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Analysis of self-heating phenomenon of piezoelectric microcomponents actuated harmonically

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Abstract This paper presents an analysis of the thermo-electro-mechanical behaviour of the piezoelectric microcomponents during their self-heating at harmonic oscilltions. The reasons of the self-heating effect are discussed. An iterational algorithm for solving the self-heating problem is suggested based on the thermo-visco-elastic model and the equations describing the piezoelectric effect. The thermal energy dissipated into piezoelectric microcomponents during their oscillations is obtained. It is shown that the characteristics of the piezoelectric microcomponents depend in great extent on the influence of the thermal fields of the self-heating. The thermo-electro-mechanical analysis is carried out by the finite element method (FEM).

1
Introduction

Piezoelectric microcomponents of various constructions are widely used as actuators in micro-electro-mechanical systems (Jendritza and Karthe, 1997). For example, they have been successfully applied as actuators in the form of monomorph piezoelements in micropositioning (Keoschkerjan et al., 2000) and microgripping systems (Salim et al., 1999). Their application areas have been further expanded and piezoactuators of bimorph construction were developed for different microfluidic components (Gravesen et al., 1993). They have been used as an actuating membrane in microfluidic components such as micropumps, microdosing systems and microvalves for generation of additional pressure in their pump chambers (Keoschkerjan and Dressler, 2000).

Depending on the application of the microsystems the piezoelectric microcomponents can be actuated statically or dynamically. As a result of the actuation at dynamic mode with high frequencies, self-heating of the piezoelectric microcomponents occurs due to the internal viscous friction. The characteristics of the microsystems depend to a great extent on internal and external thermal fields which are generated during their self-heating. Many different experiments have shown that the resonance frequencies and oscillation amplitudes of the piezoelectric elements are not stable in the initial stage of their actuation until the thermal steady-state is reached. The self-heating thermal fields lead to a change a of the electro-mechanical coefficients of the piezoelectric elements and as a result a change of their resonance frequencies and oscillation amplitudes takes place. This influence should be taken into account while constructing the whole microsystems.

This paper presents an approach for solving the coupled problem of the thermal, electrical and visco-elastic effects by taking into account the internal dissipative energy of the piezo microcomponents. The thermo-electro-mechanical analysis is carried out by the finite element method (FEM).

2
Thermo-electro-mechanical analysis

An approximation procedure for calculation of the construction elements based on the linear theory of the thermo-viscoelasticity has been proposed in (Iljusin and Rabotnow 1970). In particular for description of the thermoreological behaviour of the piezoelectric microcomponents the linear theory of the thermo-viscoelasticity (Iljusin and Pobedrja 1970) can be applied. According to this theory it is necessary to express the elastic, piezoelectric and dielectric constants of the piezomaterial in the mathematical complex form. For analysing of the self-heating effects in the simplest case, i.e. when the characteristics of the piezomaterial do not depend on the temperature, the following method of the theory of viscoelasticity can be used. First it is necessary to solve the electromechanical problem, then to determine the internal dissipative energy, and at the end to solve the energy equation with the known heat source. To analyse the thermo-electro-mechanical behaviour of the piezoelectric microcomponents in dynamic mode the dependence of the piezomaterial electro-mechanical characteristics on the temperature has to be taken into account. It is realised through the following iterative algorithm by adding an iterative step at the end of the

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above mentioned method, i.e. determining the piezoelectric characteristics and starting the calculations over again:

a. solving the problem of the mechanical vibrations: calculation of the natural modes, the frequencies and the forced harmonic oscillations
b. calculation of the thermal energy dissipated through mechanical vibrations into the piezomaterial
c. solving the problem of non-steady thermoconductivity with dissipative function as a source of thermal energy
d. correction of the electro-mechanical coefficients of the piezomaterial depending on the calculated thermal fields
e. determination of the piezoelectric microcomponents characteristics.

