Microsyst Technol (2018) 24:1713–1731
https://doi.org/10.1007/s00542-017-3545-z

TECHNICAL PAPER

Size-dependent hygro–thermo–electro–mechanical vibration analysis of functionally graded piezoelectric nanobeams resting on Winkler–Pasternak foundation undergoing preload and magnetic field

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Received: 20 June 2017 / Accepted: 24 August 2017 / Published online: 5 September 2017

Abstract In this study, nonlocal beam theory is utilized for vibration analysis of hygro–electro–thermo–mechanical of functionally graded material (FGM) nanobeam by consideration of magnetic field and preload. Moreover, the material properties are considered to vary corresponding to the thickness of nanobeam in the framework of power-law distribution. Differential equations are derived by means of Hamilton principle in the framework of Euler–Bernoulli beam theory. The derived governing differential equations are solved by differential transformation method (DTM) which demonstrates to have high precision and computational efficiency in the vibration analysis of nanobeams. Numerical results are presented for various boundary conditions. A detailed parametric study is conducted temperature to examine the effects of the nonlocal parameter, voltage and, elastic mediums, power-law index, aspect ratio, preload, magnetic field and moisture effect on vibration characteristics of functionally graded nanobeam. Numerical results are presented in this paper to serve as benchmarks for future analyses of nanotubes.

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List of symbols

A_{xx}, B_{xx}, C_{xx}	Cross-sectional rigidities
b	Width of beam
D	Electric field
$E_c.E_m$	Young's modulus (C:ceramic, m: metal)
e_{31}	Piezoelectric coefficient
h	Height of beam
H	Moisture effect
k_p	Pasternak parameter
L	Length of beam
M	Bending moment
N	Axial force
N^T	Thermal load
N^P	Preload
P	Power-law index
$P_{electric}$	Piezoelectric load
$P_0, P_{-1}, P_1, P_2, P_3$	Temperature dependent coefficient
t	Time
T	Kinetic energy
u	Axial displacement
U	Strain energy
V	Volume fraction
W_{ext}	Work done
x	x coordinate
z	z coordinate
η	Magnetic property
λ_{33}	Dielectric constants
β	Moisture expansion coefficient
ρ	Density
ω	Non-dimensional natural frequency
μ	Nonlocal parameter
v	Poison ratio

1 Introduction

In recent years, numerous researchers are interested in the study and survey of smart structures. It is well known that magneto–electro-elastic materials reveal a particular ability in order to convert the energy of magnetism to electricity or inverse, which does not appear in a single-phase piezoelectric or piezo-magnetic material. For the great coupling effects of magneto–electro-elastic materials, they open towards new interesting and effective applications in many technological fields such as sensing and actuator, vibrations control, energy harvesting and smart structure technologies (Liu et al. 2016b). Hence, it is essential to analyze the behavior of a magneto–electro-elastic beam under mechanical, electrical or magnetic loads. The piezoelectric materials are considered to be a class of smart structure materials which are used widely as sensors and actuators for their fantastic electromechanical properties. It can be found that there are a lot of works about their applications and analysis of piezoelectric materials in fields of vibration, buckling, postbukling and wave propagation for plate (GhorbanpourArani et al. 2015; Kiani 2016; Liu et al. 2016a; Wang et al. 2015; Yao and Li 2016; Zhou et al. 2014), beam (Ebrahimi et al. 2014, 2015a; Farokhi et al. 2016; Marzbanrad et al. 2016; Shaghaghi 2015; Zidour et al. 2012) and wire (Bo and Ronca 1970; Gheshlaghi and Hasheminejad 2012; Haghpanahi et al. 2013).

Functionally graded materials (FGMs) are composite materials whose composition varies continuously along the thickness of the structure. They are usually composed of two different parts ceramic with great properties in heat and corrosive resistances and metal with toughness. For this reason, these materials have incredible properties such as high performance, novel thermos-mechanical properties and resistance under ultra-high temperature. For their novel properties, FGMs have received the significant attention which enables them to be widely used in various researches, which were mainly focused on studying their static, dynamic and vibration characteristics of FG structures (Brischetto et al. 2016; Ebrahimi et al. 2009; Hosseini-Hashemi and Nazemnezhad 2013; Natarajan et al. 2012; Şimşek 2012; Wattanasakulpong and Ungbhakorn 2014).

On the other hand, after discovering carbon nanotubes (CNT) by Iijima (1991), nanoscale engineering materials have attracted more interest in science societies. They have great mechanical, electrical and thermal performances promotion in comparing with conventional materials. Attention is sought toward the development of nanodevices and nanomachines such as beams which are one of the basic components for micro/nano electromechanical systems, biomedical sensors, actuators, transistors, probes, and resonators. Hence understanding and studying the mechanical and physical behaviors of nanobeams made of piezoelectric material is important in the design of nanodevices and nanomachines.

Besides, for the fact that running experiments at the nanoscale is hard, and atomistic modeling is limited to small-scale systems owing to computer resource limitations, continuum mechanics suggest an easy and useful tool for the analysis of MEMs/NEMs. However, the classical continuum models must be developed to consider the nanoscale effects which can be achieved through the nonlocal elasticity theory proposed by Eringen (1972) in order to consider the size-dependent effect. This theory states that the stress at a reference point is a function of strain at all of other points in the body (Eringen 1983). In the field of nonlocal elasticity theory, Ansari et al. (2016) investigated free vibration behavior of piezoelectric Timoshenko nanobeams in the vicinity of postbuckling domain based on the nonlocal elasticity theory. Togun (2016) studied nonlinear free and forced vibration of nanobeam using Euler–Bernoulli theory with attached nanoparticle at the free end based on nonlocal elasticity theory. Moreover, the simplicity in the application of Eringen's nonlocal elasticity resulted in rapid extensions of this theory in various static and dynamic analyses of nanostructures, such as Ebrahimi et al. (2015c), Malekzadeh and Shojaee (2013), Murmu et al. (2013) and Reddy 2007).

in recent years FGMs and their application have received a considerable attention within the nanotechnology community. Actually, FGM's applications have been increased in micro and nanoscale systems such as atomic force microscopes (AFMs), micro sensors, nanomotors, and nanomachines. In all of these applications, small-scale effect plays a major roll and must be considered in order to study and investigate mechanical behaviors. Afterward, Chaht et al. (2015) examined the bending and buckling behaviors of size-dependent nanobeams made of FGM including the thickness stretching effect on the basis of the nonlocal continuum model. Moreover, Ebrahimi et al. (2015b) investigated vibrational characteristics of size-dependent FG nanobeams using differential transformation method. Furthermore, free vibration analysis of size-dependent functionally graded rotating nanobeams with considering all surface effects base on the nonlocal continuum model is studied by Ghadiri et al. (2016).

In addition, for the wide range of application of FGMs in sensors and actuators, researchers tend to investigate FG piezoelectric material properties. As, Hashemi-Hosseini et al. (2014) investigated free vibration of FGM nanobeams by considering surface effects as well as the piezoelectric field using nonlocal elasticity theory. In an effort Sabzikar et al. (Boroujerdy and Eslami 2015) developed the buckling analysis of functionally graded piezoelectric spherical shells by considering thermal loading.

On the other hand, thermal effects have considerable influence on mechanical characteristics of FGMs. So, many investigations have been carried out in the open literature dealing with studying the thermal influence on mechanical characteristics of nanostructures such as those in (Ebrahimi and Salari 2015b; Ebrahimi et al. 2016; Zhang et al. 2015). Further, investigating the thermo–electrical buckling of FG piezoelectric Timoshenko nanobeams subjected to in-plane thermal loads and applied electric voltage is presented by Ebrahimi and salari (2015a). The nonlinear thermo-electro-mechanical response of FGM piezoelectric actuators is investigated by Reddy et al. (Komijani et al. 2014). In this study the theoretical formulation is based on the Timoshenko beam theory with the von Kármán nonlinearity and a microstructural length scale is incorporated by means of the modified couple stress theory. And also, Ansari et al. (2015) proposed an exact solution for the nonlinear forced vibration analysis of nanobeams made of FGM subjected to the thermal environment including the effect of surface stress.

To date, one of the basic terms in MEMs/NEMs is consideration of elastic medium that tends to increase the natural frequency and buckling loads. Therefore, buckling of higher-order shear deformable nanobeams made of functionally graded piezoelectric materials embedded in an elastic medium is examined by Ebrahimi and Barati (2016). After that, Jandaghian and Rahmani (2016) investigated free vibration analysis of magneto–electro–thermo-elastic nanobeams resting on a Pasternak medium based on nonlocal theory and Timoshenko beam theory. Most recently, Marzbanrad et al. (2016) studied the thermo–electro–mechanical vibration analysis of nanobeam rested in Winkler–Pasternak elastic medium by considering surface effects.

Moreover, although the dynamic analysis of piezoelectric FG nanobeams by considering thermal effects is studied, whereas investigating the moisture and preload effects in presence of magnetic field for various boundary conditions on the natural frequencies is rather limited. In the perspective of the above discussion, the current manuscript is concerned with the hygro–thermo–electro–mechanical vibration of nanobeam made of FGM in presence of magnetic field resting on Pasternak medium undergoing preload. The approximate expressions of natural frequencies based on Hamilton's method in the framework of Euler–Bernoulli beam theory are obtained. A new semi-analytical method called differential transformation method (DTM) is utilized for vibration analysis of size-dependent FG nanobeams. However, implementing the DTM in order to solve similar studies is also rather limited. Comparisons with the results from the well-known references with good agreement between the results of the DTM method and those available in literature validated the presented approach. It is demonstrated that the DTM has high accuracy and precision in dynamic analysis of nanotubes. Finally, through some numerical examples, the influence of various parameters such as a nonlocal parameter, voltage and temperature, elastic medium, power-law index, aspect ratio, preload, magnetic field and moisture effect is investigated.

