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TECHNICAL PAPER

Effect of rotation and gravity on the reflection of P-waves from thermo-magneto-microstretch medium in the context of three phase lag model with initial stress

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Abstract

In this paper the linear theory of the thermoelasticity has been employed to study the effect the reflection of plane harmonic waves from a semi-infinite elastic solid under the effect of the magnetic field , rotation, initial stress and gravity. The medium under consideration is traction free, homogeneous, isotropic, as well as with three-phase-lag. The normal mode analysis is used to solve the resulting non-dimensional coupled equations. The expressions for the reflection coefficients, which are the relations of the amplitudes of the reflected waves to the amplitude of the incident waves, are obtained similarly, the reflection coefficient ratio variations with the angle of incident under different conditions are shown graphically. Comparisons are made with the results predicted by different theories Lord-Shulman theory (L-S), the Green-Naghdi theory of type III (G-N III) and the three-phase-lag model in the absence and presence of a magnetic field, rotation, initial stress and gravity. The results indicate that the effect of rotation, magnetic field, initial stress and gravity field are very pronounced.

1 Introduction

In recent decades, the influences of magnetic field, thermal field have too pronounced in diverse fields, especially, Engineering, Geophysics, Geology, Acoustics, Plasma and Physics because of its utilitarian aspects in these fields. The generalized theories of thermoelasticity, which admit the finite speed of a thermal signal, have been the center of interest of active research. Othman and Song (2008) studied the reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic module. Abo-Dahab and Biswas (2017) investigated the effect of rotation on Rayleigh waves in magneto-thermoelastic transversely isotropic medium with thermal relaxation times. Said (2016a) discussed the two-temperature generalized magneto-thermoelastic medium for dual-phase-lag model under the effect of the gravity field and hydrostatic initial stress. Xiong and Guo (2016) investigated the effect of variable properties and moving heat Source on magnetothermoelastic Problem under fractional order thermoelasticity. Othman et al. (2010) investigated the generalized thermo-microstretch elastic medium with temperature dependent properties for the different theories. Chakraborty and Singh (2011a) studied the reflection and refraction of a plane thermoelastic wave at a solid–solid interface under perfect boundary condition, in the presence of normal initial stress. Singh (2008) studied the effect of hydrostatic initial stresses on waves in a thermoelastic solid half-space. Abo-Dahab et al. (2017) investigated a two-dimensional problem with rotation and magnetic field in the context of four thermoelastic theories. Kumar and Kaur (2014) investigated the reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures.

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Elsagheer and Abo-Dahab (2016) studied the reflection of thermoelastic waves from insulated boundary fibre-reinforced half-space under the influence of rotation and magnetic field. Abo-Dahab and Abd-Alla (2016) investigated the Green Lindsay model on reflection and refraction of p- and SV-waves at the interface between solid and liquid media presence in a magnetic field and initial stress. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in Abd-Alla et al. (2016a, b, 2017a, b, c), Ailawalia and Narah (2009), Lykotrafitis et al. (2001), Chakraborty and Singh (2011b) and Said (2015, 2016b).

The objective of the present investigation is to determine the reflection of P-waves from thermo-magneto-microstretch medium in the context of three phase lag model under the effect of rotation and gravity with initial stress. The reflection coefficient ratios of various reflected waves with the angle of incidence have been obtained from (3PHL) model and LS theory. Also the effect of the magnetic field and gravity is discussed numerically and illustrated graphically. The physical quantities are obtained and tested by a numerical study using the parameters of Cu as a target and presented graphically. The distribution of these quantities is represented graphically in the presence and absence of magnetic and gravity field.

2 Formulation of the problem

The equations of generalized thermo-micro-stretch in a rectangular coordinate system $(x,\ y,\ z)$ with z-axis directed in the media are used. Constant magnetic field intensity $\boldsymbol{H} = (0,\ H_0,\ 0)$ is taken as the direction of the y-axis. We begin our consideration with linearized equations of electro-dynamics of slowly moving media (Xiong and Guo 2016)

$$\boldsymbol{J} = \mathrm{curl}\boldsymbol{h} - \varepsilon_0 \dot{\boldsymbol{E}}, \tag{1}$$

$$\mathrm{curl}\boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}}, \tag{2}$$

$$\boldsymbol{E} = -\mu_0(\dot{\boldsymbol{u}} \times \boldsymbol{H}), \tag{3}$$

$$\nabla.\boldsymbol{h} = 0. \tag{4}$$

The constitutive equation in the presence of gravitational and magnetic field (Othman and Song 2008)

$$\sigma_{ji,i} + F_i + G_i = \rho[\ddot{\boldsymbol{u}} + \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{u} + 2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}]_i, \tag{5}$$

$$F_i = \mu_0(\boldsymbol{J} \times \boldsymbol{H})_i, \quad G_i = \rho g\left(\frac{\partial w}{\partial x}, 0, -\frac{\partial u}{\partial x}\right). \tag{6}$$

where, $\boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{u}$ is the centripetal acceleration due to the time varying motion only and $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$ is the Coriolis acceleration.

From the above equations, we can obtain

$$\boldsymbol{E} = \mu_0 H_0(\dot{w},\ 0,\ -\dot{u}), \tag{7}$$

$$\boldsymbol{h} = (0,\ -H_0 e,\ 0), \tag{8}$$

$$\boldsymbol{J} = (H_0\, e_{,z} - \varepsilon_0\mu_0 H_0\, \ddot{w},\ \ 0,\ \ -H_0\, e_{,x} + \varepsilon_0\mu_0 H_0\, \ddot{u}). \tag{9}$$

From Eqs. (6) and (9), we obtain

$$\begin{aligned} F &= (F_x, F_y, F_z) \\ &= (\mu_0 H_0^2 e_{,x} - \varepsilon_0\mu_0^2 H_0^2 \ddot{u},\ 0,\ \mu_0 H_0^2 e_{,z} - \varepsilon_0\mu_0^2 H_0^2 \ddot{w}). \end{aligned} \tag{10}$$

So the displacement vector $\boldsymbol{u}$ has the components $u_x = u(x,z,t), \quad u_y = 0, \quad u_z = w(x,z,t).$

$$\begin{aligned} \sigma_{ij} = {} & (\lambda_0\varphi^* - P + \lambda\, u_{k,k})\, \delta_{ij} + (\,\mu + k\,)\, u_{j,i} + \mu\, u_{i,j} \\ & - k\, \varepsilon_{ijk}\varphi_k - \hat{\gamma}\, T\, \delta_{ij} - P\omega_{ij}, \end{aligned} \tag{11}$$

$$m_{ij} = \alpha\varphi_{k,k}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i}, \tag{12}$$

$$\lambda_i = \alpha_0\varphi^*_{,i}. \tag{13}$$

where, $\hat{\gamma} = (3\lambda + 2\mu + k)\alpha_{t_1}, \quad \hat{\gamma}_1 = (3\lambda + 2\mu + k)\alpha_{t_2}$, $\omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j})$, T is the temperature above the reference temperature T_0, λ, μ and t are the Lame' constants and time, u_i, σ_{ij}, e_{ij}, m_{ij}, P are the components of the displacement vector, of the stress tensor, strain tensor, couple stress tensor and the initial stress, respectively, j is the micro inertia moment, k, α, β, γ and α_0, λ_0, λ_1 are the micropolar and microstretch constants, respectively, φ^* is the scalar microstretch, $\boldsymbol{\varphi}$ is the rotation vector and ρ. is the mass density

The constitutive relation can be written as

$$\begin{aligned} \sigma_{xx} &= (\lambda + 2\mu + k)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} + \lambda_0\varphi^* - \hat{\gamma}T - P, \\ \sigma_{zz} &= (\lambda + 2\mu + k)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} + \lambda_0\varphi^* - \hat{\gamma}T - P, \\ \sigma_{xz} &= \left(\mu + \frac{P}{2}\right)\frac{\partial u}{\partial z} + \left(\mu + k - \frac{P}{2}\right)\frac{\partial w}{\partial x} + k\varphi_2, \\ \sigma_{zx} &= \left(\mu + \frac{P}{2}\right)\frac{\partial w}{\partial x} + \left(\mu + k - \frac{P}{2}\right)\frac{\partial u}{\partial z} - k\varphi_2, \end{aligned} \tag{14}$$

$$m_{xy} = \gamma\frac{\partial\varphi_2}{\partial x}, \quad m_{zy} = \gamma\frac{\partial\varphi_2}{\partial z}, \tag{15}$$

$$\lambda_x = \alpha_0\frac{\partial\varphi^*}{\partial x}, \quad \lambda_z = \alpha_0\frac{\partial\varphi^*}{\partial z}. \tag{16}$$