In order to solve the electro-mechanical problem, the equation of the piezomaterial can be written in the following form:

$$\begin{aligned} \sigma_x &= c_{11}^D \varepsilon_x + c_{12}^D \varepsilon_y - h_{31} D_z \\ \sigma_y &= c_{12}^D \varepsilon_x + c_{11}^D \varepsilon_y - h_{31} D_z \\ \tau_{xy} &= c_{33}^D \gamma_{xy} \\ D_z &= e_{31}^D \varepsilon_x + e_{31}^D \varepsilon_y + \varepsilon_{33}^S E_z \end{aligned} \tag{1}$$

where $c_{11}^D, c_{12}^D, c_{33}^D$ are the elastic constants (open circuit), ε_{33}^S is the dielectric constant, $h_{31} = e_{31}/\varepsilon_{33}^s$ with e_{31} is the piezoelectric charge constant, σ_x, σ_y, τ_{xy}, ε_x, ε_y, γ_{xy} are the mechanical stresses and strains, E_z and D_z are the strength and the induction of the applied electrical field.

The dissipative function of the energy for one cycle of vibrations of the piezoelement microcomponent is determined in the following form:

$$Q = \frac{\omega}{2} \int_t^{t+T} (\mathrm{Re}\sigma_x \, \mathrm{Re}\dot{\varepsilon}_x + \mathrm{Re}\sigma_y \, \mathrm{Re}\dot{\varepsilon}_y + \mathrm{Re}\tau_{xy} \, \mathrm{Re}\dot{\gamma}_{xy} + \mathrm{Re}E_z \, \mathrm{Re}\dot{D}_z) \mathrm{d}t \ , \tag{2}$$

where ω is the actuating frequency of the piezoelement, T is the period of the piezoelement oscillations. In (2) the real parts of the complex values of the mechanical stresses and strains, and the complex values of the strength and the induction of the electrical field are used. Using the basic equations of the piezomaterial (1), the following expression for the dissipative function is obtained:

$$Q = \frac{\omega}{2} [\underbrace{C_1(\varepsilon_{xo}^2 + \varepsilon_{yo}^2) + 2C_2 \varepsilon_{xo}\varepsilon_{yo} + c_{33}'' \gamma_{xyo}^2}_{\text{mechanical}} \underbrace{- 2e_{31} E_{zo}(\varepsilon_{xo} + \varepsilon_{yo})}_{\text{elec.-mech.}} - \underbrace{\varepsilon_{33}'' E_{zo}^2}_{\text{elec.}}] \ , \tag{3}$$

$$\begin{aligned} &\text{where } C_1 = c_{11}' - h_{31}' e_{31}'' - h_{31}'' e_{31}', \\ &\text{with } h_{31}' = \frac{e_{31}' \varepsilon_{33}' + e_{31}'' \varepsilon_{33}''}{(\varepsilon_{33}')^2 + (\varepsilon_{33}'')^2} \\ &C_2 = c_{12}' - h_{31}' e_{31}'' - h_{31}'' e_{31}', \\ &\text{with } h_{31}'' = \frac{e_{31}' \varepsilon_{33}'' - e_{31}'' \varepsilon_{33}'}{(\varepsilon_{33}')^2 + (\varepsilon_{33}'')^2} \end{aligned} \tag{4}$$

where ε_{xo}, ε_{yo}, γ_{xyo}, E_{zo} are the magnitudes of the complex values of the mechanical strains and the electrical field strength. In (3) and (4) complex elastic, piezoelectric and dielectric constants of the piezomaterial are used:

$$\begin{aligned} c_{ij}^D &= c_{ij}' + ic_{ij}'' = c_{ij}'(1 + i\delta_{ij}^D), \ \ ij = 11, \ 12, \ 33 \\ e_{31} &= e_{31}' + i\, e_{31}'' = e_{31}'(1 + i\delta_{31}^e) \\ \varepsilon_{33}^s &= \varepsilon_{33}' + i\varepsilon_{33}'' = \varepsilon_{33}'(1 + i\delta_{33}^S) \end{aligned} \tag{5}$$

where δ_{ij}^D, δ_{31}^e, δ_{33}^S are the loss factors of the piezomaterial. The analysis of (3) shows that the dissipative energy saved in the piezomaterial consists of three different energy parts coming from the separate influence of the mechanical and electrical fields and the combined influence of the both fields.

