophysiology. When faced with two mismatched visual targets, people need to concentrate more energy. Of course, the concentration of attention to the two targets is significantly greater than that to one target, and thus there will be more neurons in the human brain for clustering. At the same time, the normal people's brain networks are more concentrated than the patients', and the small world feature is more obvious, which seems to show that normal people can better focus on dealing with things than the patients. The results suggest that alcohol addiction has an effect on the function of the brain network, and the clustering coefficient can be used as an indicator of the brain damage.

3.3 Information entropy of the brain neural network

Entropy is an important statistical parameter in the thermodynamic system. Since the milestone was established by Clausius and Boltzmann, entropy has been widely used in many fields, such as physics, chemistry, life science, information science and so on. In 1943, Schrödinger put forward the idea that life feeds on "negative entropy", and for the first time, entropy was introduced into the field of life science. After that, in 1948, when Shannon was studying the uncertainty of the information transmission process, he proposed the concept of information entropy, which was defined as a reduction in uncertainty, and such uncertainty could be measured by entropy. It is assumed that the uncertainty of a probability information system before the information is obtained is H_0. After information is obtained, a part of the uncertainty is eliminated and its uncertainty is reduced to H_1. Therefore, from the

Table 5 Clustering coefficient of brain network and corresponding random network for patients

Clustering coefficient	Brain network	Corresponding random network
C_{S1}	0.2043	0.0216
C_{S2-M}	0.3068	0.0308
C_{S2-N}	0.3126	0.0328

Table 6 Clustering coefficient of brain network and corresponding random network for normal persons

Clustering coefficient	Brain network	Corresponding random network
C_{S1}	0.2052	0.0339
C_{S2-M}	0.3657	0.0682
C_{S2-N}	0.3829	0.0576
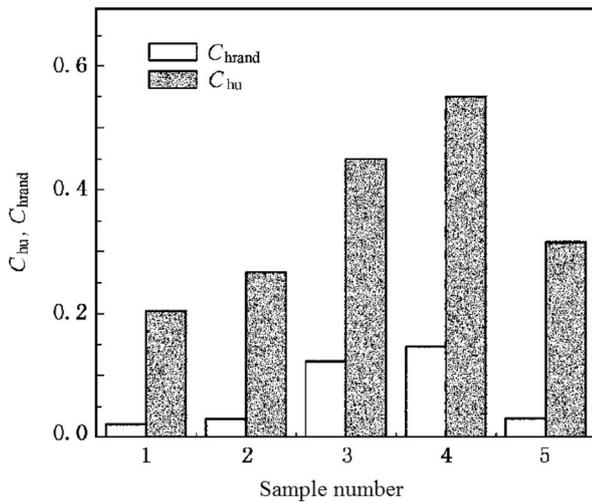
(a) Under $S1$ condition
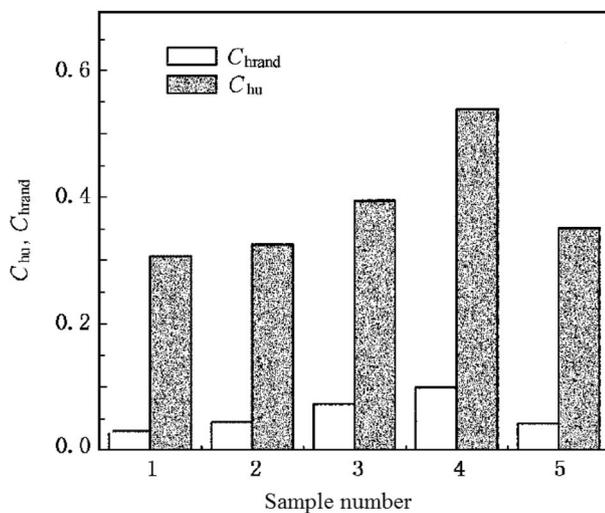
(b) Under $S2 - M$ condition
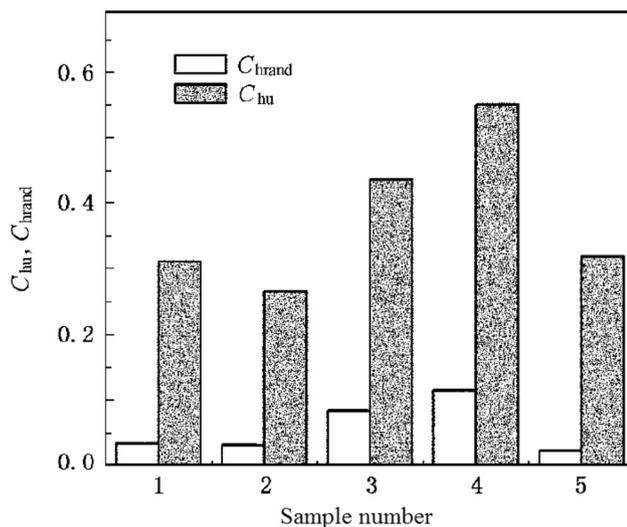
(c) Under $S2 - N$ condition