In component form the basic governing equations become,

$$\begin{aligned} & \left(\mu + k - \frac{P}{2}\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ & + \left(\lambda + \mu + \frac{P}{2} + \mu_0 H_0^2\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + \lambda_0\frac{\partial\varphi^*}{\partial x} \\ & - k\frac{\partial\varphi_2}{\partial z} - \hat{\gamma}\frac{\partial T}{\partial x} \\ & = \rho\left[\left(1 + \frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{u} - \Omega^2 u + 2\Omega\dot{w} - g\frac{\partial w}{\partial x}\right], \end{aligned} \tag{17}$$

$$\left(\mu+k-\frac{P}{2}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$+\left(\lambda+\mu+\frac{P}{2}+\mu_0 H_0^2\right)\left(\frac{\partial^2 u}{\partial x \partial z}+\frac{\partial^2 w}{\partial z^2}\right)+\lambda_0\frac{\partial \varphi^*}{\partial z}$$
$$+k\frac{\partial \varphi_2}{\partial x}-\hat{\gamma}\frac{\partial T}{\partial z}=\rho\left[\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{w}-\Omega^2 w-2\Omega\dot{u}+g\frac{\partial u}{\partial x}\right], \tag{18}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\boldsymbol{\varphi})-\gamma\nabla\times(\nabla\times\boldsymbol{\varphi}) + k(\nabla\times\boldsymbol{u})-2k\boldsymbol{\varphi}=\rho j\boldsymbol{\varphi}, \tag{19}$$

$$\alpha_0\nabla^2\varphi^*-\frac{1}{3}\lambda_1\varphi^*-\frac{1}{3}\lambda_0(\nabla.u)+\frac{1}{3}\hat{\gamma}_1 T=\frac{3}{2}\rho j\ddot{\varphi}^*, \tag{20}$$

$$\left(K^*+\tau_v^*\frac{\partial}{\partial t}+K\tau_T\frac{\partial^2}{\partial t^2}\right)\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\rho C_e\ddot{T}+\hat{\gamma}T_0\ddot{e}+\hat{\gamma}_1 T_0\ddot{\varphi}^*]. \tag{21}$$

Such that $\tau_v^*=K+K^*\tau_v$, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$.

To facilitate the solution, the following non-dimensions, quantities are introduced

$$x_i'=\frac{\omega^*}{c_0}x_i,\quad u_i'=\frac{\rho c_0\omega^*}{\hat{\gamma}T_0}u_i,$$
$$\Theta'=\frac{\hat{\gamma}}{\rho c_0^2}(T-T_0),\{t',\tau_T',\tau_v',\tau_q'\}=\omega^*\{t,\tau_T,\tau_v,\tau_q\},$$
$$\sigma_{ij}'=\frac{\sigma_{ij}}{\hat{\gamma}T_0},\quad m_{ij}'=\frac{\omega^*}{c_0\hat{\gamma}T_0}m_{ij},\quad \lambda_i'=\frac{\omega^*}{c_0\hat{\gamma}T_0}\lambda_i,\quad \varphi'^*=\frac{\rho c_0^2}{\hat{\gamma}T_0}\varphi^*,$$
$$\varphi_2'=\frac{\rho c_0^2}{\hat{\gamma}T_0}\varphi_2,\quad g'=\frac{g}{c_0\omega^*},\quad \omega^*=\frac{\rho C_e c_0^2}{K},$$
$$h'=\frac{h}{H_0},\quad \Omega'=\frac{\Omega}{\omega^*}\quad\text{and}\quad c_0^2=\frac{\lambda+2\mu+k}{\rho},\quad i,j=1,2,3. \tag{22}$$

Using Eq. (22) the governing Eqs. (17–21) recast in the following form (after suppressing the primes)

$$\left(\frac{\mu+k-P/2}{\rho c_0^2}\right)\nabla^2 u+\left(\frac{\lambda+\mu+P/2}{\rho c_0^2}+R_H\right)\frac{\partial e}{\partial x} -\frac{k}{\rho c_0^2}\frac{\partial\varphi_2}{\partial z}+\frac{\lambda_0}{\rho c_0^2}\frac{\partial\varphi^*}{\partial x}-\frac{\rho c_0^2}{\hat{\gamma}T_0}\frac{\partial\Theta}{\partial x}+g\frac{\partial w}{\partial x} =\beta^2\ddot{u}-\Omega^2 u+2\Omega\dot{w}, \tag{23}$$

$$\left(\frac{\mu+k-P/2}{\rho c_0^2}\right)\nabla^2 w+\left(\frac{\lambda+\mu+P/2}{\rho c_0^2}+R_H\right)\frac{\partial e}{\partial z} +\frac{k}{\rho c_0^2}\frac{\partial\varphi_2}{\partial x}+\frac{\lambda_0}{\rho c_0^2}\frac{\partial\varphi^*}{\partial z}-\frac{\rho c_0^2}{\hat{\gamma}T_0}\frac{\partial\Theta}{\partial z}-g\frac{\partial u}{\partial x} =\beta^2\ddot{w}-\Omega^2 w-2\Omega\dot{u}, \tag{24}$$

$$\nabla^2\varphi_2-\frac{2kc_0^2}{\gamma\omega^{*2}}\varphi_2+\frac{kc_0^2}{\gamma\omega^{*2}}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)=\frac{\rho jc_0^2}{\gamma}\frac{\partial^2\varphi_2}{\partial t^2}, \tag{25}$$

$$\left(\frac{C_1^2}{c_0^2}\nabla^2-\frac{C_2^2}{\omega^{*2}}-\frac{\partial^2}{\partial t^2}\right)\varphi^*-\frac{C_3^2}{\omega^{*2}}e+a_0\Theta+C=0, \tag{26}$$

$$\left(C_k+C_v\frac{\partial}{\partial t}+C_T\frac{\partial^2}{\partial t^2}\right)\nabla^2\Theta =\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\ddot{\Theta}+\varepsilon_1\ddot{e}+\varepsilon_2\ddot{\varphi}^*], \tag{27}$$

$$\sigma_{xx}=u_{,x}+a_1 w_{,z}+a_2\varphi^*-a_3\Theta-a_4,$$
$$\sigma_{zz}=w_{,z}+a_1 u_{,x}+a_2\varphi^*-a_3\Theta-a_4,$$
$$\sigma_{xz}=a_5 u_{,z}+a_6 w_{,x}+a_7\varphi_2,\quad \sigma_{zx}=a_5 w_{,x}+a_6 u_{,z}-a_7\varphi_2,$$
$$m_{xy}=a_8\frac{\partial\varphi_2}{\partial x},\quad m_{zy}=a_8\frac{\partial\varphi_2}{\partial z},\quad \lambda_x=a_9\frac{\partial\varphi^*}{\partial x},\quad \lambda_z=a_9\frac{\partial\varphi^*}{\partial z}. \tag{28}$$

where

$$R_H=\frac{\mu_0 H_0^2}{\rho c_0^2},\quad \beta^2=\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right),$$
$$(C_1^2,C_1^2,C_3^2)=\frac{2}{9j\rho}(3\alpha_0,\lambda_1,\lambda_0),\quad a_0=\frac{2\rho\, c_0^4\hat{\gamma}_1}{9j\hat{\gamma}^2 T_0\omega^{*2}},$$
$$C=\frac{2c_0^2\hat{\gamma}_1}{9j\hat{\gamma}\omega^{*2}},\quad (C_k,C_v,C_T)=\frac{1}{\rho\, c_0^2 C_e}(K^*,\tau_v^*,K\tau_T\omega^*),$$
$$\varepsilon_1=\frac{\hat{\gamma}^3 T_0^2}{\rho^3\, c_0^4 C_e},\quad \varepsilon_2=\frac{\hat{\gamma}_1\hat{\gamma}^2 T_0^2}{\rho^3\, c_0^4 C_e},\quad a_3=\frac{\rho\, c_0^2}{\hat{\gamma}T_0},$$
$$a_4=\left(\frac{P}{\hat{\gamma}T_0}+1\right),$$
$$(a_1,a_2,a_5,a_6,a_7)=\frac{1}{\rho c_0^2}\left(\lambda,\lambda_0,\mu+\frac{P}{2},\mu+k-\frac{P}{2},k\right),$$
$$(a_8,a_9)=\frac{\omega^{*2}}{\rho c_0^4}(\gamma,\alpha_0). \tag{29}$$

The displacement components $u(x,z,t)$ and $w(x,z,t)$ may be written in terms of potential functions $\Phi(x,z,t)$ and $\Psi(x,z,t)$ as

$$u=\Phi_{,x}+\Psi_{,z},w=\Phi_{,z}-\Psi_{,x},\quad \overrightarrow{\Psi}=(0,-\Psi,0). \tag{30}$$