2 Theory and formulation

2.1 Material gradient of FG nanobeams

A FG nanobeam with length L, uniform thickness h, and width b having rectangular cross-section is considered as shown in Fig. 1. The coordinate system of FG nanobeam is considered to be located on the central axis of the beam while the x-axis is taken along the central axis, y-axis in the width direction and z-axis in the thickness direction. The effective material properties of FG nanobeam which is made of ceramic and metal such as Young's modulus E, mass density ρ, poison ratio v and thermal expansion λ are assumed to change continuously in the thickness direction. According to the mixture rule, the effective material properties for FG material, P_f, can be defined as (Şimşek 2010a, b):

$$P_f = P_c V_c + P_m V_m \tag{1}$$

where P_m, P_c, V_m and V_c are the material properties, accordingly the volume fractions of the metal and the ceramic constituents as (Şimşek 2010b):

$$V_c + V_m = 1 \tag{2}$$

The variation of volume fraction is described by a simple power law function which is described as follows:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^P \tag{3}$$

which p denotes the non-negative variable parameter of the power-law exponent in order to dictate the material distribution through the thickness of the beam. Also, z is the distance to the mid-plane of the FG beam. When p is assumed to be zero, the FG beam becomes a fully ceramic beam. So from Eqs. (1) to (3), the effective material properties of the FG nanobeam such as Young's modulus E, mass density ρ, poison ratio v and thermal expansion λ can be obtained as follows:

$$\begin{aligned} E(z) &= (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_m \\ \rho(z) &= (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_m \\ \lambda(z) &= (\lambda_c - \lambda_m)\left(\frac{z}{h} + \frac{1}{2}\right)^2 + \lambda_m \\ v(z) &= (v_c - v_m)\left(\frac{z}{h} + \frac{1}{2}\right)^2 + v_m \end{aligned} \tag{4}$$

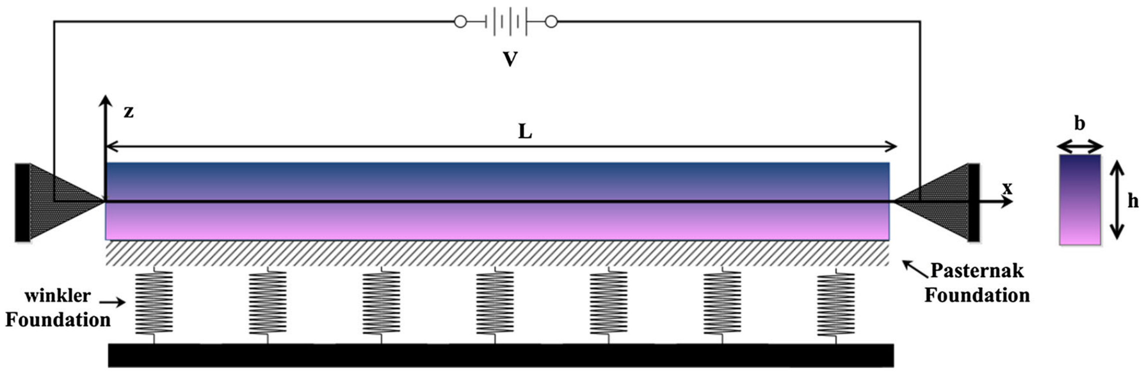

Fig. 1 FG nanobeam with Cartesian coordinate and elastic medium

In order to model the behavior of FGMs under high temperature effectively, in the material properties the temperature dependency must be considered as follows (Reddy and Chin 1998):

$$P = P_0\left(\frac{P_{-1}}{T} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right) \tag{5}$$

where P_0, P_{-1}, P_1, P_2 and P_3 denote the temperature dependent coefficients that are listed in Table 1 for steel and alumina (Al_2O_3). The bottom surface of FG nanobeam is considered as pure metal (steel) and the top surface is considered to be pure ceramic (alumina).

2.2 Nonlocal elasticity theory via piezoelectric materials

The nonlocal elasticity theory expresses that the stress field at a reference point x in an elastic medium is considered to depend on not only the strain at that point but also the strains at all other points in the domain(Eringen 1972). This assumption can explain some experimental observations of atomic and molecular scales for example high-frequency vibration and wave dispersion (Ke et al. 2012). The basic equations for stress-tensor and electric displacement for a homogeneous and nonlocal piezoelectric solid at any point x in the bulk of material by ignoring the body forces obtained as (Wang and Wang 2012):

Table 1 Some basic theorems of DTM for equations of motion

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F(K) = G(K) \pm H(K)$
$f(x) = \lambda g(x)$	$F(K) = \lambda G(K)$
$f(x) = g(x)h(x)$	$F(K) = \sum_{l=0}^{K} G(K-l)H(l)$
$f(x) = \frac{d^n g(x)}{dx^n}$	$F(K) = \frac{(k+n)!}{k!} G(K+n)$
$f(x) = x^n$	$F(K) = \delta(K-n) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$

$$\sigma_{ij} - \mu^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T \tag{6}$$

$$D_i - \mu^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}E_k + p_i \Delta T \tag{7}$$

where σ_{ij}, D_i denote the component of the stress, electric field, and also the $\varepsilon_{kl}, C_{ijkl}, e_{ikl}, \lambda_{ij}$ are the strain, elastic constant, piezoelectric constants, and thermal module. ΔT and p_i are the temperature change and piezoelectric constants, while the $\mu = (e_0 a)^2$ is the nonlocal parameter furthermore $e_0 a$ is the scale length to incorporate the size effect for the response of nanostructures.

2.2.1 Euler–Bernoulli beam theory

Component of displacement vector for Euler–Bernoulli beam theory at an arbitrary point can expressed as following:

$$u_1 = u(x,t) - z\frac{\partial w(x,t)}{\partial x}, \quad u_2 = 0, \quad u_3 = w(x,t) \tag{8}$$

where $u(x,t)$ and $w(x,t)$ express axial and transverse displacement components along midplane of the FG nanobeam respectively. The nonzero strains of the EBT are obtained as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial^2 x} \tag{9}$$

Besides the displacement field, in order to satisfy Maxwell's equation, The electric displacement can be defined as (Samaei et al. 2012):

$$\begin{aligned} &E_x = -\frac{\partial \phi}{\partial x}; \quad E_z = -\frac{\partial \phi}{\partial z}, \\ &D_x = \lambda_{11}E_x; \quad D_z = e_{31}\varepsilon_x + \lambda_{33}E_z, \\ &\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0 \end{aligned} \tag{10}$$

where λ_{11} and λ_{33} are the dielectric constants, while D_x and D_z denote the electric displacements. Because the λ_{11} and λ_{33} are in the same order and also by considering

$E_x << E_z$, D_x can be neglected in compare with D_z (Hosseini-Hashemi et al. 2014). By substituting Eq. (9) into Eq. (11) and the electrical boundary conditions are $\phi(x, -h/2) = 0, \quad \phi(x, h/2) = 2V$, the electrical potential is obtained as (Gheshlaghi and Hasheminejad 2012):

$$\phi(x,z) = -\frac{e_{31}}{\lambda_{33}}\left(\frac{z^2-h^2}{2}\right)\frac{\partial^2 w}{\partial x^2} + \left(1+\frac{z}{h}\right)V \tag{11}$$

So, it is assumed that the equivalent load is applied to the piezoelectric layers, this load can be obtained as (Gheshlaghi and Hasheminejad 2012):

$$P_{electric}(x,t) = b\int_{-h}^{h} \sigma_x^*\, dz = 2\,V\,b\,e_{31} \tag{12}$$

The Hamilton's principle is utilized to derive governing equation of motion and boundary conditions in the framework ok EBT as (Reddy 2007):

$$\int_0^t \delta(U - T + W_{ext})\,dt = 0 \tag{13}$$

Here U expresses the strain energy; T denotes kinetic energy and W_{ext} is the work done by external forces. The first variation of the piezoelectric nanobeams strains energy is obtained as:

$$\delta U = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz} - D_x\delta E_x - D_z\delta E_z\right)dzdx \tag{14}$$

Substituting Eq. (9) into Eq. (14) gives:

$$\delta U = \int_0^l \left[N\delta u - M\delta\left(\frac{\partial^2 w}{\partial x^2}\right) + D_x\delta\left(\frac{\partial\phi}{\partial x}\right) + D_z\delta\left(\frac{\partial\phi}{\partial z}\right)\right]dx \tag{15}$$

In which the axial force N and bending moment force M are determined as:

$$N = \int \sigma_{xx}dz\,, M = \int \sigma_{xx}zdz \tag{16}$$

The kinetic energy for FG nanobeam is expressed as:

$$T = \frac{1}{2}\iint \rho(z)(\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2)\,dA.dx \tag{17}$$

where the first variation of Eq. (17) can be written as:

$$\begin{aligned}\delta T = -\int_0^l \Bigg(& I_0\left(\frac{\partial^2 w}{\partial t^2}\right)\delta(w) + I_0\left(\frac{\partial^2 u}{\partial t^2}\right)\delta(u) \\ & - I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2}\right)\delta(w) - I_1\left(\frac{\partial^3 w}{\partial x.\partial t^2}\right) + I_1\left(\frac{\partial^3 u}{\partial x.\partial t^2}\right)\Bigg)dx\end{aligned} \tag{18}$$

where

$$\{I_0, I_1, I_2\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\{1, z, z^2\}\,dz \tag{19}$$

