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Structural modelling and integrative analysis of microelectromechanical systems product using graph theoretic approach

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Abstract Micro-electromechanical systems (MEMS) as an enabling technology is seen to play a more and more important role for the main stream of industry of the future by broadening its applications to information, communications and bio technologies. Development of MEMS devices, however, still relies on knowledge and experience of MEMS experts due to the design and fabrication process complexity. It is difficult to understand the trade-offs inherent in the system and achieve an optimal structure without any MEMS-related insight. An attempt is made to develop an integrated systems model for the complete structure of the MEMS product system in terms of its constituents and interactions between the constituents. The hierarchical tree structures of the MEMS system and its subsystems are presented up to component level. For characterization, analysis and identification of MEMS product system, three different mathematical models say graph theoretic model, matrix model and permanent model are presented. These models are associated with graph theory, matrix method and variable permanent function by considering the various subsystems, subsubsystems up to component level, their connectivity and interdependency of the MEMS product system. The developed methodology is explained with an example. The proposed modeling and analysis is extendable to the subsystems and the component level. An overall structural analysis can be carried out by following a 'top-down' approach or 'bottom-up' approach. Understanding of MEMS product structure will help in the improvement of performance, cost, design time, and so on.

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1 Introduction

Micro-electromechanical systems (MEMS) are an emerging field, which attracted attention in various fields such as the automobile, information, communications and biology (Lin 2001; Peterson 1982; Mamiya et al. 2004; Eapen et al. 2006; Lee et al. 2001). MEMS are a hybrid of electronics, mechanical elements, sensors, and actuators on a common silicon substrate through the utilization of microfabrication technique with common package. It promises to revolutionize nearly every product category by bringing together silicon-based microelectronics with manufacturing technology, their by making possible the realization of complete Systems-on-a-Chip (SOC) (Kovacs 1998; Gardner 1994). Since the complexity of MEMS increases, it is becoming increasingly important to design and optimize the coupling between the micromachined elements, the microelectronics circuits that control them and condition the signal and the constraints due to packaging (Schropfer et al. 2004). MEMS design must be separated from the complexities of the fabrication sequence and packaging process with consideration of different materials, process and environment (MEMS-Exchange 2008; Reithel 2008).

The package IC die is part of the complete system and should be designed as the MEMS chip is designed, with the specific and many times custom package in mind. The chip, package, and environment all must function together and must be compatible with each other (Yufeng et al. 2005). This determines which materials and what design considerations and limitations become important. One of the main scientific challenges of MEMS is the issue of material properties. The properties of the materials depend on how they are used, processed, the heat treatments to which the materials are subjected, and even the specific pieces of equipment used during fabrication. Not all the materials used react the same to these parameters, so compromises must be made. Some materials may be hard to obtain with R&D production run numbers. Low quantities of materials are used, and suppliers are reluctant to sell small quantities or develop new products for limited markets (Monk and Shah 1996). One good point about the materials used in microsystems is that the material properties generally get better at the microscale. This is due to a decrease in the number of defects encountered in the materials. The defect density remains about the same as in macroscale devices, but since the MEMS devices are so small, the chance of a killer defect occurring in a device is reduced (O'Neal et al. 1999).

Researcher Lucyszyn (2004) has given a unique roadmap that shows how the enabling technologies, RF MEMS components, RF MEMS circuits and RF Microsystems packaging are linked together; leading towards enhanced integrated subsystems. Steve Mechels et al. (2003) discussed the 1-D MEMS based wavelength switching subsystem. Due to design and fabrication complexity the development of MEMS devices still relies on knowledge and experience of MEMS experts. It is difficult to understand the trade-offs inherent in the system and achieve an optimal structure without any MEMS-related insight. MEMS product development is too slow because iterative structural analysis, layout and testing are necessary to achieve complete structures. Also MEMS researchers need to design structures under some MEMS fabrication limitations. Thus MEMS designers tend to pursue just a few primary characteristics without consideration of the total system performance. This time consuming product development process is unavoidable to bring MEMS to market (Mamiya et al. 2004).

From the above literature review, we can say that no one has considered the structural constituents for analyzing a MEMS product system along with their interactions at the conceptual design stages. There is no methodology proposed for an integrated system approach for analyzing MEMS products, i.e., by considering the subsystems and their interactions/interdependence and connectivities. To fill this research gap, a graph theory based MEMS product system model is proposed, which has the capability to consider the interactions/interdependence and connectivities in an integrated way. This tool has so far been applied extensively to analyze complex systems such as power plants (Mohan et al. 2003), composite materials (Prabhakaran et al. 2006) and manufacturing (Singh and Agrawal 2008), but it has not been used to model and analyze a MEMS product system.

2 Identification of structural constituents of MEMS product system

To develop the systems mathematical model of the structure of the MEMS products, it is very much necessary to identify the structural components, manufacturing process, process parameters, materials and the application/usage. Five subsystems have been identified namely design, fabrication, material, packaging and environment subsystems. These subsystems may vary and depend on the product and the process of manufacturing. The proposed methodology is capable of considering any such variation and is suitable for modeling any particular MEMS product structure. The importance of the identified subsystems is discussed in the following section.