Using Eq. (30) in Eqs. (23–27), we obtain

$$\left(\nabla^2+\zeta_0-\zeta_1\frac{\partial^2}{\partial t^2}\right)\Phi-\left(\zeta_2\frac{\partial}{\partial x}-\zeta_3\frac{\partial}{\partial t}\right)\Psi-\zeta_4\Theta+\zeta_5\varphi^*=0, \tag{31}$$

$$\left(\zeta_6\frac{\partial}{\partial x}-\zeta_7\frac{\partial}{\partial t}\right)\Phi+\left(\nabla^2+\zeta_8-\zeta_9\frac{\partial^2}{\partial t^2}\right)\Psi-\zeta_{10}\varphi_2=0, \tag{32}$$

$$\frac{kc_0^2}{\gamma\omega^{*2}}\nabla^2\Psi+\left(\nabla^2-\frac{2kc_0^2}{\gamma\omega^{*2}}-\frac{j\rho c_0^2}{\gamma}\frac{\partial^2}{\partial t^2}\right)\varphi_2=0, \tag{33}$$

$$\frac{C_3^2}{\omega^{*2}}\nabla^2\Phi - a_0\Theta - \left(\frac{C_1^2}{c_0^2}\nabla^2 - \frac{C_2^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2}\right)\varphi^* = 0, \qquad (34)$$

$$\left(C_k + C_v\frac{\partial}{\partial t} + C_T\frac{\partial^2}{\partial t^2}\right)\nabla^2\Theta = \left(1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\ddot{\Theta} + \varepsilon_1\nabla^2\ddot{\Phi} + \varepsilon_2\ddot{\varphi}^*], \qquad (35)$$

where

$$(\zeta_0, \zeta_1, \zeta_2, \zeta_3) = \frac{1}{1 + R_H}(\Omega^2, \beta^2, g, 2\Omega),$$

$$\zeta_3 = \frac{\rho^2 c_0^4}{\hat{\gamma}T_0(\mu_0 H_0^2 + \rho c_0^2)}, \quad \zeta_5 = \frac{\lambda_0}{\mu_0 H_0^2 + \rho c_0^2},$$

$$(\zeta_6, \zeta_7, \zeta_8, \zeta_9) = \frac{\rho c_0^2}{\mu + k - P/2}(g, 2\Omega, \Omega^2, \beta^2),$$

$$\zeta_{10} = \frac{k}{\mu + k - P/2}.$$

$$(\imath\zeta_6\xi\sin\theta + \imath\zeta_7\omega)\bar{\Phi} + (-\xi^2 + \zeta_8 + \zeta_9\omega^2)\bar{\Psi} - \zeta_{10}\bar{\varphi}_2 = 0, \qquad (38)$$

$$\frac{kc_0^2\xi^2}{\gamma\omega^{*2}}\bar{\Psi} + \left(\xi^2 + \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2\omega^2}{\gamma}\right)\bar{\varphi}_2 = 0, \qquad (39)$$

$$\frac{C_3^2}{\omega^{*2}}\xi^2\bar{\Phi} + a_0\bar{\Theta} - \left(\frac{C_1^2}{c_0^2}\xi^2 + \frac{C_2^2}{\omega^{*2}} - \omega^2\right)\bar{\varphi}^* = 0, \qquad (40)$$

$$\varepsilon_1\tau_q^*\xi^2\omega^2\bar{\Phi} + \left[(C_k - \imath C_v\omega - C_T\omega^2)\xi^2 - \tau_q^*\omega^2\right]\bar{\Theta} - \varepsilon_2\tau_q^*\omega^2\bar{\varphi}^* = 0, \qquad (41)$$

where, $\tau_q^* = 1 - \imath\tau_q\omega - \frac{\tau_q^2\omega^2}{2}$.

To find the nontrivial solution, it must Substitute from Eq. (36) into Eqs. (37–41), we get

$$\begin{vmatrix} -\xi^2 + \zeta_0 + \zeta_1\omega^2 & -\imath\zeta_2\xi\sin\theta - \imath\zeta_3\omega & -\zeta_4 & \zeta_5 & 0 \\ \imath\zeta_6\xi\sin\theta + \imath\zeta_7\omega & -\xi^2 + \zeta_8 + \zeta_9\omega^2 & 0 & 0 & -\zeta_{10} \\ 0 & \frac{kc_0^2\xi^2}{\gamma\omega^{*2}} & 0 & 0 & \xi^2 + \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2\omega^2}{\gamma} \\ \frac{C_3^2}{\omega^{*2}}\xi^2 & 0 & a_0 & -\frac{C_1^2}{c_0^2}\xi^2 - \frac{C_2^2}{\omega^{*2}} + \omega^2 & 0 \\ \varepsilon_1\xi^2\omega^2\tau_q^* & 0 & ()\xi^2 - \omega^2\tau^*_{C_k - \imath C_v\omega - C_T\omega^2 q} & -\varepsilon_2\omega^2\tau_q^* & 0 \end{vmatrix} = 0, \qquad (42)$$

3 Solution of the problem

We assume now the solution of Eqs. (31–34) takes the following form

$$\{\Phi, \Psi, \Theta, \varphi^*, \varphi_2\} = (\bar{\Phi}, \bar{\Psi}, \bar{\Theta}, \bar{\varphi}^*, \bar{\varphi}_2)\exp[i\xi(x\sin\theta + z\cos\theta) - i\omega t], \qquad (36)$$

where ξ is the wave number, ω is the complex frequency and $v = \frac{\omega}{\xi}$ is the velocity of the coupled waves.

Substitute from Eq. (36) into Eqs. (31–34), we get

$$(-\xi^2 + \zeta_0 + \zeta_1\omega^2)\bar{\Phi} - (\imath\zeta_2\xi\sin\theta + \imath\zeta_3\omega)\bar{\Psi} - \zeta_4\bar{\Theta} + \zeta_5\bar{\varphi}^* = 0, \qquad (37)$$

which tends to

$$A\xi^{10} + B\xi^8 + C\xi^6 + D\xi^4 + E\xi^2 + F = 0. \qquad (43)$$

The coefficients A, B, C, D, E and E of Eq. (43) has been shown in "Appendix".

From Eqs. (43) there are five waves with five different velocities Then $\Phi, \Psi, \Theta, \varphi^*, \varphi_2$ will take the following forms:

$$\{\Phi, \Psi, \Theta, \varphi^*, \varphi_2\} = \{1, \eta_0, \kappa_0, \chi_0, \vartheta_0\}A_0 \times \exp[i\xi(x\sin\theta_0 + z\cos\theta_0) - i\omega t] + \sum_{j=1}^{5}\{1, \eta_j, \kappa_j, \chi_j, \vartheta_j\}A_j \times \exp\left[i\xi_j(x\sin\theta_j - z\cos\theta_j) - i\omega t\right]. \qquad (44)$$

From Eqs. (38) and (39) we get

$$\eta_j = \frac{-(\imath\zeta_6\xi_j \sin\theta + i\zeta_7\omega)\left(\xi_j^2 + \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2\omega^2}{\gamma}\right)}{(-\xi_j^2 + \zeta_8 + \zeta_9\omega^2)\left(\xi_j^2 + \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2\omega^2}{\gamma}\right) + \zeta_{10}\xi_j^2 \frac{kc_0^2}{\gamma\omega^{*2}}},$$

$$\kappa_j = \frac{\xi_j^2 \frac{kc_0^2}{\gamma\omega^{*2}}(\imath\zeta_6\xi_j \sin\theta + i\zeta_7\omega)}{(-\xi_j^2 + \zeta_8 + \zeta_9\omega^2)\left(\xi_j^2 + \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2\omega^2}{\gamma}\right) + \zeta_{10}\xi_j^2 \frac{kc_0^2}{\gamma\omega^{*2}}}. \tag{45}$$

Also, from Eqs. (40) and (41) we get

$$\chi_j = \frac{\varepsilon_2\tau_q^*\omega^2 \frac{C_3^2}{\omega^{*2}}\xi_j^2 + \varepsilon_1\tau_q^*\xi_j^2\omega^2\left(\frac{C_1^2}{c_0^2}\xi_j^2 + \frac{C_2^2}{\omega^{*2}} - \omega^2\right)}{-a_0\varepsilon_2\tau_q^*\omega^2 - \left(\frac{C_1^2}{c_0^2}\xi_j^2 + \frac{C_2^2}{\omega^{*2}} - \omega^2\right)\left[(C_k - iC_v\omega - C_T\omega^2)\xi_j^2 - \tau_q^*\omega^2\right]},$$

$$\vartheta_j = \frac{-a_0\varepsilon_1\tau_q^*\xi_j^2\omega^2 + \frac{C_3^2}{\omega^{*2}}\xi_j^2\left[(C_k - iC_v\omega - C_T\omega^2)\xi_j^2 - \tau_q^*\omega^2\right]}{-a_0\varepsilon_2\tau_q^*\omega^2 - \left(\frac{C_1^2}{c_0^2}\xi_j^2 + \frac{C_2^2}{\omega^{*2}} - \omega^2\right)\left[(C_k - iC_v\omega - C_T\omega^2)\xi_j^2 - \tau_q^*\omega^2\right]}. \tag{46}$$