◀ **Fig. 6** Comparison of clustering coefficients between EEG network and corresponding random network for patients

uncertainty eliminated $(H_0 - H_1)$, the amount of information I obtained by the system can be calculated, for which the mathematical expression is

$$I = H_0 - H_1 = -(H_1 - H_0). \tag{5}$$

The amount of information derived by the system from the outside world is equal to the negative value of the system's entropy increment, which is referred to as negative entropy. It embodies the meaning of "information is negative entropy" and its process quantity characteristics. In this way, it is also consistent with the meaning of "life feeds on negative entropy".

Shannon further gave the formula for calculating the amount of information I:

$$I = k_B \ln \Omega = -\sum_{j=1}^{N} k_B P(j) \ln P(j), \tag{6}$$

where Ω is the information to be selected, and the larger the Ω, the greater the amount of information will be, and $P(j) = 1/\Omega$, k_B is the Boltzmann constant.

In fact, the main function of the brain neural network is to divide and integrate information. Therefore, we derive and define a computational rule for the information entropy of the brain neural network by calculating the information entropy. The calculation and comparison of the different entropy values between neural networks of the patients and the normal persons provides a new criterion for the diagnosis of the disease and the effectiveness of the medication or operation.

When it comes to the information entropy of the brain neural network, PC actually refers to the probability, and for a specific node, the importance of node is defined as the node degree divided by the total node degree of the network. Therefore, the node importance can be considered as the probability of the transmission of the node (for the specific functional area). Based on mathematical knowledge, it can be known that when the degree increases, the node importance increases and the information transfer ability of the functional area also increases. When the degree is zero, the node importance is also zero, and in this case the function area cannot carry out the information transfer.