2.1 Design subsystem

MEMS design is a complex process, because it is important to design and optimize the coupling between the micromachined elements and the microelectronics circuits that control them and condition the signal (Schropfer et al. 2004). The micromachined element designer should be an expert from the design type/domain in which he is working and the sensing & actuation technique used for the design (Liu 2006; Senturia 2001). For example if the design is an RF switch the micromachined element designer should be an RF domain expert and the possible actuation technique is electrostatic. Microelectronics circuit designer has to consider the design of signal conditioning circuit and signal processing circuits in a structured way. On the basis of critical literature review (Girbau et al. 2006; Jeong et al. 2004; Museau et al. 2007) different subsystems for MEMS product system are identified. Subsystems of a typical MEMS product system and sub-subsystems of design subsystem are shown in Fig. 1.

2.2 Fabrication subsystem

While the microelectronic circuits are fabricated using IC process sequences, the micromechanical components are fabricated using compatible micromachining process that selectively etch away parts of the silicon wafer or add new structural layers to form the mechanical and/or electromechanical products (Zha and Du 2003). The fabrication subsystem experts have to identify the process requirements, fabrication technique and the fabrication process equipment/tools. Different sub-subsystems identified for fabrication subsystem are shown in Fig. 2 (Museau et al. 2007; Zha and Du 2003; Kovacs et al. 1998; Bustillo et al. 1984; Schmidt 1998).

2.3 Material subsystem

The material property can change the performance of the MEMS product. Since MEMS devices are essentially mechanical, it is required to characterize all the material properties. The material property characteristics and



Fig. 1 Subsystems of a typical MEMS product system and sub-subsystems of design subsystem



Fig. 2 Sub-subsystems of fabrication subsystem

structure help us to understand the electrical, mechanical, optical or magnetic properties of the materials. The usage level of each material can also be structured in a systematic way to understand the use of it in substrate level or package level or doping level etc. the material subsystem may also have the information about the type and availability of materials. Different sub-subsystems identified for materials subsystem are shown in Fig. 3 (Schropfer et al. 2004; Liu 2006; Zha and Du 2003; Judy and Myung 2002; Gilleo 2005).

2.4 Packaging subsystem

Understanding of the package level is required, it may be system level (micromechanical elements and microelectronic circuits), device level (Transduction element and signal mapping circuits) or die level (sensing and/or actuation element). The package may give protection to the MEMS product from the environment or electrical and mechanical disturbances. Packaging process is again a complex structure; it is to be done in coordination with the



Fig. 3 Sub-subsystems of materials subsystem



Fig. 4 Sub-subsystems of packaging subsystem

design and fabrication experts to have better compatibility. Different sub-subsystems are identified for the packaging subsystem and are shown in Fig. 4 (Yufeng et al. 2005; O'Neal et al. 1999; Gilleo 2005; Hsu 2004).

2.5 Environment subsystem

Environment is also a part of the MEMS product system (Mir et al. 2006; Persson and Boustedt 2002). The micromachined element is required to interact with the environment to sense/activate, but at the same time it is to be protected from the environment. The microelectronics circuits are also very sensitive to the environment. It is very much necessary to understand the MEMS product interface domain, the possible noise/disturbance and the compatibility of the chip for the designed environment. Different subsubsystems are identified for the environment subsystem and are shown in Fig. 5 (Mir et al. 2006; Persson and Boustedt 2002; Tanner et al. 2000; Shea 2006).

3 Hierarchical structure of MEMS product system

The hierarchical tree structure helps to understand and analyse the MEMS product system in top-down or bottomup fashion. Different elements of the system, subsystems, sub-subsystems etc., at each level are identified up to component level. This tree structure helps the industry to develop the product from component level to the system level in the hierarchical order. Normally the tree structure may have (n + 1) level as given below.

Level 0: Total system Level 1: Subsystems Level 2: Sub-subsystems





Environment Subsystem

Level 3: Sub-sub-subsystems

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Level n: Components

A five level tree structure of MEMS product is pro-

posed in Fig. 1 can be described as:

Level 0: MEMS product system (Total)

Level 1: Design, Fabrication etc. (Subsystems)

As an example for design subsystem (Fig. 1)

Level 2: Micromachined element design, Microelectronics circuit design (Sub-subsystems)

Level 3: Design type, Transduction technique etc.

(Sub-sub-subsystems)

Level 4: RF, Optical etc. (Components)

4 Interaction in the subsystems of the MEMS product system

The subsystems are interrelated and interdependent on each other in number of ways. For example the product designer should consider the environment in which the product is going to be used, the suitable fabrication technique for the design and the package compatible for the product. Materials used in the product fabrication and package should retain its properties under environmental conditions. Though the tree diagram in Figs. 1, 2, 3, 4 and 5 developed above represents all the subsystems of the MEMS product system, it fails to include the interdependencies and connectivity among different subsystems. So, a schematic diagram is developed as shown in Fig. 6. The authors cannot claim that this is an exhaustive schematic diagram. It is developed to explain the new methodology.

This diagram shows all such interactions and is in good accordance with recent findings about interrelationship between subsystems. The design system depends on the kind of environment the product is going to sense or give output. The design team has to bother about the fabrication limitation. All the designs may not be able to fabricate with the required tolerance. The package also has to be designed along with the product design (O'Neal et al. 1999; Velten et al. 2005).

The fabrication subsystem directly depends on the design, from the design specifications only fabrication process can be selected. Packaging schemes should be designed and incorporated into the device fabrication process itself (Chiao and Lin 2004).

The materials used are to be stable for the operating environmental conditions. The materials used in fabricatin and packaging are to be investigated for compatibility, for example in Bio-chip design the materials used need to be biocompatible (Grayson et al. 2004). The package plays a key role in ensuring the long-term reliability of a MEMS product and it is to be designed along with the MEMS design. The materials used for package are studied to give the required protection and isolation from the environment.