4 The boundary conditions

The boundary conditions for the problem be taken as

$$\sigma_{zz} + \tau_{zz} = 0, \quad \sigma_{xz} + \tau_{xz} = 0, \quad \frac{\partial T}{\partial z} = 0, \quad m_{zy} = 0, \quad \lambda_z = 0 \quad \text{at} \quad z = 0, \tag{47}$$

$$\sum_{j=1}^{5}\left[a_2\vartheta_j - a_3\chi_j - \mu_0H_0^2\xi_j^2 - \xi_j^2\left(\cos^2\theta_j + \frac{\eta_j}{2}\sin 2\theta_j\right) - a_1\xi_j^2\left(\sin^2\theta_j - \frac{\eta_j}{2}\sin 2\theta_j\right)\right] A_j = -\left[a_2\vartheta_1 - a_3\chi_j - \mu_0H_0^2\xi_1^2 - \xi_1^2\left(\cos^2\theta_0 - \frac{\eta_1}{2}\sin 2\theta_0\right) - a_1\xi_1^2\left(\sin^2\theta_0 + \frac{\eta_1}{2}\sin 2\theta_0\right)\right]A_0, \tag{48}$$

$$\sum_{j=1}^{5}\left[a_7\kappa_j + a_5\xi_j^2\left(\frac{1}{2}\sin 2\theta_j - \frac{\eta_j}{2}\cos^2\theta_j\right) + a_6\xi_j^2\left(\frac{1}{2}\sin 2\theta_j + \frac{\eta_j}{2}\sin^2\theta_j\right)\right] A_j = -\left[a_7\kappa_1 + a_5\xi_1^2\left(\frac{1}{2}\sin 2\theta_0 + \frac{\eta_1}{2}\cos^2\theta_0\right) + a_6\xi_1^2\left(\frac{1}{2}\sin 2\theta_0 - \frac{\eta_j}{2}\sin^2\theta_0\right)\right]A_0, \tag{49}$$

$$\sum_{j=1}^{5}\chi_j\cos\theta_j A_j = \chi_1\xi_1\cos\theta_0 A_0, \tag{50}$$

$$\sum_{j=1}^{5}\kappa_j\xi_j\cos\theta_j A_j = \kappa_1\xi_1\cos\theta_0 A_0. \tag{51}$$

Finally,

$$\sum_{j=1}^{5}\vartheta_j\xi_j\cos\theta_j A_j = \vartheta_j\xi_1\cos\theta_0 A_0. \tag{52}$$

From Eqs. (48–52), we get

$$a_{ij}Z_j = B_i, \quad i,j = (1, 2, \ldots, 5), \quad Z_j = \frac{A_j}{A_0}, \tag{53}$$

$$a_{1j} = a_2\vartheta_j - a_3\chi_j - \mu_0H_0^2\xi_j^2 - \xi_j^2\left(\cos^2\theta_j + \frac{\eta_j}{2}\sin 2\theta_j\right) - a_1\xi_j^2\left(\sin^2\theta_j - \frac{\eta_j}{2}\sin 2\theta_j\right),$$
$$a_{2j} = a_7\kappa_j + a_5\xi_j^2\left(\frac{1}{2}\sin 2\theta_j - \frac{\eta_j}{2}\cos^2\theta_j\right) + a_6\xi_j^2\left(\frac{1}{2}\sin 2\theta_j + \frac{\eta_j}{2}\sin^2\theta_j\right),$$
$$a_{3j} = \chi_j\xi_j\cos\theta_j,$$
$$a_{4j} = \kappa_j\xi_j\cos\theta_j,$$
$$a_{5j} = \vartheta_j\xi_j\cos\theta_j.$$

5 Numerical results and discussion

With the view of illustrating results obtain in the preceding sections and comparing these in various cases, we now study some numerical results. The materials chosen for this purpose are (Fig. 1)

$$i = \sqrt{-1}, \quad \alpha_0 = 0.779 \times 10^{-4}, \quad \lambda_0 = 0.5 \times 10^{11},$$
$$\lambda_1 = 0.5 \times 10^{11}, \quad j = 0.2 \times 10^{-15}, \quad \rho = 8954,$$
$$C_e = 383.1, \quad k = 386, \quad T_0 = 293, \quad \lambda = 7.76 \times 10^{10},$$
$$\mu = 3.86 \times 10^{10}, \quad \gamma = 0.779 \times 10^{-4},$$
$$\mathrm{K}^* = 2.97 \times 10^{13}, \quad \mu_0 = 0.1, \quad \varepsilon_0 = 0.1, \quad \omega = 0.034,$$
$$\tau_T = 0.2, \quad \tau_v = 0.1, \quad \tau_q = 0.5.$$

Figure 2 shows the variation of the amplitudes $|z_1|$ with respect to the angle of incidence θ of P-waves for the different values of gravity g and rotation Ω, which it has oscillatory behavior in the whole range of angle θ. It is notice that the amplitudes of P-wave increases with increasing of gravity field in the absence of rotation, while it decreases with increasing of gravity in the presence of rotation, while it equal zero at $\theta = 90^0$.

Figure 3 shows the variation of amplitudes $|z_2|$ with respect to the angle of incidence θ of P-waves for the different values of gravity g and rotation Ω, which it has oscillatory behavior in the whole range of angle θ. It is obvious that the amplitudes of P-wave decreases with

Fig. 1 Geometry of the problem

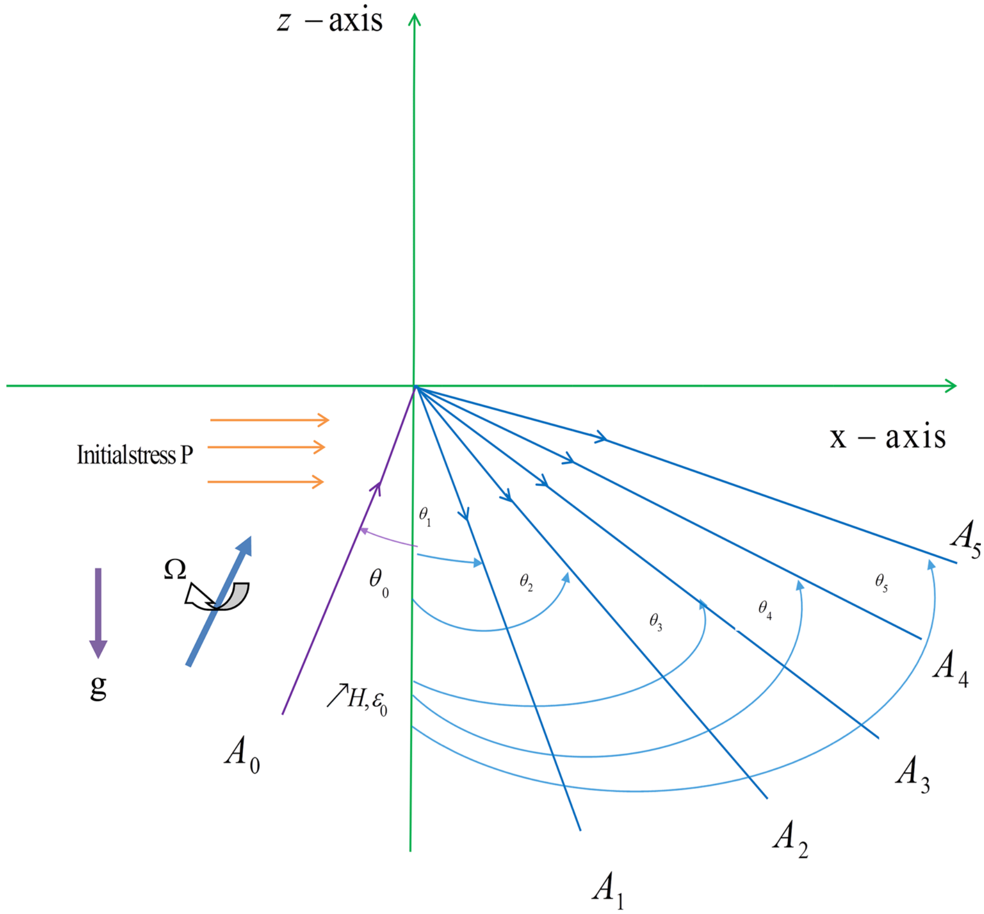
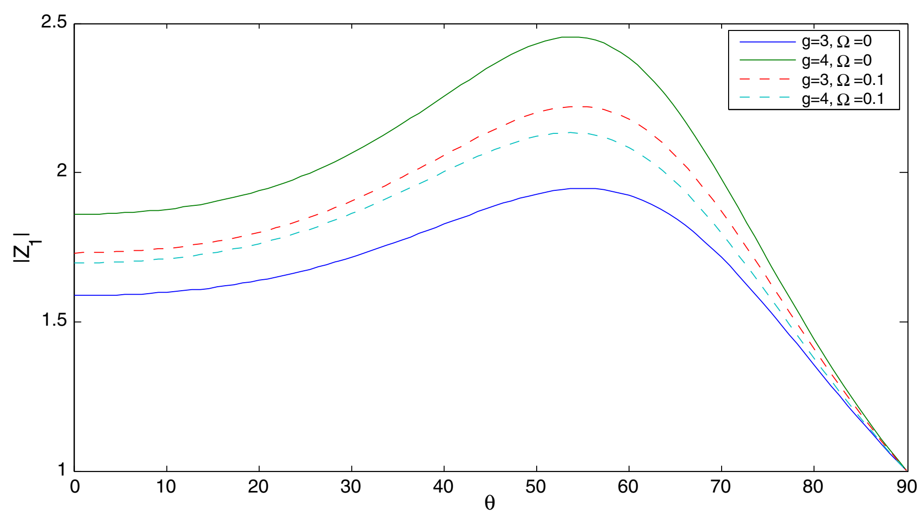


Fig. 2 Variation of the amplitudes $|Z_1|$ with the angle of incidence of P-waves for variation of gravity: $g = 3$, 4, $\Omega = 0$ ____, $\Omega = 0.1$ − −−



increasing of the gravity field in the absence or presence of rotation, while it equal zero at $\theta = 90^0$.