Then, the first variation of the work done by external forces can be expressed as:

$$\delta W_{ext} = \int_0^t (f\delta(u) + q\delta(w))dt \tag{20}$$

where f is calculated as:

$$f = (N_p + N_T + P_{electric} + K_P + q_z + N^H)\left(\frac{\partial^2 W}{\partial x^2}\right) - K_w\,W \tag{21}$$

where $N_p, N_T, P_{electric}, K_P, K_w, q_z$ and N^H are a normal force, thermal force, external electric potential, Pasternak foundation, Winkler foundation, magnetic field and moisture effect. This can be expressed as:

$$\begin{aligned} N_P &= P_0 \\ N_T &= -\int_{-h/2}^{h/2} E(z)\lambda(z)\,b\,\Delta T\,dz \\ q_z &= \int_A f_z\,dz = \eta\,A\,H_x^2\left(\frac{\partial^2 w}{\partial x^2}\right) \\ N^H &= -\int_A E\,\beta\,\Delta H\,dA = -E\,\beta\,\Delta H\,dA \end{aligned} \tag{22}$$

By utilizing Hamilton's principle and substituting Eqs. (15), (18) and (20) into Eq. (13), the equations of motion are obtained as following:

$$\frac{\partial N}{\partial x} + f - I_0\left(\frac{\partial^2 u}{\partial t^2}\right) + I_1\left(\frac{\partial^3 w}{\partial x.\partial t^2}\right) = 0 \tag{23a}$$

$$\frac{\partial^2 M}{\partial x^2} + q - I_0\left(\frac{\partial^2 w}{\partial t^2}\right) - I_1\left(\frac{\partial^3 u}{\partial x.\partial t^2}\right) + I_2\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) = 0 \tag{23b}$$

For FG nanobeam with piezoelectric properties, nonlocal equations can be developed as following (Jandaghian and Rahmani 2016):

$$N - \mu\frac{\partial^2 N}{\partial x^2} = A_{xx}\frac{\partial u}{\partial x} - B_{xx}\frac{\partial^2 w}{\partial x^2} + 2Vb \tag{24a}$$

$$M - \mu\frac{\partial^2 M}{\partial x^2} = B_{xx}\frac{\partial u}{\partial x} - C_{xx}\frac{\partial^2 w}{\partial x^2} + \frac{e_{31}^2}{\lambda_{33}}I\frac{\partial^2 w}{\partial x^2} \tag{24b}$$

where

$$\{A_{xx}, B_{xx}, C_{xx}\} = \int_A E(z)\,\{1, z, z^2\}\,dA \tag{25}$$

The explicit relations for axial force and bending moment force can be written by substituting Eqs. (23a, 23b) into Eqs. (24a, 24b) as:

$$N = A_{xx}\frac{\partial u}{\partial x} - B_{xx}\frac{\partial^2 w}{\partial x^2} + 2Vb + \mu\left(\frac{\partial f}{\partial x} + I_0\left(\frac{\partial^3 u}{\partial x.\partial t^2}\right) - I_1\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right)\right) \quad (26a)$$

$$M = B_{xx}\frac{\partial u}{\partial x} - C_{xx}\frac{\partial^2 w}{\partial x^2} + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^2 w}{\partial x^2} + \mu\left(q + I_0\left(\frac{\partial^2 w}{\partial t^2}\right) + I_1\left(\frac{\partial^3 u}{\partial x.\partial t^2}\right) - I_2\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right)\right) \quad (26b)$$

Accordingly, by substituting Eqs. (26a, 26b) into Eqs. (24a, 24b), the nonlocal governing equations of the Euler–Bernoulli FG nanobeam can be derived:

$$A_{xx}\frac{\partial^2 u}{\partial x^2} - B_{xx}\frac{\partial^3 w}{\partial x^3} + \mu\left(\frac{\partial^2 f}{\partial x^2} + I_0\left(\frac{\partial^4 u}{\partial x^2.\partial t^2}\right) - I_1\left(\frac{\partial^5 w}{\partial x^3.\partial t^2}\right)\right) - f - I_0\left(\frac{\partial^2 u}{\partial t^2}\right) + I_1\left(\frac{\partial^3 w}{\partial x.\partial t^2}\right) = 0 \quad (27a)$$

$$B_{xx}\frac{\partial^3 u}{\partial x^3} - C_{xx}\frac{\partial^4 w}{\partial x^4} + \frac{e_{31}^2}{\lambda_{33}} I \frac{\partial^4 w}{\partial x^4} + \mu\left(\frac{\partial^2 q}{\partial x^2} + I_0\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) + I_1\left(\frac{\partial^5 u}{\partial x^3.\partial t^2}\right) - I_2\left(\frac{\partial^6 w}{\partial x^4.\partial t^2}\right)\right) - q - I_0\left(\frac{\partial^2 w}{\partial t^2}\right) - I_1\left(\frac{\partial^3 u}{\partial x.\partial t^2}\right) + I_2\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) = 0 \quad (27b)$$

2.3 Differential transformation method

Among all numerical methods which are used to solve the resultant motion equations such as finite element method, Galerkin method or analytical methods, DTM is one of the useful techniques. Differential transformation (DT) technique is a method which utilized to solve ordinary differential equations. The polynomial form is used as an approximation to the exact solution which comes to the Taylor series expansion. But the main difference between the DTM and Taylor series method is that the Taylor series requires computations of higher order derivatives, while DTM involves iterative procedures instead. Applying the DTM in solving free vibration problems generally requires two transformations, namely, differential transformation (DT) and inverse differential transformation (IDT). The definitions of DT and IDT are expressed as (Hassan 2002):

$$\mathrm{DT{:}Y(k)} = \frac{1}{\mathrm{k!}}\left[\frac{\mathrm{d^k}}{\mathrm{dx^k}}\mathrm{y(x)}\right]_{\mathrm{x}=0} \quad (29)$$

$$IDT : y(x) = \sum_{k=0}^{\infty} Y_k (x - x_0) \quad (30)$$

Some of the basic transformation functions which are used to transform the constitutive equations and also boundary conditions into algebraic equations are listed in Tables 1 and 2, respectively.

After applying transformation operations introduced in Table 1, the Eq. (28) will be transformed as follows:

$$W[k+4] = \frac{\begin{pmatrix} \mu\left(-K_w + K_p - I_0\omega^2\right) - \left(N_p + N_T + P_{electric}\right) \\ -K_p - \eta A H_x^2 + I_1\omega^2 \dfrac{B_{xx}}{A_{xx}} - I_2\omega^2 \end{pmatrix} \frac{(k+2)!}{k!} W[k+2] + \left(K_w + I_0\omega^2\right) W[k]}{\left(\frac{B_{xx}^2}{A_{xx}} - C_{xx} + \frac{e_{31}^2}{\lambda_{33}} I + \mu\left(\left(N_p + N_T + P_{electric}\right) + \eta A H_x^2 - I_1\omega^2 \frac{B_{xx}}{A_{xx}} + I_2\omega^2\right)\right) \frac{(k+4)!}{k!}} \quad (31)$$

By neglecting terms of time-dependent and forces and nonlocal parameter in Eq. (27a) and also by substituting Eq. (27a) into Eq. (27a), governing equation is obtained as follows:

$$\left(\frac{B_{xx}^2}{A_{xx}} - C_{xx} + \frac{e_{31}^2}{\lambda_{33}} I\right)\frac{\partial^4 w}{\partial x^4} + \mu\left(\frac{\partial^2 q}{\partial x^2} + I_0\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) + I_1\frac{B_{xx}}{A_{xx}}\left(\frac{\partial^6 w}{\partial x^4.\partial t^2}\right) - I_2\left(\frac{\partial^6 w}{\partial x^4.\partial t^2}\right)\right) - q - I_0\left(\frac{\partial^2 w}{\partial t^2}\right) - I_1\frac{B_{xx}}{A_{xx}}\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) + I_2\left(\frac{\partial^4 w}{\partial x^2.\partial t^2}\right) = 0 \quad (28)$$

By applying DTM to various boundary conditions according to Table 2, following equations will be obtained:

- Simply–simply supported:

$$W[0] = 0, \quad W[2] = 0$$
$$\sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} k(k-1)W[k] = 0 \quad (32a)$$

- Clamped–clamped:

Table 2 Transformed boundary conditions (BC) based on DTM

X = 0		X = L	
Original BC	Transformed B.C.	Original BC	Transformed BC
$f(0) = 0$	$F[0] = 0$	$f(L) = 0$	$\sum_{k=0}^{\infty} F[k] = 0$
$\frac{df(0)}{dx} = 0$	$F[1] = 0$	$\frac{df(L)}{dx} = 0$	$\sum_{k=0}^{\infty} kF[k] = 0$
$\frac{d^2f(0)}{dx^2} = 0$	$F[2] = 0$	$\frac{d^2f(L)}{dx^2} = 0$	$\sum_{k=0}^{\infty} k(k-1)F[k] = 0$
$\frac{d^3f(0)}{dx^3} = 0$	$F[3] = 0$	$\frac{d^3f(L)}{dx^3} = 0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)F[k] = 0$

$$\begin{gathered} W[0] = 0, \quad W[1] = 0 \\ \sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} kW[k] = 0 \end{gathered} \tag{32b}$$

- Clamped–simply:

$$\begin{gathered} W[0] = 0, \quad W[1] = 0 \\ \sum_{k=0}^{\infty} W[k] = 0, \quad \sum_{k=0}^{\infty} k(k-1)W[k] = 0 \end{gathered} \tag{32c}$$

3 Numerical results

The FG material properties for steel as metal and alumina as ceramic are written in Table 3 (Reddy and Chin 1998).