The MEMS product communicates with the environment to sense or activate. The environment is more transparent for the required parameter monitoring and it should not be harmful to the package. Since environment can change the materials property and product performance.

The interconnection and interaction between these subsystems distinguish one MEMS product from the others and is the cause for their performance variations.

5 Graph theoretic modeling and analysis of the MEMS product system

The MEMS product subsystems are connected with each other through different forms of bonding and interactions. The constituents and interactions forming a MEMS product are shown in Fig. 6. Blocks show the constituents, lines show the connectivity and arrows show the direction of dependency. Though, the schematic diagram is a good representation of the MEMS product structure, it is not a mathematical entity. Hence it is not possible to derive/ develop different results as no mathematical operation can be carried out. Mathematical modeling is done using graph theory for systems like power plants, composite industry and manufacturing system (Mohan et al. 2003; Prabhakaran et al. 2006; Singh and Agrawal 2008). Thus, for modeling of the MEMS product systems it is meaningful to select the graph theory and matrix algebra (Jurkat and Ryser 1966).

A MEMS product system may be considered to be a system [M, I] of its constituent set $\{M\} = \{M_1, M_2, ..., M_n\}$ and interconnection set $\{I\} = \{I_1, I_2, ..., I_n\}$, where M_i represents *i*th constituent while I_j corresponds to *j*th interconnection between two corresponding constituents of the MEMS product (Deo 2004).

A graph G has been defined as a function of vertex set and edge set as $G = f\{V, E\}$, where V corresponds to a set of vertices $\{V\} = \{V_1, V_2, ..., V_n\}$ and E corresponds to a set of edges $\{E\} = \{E_1, E_2, ..., E_n\}$ joining different vertices.

For MEMS product system, let vertices corresponds to subsystems (S_i) and the edges (e_{ij}) corresponds to interconnection/connectivity from subsystem S_i to S_j . If we assume that all the five subsystems are interacting with each other and have general directional characteristics, the MEMS product has a graph theoretic representation with $e_{ij} \neq e_{ji}$. If the directional property is not significant, the MEMS product is represented by an undirected graph, in this case $e_{ij} = e_{ji}$. The MEMS product graph developed is shown in Fig. 7. This graph is a useful mathematical entity and is highly useful for the total understanding of the MEMS product through for visual analysis. To have a better mathematical representation and information storage MEMS product graph can be represented in the form of various matrix models as discussed in the next section.

5.1 Matrix representation for the MEMS product system

5.1.1 Adjacency matrix (AM-MP)

An alternative to the incidence matrix, it is more convenient to represent a graph by its adjacency matrix/connectivity matrix (Bonchev 1983). The adjacency matrix of the graph G with five nodes is a five order binary (0, 1) square matrix, $A = [a_{ij}]$ such that:

 $a_{ij} = \begin{cases} 1, \text{ if subsystem } i \text{ have an influence on subsystem } j \\ 0, \text{ if } i \text{ and } j \text{ are not connected} \end{cases}$

where $i, j \in \{1, 2, 3, 4, 5\}$ and $i \neq j$.

The MEMS product system adjacency matrix (AM-MP), A for the Graph can be written as Eq. 1





Fig. 7 MEMS product system graph

5.1.2 Characteristic matrix (CM-MP)

Since the adjacency matrix represents the interrelationships only, in order to represent the MEMS product system characteristic also another matrix B called characteristics matrix (Deo 2004) is derived and is given in Eq. 2.

$$B = [\lambda I - A] = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \text{subsystems} \\ \lambda & -1 & 0 & -1 & -1 \\ -1 & \lambda & 0 & -1 & 0 \\ 0 & -1 & \lambda & -1 & -1 \\ -1 & -1 & 0 & \lambda & -1 \\ -1 & 0 & -1 & -1 & \lambda \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$
(2)

where λ represents invariant eigen values of the system; *I* is the identity matrix of same order as *A*. The determinant of the MEMS product system characteristics matrix *B* will lead to an invariant of this matrix and is given in Eq. 3.

$$Det(B) = \lambda^5 - 6\lambda^3 - 5\lambda^2 \tag{3}$$

The solution of Eq. 3 will give eigen spectrum i.e. invariant eigen values.

Interdependencies between the subsystems have been assigned values of 0 and 1 depending on whether it is there or not. But this does not represent varying degree of influence of one subsystem over the other subsystems. To consider this, another matrix called the MEMS product system variable characteristic matrix (VCM-MP) is proposed.

5.1.3 Variable characteristic matrix (VCM-MP)

From matrix *B*, another matrix *C*, called as MEMS product system variable characteristic matrix is developed as given in Eq. 4.

Consider a five order square matrix E with off-diagonal elements e_{ij} representing levels of interactions between the subsystems is known. Another matrix D, a diagonal matrix with diagonal elements representing five different subsystems is defined. The matrix C (VCM-MP) can be given as below:

$$C = [D - E] = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \text{subsystems} \\ S_1 & -e_{12} & 0 & -e_{14} & -e_{15} \\ -e_{21} & S_2 & 0 & -e_{24} & 0 \\ 0 & -e_{32} & S_3 & -e_{34} & -e_{35} \\ -e_{41} & -e_{42} & 0 & S_4 & -e_{45} \\ -e_{51} & 0 & -e_{53} & -e_{54} & S_5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$
(4)

The above matrix C permits us to represent complete information about all the five subsystems and interactions

amongst them of any industrially useful MEMS product. This information is useful for analysis, design, and development of new MEMS products at conceptual stage itself or for optimization purposes.