$P(i)$ is defined as the importance of node i

$$P(i) = \frac{k_i}{\sum_{i=1}^{n} k_i}. \tag{7}$$

(a) Under $S1$ condition

(b) Under $S2 - M$ condition

(c) Under $S2 - N$ condition
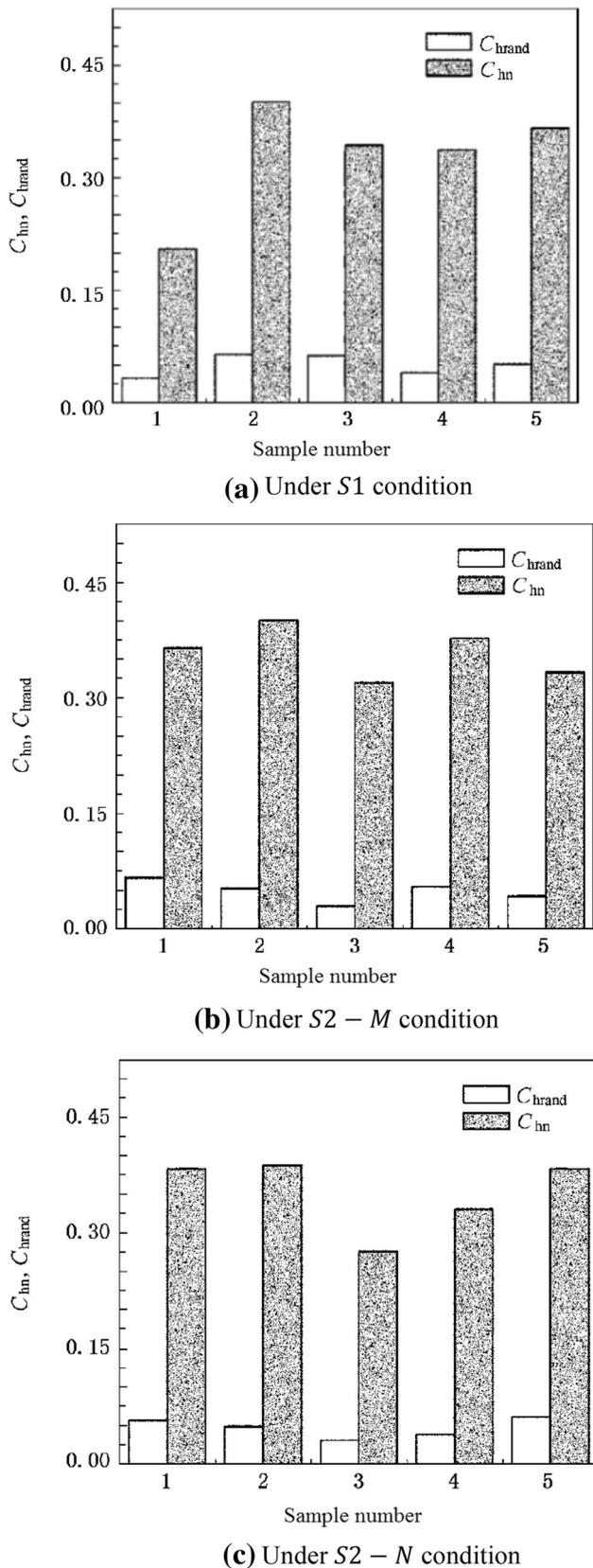
Fig. 7 Comparison of clustering coefficients between EEG network and corresponding random network for normal persons

Table 7 Comparison of clustering coefficients between various biological neural networks and corresponding random network

Type of biological neural network	N	C	C_{rand}
Nematode's neural network	282	0.29	0.026
Ape's visual cortex network	32	0.56	0.321
Cat's visual cortex network	66	0.55	0.276
Nematode's neural network	282	0.29	0.026

where n is the number of nodes in the network, and k_i is the connection degree of the node i. When $k = 0$, the node is meaningless, so let $k > 0$, and then $P(i) > 0$.

E is the information entropy of the brain neural network, which refers to the amount of information obtained by the brain neural network, and is also called the increment of network negative entropy.

$$E = -\sum_{i=1}^{n} k_B P(i) \ln P(i). \tag{8}$$

In order to simplify the calculation, the Boltzmann constant k_B can be omitted, that is

$$E = -\sum_{i=1}^{n} P(i) \ln P(i). \tag{9}$$

Obviously, when the network is a fully connected rule network, it can be represented as

$$E_{max} = \ln n. \tag{10}$$

When the network is a star network with only one central node, it can be represented as

$$E_{min} = \frac{\ln 4(n - 1)}{2}. \tag{11}$$

In order to eliminate the influence of the node number n on E, and carry out normalization calculation of them, the standard entropy of brain neural network can be defined as follows:

$$E_a = \frac{E - E_{min}}{E_{max} - E_{min}}. \tag{12}$$

Through the calculation of the brain functional network, the results are shown in Table 8 and Fig. 8.