Figure 4 shows the variation of amplitudes $|z_3|$ with respect to the angle of incidence θ of P-waves for the different values of gravity g and rotation Ω, which it decreases with increasing of angle θ. It is obvious that the amplitudes of P-wave decreases with increasing of the gravity field in the absence or presence of rotation, while it equal zero at $\theta = 90$.

Figure 5 shows the variation of amplitudes $|z_4|$ with respect to the angle of incidence θ of P-waves for the different values of gravity g and rotation Ω, which it has oscillatory behavior in the whole range of angle θ. It is obvious that the amplitudes of P-wave decreases with

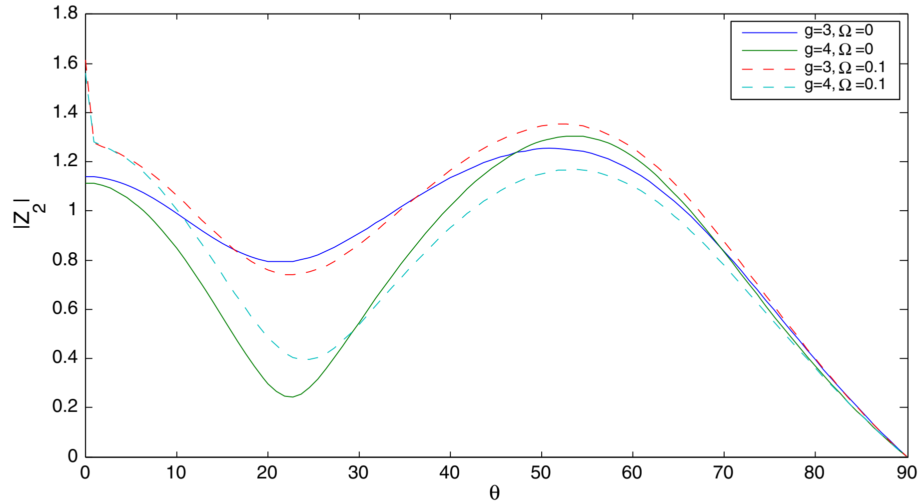


Fig. 3 Variation of the amplitudes $|Z_2|$ with the angle of incidence of P-waves for variation of gravity: $g = 3, 4$, $\Omega = 0$ ____, $\Omega = 0.1$ – ––

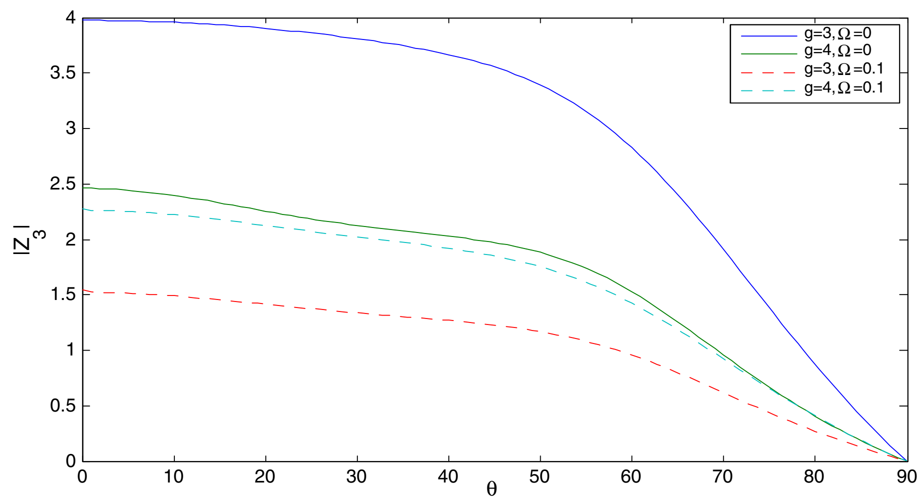


Fig. 4 Variation of the amplitudes $|Z_3|$ with the angle of incidence of P-waves for variation of gravity: $g = 3, 4$, $\Omega = 0$ ____, $\Omega = 0.1$ – ––

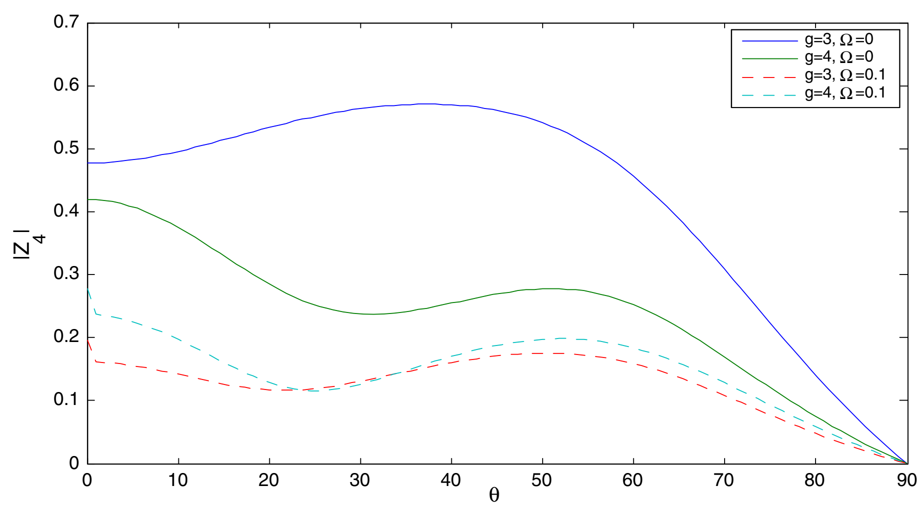


Fig. 5 Variation of the amplitudes $|Z_4|$ with the angle of incidence of P-waves for variation of gravity: $g = 3, 4$, $\Omega = 0$ ____, $\Omega = 0.1$ – ––

increasing of the gravity field in the absence of rotation, while it increases with increasing of gravity in the presence of rotation, as well it equal zero at $\theta = 90°$.

Figure 6 shows the variation of amplitudes $|z_5|$ with respect to the angle of incidence θ of P-waves for different values of gravity g and rotation Ω, which it decreases with

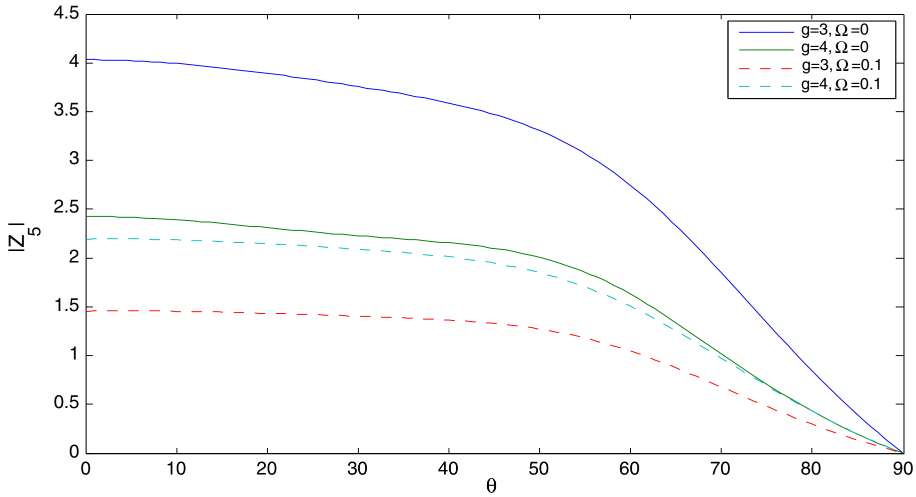


Fig. 6 Variation of the amplitudes $|Z_5|$ with the angle of incidence of P-waves for variation of gravity: $g = 3,\ 4,$ $\Omega = 0$ ____, $\Omega = 0.1$ – ––