The determinant of the matrix C, is the variable characteristic MEMS product multinomial. It carries both positive and negative signs with some of its terms. The symbolic terms in the multinomial has complete information of the MEMS product system. The complete information in the MEMS product system will not be obtained as some will be lost due to the addition and subtraction of numerical values of the diagonal and off-diagonal elements. Thus the multinomial of the matrix, C in Eq. 4 does not provide complete information concerning the MEMS product system under certain conditions i.e. when numerical values of e_{ij} and S_i are substituted.

In order to avoid the loss of structural information during mathematical processing, another matrix MEMS product variable permanent matrix (VPM-MP) is proposed.

5.1.4 Variable permanent matrix (VPM-MP)

Let the permanent matrix of five-subsystem MEMS product be defined as

$$F = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \text{subsystems} \\ S_1 & e_{12} & e_{13} & e_{14} & e_{15} \\ e_{21} & S_2 & e_{23} & e_{24} & e_{25} \\ e_{31} & e_{32} & S_3 & e_{34} & e_{35} \\ e_{41} & e_{42} & e_{43} & S_4 & e_{45} \\ e_{51} & e_{52} & e_{53} & e_{54} & S_5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$
(5)

Above matrix is the most general matrix representation for a MEMS product modeled as five-subsystem MEMS product system. Thus the VPM-MP corresponds to the five variable subsystems as shown in Fig. 7 is given in Eq. 6.

$$F = \begin{bmatrix} S_1 & e_{12} & 0 & e_{14} & e_{15} \\ e_{21} & S_2 & 0 & e_{24} & 0 \\ 0 & e_{32} & S_3 & e_{34} & e_{35} \\ e_{41} & e_{42} & 0 & S_4 & e_{45} \\ e_{51} & 0 & e_{53} & e_{54} & S_5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$
(6)

In the matrix, the diagonal elements represent the contribution of the five subsystems and off-diagonal elements represent interdependencies of subsystems in producing a MEMS product. This model permits the representation of the contribution of each subsystem and interconnection quantitatively without any loss of information in multinomial representation as permanent function.

5.2 Permanent function representation for the MEMS product system

Both diagraph and matrix representations are not unique as these models change by changing the labeling of nodes. In order to develop a unique representation of MEMS products, a permanent function of the matrix VPM-MP is proposed. Permanent is a standard matrix function and is used in combinational mathematics (Jurkat and Ryser 1966; Marcus and Minc 1965). Procedure for deriving determinant and permanent from the respective matrices are identical except no negative signs appearing in permanent at any stage of its calculation. Thus, permanent function is a unique and complete structural representation of the MEMS product system with the added advantage of using numerical values of each term without any chance of loosing important information in the total numerical index.

The VPM for a MEMS product modeled as five subsystem MEMS product system can be derived from matrix representation shown in Eq. 6 is given in Eq. 7.

The permanent function for the matrix Eq. 6 corresponds to Fig. 7, has 28 terms. Because the variables e_{13} , e_{23} , e_{25} , e_{31} , e_{43} , and e_{52} values are 0 which means interactions are absent, so terms reduce from 120 (5!) to 28. The terms are arranged in six groups in standard manner. Second group is always absent as self group is not present.

 $per(F) = S_1 S_2 S_3 S_4 S_5 + (S_1 S_2 S_3 e_{45} e_{54} + S_1 S_2 S_4 e_{53} e_{35} + S_1 S_3 S_5 e_{42} e_{24} + S_3 S_4 S_5 e_{21} e_{12} + S_2 S_3 S_5 e_{41} e_{14} + S_2 S_3 S_4 e_{51} e_{15}) + (S_1 S_2 e_{53} e_{34} e_{45} + S_3 S_5 e_{21} e_{42} e_{14} + S_3 S_5 e_{41} e_{12} e_{24} + S_2 S_3 e_{41} e_{15} e_{54} + S_2 S_3 e_{51} e_{14} e_{45}) + (S_1 e_{32} e_{53} e_{24} e_{45} + S_4 e_{21} e_{22} e_{53} e_{24} e_{35} + S_3 e_{21} e_{12} e_{45} e_{54} + S_2 e_{41} e_{53} e_{24} e_{35} + S_3 e_{21} e_{42} e_{15} e_{54} + S_2 e_{41} e_{53} e_{15} e_{34} + S_3 e_{21} e_{42} e_{15} e_{54} + S_2 e_{41} e_{53} e_{14} e_{35} + S_4 e_{21} e_{32} e_{53} e_{15} + S_3 e_{21} e_{42} e_{15} e_{54} + S_2 e_{41} e_{53} e_{14} e_{35} + S_2 e_{41} e_{53} e_{15} e_{34} + S_3 e_{51} e_{12} e_{24} e_{45} + S_2 e_{41} e_{53} e_{14} e_{35} + S_2 e_{41} e_{53} e_{15} e_{34} + S_3 e_{51} e_{12} e_{24} e_{45} + S_3 e_{51} e_{42} e_{15} e_{24}) + (e_{21} e_{12} e_{53} e_{34} e_{45} + e_{21} e_{32} e_{53} e_{14} e_{45} + e_{21} e_{42} e_{53} e_{14} e_{35} + e_{21} e_{42} e_{53} e_{15} e_{34} + e_{41} e_{12} e_{53} e_{24} e_{35} + e_{41} e_{32} e_{53} e_{15} e_{24})$ (7)