The numerical results are investigated to examine the influence of nonlocal parameter (μ), power-law index, temperature change (ΔT), external electric voltage (V_0), preload (N_p), length effect (L), moisture effect and magnetic field with various boundary conditions including simply–simply supported (S–S), clamped–simply (C–S) and clamped–clamped (C–C).

In the calculation of natural frequencies by utilizing DTM, the value of natural frequency converges to a constant value after some iteration. First three natural frequencies of piezoelectric FG nanobeam for different boundary conditions are given in Table 4 for different iteration. As indicated, the first frequencies of nanobeam converge after 21st, 19th and 15th iterations with four digit precisions, while the second frequencies converge after 31th, 29th, and 25th itereations and the third frequencies after 37th, 39th and 31th converge to a constant value for C–C, C–S, and S–S, respectively. Therefore, a number of iterations are selected as k = 30 for results reported here for first natural frequencies for all boundary conditions.

After that, in order to validate and check the accuracy of resultant natural frequencies and procedure in the FG nanobeam, the results obtained from present work are compared with results given by Eltaher et al. (Eltaher et al. 2012) for various boundary conditions. Table 5 presents

Table 3 Temperature dependent coefficients of Young's modulus, density, poison ratio, thermal expansion (Reddy and Chin 1998)

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Alumina	E (Pa)	349.55×10^9	0	-3.853×10^{-4}	4.027×10^{-7}	-1.673×10^{-10}
	ρ (kg/m^3)	3960				
	v	0.26	0	0	0	0
	$\lambda(1/k)$	6.8269×10^{-6}	0	1.838×10^{-4}	0	0
	β	0.00001	0	0	0	0
Steel	E (MPa)	201.04×10^9	0	3.079×10^{-4}	-6.534×10^{-7}	0
	ρ (kg/m^3)	7800	0	0	0	0
	v	0.3262	0	-2.002×10^{-4}	3.979×10^{-7}	0
	$\lambda(1/k\times18^{-6})$	12.330×10^{-6}	0	8.086×10^{-4}	0	0
	β	0.0005	0	0	0	0
Magnetic property	η (H/m)	$4\,\pi\times10^{-7}$	0	0	0	0
Piezoelectric coefficient	e_{31}	-0.51	0	0	0	0
Dielectric constants	λ_{33}	-7.88×10^{-11}	0	0	0	0

Table 4 Convergence study of nanobeam for the first three natural frequencies of FG nanobeam ($L/h = 20$, $L = 10$ nm, $\mu = 2\ \text{nm}^2, p = 0, b = h$)

k	C–C			C–S			S–S		
	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$
11	18.9135			13.5825			9.0292		
13	20.4186			13.9505			9.0090		
15	19.9537			13.8719			9.0102	27.8714	
17	19.9999			13.8774			9.0102	29.8781	
19	19.9951			13.8770			9.0102	29.3430	
21	19.9954	43.3310		13.8770	36.3018		9.0102	29.3952	
23	19.9954	44.2576		13.8770	36.4887		9.0102	29.3901	
25	19.9954	44.0858		13.8770	36.4653		9.0102	29.3905	52.2586
27	19.9954	44.1051		13.8770	36.4676	59.1424	9.0102	29.3905	52.9134
29	19.9954	44.1032		13.8770	36.4674	61.3167	9.0102	29.3905	52.8115
31	19.9954	44.1033	68.7903	13.8770	36.4674	60.7378	9.0102	29.3905	52.8223
33	19.9954	44.1033	69.2441	13.8770	36.4674	60.8000	9.0102	29.3905	52.8213
35	19.9954	44.1033	69.1756	13.8770	36.4674	60.7928	9.0102	29.3905	52.8213
37	19.9954	44.1033	69.1833	13.8770	36.4674	60.7935	9.0102	29.3905	52.8213
39	19.9954	44.1033	69.1833	13.8770	36.4674	60.7934	9.0102	29.3905	52.8213
41	19.9954	44.1033	69.1826	13.8770	36.4674	60.7934	9.0102	29.3905	52.8213
43	19.9954	44.1033	69.1826	13.8770	36.4674	60.7934	9.0102	29.3905	52.8213

Table 5 Comparison of non-dimensional fundamental natural frequencies of simply supported beams (L = 10 nm, h = b)

L/h	$\mu \times 10^{-9}$	P = 0.1			P = 5		
		FEM eltaher et al. (Eltaher et al. 2012)	Ebrahimi et al. (Ebrahimi et al. 2015b)	Present	FEM eltaher et al. (Eltaher et al. 2012)	Ebrahimi et al. (Ebrahimi et al. 2015b)	Present
20	0	9.2129	9.1887	9.2103	6.0025	5.9373	6.0673
	1	8.7879	8.7663	8.7869	5.7256	5.6643	5.7884
	2	8.4166	8.3972	8.4170	5.4837	5.4258	5.5447
	3	8.0887	8.0712	8.0902	5.2702	5.1252	5.3294
	4	7.7964	7.7804	7.7987	5.0797	5.0273	5.1374
	5	7.5336	7.5189	7.5366	4.9086	4.8583	4.9647
50	0	9.2045	9.1968	9.2185	5.9990	5.9421	6.0723
	1	8.7815	8.7740	8.7947	5.7218	5.6690	5.7931
	2	8.4116	8.4047	8.4244	5.4808	5.4303	5.5493
	3	8.0848	8.0783	8.0973	5.2679	5.2195	5.3338
	4	7.7934	7.7873	7.8056	5.0780	5.1314	5.1416
	5	7.5313	7.5256	7.5433	4.9072	4.8623	4.9688
100	0	9.2038	9.1980	9.2196	5.9970	5.9428	6.0730
	1	8.7806	8.7752	8.7958	5.7212	5.6696	5.7938
	2	8.4109	8.4057	8.4255	5.4803	5.4309	5.5499
	3	8.0842	8.0793	8.0983	5.2675	5.2201	5.3344
	4	7.7929	7.7883	7.8066	5.0777	5.0320	5.1422
	5	7.5310	7.5265	7.5442	4.9071	4.8629	4.9694

Table 6 First natural frequency for piezoelectric FG nanobeam ($L = 20$ nm, b = 0.5h, V = -0.5)

L/h	$\mu(nm^2)$	p = 0	p = 0.1	p = 0.2	p = 0.5	p = 1	p = 2	p = 5	p = 10
S–S									
10	0	9.8305	9.1827	8.6986	7.7892	7.0718	6.5202	6.0512	5.7945
	1	9.7114	9.0715	8.5932	7.6948	6.9862	6.4412	5.9780	5.7244
	2	9.5966	8.9643	8.4916	7.6039	6.9036	6.3651	5.9073	5.6567
	3	9.4857	8.8607	8.3936	7.5161	6.8239	6.2916	5.8391	5.5914
	4	9.3787	8.7607	8.2988	7.4312	6.7468	6.2206	5.7732	5.5283
	5	9.2751	8.6640	8.2072	7.3492	6.6724	6.1519	5.7095	5.4673
20	0	9.8794	9.2310	8.7463	7.8354	7.1164	6.5630	6.0931	5.8371
	1	9.7602	9.1197	8.6409	7.7411	7.0308	6.4841	6.0199	5.7671
	2	9.6453	9.0123	8.5392	7.6501	6.9482	6.4080	5.9493	5.6995
	3	9.5343	8.9087	8.4410	7.5622	6.8685	6.3346	5.8812	5.6343
	4	9.4271	8.8086	8.3462	7.4774	6.7915	6.2636	5.8154	5.5714
	5	9.3235	8.7118	8.2545	7.3953	6.7171	6.1950	5.7518	5.5105
50	0	10.6217	9.9954	9.5300	8.6607	7.97508	7.4440	7.0097	6.8027
	1	10.5107	9.8925	9.4332	8.5753	7.8986	7.3744	6.9461	6.7426
	2	10.4039	9.7935	9.3400	8.4931	7.8250	7.3075	6.8849	6.6848
	3	10.3009	9.6980	9.2502	8.4139	7.7542	7.2431	6.8260	6.6292
	4	10.2016	9.6060	9.1636	8.3375	7.6860	7.1810	6.7693	6.5757
	5	10.1058	9.5171	9.0800	8.2639	7.6201	7.1211	6.7146	6.5241
C–S									
10	0	15.3457	14.3342	13.5782	12.1584	11.0386	10.1778	9.4458	9.0449
	1	15.1278	14.1306	13.3854	11.9857	10.8819	10.0333	9.3118	8.9166
	2	14.9187	13.9353	13.2004	11.8201	10.7315	9.8946	9.1831	8.7934
	3	14.7179	13.7477	13.0226	11.6609	10.5870	9.7615	9.0596	8.6752
	4	14.5249	13.5674	12.8518	11.5079	10.4482	9.6335	8.9409	8.5615
	5	14.3391	13.3938	12.6874	11.3607	10.3145	9.5103	8.8266	8.4520
20	0	15.4147	14.4013	13.6437	12.2199	11.0960	10.2311	9.4961	9.0950
	1	15.1978	14.1987	13.4518	12.0482	10.9402	10.0875	9.3630	8.9676
	2	14.9896	14.0043	13.2677	11.8834	10.7907	9.9498	9.2353	8.8454
	3	14.7897	13.8176	13.0908	11.7251	10.6471	9.8175	9.1126	8.7280
	4	14.5974	13.6380	12.9207	11.5729	10.5090	9.6902	8.9946	8.6151
	5	14.4123	13.4651	12.7570	11.4264	10.3761	9.5678	8.8810	8.5064
50	0	15.9881	14.9922	14.2499	12.8592	11.7623	10.9158	10.2101	9.8490
	1	15.7919	14.8110	14.0799	12.7105	11.6304	10.7968	10.1025	9.7485
	2	15.6039	14.6373	13.9170	12.5681	11.5042	10.6829	9.9995	9.6523
	3	15.4235	14.4707	13.7608	12.4316	11.3832	10.5738	9.9009	9.5602
	4	15.2504	14.3108	13.6109	12.3005	11.2670	10.4690	9.8062	9.4718
	5	15.0839	14.1571	13.4668	12.1746	11.1555	10.3685	9.7154	9.3870
C–C									
10	0	22.2601	20.7926	19.6959	17.6363	16.0119	14.7633	13.7017	13.1202
	1	21.9175	20.4724	19.3925	17.3645	15.7654	14.5362	13.4911	12.9185
	2	21.5898	20.1662	19.1024	17.1047	15.5295	14.3189	13.2897	12.7256
	3	21.2760	19.8730	18.8245	16.8559	15.3037	14.1109	13.0967	12.5409
	4	20.9752	19.5918	18.5581	16.6173	15.0872	13.9115	12.9118	12.3638
	5	20.6865	19.3220	18.3025	16.3884	14.8795	13.7200	12.7343	12.1938
20	0	22.3557	20.8849	19.7852	17.7186	16.0873	14.8319	13.7649	13.1821
	1	22.0185	20.5699	19.4869	17.4517	15.8452	14.6089	13.5581	12.9842
	2	21.6958	20.2685	19.2014	17.1962	15.6134	14.3954	13.3602	12.7948
	3	21.3865	19.9796	18.9278	16.9514	15.3913	14.1908	13.1705	12.6133
	4	21.0898	19.7025	18.6654	16.7165	15.1782	13.9945	12.9886	12.4392
	5	20.8048	19.4364	18.4133	16.4909	14.9736	13.8060	12.8138	12.2720