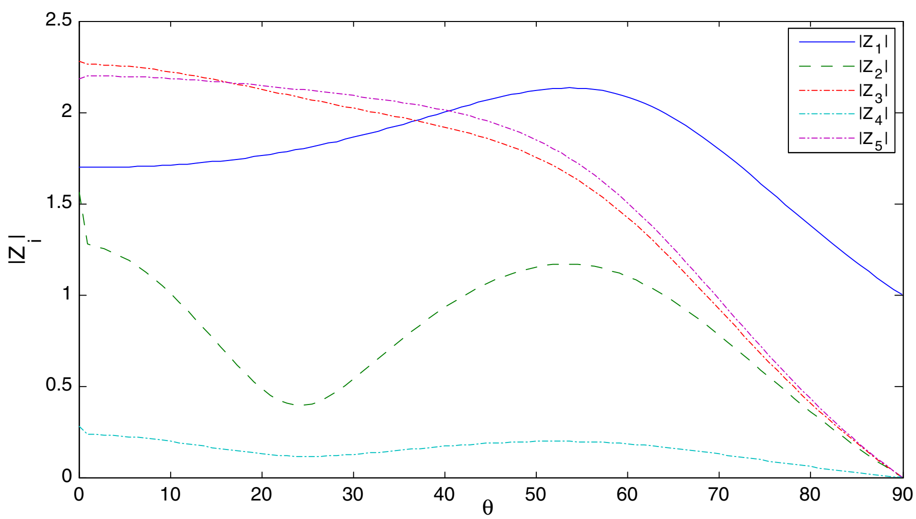


Fig. 7 Variation of the amplitudes $|Z_i|\ (i = 1, 2, \ldots, 5)$ with the angle of incidence of P-waves

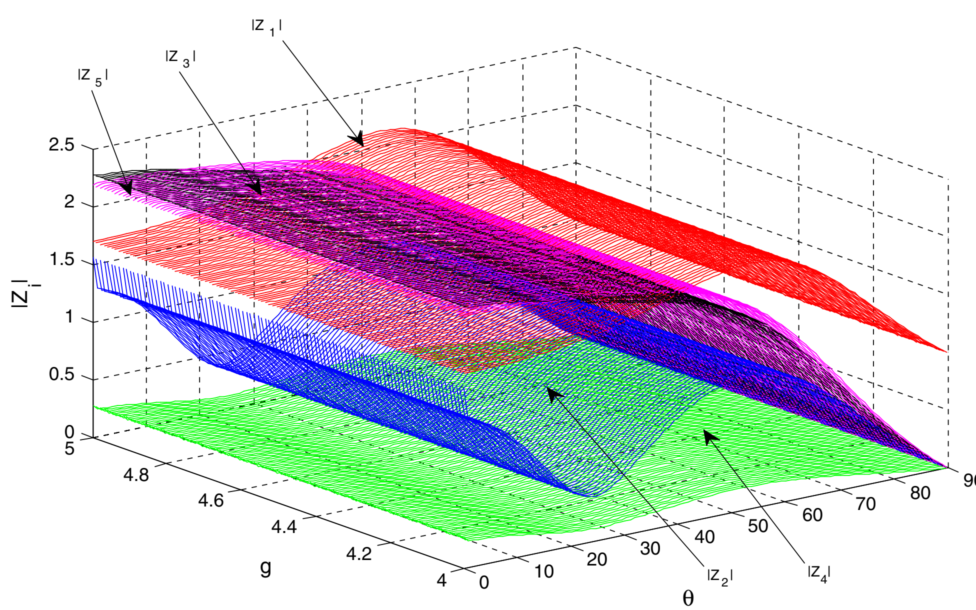


Fig. 8 3D variation of the amplitudes $|Z_i|\ (i = 1, 2, \ldots, 5)$ respect to the angle of incidence and gravity g of P-waves

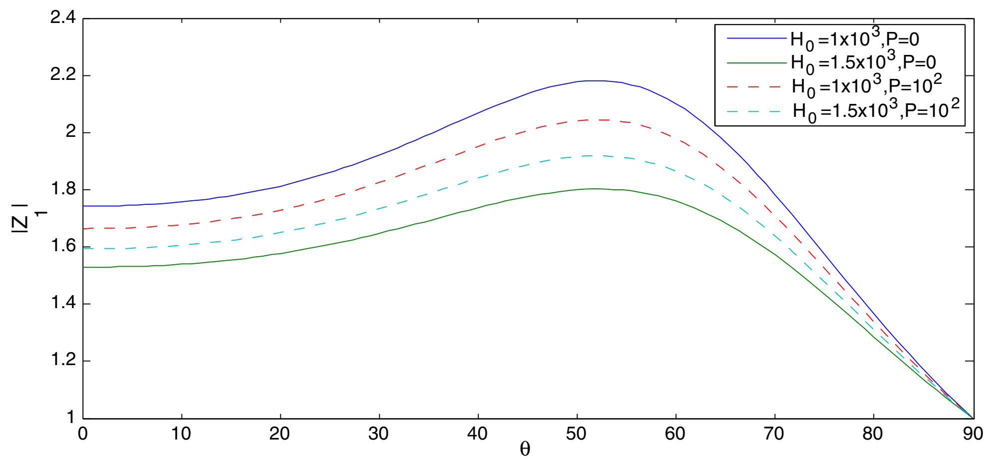


Fig. 9 Variation of the amplitudes $|Z_1|$ with the angle of incidence of P-waves for variation of magnetic field: $H_0 = (1,\ 1.5) \times 10^3$, $P = 0$ ____, $P = 10^2$ – – –

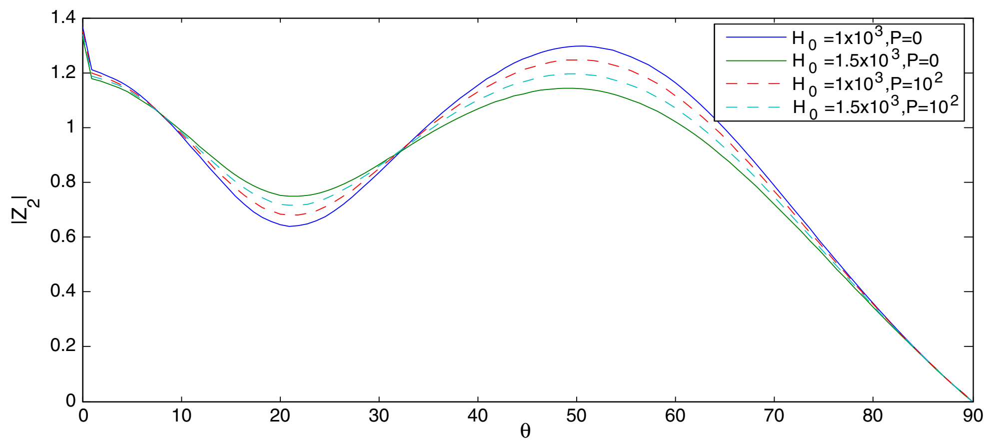


Fig. 10 Variation of the amplitudes $|Z_2|$ with the angle of incidence of P-waves for variation of magnetic field: $H_0 = (1,\ 1.5) \times 10^3$, $P = 0$ ____, $P = 10^2$ – – –

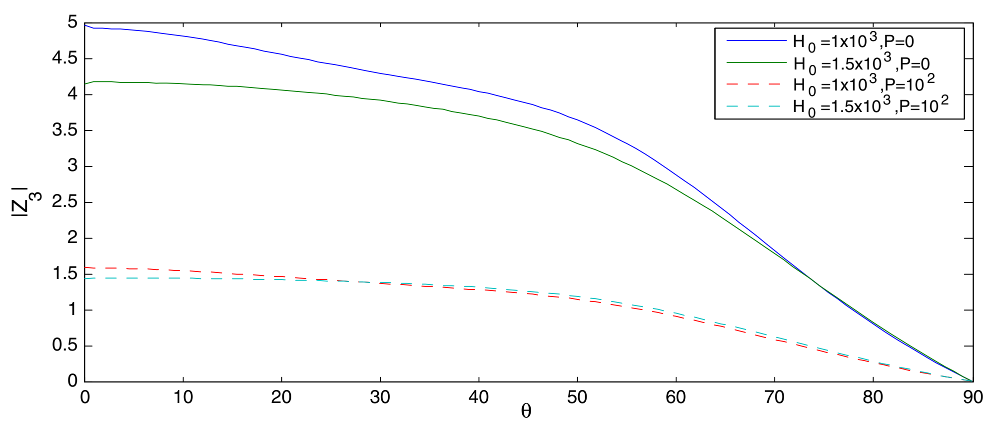


Fig. 11 Variation of the amplitudes $|Z_3|$ with the angle of incidence of P-waves for variation of magnetic field: $H_0 = (1,\ 1.5) \times 10^3$, $P = 0$ ____, $P = 10^2$ – – –

increasing of angle θ. It is obvious that the amplitudes of P-wave decreases with increasing of the gravity field in the absence of rotation, while it increases with increasing of gravity in the presence of rotation, as well it equal zero at $\theta = 90$.

