ORIGINAL PAPER

S. Straub Predictability of long runout landslide motion: implications from granular flow mechanics

Received: 6 July 1996 / Accepted: 25 November 1996

Abstract Quality of landslide motion prediction is directly linked to the understanding of the basic flow mechanisms. Although it is known that landslides are granular mass flows and granular flow mechanics is an established area of research, hypotheses on landslide motion are still based on simple geometrical relations and heuristic assumptions. New insights into the development of flow properties of high-speed, high-concentration granular flows are given by results of discrete particle simulations: rapid granular flows are self-organizing dynamic systems that are forced to develop a plastic body rheology. This behaviour must be described by a coefficient of internal friction μ_{CM} that refers to the center of mass of a flow. Coefficients of rapid granular flows of inelastic and rough particles, which are typical for common rock materials, do not vary significantly around $\mu_{CM} \approx 0.45$ that is definitively smaller than the friction coefficient of soil creep (≈ 0.6) . The motion of the center of mass is superimposed by the spreading of the granular mass that is controlled by the same plastic body rheology. This combined motion is a scale-invariant self-similar process that depends only on the drop height of a landslide and its volume. This allows specification of implications that must be given special attention in the development of future models for landslide prediction.

Key words Long runout landslide · Debris flow · Mass movement · Prediction · Geological hazard · Granular material · Rapid granular flow · Quasi-static regime · Grain-inertia regime · Self-organization · Attractor · Dynamic system · Self-similarity · Scale invariance

S. Straub

Freie Universität Berlin, Institut für Geologie, Geophysik und Geoinformatik, Malteserstr. 74*—*100,

D-12249 Berlin, Germany

E-mail: straub@zedat.fu-berlin.de

Introduction

Long runout landslides are disastrous events that can cause considerable damage. Triggered by seismic or volcanic activity, a large mass of rock up to several cubic kilometres is released from a mountain side, accelerates down into the valley and moves along a more or less flat ground until it comes to rest. It exhibits a mode of motion that can be characterized as coherent flowage of loose granular material of various grain size. This kind of debris stream has been termed ''sturzstrom" (Heim 1932; Hsü 1975). A long runout landslide has a very long horizontal runout path L_{max} , measured from the upper point of release to the farthest flow front. Compared with this the effective vertical drop height *Hmax* is relatively small (Fig. 1). This principal behaviour is independent of the initial failure process and the type of rock material. The only compelling condition is that a minimum flow volume has to be exceeded (Heim 1932). Below this minimum volume the mode of motion is that of a steep-slope short-distance rockfall: a number of single blocks bounce, roll and slide independently downslope and deposit on a debris fan below.

It was Heim (1932) who first applied a simple model of a rigid block sliding with a coulomb-type friction to describe landslide motion. According to the geometrical relations of his model he defined an apparent coefficient of friction μ_{app} as ratio of the maximum vertical drop height *Hmax* to the maximum horizontal runout distance L_{max} :

$$
\mu_{app} = H_{max}/L_{max} \tag{1}
$$

 μ_{app} can also be determined by connecting the uppermost edge of the initial position with the farthest flow front. The inclination of this "energy line" is the apparent angle of friction α_{app} and its tangents is the apparent coefficient of friction μ_{app} . The inclination angle α_{app} and the coefficient μ_{app} are denoted as "apparent" because

Fig. 1 Geometrical relations of a landslide flow path as defined by Heim (1932)

they refer to the idealized sliding motion of a single block and not to the actual flow of granular material.

Moderate to extremely low μ_{app} down to 0.06 have been reported from landslides on Earth (Heim 1932; Harrison and Falcon 1938; Crandell and Fahnestock 1964; Shreve 1966, 1968b; Johnson 1978; Voight and Pariseau 1978; Voight et al. 1981, 1983; Siebert 1984), moon (Guest 1971; Howard 1973) and Mars (Lucchitta 1979; McEwen 1989). This is associated with a clearly negative correlation between the apparent coefficient of friction μ_{app} and an increasing flow volume. The larger a landslide is the relatively longer is the horizontal runout distance L_{max} and, thus, the lower is the apparent coefficient of friction (Heim 1932; Scheidegger 1973; Hsü 1978; Davies 1982). This correlation seems to follow a simple logarithmic function (see Fig. 2; Scheidegger 1973).