Table 6 continued

L/h	$\mu(nm^2)$	p = 0	p = 0.1	p = 0.2	p = 0.5	p = 1	p = 2	p = 5	p = 10
50	0	22.7945	21.3366	20.2482	18.2062	16.5948	15.3531	14.3082	13.7562
	1	22.4942	21.0596	19.9888	17.9802	16.3953	15.1738	14.1470	13.6064
	2	22.2070	20.7948	19.7410	17.7643	16.2047	15.0026	13.9931	13.4635
	3	21.9323	20.5415	19.5038	17.5578	16.0225	14.8389	13.8460	13.3269
	4	21.6690	20.2988	19.2766	17.3600	15.8480	14.6823	13.7053	13.1963
	5	21.4164	20.0660	19.0587	17.1705	15.6809	14.5322	13.5705	13.0712

Table 7 First natural frequency for piezoelectric FG nanobeam ($L = 10$ nm, h = L/20, b = 0.5h, $p = 1$)

T (°C)	$\mu = 0$ nm^2			$\mu = 2$ nm^2			$\mu = 4$ nm^2		
	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S									
0	7.1872	7.0927	6.9969	6.5850	6.4817	6.3768	6.1170	6.0056	5.8921
10	7.0327	6.9356	6.8370	6.4140	6.3074	6.1989	5.9309	5.8154	5.6976
20	6.8715	6.7716	6.6701	6.2349	6.1246	6.0122	5.7352	5.6151	5.4923
30	6.7033	6.6003	6.4956	6.0470	5.9326	5.8160	5.5288	5.4034	5.2750
40	6.5275	6.4211	6.3129	5.8495	5.7305	5.6091	5.3104	5.1791	5.0443
50	6.3434	6.2333	6.1213	5.6413	5.5172	5.3903	5.0785	4.9404	4.7982
60	6.1503	6.0362	5.9199	5.4213	5.2914	5.1583	4.8313	4.6851	4.5343
70	5.9475	5.8288	5.7077	5.1879	5.0514	4.9112	4.5662	4.4105	4.2492
80	5.7339	5.6101	5.4835	4.9394	4.7952	4.6465	4.2800	4.1127	3.9384
90	5.5082	5.3786	5.2458	4.6734	4.5199	4.3611	3.9682	3.7863	3.5952
100	5.2689	5.1327	4.9927	4.3866	4.2220	4.0507	3.6242	3.4231	3.2095
C–S									
0	11.1490	11.0782	11.0069	10.0732	9.9827	9.8914	9.2623	9.1551	9.0467
10	11.0425	10.9706	10.8981	9.9305	9.8382	9.7450	9.0889	8.9791	8.8679
20	10.9323	10.8592	10.7856	9.7825	9.6883	9.5932	8.9085	8.7958	8.6816
30	10.8182	10.7439	10.6691	9.6291	9.5328	9.4356	8.7207	8.6049	8.4875
40	10.7003	10.6247	10.5486	9.4699	9.3715	9.2720	8.5249	8.4057	8.2849
50	10.5783	10.5014	10.4240	9.3047	9.2039	9.1021	8.3206	8.1979	8.0732
60	10.4522	10.3739	10.2950	9.1331	9.0299	8.9255	8.1072	7.9805	7.8518
70	10.3217	10.2420	10.1616	8.9548	8.8490	8.7418	7.8840	7.7530	7.6197
80	10.1868	10.1056	10.0236	8.7694	8.6607	8.5506	7.6501	7.5143	7.3760
90	10.0472	9.9643	9.8808	8.5765	8.4647	8.3514	7.4045	7.2634	7.1195
100	9.9028	9.8182	9.7329	8.3755	8.2604	8.1436	7.1461	6.9990	6.8488
C–C									
0	16.1265	16.0742	16.0218	14.4690	14.3840	14.2983	13.2393	13.1269	13.0135
10	16.0601	16.0074	15.9545	14.3427	14.2564	14.1695	13.0632	12.9486	12.8330
20	15.9914	15.9381	15.8846	14.2121	14.1245	14.0364	12.8807	12.7639	12.6460
30	15.9200	15.8662	15.8122	14.0771	13.9882	13.8988	12.6916	12.5725	12.4522
40	15.8461	15.7918	15.7372	13.9377	13.8474	13.7566	12.4957	12.3740	12.2412
50	15.7697	15.7147	15.6596	13.7936	13.7020	13.6097	12.2926	12.1683	12.0427
60	15.6906	15.6351	15.5793	13.6448	13.5517	13.4579	12.0820	11.9549	11.8263
70	15.6088	15.5527	15.4964	13.4912	13.3965	13.3011	11.8635	11.7334	11.6017
80	15.5243	15.4676	15.4107	13.3325	13.2361	13.1391	11.6367	11.5033	11.3684
90	15.4371	15.3798	15.3222	13.1686	13.0705	12.9718	11.4011	11.2643	11.1257
100	15.3471	15.2892	15.2309	12.9992	12.8994	12.7988	11.1562	11.0156	10.8732

Table 8 First natural frequency for piezoelectric nanobeam ($L = 50$ nm, h = L/20, b = 0.5h, μ = 2 nm^2)

N_p	μ = 2nm^2						μ = 4nm^2					
	p = 0.1			p = 1			p = 0.1			p = 1		
	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S												
−10	6.2428	6.2379	6.2330	4.0990	4.0924	4.0858	6.1901	6.1852	6.1803	4.0514	4.0448	4.0381
−5	7.8485	7.8446	7.8407	5.7779	5.7732	5.7685	7.8067	7.8028	7.7989	5.7442	5.7395	5.7348
0	9.1775	9.1742	9.1709	7.0687	7.0648	7.0610	9.1418	9.1385	9.1351	7.0412	7.0373	7.0335
5	10.3371	10.3341	10.3312	8.1577	8.1544	8.1511	10.3054	10.3024	10.2994	8.1339	8.1306	8.1272
10	11.3790	11.3763	11.3737	9.1176	9.1146	9.1116	11.3502	11.3476	11.3449	9.0963	9.0933	9.0903
C–S												
−10	12.3170	12.3141	12.3111	9.0814	9.0778	9.0743	12.2256	12.2226	12.2196	9.0048	9.012	8.9976
−5	13.3598	13.3571	13.3544	10.1059	10.1028	10.0996	13.2828	13.2801	13.2773	10.0442	10.0410	10.0378
0	14.3224	14.3199	14.3174	11.0303	11.0275	11.0246	14.2574	14.2549	14.2523	10.9803	10.9774	10.9745
5	15.2204	15.2180	15.2157	11.8785	11.8759	11.8732	15.1657	15.1633	15.1609	11.8382	11.8355	11.8328
10	16.0649	16.0627	16.0605	12.6663	12.6638	12.6614	16.0193	16.0171	16.0148	12.6343	12.6318	12.6292
C–C												
−10	19.3299	19.3279	19.3258	14.6109	14.6086	14.6062	19.1883	19.1862	19.1842	14.4938	14.4914	14.4889
−5	20.0654	20.0635	20.0615	15.3212	15.3190	15.3167	19.9449	19.9429	19.9409	15.2247	15.2224	15.2201
0	20.7727	20.7709	20.4690	15.9974	15.9952	15.9930	20.6716	20.6697	20.6678	15.9195	15.9173	15.9151
5	21.4547	21.4529	21.4510	16.6436	16.6416	16.6395	21.3716	21.3698	21.3679	16.5828	16.5807	16.5786
10	22.1137	22.1119	22.1102	17.2635	17.2616	17.2596	22.0475	22.0457	22.0439	17.2185	7.2165	17.2144

non-dimensional natural frequency, $\hat{\omega} = \omega L^2 \sqrt{\rho_c \mathrm{A}/\mathrm{E}_c \mathrm{I}}$, of piezoelectric FG nanobeams for power-law index equal to 0.1 and 5 by varying aspect ratio and nonlocal parameter. It is clear that the obtained results are in good agreement with results given by Eltaher et al. (2012).