The loops and dyads (interaction loop between two subsystems) in Eq. 7 are written in a more convenient way in Eq. 8. Here, in place of e_{ij} , e_{ji} the dyad (two subsystem loop) between subsystems S_i and S_j is represented as a loop L_{ij} . A loop between subsystems S_i , S_j and S_k i.e. $e_{ij}e_{jk}e_{kl}$ has been represented as L_{ijk} and the loops $e_{ij}e_{jk}e_{kl}e_{li}$, and $e_{ij}e_{jk}e_{kl}e_{lm}e_{mi}$ are represented by L_{ijkl} and L_{ijklm} respectively. Thus, Eq. 7 has been arranged and written as shown in Eq. 8.

$$per(F) = [S_{1}S_{2}S_{3}S_{4}S_{5}] + [S_{1}S_{2}S_{3}L_{45} + S_{1}S_{2}S_{4}L_{35} + S_{1}S_{3}S_{5}L_{24} + S_{3}S_{4}S_{5}L_{12} + S_{2}S_{3}S_{5}L_{14} + S_{2}S_{3}S_{4}L_{15}] + [S_{1}S_{2}L_{345} + S_{3}S_{5}L_{214} + S_{3}S_{5}L_{412} + S_{2}S_{3}L_{415} + S_{2}S_{3}e_{514}] + [\{S_{1}L_{24}L_{35} + S_{3}L_{12}L_{45} + S_{4}L_{12}L_{35} + S_{2}L_{14}L_{35} + S_{3}L_{15}L_{24}\} + \{S_{1}L_{3245} + S_{4}L_{2153} + S_{3}L_{2154} + S_{2}L_{4153} + S_{3}L_{5124}\}] + [\{L_{12}L_{534} + L_{35}L_{214} + L_{35}L_{412}\} + \{L_{21453} + L_{21534} + L_{41532}\}]$$

$$(8)$$

The above multinomial consists of distinct subsystems S_i , dyads L_{ij} and loops $e_{ij}e_{jk}...e_{mi}$. The complete permanent function has been written in a systematic manner for the unique representation. In short, it can be represented as:

$$per(F) = f(S_i, L_{ij}, L_{ijkl}, L_{ijkl}, L_{ijklm})$$

= f(Vertices, dyads, loops)
= f(structural components)

The multinomial Eq. 8 is the structural model of the MEMS product system, Fig. 7 consists of various structural components such as S_i , which represents the characteristic structural features of the *i*th unconnected subsystem. Similarly L_{ii} is interpreted as 2-subsystem structural dyad and L_{iik} is 3-subsystem interaction loop. Each term of the multinomial is considered as a set of different structural components. The terms S_1 , S_2 , S_3 , S_4 and S_5 are considered as a set of five $S'_i s$ and the term $S_1 S_2 S_3$ and L_{45} is read as a collection of three S'_{is} and one L_{ij} . The terms of the multinomial are expressed in (N + 1) groups with N = 5in the example; present an exhaustive way of analysis of a MEMS product at different levels. It helps in identifying different constituents, process parameters, design attributes, and the interaction among various subsystems of MEMS product system up to component level.

The terms in the permanent are grouped in to six groups as follows:

- Group 1: 1 term Group 2: 0 term Group 3: 6 terms Group 4: 5 terms Group 5: (5 + 5) = 10 terms Group 6: (3 + 3) = 6 terms
- (1) The first group consists of a single term representing a set of five subsystems singularly representing each subsystem and that is S_1 , S_2 , S_3 , S_4 and S_5 .
- (2) The second group terms if exist should have four singular subsystems and a subsystem dependent on itself (self loop). Such condition is non-existent in the MEMS product system that's why the second group is absent. The second group will appear in the

presence of self loops i.e. a subsystem connects itself.

- (3) The third group has six terms, each term is a set of three singular subsystems and a dyad (L_{ij}) .
- (4) The fourth group consists of five terms, which is a set of three subsystem interaction loops (L_{ijk}) and 2-subsystem characteristic structural features.
- (5) The fifth group has two subgroups. Each term of the first subgroup is a collection of two 2-subsystem interaction dyads $(L_{ij} \text{ and } L_{kl})$ and 1-subsystem characteristic structural features. Each term of the second subgroup is a collection of 4-subsystem interaction loop (L_{ijkl}) and 1-subsystem characteristic structural features.
- (6) The sixth group also has two subgroups. Each term of the first subgroup is a product of two 2-element MEMS product subsystem interaction loop (L_{ij}) and a product of two 3-element MEMS product subsystem interaction loop (L_{klm}) . Each term of the second subgroup consists of a 5-component MEMS product subsystem interaction loop (L_{ijlkm}) .

The diagonal elements (S_i) are obtained from the subsystem structure graphs. The above procedure analyses the system thoroughly from the perspective of its structure. Since the performance of the MEMS product is dependent on its structure, we can claim that its structural analysis and modeling is an indirect way of performance analysis. It is therefore possible for the designer as well as the manufacturer to carry out SWOT (strength-weakness-opportunities-threats) analysis of their MEMS product system and take strategic decisions to their advantage as per policy.