For description of the thermoconductivity effects in the piezomaterial during their self-heating, the general equation for non-steady processes (Segerlind, 1976) can be written in the following form:

$$k_x \frac{\partial^2 \Theta}{\partial x^2} + k_y \frac{\partial^2 \Theta}{\partial y^2} + k_z \frac{\partial^2 \Theta}{\partial z^2} + Q = \rho C \frac{\partial \Theta}{\partial t} \ , \tag{6}$$

where k_x, k_y, k_z are the thermoconductivity coefficients in the x, y, z directions; ρ is the density of the piezomaterial; C is the specific heat of the piezomaterial; Q is the heat source inside the piezomaterial (the dissipative function); and Θ is the temperature of the piezomaterial. There are two different boundary conditions for the (6):

The first boundary condition for a given temperature on the surface of the piezoelement is

$$\Theta = \Theta_B(S) \ . \tag{7}$$

the second boundary condition for the heat convection on the surface of the piezoelement described with the value $\alpha(\Theta - \Theta_\infty)$ is

$$k_x \frac{\partial \Theta}{\partial x} l_x + k_y \frac{\partial \Theta}{\partial y} l_y + k_z \frac{\partial \Theta}{\partial z} l_z + \alpha(\Theta - \Theta_\infty) = 0 \ , \tag{8}$$

where α is the coefficient of the heat convection between the piezoelement and the environment, Θ_∞ is the temperature of the environment and l_x, l_y, l_z are the cosinuses of the normals to the body surface. The initial condition for (6) is $\Theta = \Theta_0$ for $t = 0$. It could be assumed that: $\Theta_0 = \Theta_\infty$. Using the conventional procedure of the FEM, the Eq. (6) of the non-steady thermoconductivity with the above mentioned boundary conditions takes the following matrix form (Segerlind, 1976):

$$[C_\Theta]\{\dot{\Theta}\} + [K_\Theta]\{\Theta\} + \{F_\Theta\} = 0 \ , \tag{9}$$

where $[C_\Theta]$ and $[K_\Theta]$ are matrixes of the specific heat and the heat conductivity respectively; $\{F_\Theta\}$ is the vector of the thermal loads and $\{\Theta\}$ is the temperature vector. Equations (6) and (8) can be used also for solving the one- or two-dimensional problems with excluding the terms corresponding to the not-relevant coordinates. For definition of the $\{\Theta\}$ values for each time point, the linear differential equation (9) must be solved. For this solution the finitedifferences-method is used. The application of this method to (9) leads to the following system of linear equations:

$$\left(\frac{2}{\Delta t}[C_\Theta]+[K_\Theta]\right)\{\Theta\}_{i+1}=\left(\frac{2}{\Delta t}[C_\Theta]-[K_\Theta]\right)\{\Theta\}_{i+1} -2\{F_\Theta\}\,, \qquad (10)$$

where Δt is the time step. The solving of (10) gives the distribution of the temperature over the surface of the piezoelectric element.