In order to make the analysis more accurate, the neural network standard entropy E_a is used to calculate the value in Fig. 8.

It can be easily known from the above calculation that the information entropy of the brain neural network and the standard information entropy of the brain neural network under the condition of S2 are obviously larger than those of S1, but there is no significant difference between the values of S2-M and S2-N. When faced with two visual objects, the amount of information obtained by the functional brain

Table 8 Comparison of the brain neural network information entropy and the neural network standard information entropy between normal people and patients based on EEG

	Stimulus condition	Patients	Normal person
E	S1	3.57	3.63
E_a	S1	0.59	0.62
E	S2-M	3.65	3.68
E_a	S2-M	0.67	0.64
E	S2-N	3.62	3.68
E_a	S2-N	0.61	0.66
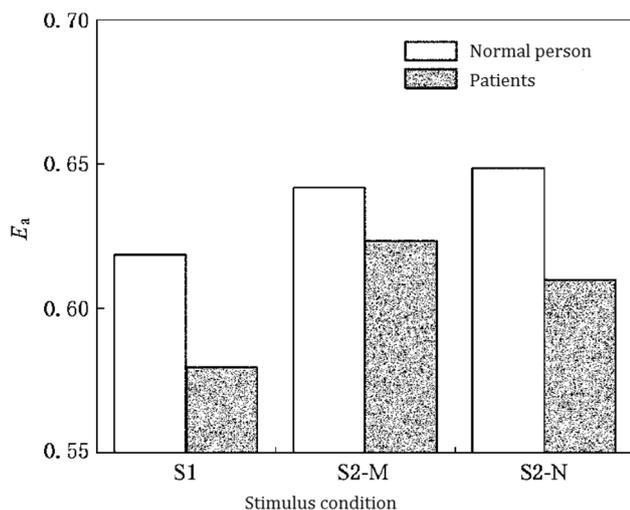
Fig. 8 Comparison of EEG network standard information entropy between normal persons and alcoholism patients

network is increased, which means that the negative entropy increment is greater at this time. However, there is no significant difference in the amount of information obtained by the network when there is no significant difference in visual objects. Under the same condition, the information entropy of the brain network and the standard information entropy of the brain neural networks of the patients are significantly smaller than those of the normal people, which means that the negative entropy increment of the brain networks of patients is significantly smaller than that of the normal people, and thus it can be concluded that encephalopathy (alcohol addiction) obviously affects the brain's reception of information.

4 Conclusion

Through the calculation of the functional brain network based on EEG data and the analysis of its complex network characteristics, it is found that its clustering coefficient is far larger than that of the corresponding random network,

and that it has the characteristics of a small world network, which further confirms the small world characteristics of the human brain. The degree distribution of the network is also close to that of a scale-free network. Of course, it is necessary to carry out further calculation of large-scale networks to verify the pattern. Actually, the degree distribution and clustering coefficient of the functional brain network for alcohol addicts are different from those for normal people, which suggest that we can find a new criterion for the diagnosis of the disease and the effectiveness of the treatment.

The calculation based on the information entropy of the brain neural network suggests that the brain neural network has a series of characteristic tendencies, such as the formation of small world, the scale-free degree, and so on. It is very likely that this kind of structure helps absorb "negative entropy", so as to maintain the survival of the network as an organism. The information entropy of the brain neural networks of patients is obviously different from that of normal people, that is to say, the amount of information obtained by the patients is obviously smaller than that by the normal persons. As they cannot receive some information, the brain network is too orderly, making the brain inclined to die as a system, which is consistent with the theory of "life feeds on negative entropy". Therefore, further research and application of information entropy of brain neural network will provide a good means for diagnosis and treatment of encephalopathy. In short, the above calculations and research are only the beginning, and our next research will work on the functional brain networks based on brain functional imaging. The theory is expected to be applied in clinical practice to predict the condition and verify the effectiveness of drugs and surgery.

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