The diagonal elements of the matrix in Eq. 6 correspond to the five subsystems that constitute a MEMS product system. The values of these diagonal elements S_1 , S_2 , S_3 , S_4 and S_5 are calculated as

$$S_1 = per(FS_1); S_2 = per(FS_2); S_3 = per(FS_3); S_4 = per(FS_4); S_5 = per(FS_5)$$

where FS_1 , FS_2 , FS_3 , FS_4 and FS_5 are the variable permanent matrices (functions or function values) for five subsystems of the MEMS product system. The procedure for calculating S_1 , S_2 , S_3 , S_4 and S_5 is the same as for calculating *per*(*F*) of Eq. 6. For this purpose the subsystems of MEMS product system are considered, and the procedure given below is followed (Prabhakaran et al. 2006).

- (1) The schematic of these subsystems are drawn separately by considering their values subsystems.
- (2) Identify the degree of interactions, interconnections, dependencies, connectivity etc., between different sub-subsystems.

Digraph representations like Fig. 7 of five subsystems are drawn first separately to obtain their matrix equations

like Eq. 6. The permanent function of these variable permanent structure matrices will give the values of the corresponding S_i . The off-diagonal elements of the matrix give the interactions between systems. For getting exact degree of dependencies, connectivity, interactions etc., between subsystems or sub-subsystems, we may have to consider the views of experts from design, fabrication, packaging, material science, chemistry etc. Thus, the methodology can be applied in a bottom-up approach where in the analysis is proceeded from the lowest level to the total MEMS product system level and gives the complete structural evaluation of the MEMS product system as a single index.

5.3 Graphical representation of permanent function

The multinomial in Eq. 8 models the structure of MEMS product system completely. The representation can be related to set theory. The complete structure represents a full set; every term of the permanent therefore represents one subset of the full set and has a physical meaning. So every term represents a collection of subsystems of MEMS product system. Graphical representation of the terms of the permanent is given in Fig. 8. Terms of any group or subgroup represent all possible subsets of the given type shown in Fig. 8. Each structural subset can be used to develop tests for analyzing structure, design, performance, reliability, quality of the given MEMS product system. The analysis based on this will lead to better understanding and the development of high performance MEMS product system.

5.4 Generalization of the methodology

For a general MEMS product system with N subsystems, the MEMS product system characteristic and interdependence permanent matrix, G may be written as shown in Eq. 9.

$$G = \begin{bmatrix} 1 & 2 & 3 & \dots & N & \text{subsystems} \\ S_1 & e_{12} & e_{13} & \dots & e_{1N} \\ e_{21} & S_2 & e_{23} & \dots & e_{2N} \\ e_{31} & e_{32} & S_3 & \dots & e_{3N} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ e_{N1} & e_{N2} & e_{N3} & \dots & S_N \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ N \end{bmatrix}$$
(9)

For a general N subsystem with all the subsystems linked together, the total number of terms of the permanent function shall be equal to N!. Permanent for the above matrix per(E) can be written in sigma form as shown in Eq. 10.





$$per(G) = \prod_{a=1}^{N} S_a + \sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{ji})S_kS_lS_m \cdots S_N$$
$$+ \sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{jk}e_{ki} + e_{ik}e_{kj}e_{ji})S_lS_m \cdots S_N$$
$$+ \left[\sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{ji})(e_{kl}e_{lk})S_mS_n \cdots S_N$$
$$+ \sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{jk}e_{kl}e_{li} + e_{il}e_{lk}e_{kj}e_{ji})S_mS_n \cdots S_N\right]$$
$$+ \left[\sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{jk})(e_{kl}e_{lm}e_{mk} + e_{km}e_{ml}e_{lk})S_nS_o \cdots S_N$$
$$+ \sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{jk}e_{kl}e_{lm}e_{mi} + e_{im}e_{ml}e_{lk}e_{kj}e_{ji})S_nS_o \cdots S_N$$
$$+ \sum_i \sum_j \sum_k \cdots \sum_N (e_{ij}e_{jk}e_{kl}e_{lm}e_{mi} + e_{im}e_{ml}e_{lk}e_{kj}e_{ji})S_nS_o \cdots S_N$$
$$+ \ldots$$

This form of permanent multinomial has been derived from the fact that the terms in the permanent multinomial observe a regular pattern. It may be noted that a permanent function will contain N! terms only, provided if e'_{ij} sare not zero.

6 Structural identification and comparison of MEMS product system

We have represented the MEMS product as a system consisting of five subsystems, which affect property and performance of the final product. This five-subsystem product is represented/modeled as a multinomial, a permanent function. MEMS products are manufactured and used for different applications and will have a different number of terms in different groups and subgroups of their permanent function. By comparing their permanents, similarity and dissimilarity between different MEMS product system can be obtained. Using the proposed methodology, identification of a MEMS product and its comparison with other MEMS product is based on the analysis carried out with the help of VPF-MP. From subsystems and its interactions viewpoint, two MEMS products may be similar if their digraphs are isomorphic. Two MEMS products digraphs are isomorphic if they have identical VPF-MP. This shows that not only the terms are same but also the values are same. On this basis, MEMS product identification can be written as in Eq. 11:

$$\left[(J_1/J_2/J_3/J_4/J_{51}/J_{52}/J_{61}/J_{62}/\ldots) \right] \tag{11}$$

where J_i is the total number of terms in the *i*th grouping J_{ij} is the total number of terms in the *j*th subgroup of *i*th grouping. If there is no subgrouping, then J_{ij} is same as J_i . The subgroups are arranged in decreasing order of size i.e. based on number of elements in the loop.