3 Calculations

The algorithm described above was applied to calculate the thermal fields of the piezoelectric microcomponents and their influence on the main dynamic characteristics. The mostly used geometrical forms of the piezoelectric microcomponents, rectangular and ring, were considered and their harmonic oscillations were analysed. The overall dimensions of the rectangular and ring piezoelectric elements were $37.5 \times 15 \times 2$ mm^3 and $13.5 \times 6.5 \times 6$ mm^3 respectively. The piezoelements were actuated with a harmonical electrical signal of 100 V. The main characteristics of the piezomaterial PZT-5 at $\Theta_0 = 20$ °C are: $C_{11}^D = 1.39 \times 10^{11}$ N/m^2, $C_{12}^D = 0.778 \times 10^{11}$ N/m^2, $C_{33}^D = 1.15 \times 10^{11}$ N/m^2, $e_{31} = -5.2$ C/m^2, $h_{31} = -9.2 \times 10^8$ V/m, $\varepsilon_{33}^e = 5.62 \times 10^{-9}$ F/m, $\rho = 7500$ kg/m^3, $c = 420$ J/(kg · deg), $k_x = k_y = 1.25$ W/(m · deg) (Berlincourt et al., 1966). Some temperature dependencies of the stiffnesses, the piezoelectric charge constant and the dielectric constant are given in (Berlincourt et al., 1966). The case of conductive heat transfer from the upper and lower surfaces of the piezoelement with $\alpha = 15$ W/(m^2 · deg) was considered. The initial temperature is assumed equal to the environment temperature, i.e. 20 °C. Figure 1 shows the dependences of the steady-state maximal temperature of the self-heating of the rectangular piezoelement and the saturation time (the time for which the temperature reaches the steady-state) as a function of the frequency. The maximal temperature and the saturation time are increasing functions of the frequency provided the dissipative energy increases with increasing the frequency as shown in Fig. 2.

The analysis of the temperature fields of the piezoelement self-heating allows to define their influence on the main dynamic characteristics. The analysis of the piezoelement eigen vibrations in the temperature range from 20 to 120 °C shows that the vibration modes (Figs. 3 and 4) are not sensitive to the thermal fields, while their frequencies are temperature dependent. The frequencies are increasing due to the increasing of the elastic coefficients C_{ij} in the same temperature range (Berlincourt et al., 1966). Theoretical determination of the amplitude-frequency response was carried out by actuating the piezoelements with a homogeneous harmonic electric field. The amplitude–frequency responses for the point K (see Figs. 3 and 4) of the rectangular and ring piezoelements are shown in Figs. 5 and 6 correspondingly. The vibration amplitude of the rectangular piezoelement is represented with a longitudinal A_v and a transversal A_u component. The vibration amplitude of the ring

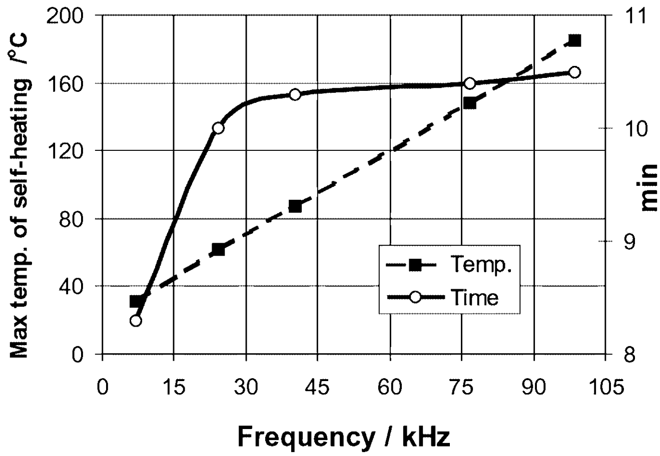

Fig. 1. Steady-state maximal temperature of the self-heating rectangular piezoelement and the saturation time versus operating frequency