After the convergence study and validation of results, the influences of the power-law index, aspect ratio, and nonlocal parameter are examined in Table 6 for various boundary conditions. It is observed when power-law index and nonlocal parameter increase, natural frequency declines while as the aspect ratio grows, natural frequency increases. In addition, it should be noted that in the case that the nonlocal parameter assumed to be zero, the resultant natural frequencies correspond to the classical beam theory.

Influences of temperature and nonlocal parameter change on natural frequency are examined while the voltage is varying between −0.5 and 0.5 for various boundary conditions and the results are listed in Table 7 for all boundary conditions. It is observed when temperature increases, natural frequency decrease, the reason is that, increasing temperature causes the stiffness to decrease which is followed by a decrease in natural frequency. On the other hand, as the nonlocal parameter grows the natural frequency decline, that its reason is similar with increasing frequency which reduces the stiffness of FG nanobeam. Also varying Voltage between −0.5 and 0.5 leads to decline natural frequency. As indicated in obtained results, it can be considered that influence of Voltage on natural frequency is less than nonlocal parameter and temperature change. And also it should be noted that the positive voltage causes the natural frequencies to decrease, while the negative voltage tends to increase the natural frequency. These phenomena happen for the fact that by applying a positive and negative voltage to the nanobeam the axial compressive and tensile forces are generated which affect the natural frequencies.

Table 8 reveals natural frequency for varying preload from −10 to 10 with considering power law index, voltage and nonlocal parameter changes for various boundary conditions. From obtained results, it can be deduced that as the preload parameter increases, natural frequency increases. With increasing comprehensive load, frequency declines and with increasing tensile preload, the natural frequency of FG nanobeam decreases. It happens for the fact that as the FG nanobeam locates under tension, it becomes stiffer and when it puts into compression, it becomes softer.

Table 9 presents the natural frequency for various boundary conditions in order to investigate the influence of Pasternak medium, temperature change and voltage on natural frequency. As seen, when the nanobeam is assumed to rest in an elastic medium, natural frequency rises; In other words, elastic medium leads to improvement stiffness

Table 9 First natural frequency for piezoelectric FG nanobeam ($L = 10$ nm, h = L/20, $p = 1$, b = 0.5h)

μ (nm^2)	k_p	$\Delta T = 0$			$\Delta T = 50$			$\Delta T = 100$		
		V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S										
0	0	7.1872	7.0927	6.9969	6.3434	5.2333	5.1213	5..2689	5.1327	4.9927
	5	9.2098	9.3162	9.0621	8.5675	8.4664	8.4044	7.8055	7.7142	7.6218
	10	10.8621	10.7998	10.7371	10.3232	10.2559	10.1882	9.7000	9.62672	9.5528
2	0	6.5850	6.4817	6.3768	5.6413	5.5172	5.3903	4.3866	4.2220	4.0507
	5	8.7480	8.6705	8.5923	8.0615	7.9752	7.8880	7.2392	7.1407	7.0408
	10	10.4734	10.4088	10.3437	9.9072	9.8371	9.7665	9.2505	9.1735	9.0960
4	0	6.1170	6.0056	5.8921	5.0785	4.9404	4.7982	3.6242	3.4231	3.2095
	5	8.4013	8.3205	8.2390	7.6782	7.5876	7.4958	6.8043	6.6994	6.5928
	10	10.1856	10.1191	10.0522	9.5979	9.5255	9.4526	8.9142	8.8344	8.7538
C–S										
0	0	11.1490	11.0782	11.0069	10.5783	10.5014	10.4240	9.9028	9.8182	9.7329
	5	12.7576	12.6964	12.6349	12.2681	12.2026	12.1367	11.6979	11.6272	11.5561
	10	14.1723	14.1177	14.0629	13.7380	13.6801	13.6219	13.2368	13.1750	13.1129
2	0	10.0732	9.9827	9.8914	9.3047	9.2039	9.1021	8.3755	8.2604	8.1436
	5	12.0799	12.0049	11.9294	11.4505	11.3692	11.2874	10.8141	10.6250	10.5351
	10	13.7920	13.7265	13.6608	13.2462	13.1762	13.1059	12.6176	12.5423	12.4666
4	0	9.2623	9.1551	9.0467	8.3206	8.1979	8.0732	7.1461	6.9990	6.8488
	5	11.5856	11.5003	11.4143	10.8497	10.7561	10.6618	9.9811	9.8768	9.7714
	10	13.5122	13.4392	13.3658	12.8878	12.8092	12.7302	12.1669	12.0817	11.9958
C–C										
0	0	16.1265	16.0742	16.0218	15.7697	15.7147	15.6596	15.3472	15.2892	15.2310
	5	17.3555	17.3074	17.2591	17.0280	16.9775	16.9269	16.6413	16.5883	16.5350
	10	18.4951	18.4503	18.4053	18.1911	18.1442	18.0972	17.8331	17.7839	17.7347
2	0	14.4690	14.3840	14.2983	13.7936	13.7020	13.6097	12.9993	12.8995	12.7989
	5	16.4191	16.3444	16.2693	15.8291	15.7495	15.6693	15.1444	15.0591	14.9733
	10	18.1576	18.0901	18.0224	17.6270	17.5557	17.4841	17.0163	16.9407	16.8646
4	0	13.2393	13.1269	13.0135	12.2926	12.1683	12.0427	11.1562	11.0157	10.8732
	5	15.7496	15.6553	15.5605	14.9637	14.8619	14.7594	14.0469	13.9358	13.8237
	10	17.9098	17.8270	17.7438	17.2233	17.1350	17.0462	16.4338	16.3390	16.2436

Table 10 First natural frequency for piezoelectric FG nanobeam for various length (h = L/20, b = 0.5h, $\Delta T = 20^{\circ}C, \mu = 2\ \mathrm{nm}^2$)

L	p = 0.1			p = 1			p = 2		
	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S									
10×10^{-9}	8.1467	8.0480	7.9513	6.2349	6.1246	6.0122	5.7436	5.6294	5.5129
20×10^{-9}	8.6697	8.6473	8.6249	6.6193	6.5935	6.5676	6.0902	6.0635	6.0367
50×10^{-9}	8.8412	8.8376	8.8341	6.7463	6.7422	6.7382	6.2054	6.2012	6.1970
80×10^{-9}	8.8618	8.8605	8.8591	6.6717	6.7601	6.7585	6.2193	6.2177	6.2161
100×10^{-9}	8.8666	8.8658	8.8649	6.7652	6.7642	6.7632	6.2226	6.2215	6.2205
1×10^{-6}	8.8751	8.8751	8.8751	6.7715	6.7715	6.7715	6.2283	6.2283	6.2283
10×10^{-6}	8.8752	8.8752	8.8752	6.7716	6.7716	6.7716	6.2283	6.2283	6.2283
100×10^{-6}	8.8752	8.8752	8.8752	6.7716	6.7716	6.7716	6.2283	6.2283	6.2283
C–S									
10×10^{-9}	12.7292	12.6472	12.5646	9.7825	9.6883	9.5932	9.0180	8.9206	8.8221
20×10^{-9}	13.7416	13.7244	13.7072	10.5522	10.5325	10.5127	9.7228	9.7024	9.6819
50×10^{-9}	14.0752	17.0726	17.0700	10.8081	10.8051	10.8022	9.9578	9.9548	9.9517
80×10^{-9}	14.1156	14.1146	14.1136	10.8391	10.8380	10.8368	9.9864	9.9852	9.9840
100×10^{-9}	14.1250	14.1243	14.1237	10.8463	10.8456	10.8449	9.9930	9.9922	9.9915
1×10^{-6}	14.1415	14.1415	14.1415	10.8590	10.8590	10.8590	10.0047	10.0047	10.0047
10×10^{-6}	14.1417	14.1417	14.1417	10.8592	10.8592	10.8592	10.0049	10.0049	10.0049
100×10^{-6}	14.1417	14.1417	14.1417	10.8592	10.8592	10.8592	10.0049	10.0049	10.0049
C–C									
10×10^{-9}	18.4627	18.3863	18.3096	14.2121	14.1245	14.0364	13.1056	13.0151	12.9240
20×10^{-9}	20.0631	20.4940	20.0358	15.4443	15.4287	15.4130	14.2391	14.2230	14.2068
50×10^{-9}	20.5937	20.5918	20.5899	15.8558	15.8536	15.8514	14.6186	14.6164	14.6141
80×10^{-9}	20.6581	20.6573	20.6566	15.9058	15.9049	15.9041	14.6648	14.6639	14.6630
100×10^{-9}	20.6730	20.6720	20.6720	15.9174	15.9169	15.9163	14.6755	14.6749	14.6744
1×10^{-6}	20.6994	20.6993	20.6993	15.9379	15.9379	15.9379	14.6944	14.6944	14.6944
10×10^{-6}	20.6996	20.6996	20.6996	15.9381	15.9381	15.9381	14.6946	14.6946	14.6946
100×10^{-6}	20.6996	20.6996	20.6996	15.9381	15.9381	15.9381	14.6946	14.6946	14.6946

Table 11 First natural frequency for piezoelectric FG nanobeam by considering magnetic field ($L = 10$ nm, h = L/20, b = 0.5h, $\Delta T = 0°C, \mu = 2$ nm^2)