A comparison is carried out on the basis of the coefficient of similarity. The coefficient is derived from the structure, i.e. VPF-MP and compares two MEMS products or a set of MEMS products on the basis of similarity and dissimilarity. If the number of distinct terms in the *j*th grouping of VPF-MP of two MEMS product system under consideration are denoted by J_{ij} and J'_{ij} , then three criteria are proposed as follows (Marcus and Minc 1965).

Criterion 1: the coefficient of dissimilarity D_{d-1} based on Criterion 1 is proposed as

$$C_{d-1} = \frac{1}{Y_1} \sum_{i} \sum_{j} \phi_{ij}$$

where $Y_1 = \max\left[\sum_{i} \sum_{j} J_{ij} \text{ and } \sum_{i} \sum_{j} J'_{ij}\right]$ (12)

When subgroupings are absent $J_{ij} = J_i$ and $J'_{ij} = J'_i$. When the subgrouping exists $\phi_{ij} = |J_{ij} - J'_{ij}|$, and when the subgroupings are absent $\phi_{ij} = |J_i - J'_i|$. Though the criterion 1 developed above present relatively simple method of quantifying the structural difference between the MEMS product system but this may cause loss of comparison information in the coefficient of dissimilarity. This is because ϕ_{ij} is difference of J_{ij} and J'_{ij} depending upon the structural difference in the MEMS product system under consideration. As a result, the subtraction operation may be involved and may cause limitation in the coefficient of similarity. To improve the differentiating power, criterion 2 is proposed.

Criterion 2: The coefficient of dissimilarity D_{d-2} based on Criterion 2 is proposed as

$$C_{d-2} = \left[\frac{1}{Y_2} \sum_i \sum_j \phi_{ij}^2\right]^{1/2}$$
where $Y_2 = \max\left[\sum_i \sum_j (J_{ij})^2 \text{ and } \sum_i \sum_j (J'_{ij})^2\right]$
(13)

When subgroupings are absent $J_{ij} = J_i$ and $J'_{ij} = J'_i$. When the subgrouping exists $\phi_{ij} = |J_{ij} - J'_{ij}|$, and when the subgroupings are absent $\phi_{ij} = |J_i - J'_i|$. To increase further the differencing power, criterion 3 is proposed.

Criterion 3: the coefficient of dissimilarity C_{d-3} based on Criterion 3 is proposed as

$$C_{d-3} = \frac{1}{Y_3} \sum_i \sum_j \phi'_{ij}$$

where $Y_3 = \max\left[\sum_i \sum_j (J_{ij})^2 \text{ and } \sum_i \sum_j (J'_{ij})^2\right]$ (14)

When subgroupings are absent $J_{ij} = J_i$ and $J'_{ij} = J'_i$. When the subgrouping exists $\phi'_{ij} = |J^2_{ij} - J'^2_{ij}|$, and when the subgroupings are absent $\phi'_{ij} = |J^2_i - J'^2_i|$. It is clear that ϕ'_{ii} is larger than ϕ_{ij} .

Using the above three equations, the coefficient of similarity is given as

$$C_{s-1} = 1 - C_{d-1};$$
 $C_{s-2} = 1 - C_{d-2};$ $C_{s-3} = 1 - C_{d-3}$
(15)

where C_{s-1} , C_{s-2} and C_{s-3} are the coefficients of similarity between two MEMS products under considerations based on Criterion 1, 2 and 3. It may be noted that the coefficient of similarity and dissimilarity lie in the range between 0 and 1.

6.1 Illustrative example

Two MEMS product systems can be compared using the coefficient of similarity/dissimilarity. Two high performance MEMS motion sensors form ColibrysTM (Colibrys 2008a) is studied for this purpose. ColibrysTM Motion sensors are ideal products for a wide range of applications in the domains of inertial and tilt/inclination sensing. The robust and low power design combined with an excellent bias stability guarantee the superior reliability of the MEMS motion sensor. The ColibrysTM MEMS motion sensor is a MEMS capacitive sensor, based upon a bulk micro-machined silicon element, a low power ASIC for signal conditioning, a micro-controller for storage of compensation values and a temperature sensor. The product is low power, calibrated, robust and stable and the electronic configuration provides a solid power on reset and a full protection against brown-out. Long-term stability of bias and scale factor are typically less than 0.1% of fullscale range. For the $\pm 2g$ version, typical bias temperature coefficient is 100 µg/°C and scale factor temperature coefficients 100 ppm/°C (Colibrys 2008a, b). Based on this study the design subsystem up to four levels is identified and is shown in Fig. 9. This helps the design experts to determine their roll in the complete MEMS product design.

6.1.1 MEMS product system 1

MEMS motion sensors are used for seismic sensing, vibration sensing, inertial sensing, tilt sensing etc. Let us consider the MEMS motion sensor FS300L from ColibrysTM designed for Seismic sensing (Colibrys 2008b). This can be used in earthquake detection, geophysics, homeland and border security, structural monitoring, strong motion and railway technology. For applications like earthquake detection the environment it is going to sense is so hazardous that the environment subsystem may affect the product. For such MEMS product the schematic developed in Fig. 6 is the appropriate representation to show the interaction between the subsystems. The structural identification of this MEMS product system can be given as:

$$J_1/J_2/J_3/J_4/J_{51}/J_{52}/J_{61}/J_{62}/ = 1/0/6/5/5/5/3/3/.$$

6.1.2 MEMS product system 2

Let us consider the MEMS motion sensor MS8000.D from ColibrysTM designed for inertial sensing (Colibrys 2008c).