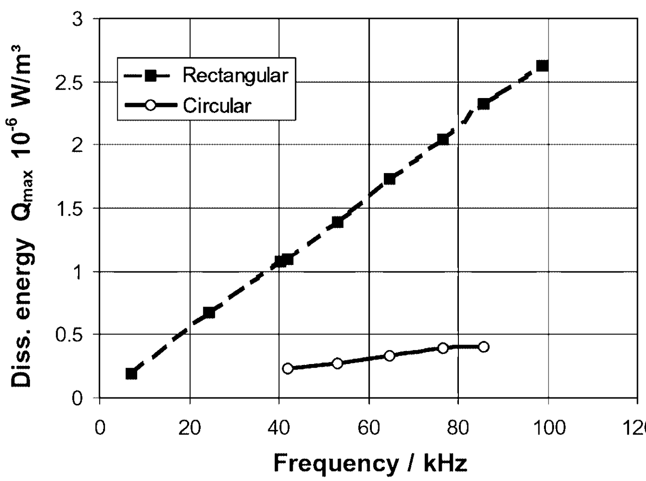

Fig. 2. Maximal values of the dissipative energy versus frequency

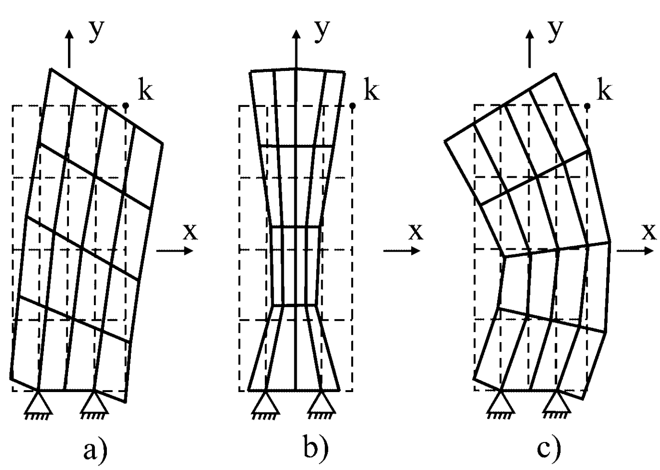

Fig. 3a–c. Modes of the rectangular piezoelement eigen vibrations at the first three frequencies: **a** $f_1^\circ = 7.1$ kHz, $f_1^\Theta = 7.3$ kHz; **b** $f_2^\circ = 24.3$ kHz, $f_2^\Theta = 25$ kHz; **c** $f_3^\circ = 40.3$ kHz, $f_1^\Theta = 41.4$ kHz

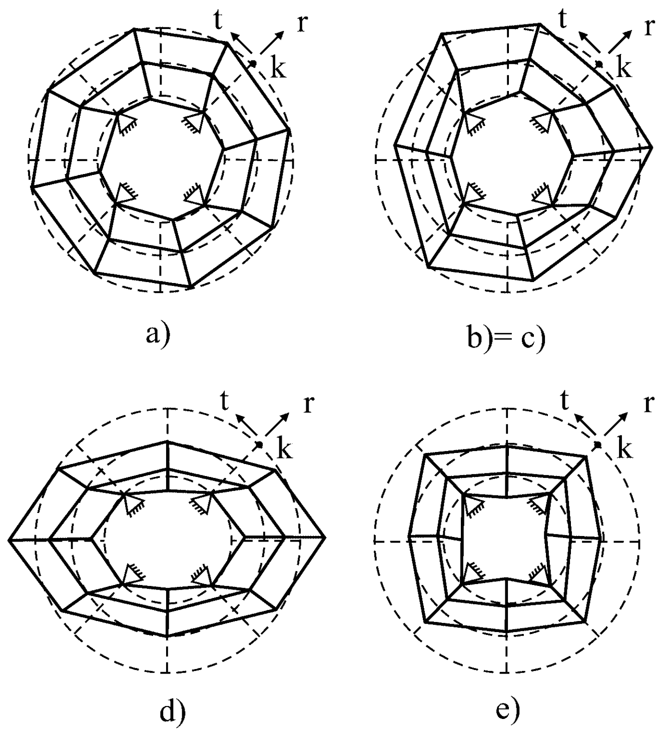

Fig. 4a–e. Modes of the ring piezoelement eigen vibrations at the first five frequencies: **a** $f_1^{\circ} = 66.9$ kHz, $f_1^{\Theta} = 71.2$ kHz; **b** = **c** $f_2^{\circ} = f_3^{\circ} = 85$ kHz, $f_2^{\Theta} = f_3^{\Theta} = 88$ kHz; **d** $f_4^{\circ} = 103$ kHz, $f_4^{\Theta} = 106$ kHz, **e** $f_5^{\circ} = 136$ kHz, $f_4^{\Theta} = 139$ kHz