Figures 7 and 8 show the variation of amplitudes $|z_1|$, $|z_2|$, $|z_3|$, $|z_4|$ and $|z_5|$ with respect to the angle of incidence θ of P-waves in the presence of the gravity field g, which it has oscillatory behavior in the whole range of angle θ. while it decreases with increasing of angle θ. It is obvious that the amplitude $|z_1|$ greater than the amplitude $|z_2|$ greater than $|z_4|$, while the dispersion curve for the amplitudes $|z_3|$ and $|z_5|$ of P-wave, as well it equal zero at $\theta = 90$ except the amplitude $|z_1| \neq 0$ at $\theta = 90$.

Figure 9 shows the variation of amplitude $|z_1|$ with respect to the angle of incidence θ of P-waves for the

Fig. 12 Variation of the amplitudes $|Z_4|$ with the angle of incidence of P-waves for variation of magnetic field: $H_0 = (1, 1.5) \times 10^3, \ldots,$ $P = 0$ ____, $P = 10^2$ – – –

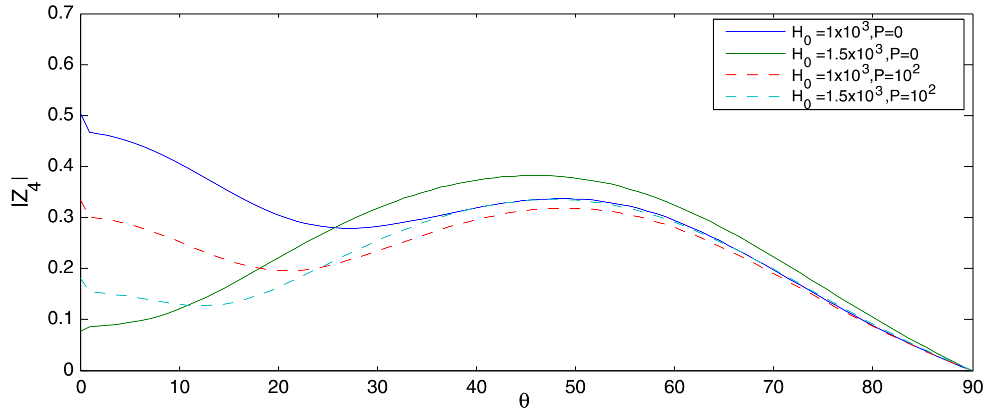
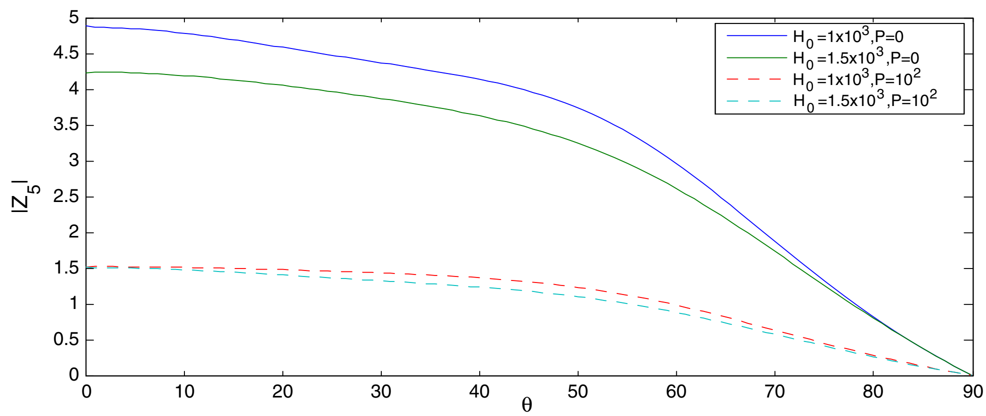


Fig. 13 Variation of the amplitudes $|Z_5|$ with the angle of incidence of P-waves for variation of magnetic field: $H_0 = (1, 1.5) \times 10^3$, $P = 0$ ____, $P = 10^2$ – – –

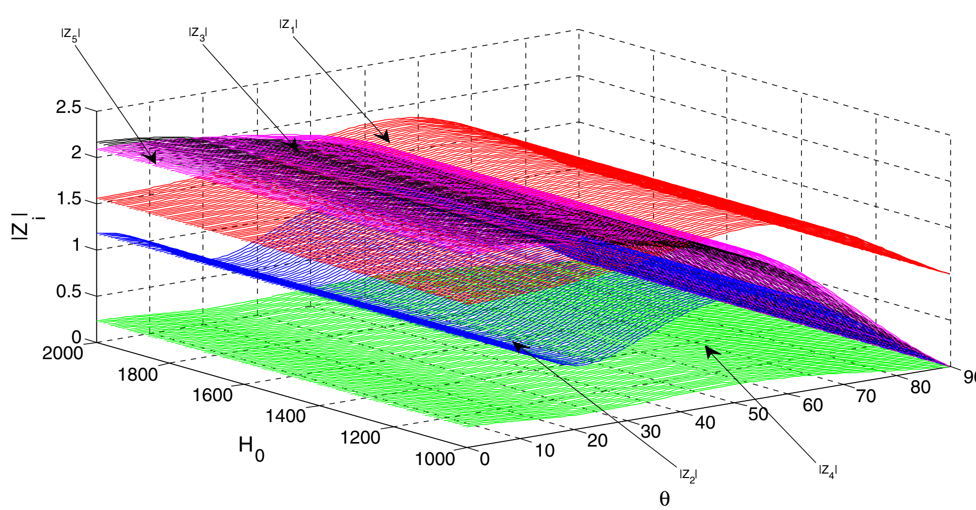


different values of magnetic field H_0 and initial stress P, which it has oscillatory behavior in the whole range of angle θ. It is obvious that the amplitude of P-wave decreases with increasing of magnetic field in the absence and presence of initial stress, while it equal zero at $\theta = 90$.

Figure 10 shows the variation of amplitude $|z_2|$ with respect to the angle of incidence θ of P-waves for the different values of magnetic field H_0 and initial stress P, which it has oscillatory behavior in the whole range of angle θ in the absence and presence of initial stress, while it equal zero at $\theta = 90$.

Figure 11 shows the variation of amplitude $|z_3|$ with respect to the angle of incidence θ of P-waves for the different values of magnetic field H_0 and initial stress P, which it decreases with increasing of the angle of incidence θ. It is obvious that the amplitude of P-wave decreases with

Fig. 14 3D variation of the amplitudes $|Z_i|(i = 1, 2, \ldots, 5)$ respect to the angle of incidence and magnetic field H_0 of P-waves



increasing of magnetic field in the absence of initial stress, while the dispersion curve for the amplitudes $|z_3|$ in the presence of initial stress, as well it equal zero at $\theta = 90$.

Figure 12 displays the variation of amplitude $|z_4|$ with respect to the angle of incidence θ of P-waves for the different values of magnetic field H_0 and initial stress P, which it has oscillatory behavior in the whole range of angle θ. It is obvious that the dispersion curve for the amplitudes of P-wave in the absence and presence of initial stress, while it equal zero at $\theta = 90$.

Figure 13 shows the variation of amplitude $|z_5|$ with respect to the angle of incidence θ of P-waves for the different values of magnetic field H_0 and initial stress P, which it decreases with increasing of angle θ. It is obvious that the amplitude of P-wave decreases with increasing of magnetic field in the absence and presence of initial stress, while it equal zero at $\theta = 90$.

Figure 14 clears the variation of amplitudes $|z_1|$, $|z_2|$, $|z_3|$, $|z_4|$ and $|z_5|$ with respect to the angle of incidence θ of P-waves in the presence of magnetic field H_0, which it has oscillatory behavior in the whole range of angle θ. It is obvious that the amplitude $|z_1|$ greater than the amplitude $|z_2|$ greater than $|z_4|$, while the dispersion curve for the amplitudes $|z_3|$ and $|z_5|$ of P-wave, as well it equal zero at $\theta = 90^0$ except the amplitude $|z_1| \neq 0$ at $\theta = 90$.

6 Conclusions

We model On Maxwell's stresses and rotation effects, initial stress and gravity field of reflection of P-waves from thermo-magneto-microstretch medium in the context of three phase lag model. The reflected of P-waves with the magnetic field, initial stress, gravity field, the rotation and the amplitude of the reflected of P-waves with the angle of incidence are obtained in the framework of dynamical coupling theory. The effects of applied magnetic field, initial stress, gravity field and rotation are discussed numerically and illustrated graphically.