H	p = 0.1			p = 1			p = 2		
	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S									
0	8.5072	8.4170	8.3259	6.5850	6.4817	6.3768	6.0820	5.9753	5.8666
1	8.5507	8.4610	8.3704	6.6353	6.5328	6.4287	6.1359	6.0302	5.9225
2	8.6801	8.5918	8.5025	6.7840	6.6837	6.5820	6.2950	6.1919	6.0871
3	8.8916	8.8054	8.7183	7.0247	6.9280	6.8299	6.5514	6.4525	6.3520
4	9.1795	9.0960	9.0118	7.3485	7.2561	7.1625	6.8944	6.8005	6.7052
5	9.5369	9.4566	9.3755	7.7450	7.6573	7.5687	7.3119	7.2233	7.1337
6	9.9563	9.8794	9.8019	8.2036	8.1209	8.0373	7.7918	7.7087	7.6248
7	10.430	10.3569	10.2830	8.7145	8.6367	8.5582	8.3233	8.2456	8.1672
8	10.951	10.8818	10.8115	9.2690	9.1959	9.1222	8.8972	8.8246	8.7514
9	11.5141	11.4477	11.3808	9.8599	9.7912	9.7220	9.5059	9.4380	9.3695
10	12.1119	12.0487	11.9852	10.4810	10.4164	10.3514	10.1431	10.0794	10.0154
C–S									
0	13.0421	12.9633	12.8839	10.0732	9.98274	9.89144	9.29636	9.20287	9.1084
1	13.0804	13.0017	12.9226	10.1174	10.0274	9.93652	9.34385	9.25086	9.15689
2	13.1944	13.1165	13.0381	10.249	10.1602	10.0705	9.48488	9.39331	9.30083
3	13.3822	13.3054	13.2282	10.4645	10.3776	10.2898	9.71525	9.62593	9.53576
4	13.6407	13.5654	13.4897	10.7588	10.6743	10.5891	10.0287	9.94223	9.85503
5	13.9658	13.8923	13.8185	11.1254	11.0438	10.9616	10.4174	10.3343	10.2506
6	14.353	14.2815	14.2097	11.5574	11.4789	11.3999	10.8733	10.7938	10.7137
7	14.7971	14.7279	14.6583	12.0474	11.9722	11.8966	11.388	11.3122	11.2359
8	15.2932	15.2263	15.1591	12.5887	12.5168	12.4446	11.9538	11.8817	11.8092
9	15.8363	15.7717	15.7069	13.1747	13.1061	13.0372	12.5637	12.4952	12.4263
10	16.4215	16.3593	16.2968	13.7997	13.7343	13.6686	13.2114	13.1463	13.0809
C–C									
0	18.7527	18.6786	18.7527	14.2983	14.3840	14.4690	13.3485	13.2605	13.1719
1	18.7888	18.7148	18.7888	14.3406	14.4259	14.5108	13.3933	13.3056	13.2174
2	18.8964	18.8228	18.8964	14.4665	14.5511	14.6352	13.5268	13.4400	13.3527
3	19.0743	19.0015	19.0743	14.6740	14.7574	14.8403	13.7464	13.6611	13.5752
4	19.3206	19.2487	19.3206	14.9595	15.0412	15.1225	14.0480	13.9645	13.8805
5	19.6327	19.5620	19.6327	15.3186	15.3984	15.4778	14.4263	14.3450	14.2633
6	20.0074	19.9380	20.0074	15.7461	15.8237	15.9010	14.8753	14.7966	14.7174
7	20.4412	20.3733	20.4412	16.2366	16.3119	16.3868	15.3888	15.3128	15.2363
8	20.9303	20.8641	20.9303	16.7845	16.8572	16.9297	15.9604	15.8872	15.8136
9	21.4710	21.4064	21.4710	17.3842	17.4544	17.5243	16.5841	16.5136	16.4429
10	22.0593	21.9965	22.0593	18.0304	18.0981	18.1655	17.2541	17.1864	17.1185

Table 12 First natural frequency for piezoelectric FG nanobeam by considering magnetic field and hygro parameter ($L = 10$ nm, h = L/20, b = 0.5h, $\Delta T = 10°C$)

μ *nm^2	%ΔH	p = 0.1			p = 1			p = 2		
		V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5	V = −0.5	V = 0	V = 0.5
S–S										
0	0	9.12949	9.0449	8.9595	7.0327	6.9356	6.8370	6.4876	6.3872	6.2852
	10	9.1177	9.0330	8.9475	6.9695	6.8714	6.7720	6.4012	6.2994	6.1960
	20	9.1059	9.0211	8.9555	6.9057	6.8067	6.7063	6.3136	6.2104	6.1055
2	0	8.3280	8.2352	8.1413	6.4140	6.3074	6.1989	5.9172	5.8070	5.6946
	10	8.3151	8.2222	8.1282	6.3447	6.2368	6.1271	5.8224	5.7103	5.5960
	20	8.3022	8.2091	8.1150	6.2745	6.1655	6.0544	5.7260	5.6120	5.4956
4	0	7.7022	7.6018	7.5000	5.9309	5.8154	5.6976	5.4718	5.3524	5.2324
	10	7.6883	7.5871	7.4857	5.8558	5.7388	5.6193	5.3691	5.2474	5.1227
	20	7.6743	7.5735	7.4714	5.7798	5.6612	5.5400	5.2644	5.1402	5.0129
C–S										
0	0	14.3275	14.2648	14.2018	11.0425	10.9706	10.8981	10.1852	10.1109	10.0360
	10	14.3187	14.2560	14.1929	10.9956	10.9234	10.8506	10.1213	10.0464	9.9710
	20	14.3100	14.2472	14.1841	10.9486	10.8760	10.8029	10.0568	9.9815	9.9055
2	0	12.8872	12.8068	12.7258	9.9305	9.8382	9.7450	9.1601	9.0647	8.9683
	10	12.876	12.7955	12.7145	9.8704	9.7775	9.6837	9.0780	8.9817	8.8844
	20	12.8648	12.7842	12.7032	9.8099	9.7164	9.6220	8.9952	8.8980	8.7997
4	0	11.7974	11.7017	11.6053	9.0889	8.9791	8.8679	8.3843	8.2708	8.1557
	10	11.7841	11.6883	11.5918	9.0174	8.9067	8.7945	8.2866	8.1717	8.0552
	20	11.7708	11.6749	11.5782	8.9453	8.8337	8.7206	8.1878	8.0714	7.9534
C–C										
0	0	20.8331	20.7871	20.7410	16.0601	16.0074	15.9545	14.8125	14.7580	14.7032
	10	20.8267	20.7807	20.7345	16.0258	15.9729	15.9198	14.7655	14.7108	14.6559
	20	20.8203	20.7742	20.7281	15.9913	15.9383	15.8851	14.7184	14.6636	14.6085
2	0	18.6089	18.5336	18.4580	14.3427	14.2564	14.1695	13.2294	13.1403	13.0504
	10	15.5984	18.5231	18.4474	14.2864	14.1998	14.1126	13.1527	13.0629	12.9726
	20	18.5879	18.5125	18.4368	14.2300	14.1430	14.0554	13.0754	12.9852	12.8942
4	0	16.9520	16.8522	16.7517	13.0632	12.9486	12.8330	12.0501	11.9317	11.8121
	10	16.9381	16.8382	16.7377	12.9886	12.8733	12.7571	11.9482	11.8288	11.7081
	20	16.9242	16.8242	16.7236	12.9135	12.7976	12.6806	11.8454	11.7249	11.6032

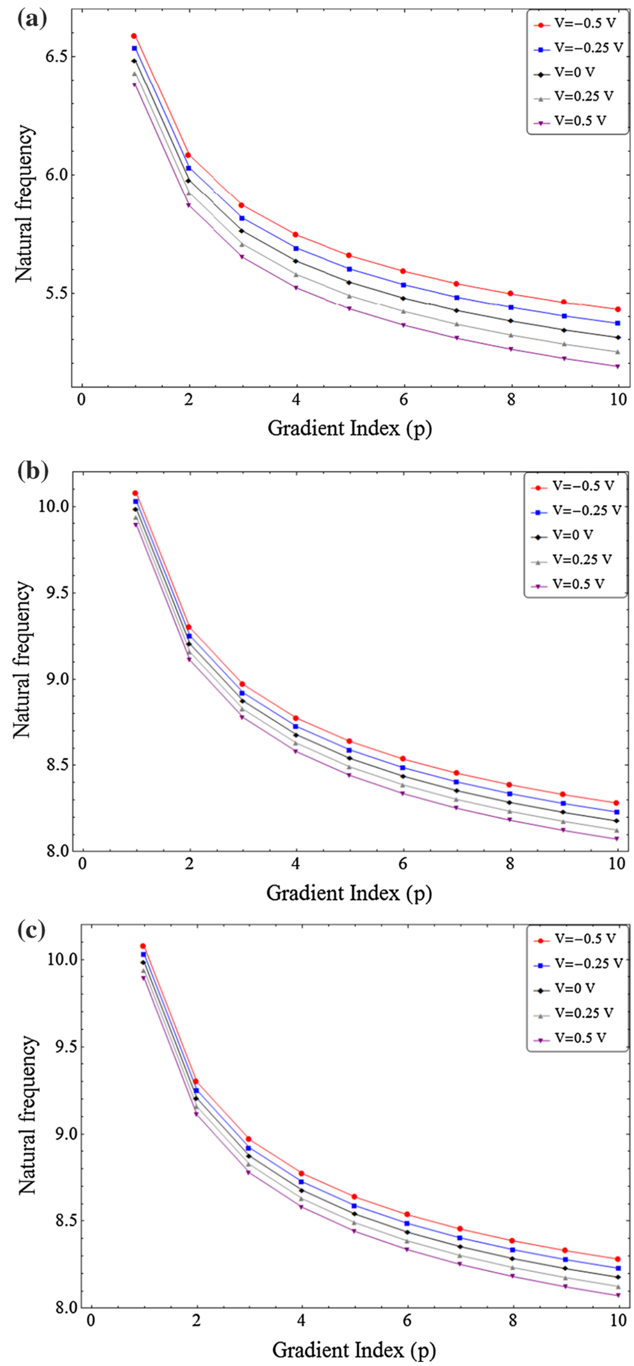

Fig. 2 Variation of natural frequency of piezoelectric FG nanobeam versus gradient index for different voltages (L = 10 nm, L/h = 20, b = 0.5 h, $\Delta T = 0$, $\mu = 2$ nm^2). **a** Simply supported–simply supported, **b** clamped–simply supported, **c** clamped–clamped