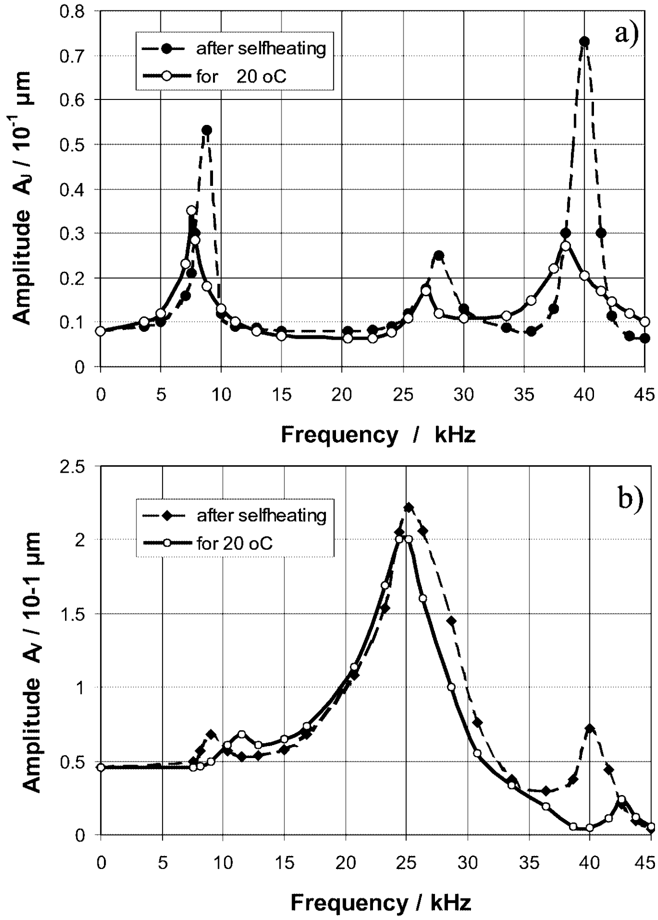

Fig. 5a, b. Theoretically calculated amplitude–frequency response of the rectangular piezoelement at 20 °C and after self-heating: **a** transversal component; **b** longitudinal component

piezoelement is represented with a radial A_r and a tangential A_t component. From the amplitude-frequency responses it is possible to conclude that as a result of the thermal field influence in the temperature range from 20 to 120 °C, an increase of the amplitude components takes place at these frequencies which modes of forced vibrations are close to the natural (eigen) modes. This means that the homogenous thermal field amplifies the homogenous electrical field and consequently the mechanical field. This phenomenon can be explained by an increase of the piezoelectric constant e_{31} relatively to its value at 20 °C in the above mentioned temperature range and by a decrease of the piezoelement damping. The increase of the vibration amplitude of the rectangular piezoelement occures due to an increase of the longitudinal component A_v (Fig. 5b) which can be explained by an increase of the longitudinal mode of vibrations (Fig. 3b) and a decrease of the bending modes (Fig. 3a, c).

For the ring piezoelement a decrease of the tangential component A_t (Fig. 6b) takes place in the whole range of the actuating frequencies. An increase of the radial component A_r is observed at the first three resonance frequencies (Fig. 6a) which corresponds to the tangential modes (Fig. 4 a, b, c). A decrease of the radial component A_r (Fig. 6a) at the fourth tangential–radial mode (Fig. 4d) and at the fifth radial mode (Fig. 4e) can be explained by amplification of the radial vibration mode and weakening of the tangential mode. The increase of the radial component A_r at the fifth resonance frequency (Fig. 6a) can be explained by an increase of the piezoelectric constant e_{31} in the temperature range from 90 to 120 °C (Berlincourt et al., 1966) where the saturation temperature of the piezoelement self-heating is located.