The following conclusions can be made:

1. The reflected of P-waves and amplitude of the reflected p-waves depend on the angle of incidence, rotation, initial stress, gravity field and magnetic field, the nature of this dependence is different for different reflected of P-waves.
2. The rotation, initial stress, gravity field and magnetic field play a significant role and the effects have the inverse trend for the reflected of P-waves and amplitude of the reflected of P-waves.
3. The rotation, initial stress, gravity field and magnetic field have a strong effect on the reflected of P-waves and amplitude of the reflected of P-waves.
4. The result provides a motivation to investigate reflection of P-waves from thermo-magneto-microstretch materials as a new class of applicable thermoelectric solids. The results presented in this paper should prove useful for researchers in material science, designers of new materials, microsystem technologies, physicists as well as for those working on the development of thermo-magneto-microstretch and in practical situations as in geophysics, optics, acoustics, geomagnetic and oil prospecting etc. The used methods in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.

It is observed that the amplitudes of reflected of P-waves changes in the presence of rotation, initial stress, gravity field and magnetic field. Hence, the presence of rotation, initial stress, gravity field and magnetic field are significantly effect on the reflection phenomena.

Appendix

We represent A, B, C, D and E in terms $L = C_k - \imath C_v \omega - C_T \omega^2$, $M = \zeta_0 + \zeta_1 \omega^2$, $N = \zeta_8 + \zeta_9 \omega^2$, $R = -\frac{C_2^2}{\omega^{*2}} + \omega^2$, $S = \frac{2kc_0^2}{\gamma\omega^{*2}} - \frac{j\rho c_0^2 \omega^2}{\gamma}$ as follow:

$$A = \frac{C_1^2}{c_0^2} L,$$

$$B=-LR-\zeta_5\frac{C_3^2}{\omega^{*2}}L-\frac{\zeta_{10}kC_1^2}{\gamma\omega^{*2}}L+\frac{C_1^2}{c_0^2}(LS-\omega^2\tau_q^*-\zeta_4\varepsilon_1\omega^2\tau_q^*-LM-LN-\zeta_2\zeta_6\sin^2\theta L),$$

$$\begin{aligned}C={}&\omega^2\tau_q^*R-a_0\varepsilon_2\omega^2\tau_q^*+\varepsilon_2\omega^2\tau_q^*\zeta_4\frac{C_3^2}{\omega^{*2}}\\&+\varepsilon_1\omega^2\tau_q^*\zeta_4R+a_0\varepsilon_1\omega^2\tau_q^*\zeta_5+LMR+\omega^2\tau_q^*\frac{C_1^2}{c_0^2}M+LNR\\&+\omega^2\tau_q^*\frac{C_1^2}{c_0^2}N-LRS-\omega^2\tau_q^*\frac{C_1^2}{c_0^2}S+\omega^2\tau_q^*\zeta_5\frac{C_3^2}{\omega^{*2}}\\&+\zeta_2\zeta_6\sin^2\theta LR+\zeta_2\zeta_6\omega^2\tau_q^*\sin^2\theta\frac{C_1^2}{c_0^2}+\zeta_5\frac{C_3^2}{\omega^{*2}}LN\\&+\varepsilon_1\omega^2\tau_q^*\zeta_4\frac{C_1^2}{c_0^2}N-\zeta_5\frac{C_3^2}{\omega^{*2}}LS-\varepsilon_1\omega^2\tau_q^*\zeta_4\frac{C_1^2}{c_0^2}S\\&+\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}LR+\zeta_{10}\omega^2\tau_q^*\frac{kC_1^2}{\gamma\omega^{*2}}-\zeta_3\zeta_7\omega^2\frac{C_1^2}{c_0^2}L\\&+\frac{C_1^2}{c_0^2}LMN-\frac{C_1^2}{c_0^2}LMS-\frac{C_1^2}{c_0^2}LNS+\zeta_5\zeta_{10}\frac{C_3^2kc_0^2}{\gamma\omega^{*4}}L\\&+\varepsilon_1\omega^2\tau_q^*\zeta_4\zeta_{10}\frac{kC_1^2}{\gamma\omega^{*2}}\\&-\zeta_2\zeta_6\sin^2\theta\frac{C_1^2}{c_0^2}LS+\zeta_{10}\frac{kC_1^2}{\gamma\omega^{*2}}LM,\end{aligned}$$

$$\begin{aligned}D={}&a_0\varepsilon_2\omega^2\tau_q^*M+a_0\varepsilon_2\omega^2\tau_q^*N-a_0\varepsilon_2\omega^2\tau_q^*S-\omega^2\tau_q^*MR\\&-\omega^2\tau_q^*NR+\omega^2\tau_q^*RS-\varepsilon_2\omega^2\tau_q^*\zeta_4\frac{C_3^2}{\omega^{*2}}N\\&+\varepsilon_2\omega^2\tau_q^*\zeta_4\frac{C_3^2}{\omega^{*2}}S+a_0\varepsilon_2\omega^2\tau_q^*\zeta_2\zeta_6\sin^2\theta\\&+a_0\varepsilon_2\omega^2\tau_q^*\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}+\zeta_3\zeta_7\omega^2LR+\zeta_3\zeta_7\omega^4\tau_q^*\frac{C_1^2}{c_0^2}\\&-\zeta_2\zeta_6\omega^2\tau_q^*\sin^2\theta R-\omega^2\tau_q^*\zeta_5\frac{C_3^2}{\omega^{*2}}N-\varepsilon_1\omega^2\tau_q^*\zeta_4NR\\&+\omega^2\tau_q^*\zeta_5\frac{C_3^2}{\omega^{*2}}S+\varepsilon_1\omega^2\tau_q^*\zeta_4RS-\omega^2\tau_q^*\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}R\\&-a_0\varepsilon_1\omega^2\tau_q^*\zeta_5N+a_0\varepsilon_1\omega^2\tau_q^*\zeta_5S-LMNR\\&-\omega^2\tau_q^*\frac{C_1^2}{c_0^2}MN+LMRS+\omega^2\tau_q^*\frac{C_1^2}{c_0^2}MS+LNRS\\&+\omega^2\tau_q^*\frac{C_1^2}{c_0^2}NS-\omega^2\tau_q^*\zeta_5\zeta_{10}\frac{C_3^2}{\omega^{*2}}\frac{kc_0^2}{\gamma\omega^{*2}}+\omega^2\tau_q^*\zeta_2\zeta_6\sin^2\theta\frac{C_1^2}{c_0^2}S\\&-\varepsilon_1\omega^2\tau_q^*\zeta_4\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}R-a_0\varepsilon_1\omega^2\tau_q^*\zeta_5\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}\\&+\zeta_2\zeta_6\sin^2\theta LRS-\zeta_3\zeta_7\omega^2\frac{C_1^2}{c_0^2}LS+\zeta_5\frac{C_3^2}{\omega^{*2}}LNS\\&+\varepsilon_1\omega^2\tau_q^*\zeta_4\frac{C_1^2}{c_0^2}NS-\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}LMR-\zeta_{10}\frac{kC_1^2}{\gamma\omega^{*2}}M\\&+\frac{C_1^2}{c_0^2}LMNS-\varepsilon_2\omega^2\tau_q^*\zeta_4\zeta_{10}\frac{C_3^2kc_0^2}{\gamma\omega^{*4}},\end{aligned}$$

$$\begin{aligned}E={}&a_0\varepsilon_2\omega^4\tau_q^*\zeta_3\zeta_7-a_0\varepsilon_2\omega^2\tau_q^*MN+a_0\varepsilon_2\omega^2\tau_q^*MS\\&+a_0\varepsilon_2\omega^2\tau_q^*NS+\omega^4\tau_q^*\zeta_3\zeta_7R+\omega^2\tau_q^*MNR\\&-\omega^2\tau_q^*MRS+\zeta_3\zeta_7\omega^2LRS+\omega^4\tau_q^*\zeta_3\zeta_7\frac{C_1^2}{c_0^2}S\\&-\omega^2\tau_q^*\zeta_2\zeta_6\sin^2\theta RS-a_0\varepsilon_2\omega^2\tau_q^*\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}M\\&-\omega^2\tau_q^*\zeta_5\frac{C_3^2}{\omega^{*2}}NS-\varepsilon_1\omega^2\tau_q^*\zeta_4NRS+\omega^2\tau_q^*\zeta_{10}\frac{kc_0^2}{\gamma\omega^{*2}}MR\\&-a_0\varepsilon_1\omega^2\tau_q^*\zeta_5NS-LMNRS-\omega^2\tau_q^*\frac{C_1^2}{c_0^2}MNS\\&-\varepsilon_2\omega^2\tau_q^*\zeta_4\frac{C_3^2}{\omega^{*2}}NS+a_0\varepsilon_2\omega^2\tau_q^*\zeta_2\zeta_6\sin^2\theta S-\omega^2\tau_q^*NRS,\end{aligned}$$

$$\begin{aligned}F={}&a_0\varepsilon_2\omega^4\tau_q^*\zeta_3\zeta_7S-a_0\varepsilon_2\omega^2\tau_q^*MNS-\omega^2\tau_q^*\zeta_3\zeta_7\omega^2RS\\&+\omega^2\tau_q^*MNRS.\end{aligned}$$

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