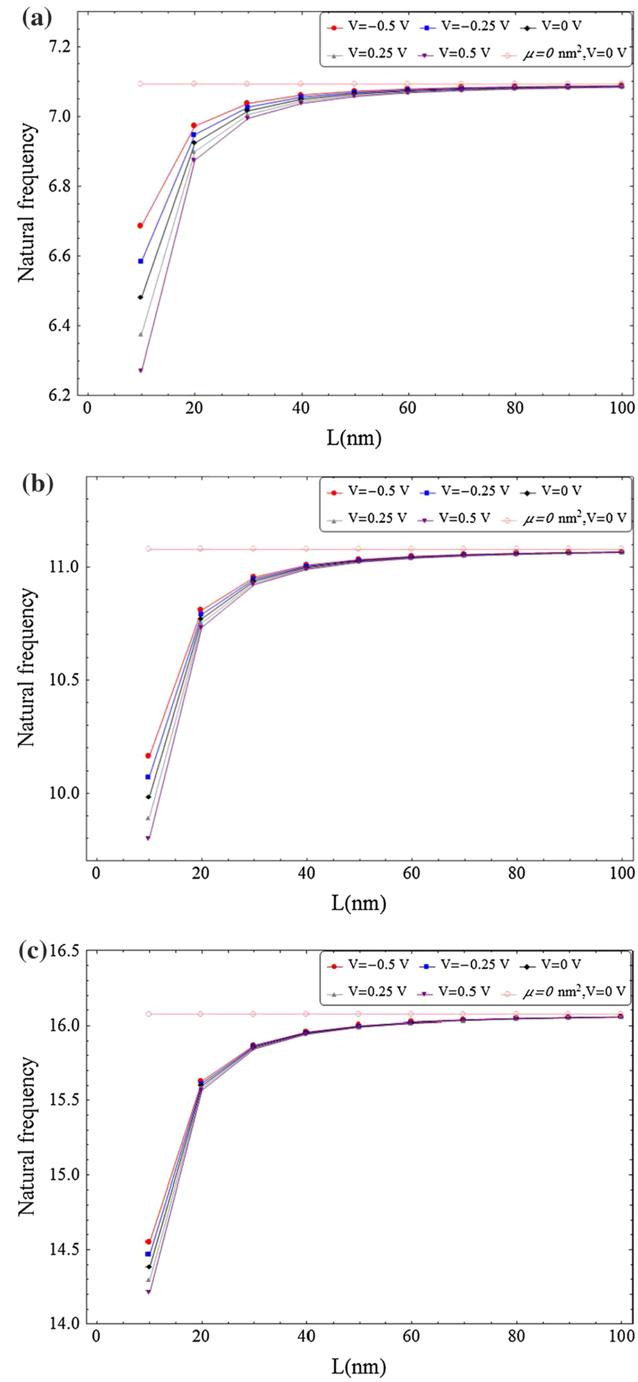

Fig. 3 Variation of natural frequency of piezoelectric FG nanobeam versus length of nanobeam for different voltages (L/h = 20, b = 0.5 h, $\Delta T = 0$, $\mu = 2$ nm^2, P = 1). **a** Simply supported–simply supported, **b** clamped–simply supported, **c** clamped–clamped

of FG nanobeam. As mentioned, rising temperature and voltage decrease natural frequency.

Moreover, the influences of length on natural frequency via power-law index and voltage for different boundary conditions are illustrated in Table 10. From the resultant natural frequencies, it can be deduced when length increase, results are converging to a certain value. Effect of various parameters such as voltage, magnetic field, and the power-law index is investigated in Table 11. It is observed when magnetic field increases, natural frequency inverse power-law index and voltage rises and FG nanobeam become stiffer.

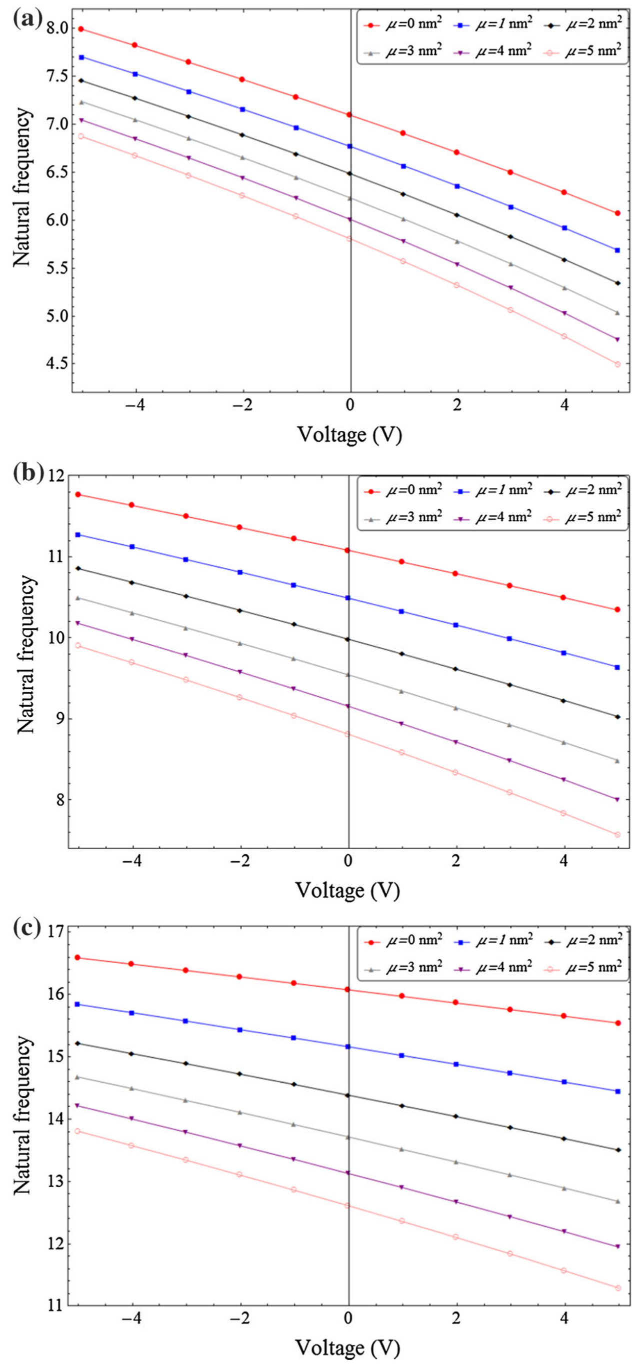

Fig. 4 Variation of natural frequency of piezoelectric FG nanobeam versus voltage external for different nonlocal parameter (L = 10 nm, L/h = 20, b = 0.5 h, ΔT = 0,p = 1). **a** Simply supported–simply supported, **b** clamped–simply supported, **c** clamped–clamped

Table 12 depicts the variation of natural frequency versus the moisture expansion coefficients and power-law index for various nonlocal parameter values. As the moisture expansion coefficient increases the natural frequency decrease which is assign that the stiffness of the nanobeam is reduced and the nanobeam becomes softer. Similar to the moisture expansion coefficients the increase in the nonlocal parameter for all values of moisture expansion coefficients results in a reduction in natural frequencies.

The variation of the natural frequency of piezoelectric FG nanobeam versus power-law index for various voltages and different boundary conditions where the temperature effects are ignored is illustrated in Fig. 2. It can be discovered that when the voltage increases, natural frequency declines for all values of voltage. And also when power-law index grows, for all values of voltage, natural frequency decreases.

Figure 3 reveals the variation of the natural frequency of piezoelectric FG nanobeam versus length for various voltages and different boundary conditions. As seen, with increasing length of FG nanobeam, natural frequency increase for all voltage values and also tends to reach its local value and nonlocal effect with voltage can be neglected in macrosize.

Finally, Fig. 4 depicts a variation of the natural frequency of FG nanobeam versus external voltage for different boundary conditions and various nonlocal parameters. According to Fig. 4, by increasing the value of external voltage from negative to positive the natural frequency tends to decrease. Because increasing external voltage decreases the stiffness of the FG nanobeam.

4 Conclusion

A piezo-nonlocal beam model is utilized to study hydro–thermo–electro–mechanical vibration response of functionally graded material nanobeam in presence of magnetic field and preload resting in the elastic medium for various boundary conditions. The governing equations are obtained with Hamilton's principle and solved with DTM. Frequency analysis is carried out for different nonlocal parameters, power-law index, temperature changes, voltage externals, magnetic fields, preload parameter, aspect ratio, moisture effect and length. Based on obtained results:

1. Increasing nonlocal parameter leads to decrease natural frequency.
2. Rising temperature decreases natural frequency.
3. Increasing external voltage causes the natural frequency to decrease.
4. Considering preload and increasing tension loads leads to increase the natural frequency and increasing compression load leads to decline natural frequency.
5. Elastic medium causes FG nanobeam becomes stiffer which tends to increase natural frequency.
6. With increasing length of FG nanobeam, results are converging to a certain value.

7. With rising moisture effect, frequencies of Piezoelectric FG nanobeam decline.

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