4
Comparison to experimental results

The results of the suggested theoretical method and calculating algorithm were compared with the results of experimental measurements carried out for the rectangular and ring piezoelectric elements with the same dimensions. The rectangular element was clamped at the smaller end side and the ring element – at the inner circle surface. The main electro-mechanical characteristics of the piezomaterial were defined as: $C_{11}^D = 4.6 \times 10^{10}$ N/m^2, $C_{12}^D = 2.6 \times 10^{10}$ N/m^2, $C_{33}^D = 1.0 \times 10^{10}$ N/m^2, $e_{31} = -15.1$ C/m^2, $h_{31} = -9.2 \times 10^8$ V/m, $\varepsilon_{33}^e = 5.62 \times 10^{-9}$ F/m, $\rho = 7500$ kg/m^3, $c = 420$ J/(kg · deg), $k_x = k_y = 1.25$ W/(m · deg). The case of conductive heat transfer from the upper and lower surfaces of the piezoelements was considered. The temperature dependencies of the stiffnesses, the piezoelectric charge constant and the dielectric constant were taken from (Berlincourt et al., 1966). The rectangular and ring piezoelectric elements were actuated with a harmonic signal of 100 V in the frequency range 5–40 kHz and 50–190 kHz respectively.

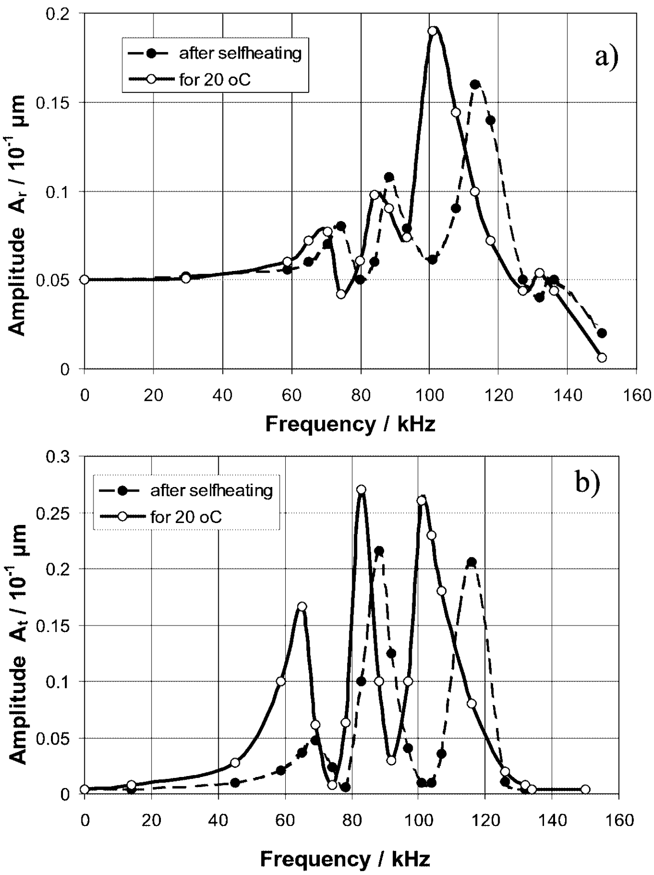

Fig. 6a, b. Theoretically calculated amplitude–frequency response of the ring piezoelement at 20 °C and after self-heating: **a** radial component; **b** tangential component

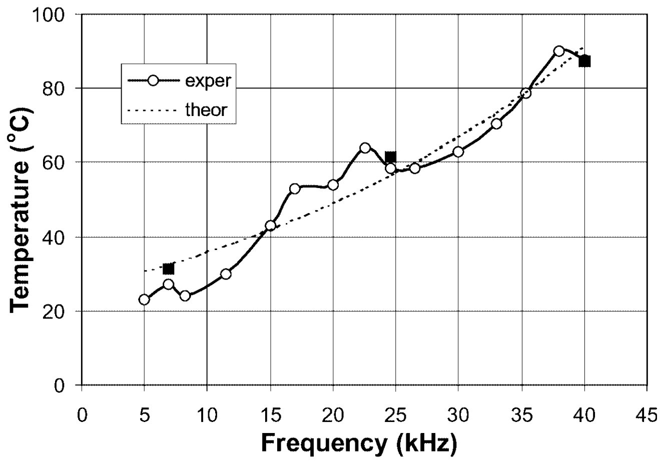

Fig. 7. Measured temperature–frequency characteristic of the rectangular piezoelectric element