Fig. 9 Subsystems of MEMS product system and sub-subsystems of design subsystem for MEMS motion sensors

This can be used in automobiles to release the airbag automatically incase of any accident. This MEMS product is not under hazardous environment and worry of the subsystem experts is less in product development. For such MEMS product the interaction between the material subsystem & the environment subsystem is negligible. The new permanent function is obtained after substituting the terms containing element e_{35} , $e_{53} = 0$. The structural identification of this MEMS product system can be given as:

$$J_1/J_2/J_3/J_4/J_{51}/J_{52}/J_{61}/J_{62}/ = 1/0/5/4/2/2/0/0/.$$

The values of the coefficient of similarity and dissimilarity based on structure by criterion 1, 2 and 3 are written below.

$$C_{d-1} = 0.5;$$
 $C_{d-2} = 0.54;$ $C_{d-3} = 0.6153$
 $C_{s-1} = 0.5;$ $C_{s-2} = 0.46;$ $C_{s-3} = 0.3847.$

The above result shows Criterion 3 has much larger value when compared with 2 and 1. When two systems are compared with same number of nodes and difference in edges leads to changes in the structural complexity. This structural complexity is directly reflected in the similarity/ dissimilarity coefficient calculated as shown in Eq. 10.

Since the coefficient lie between 0 and 1, if two systems are structurally similar they are isomorphic, their coefficient of similarity is 1 or dissimilarity is 0. Similarly, in case the two systems are completely dissimilar, their coefficient of similarity is 0 or dissimilarity is 1.

For comparison of two or more MEMS product system or a given family of systems, they are ranked based on the increasing or decreasing value of coefficient of similarity or dissimilarity. Using this, selection of the MEMS product system with desired structure is possible among different alternatives. Structural similarity/dissimilarity is an indirect measure of performance's similarity/dissimilarity. Because of this manipulation of structure is a way to develop high performance MEMS product system.

7 Usefulness to MEMS product industry

The proposed methodology is so versatile in nature and it helps the MEMS product industry to provide optimum system characteristics under different applications. The methodology is useful to analyze a MEMS product in the conceptual stage as large number of alternative solutions based on different designs of sub and sub-subsystems can be generated and evaluated without incurring any cost. The MEMS industry can select their own subsystems for the analysis of their specific product. The decisions at this stage have very large impact on the final product performance. The methodology can be used to select the optimum MEMS product based on available subsystem, sub-subsystem off-the-shelf from the global market. Since it selects the process, package type, equipment etc. according to user requirement, it is useful for the designer and manufacturer at conceptual stage, design stage and at failed stage.

The methodology also assists MEMS product industry to compare different products in terms of its characteristics and rate them for particular applications. The methodology may lead the research in new direction towards global projects of quantitative structure activity relationship (QSAR) and quantitative structure properties relationship (QSPR) (Liu et al. 2004; Katritzky et al. 1997). This procedure gives a comprehensive knowledge to the user, designer, manufacturer etc. about the MEMS product selection with right technology at right time and right cost from the market to the right environment.

8 Step-by-step procedure to develop and use graph theoretic structural model

The proposed methodology is written in the form of a stepby-step procedure and can be implemented by any existing MEMS industry in developing the graph theoretic model to have a comprehensive understanding of the MEMS product.

- Step 1: Consider the desired MEMS product system. Study the complete system and identify subsystem, sub-subsystem up to component level along with their interactions.
- Step 2: With the necessary assumptions, develop a hierarchical tree structure of the MEMS product system and interactions.
- Step 3: Develop a graph theoretic model of the total system with subsystems as nodes and edges for interaction between nodes (Fig. 7).
- Step 4: Develop the VCM-MP matrix and VCP-MP multinomial representations.
- Step 5: Develop the VPM-MP matrix and permanent functions of distinct subsystems and repeat steps from 2 to 4 for each subsystems.
- Step 6: Identify interconnections at different levels of hierarchy of the MEMS product system (i.e. systems, subsystems, sub-subsystems etc.) by grouping the terms of permanent functions.
- Step 7: Represent each term as per Fig. 8 and use it for analysis, evaluation, comparison and optimum selection.
- Step 8: Calculate the coefficients of similarity and coefficient of dissimilarity based on structure between different alternative MEMS product systems.

The above procedure is a flexible and capable of meeting the requirements of the industry.

9 Conclusions

To assist the MEMS industry, the following contributions are made:

• To understand the system better and to obtain quality products a MEMS product system consisting of subsystems and sub-subsystems is presented up to

component level in the form of hierarchical tree diagram.

- Mathematical models say graph theoretic model, matrix models and permanent models of MEMS product system are developed, which permits us to derive and exploit a number of results that are useful to designers and manufacturers of the system.
- It is brought out clearly that, how the terms of the permanent function can be represented as different subsets of MEMS product system and also can help us to generate and analyse large number of design solutions before selecting an optimum system.
- Structural identification set and the coefficients of similarity and dissimilarity are developed and are useful to select optimum set of subsystems up to component level to finally achieve high quality MEMS products in less cost and time by comparing their structures.
- Research is in progress to correlate the structure of the system with different performance parameters like reliability, quality, compatibility, cost etc.
- The proposed methodology is explained with an example to distinguish two structurally different MEMS product systems.
- In brief, the proposed structural graph theoretic methodology is comprehensive enough to deal with different structural and performance issues of MEMS product system at different levels of its life cycle.

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