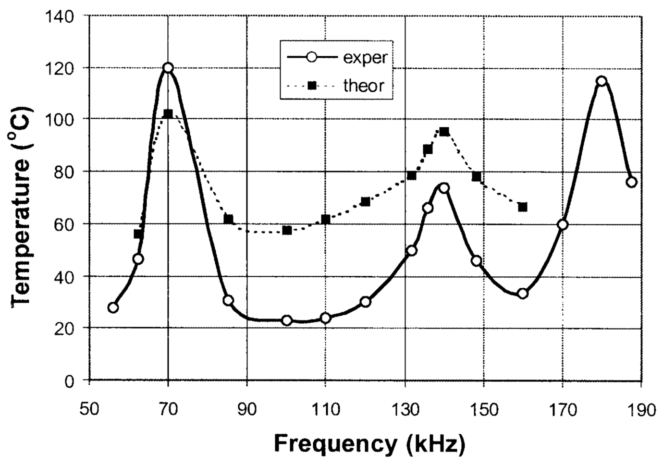

Fig. 8. Measured temperature–frequency characteristic of the ring piezoelectric element

The temperature of the self-heating was measured with a special temperature sensor based on a semiconductor microelement with temperature dependent characteristics which was mounted on the upper surface of the piezoelectric element. The sensor microdimensions (1.4×1.2 mm^2) provided negligible disturbance of the piezoelectric element oscillations. The steady-state maximal temperature of the self-heating and the saturation time were measured. The Figs. 7 and 8 present the experimental and theoretically calculated temperatures as a function of frequency for the rectangular and ring piezoelectric elements respectively. The temperature-frequency dependence for both piezoelements has maxima at frequencies close to the resonance frequencies. In the case of the ring piezoelectric element these maxima are much more pronounced which reflects the similar behaviour of the amplitude–frequency response (Fig. 6b) characterized by the lower damping. In the case of the rectangular piezoelectric element a very good agreement between the experimental and theoretical values is seen in the whole frequency range. For the ring piezoelectric element the agreement has a qualitative character: the measured and calculated temperatures are in the same order of magnitude.

Various more or less simplified models are used in practice to describe such complex viscoelastic processes. In general, it is practically impossible to find materials which behaviour can be completely described by only one theoretical model. For example, the Maxwell model is recommended for plastic materials, resins and even some metals, while the Kelvin-Voigt model describes better the energy dissipations during elastic oscillations (Iljusin and Rabotnow, 1970). Even so in most of the cases the theoretical models give only qualitative prediction of the experimental temperature-frequency dependencies. For this reason we consider that the suggested thermo-electromechanical model and the iterational algorithm describe well the self-heating phenomenon in the piezoelectric microcomponents.

5 Conclusions

The analysis presented using the above mentioned thermo-electro-mechanical model is in a good agreement with the physics of the thermo-viscous-elastic effects in the piezoelectric microcomponents during their harmonic oscillations. The results show that the components of the vibration amplitudes change in such a way that the deformable state of the piezoelement tries to attain the initial homogeneous geometrical form. The comparison between the experimental investigations and the numerical modelling confirms that the suggested theoretical algorithm describes satisfactory well the self-heating phenomenon in the piezoelectric microcomponents. It is shown that the temperature deformations in the piezoelectrical microcomponents can reach the order of magnitude of the deformations coming from the electrical actuation. Therefore, the electrical control of the piezoelectric microcomponents should be optimised so that the influence of the temperature fields on the dynamical characteristics is compensated.

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