Hypotheses on the mechanics of landslide motion

There have been many hypotheses proposed to explain flow behaviour of landslides. Most of them primarily center on the reduction of friction between the flow and the ground to explain the low apparent coefficients of friction. Shreve (1966, 1968a, 1968b) suggests an ''air layer'' beneath the flow, whereas Kent (1966) proposes fluidization by air passing upward through the slide body. A modification of this is presented by Krumdieck (1984) who infers the existence of an additional aerodynamic lift from the aerodynamic profile of the moving slide. Erismann et al. (1977) and Erismann (1979, 1986) present an example where molten rock produced by frictional heat may have played the role of a lubricant layer. Melosh (1979, 1983, 1986) assumes that low acoustic frequencies can break the frictional contact between particles. Inspired by the kinematic descriptions of Heim (1882, 1932) and based on granular flow theory by Bagnold (1954) Hsu¨ (1975, 1978, 1989) deduces that dispersive pressure forces, as an effect of high shear rates, supported by interstitial dust, can cause a self-fluidization of the granular mass. Davies (1982),

Fig. 2 Apparent coefficient of friction μ_{app} related to landslide volume. Data from Plafker and Ericksen (1978), Voight and Pariseau (1978), Ui (1983), Ui et al. (1986), Siebert (1984), Siebert et al. (1987), Shaller (1991). The dashed lines are explained in the text

extending the ideas of Hsu, proposes dilation of the flow as the main factor to reduce internal friction. Dent (1986) and Campbell (1989a) assume a layer of highly agitating particles beneath the densely packed main body where mechanical fluidization reduces the frictional forces between the slide and the ground.

None of these controversial hypotheses have been universally accepted and only few have been accompanied by detailed computations. Therefore, most of them have only restricted value for the prediction of landslide motion. Only the mechanical fluidization approach has been followed and extended successfully. This approach corresponds very well with analyses performed to examine the physical background of the fluid-like flow of granular materials in industrial processes (Ridgeway and Rupp 1970).

Granular flow mechanics

The behaviour of granular material depends on the acting deformation rate. Undergoing slow deformation it is dominated by long-termed coulomb-type frictional contacts between the particles and short-time contact failure. This ''quasi-static'' regime of granular flow has been intensively studied in soil mechanics.

Highly sheared high-concentration granular flows, the other extreme, are governed by collisional particle interactions. Stresses, produced by relative shear between adjacent layers of different velocities, are conducted by particle momentum, either by momentum transfer during particle collisions or transported by particles changing to another layer of a flow. Collisions are the fastest way of stress conduction. Once a particle is hit the transmitted momentum is immediately

available at the opposite side. This transmission of stress is many times faster than the transmission via an interstitial fluid, such as water or air. Therefore, interstitial fluids play a negligible role in this ''rapid flow'' or "grain-inertia" regime.

Strong similarities on a molecular scale between the rapid granular flow regime and the flow of gases allow to derive theories based on the kinetic theory of gases to characterize rapid granular flow mechanics (Savage and Jeffrey 1981; Jenkins and Savage 1983; Lun et al. 1984). Computer modelling techniques originally developed for the investigation of discrete molecular particle flow (Metropolis et al. 1953; Alder and Wainwright 1960) now serve as tools for granular flow investigation (Campbell 1982, 1986, 1989b, 1990; Campbell and Brennen 1985a, 1985b; Campbell and Gong 1986; Hopkins and Shen 1986; Walton and Braun 1986a, 1986b; Zhang and Campbell 1992). Together with experimental results (Bagnold 1954; Savage 1979; Savage and Sayed 1984; Johnson and Jackson 1987; Ahn et al. 1988; Sanders et al. 1988; Hutter and Koch 1991) this contributes to an understanding of the constitutive behaviour of the rapid granular flow regime.

Developing a model that considers the quasi-static as well as the rapid flow regime has its difficulties because there is no comprehensive theory that covers both. A model by Hutter et al. (1986a, 1986b) and Savage and Hutter (1989) assumes a shallow flow of an incompressible continuous mass of uniform density with a mohrcoulomb plastic behaviour and a mohr-coulomb-type basal friction. The internal angle of friction ϕ and the bed friction angle δ as measures for the internal frictional dissipation and the dissipation between flow and bed are supposed to be constant, with $\delta < \phi$. The shallowness assumption (Hutter et al. 1986a) is based on the observation that thin sheet flows seem to move as a plug on a basal shear zone. This allows use of depth-averaged equations for mass and momentum balances. Although the Hutter*—*Savage model is a simplified view of landslide motion, the comparison between its theoretical prediction and laboratory experiments shows good agreement (Hutter et al. 1986b; Savage and Hutter 1989; Hutter and Koch 1991).

Norem et al. (1987) combine a non-newtonian viscous behaviour for the basal shear zone with a plastic behaviour for the quasi-static regime of the plug in a dense snow avalanche. This model as well as the Hutter*—*Savage model does not take into account how stresses in the rapid granular flow regime of the basal shear zone are generated. This is considered by Savage (1993) who combines the plastic behaviour of the plug with the statistical approach developed by Jenkins and Savage (1983) for the rapid granular flow regime. A similar solution is proposed by Johnson and Jackson (1987) and Johnson et al. (1990) who realized a model by integrating the kinetic theory of rapid granular flow by Lun et al. (1984) instead.

It is possible to develop discrete particle simulations which model the collisional interactions between individual particles. Large-scale landslide simulations of this kind of up to one million individual particles have been performed by Campbell et al. (1995), but it took a year of cpu time on a supercomputer for their longest simulation run. Due to this computational expense, only few simulations were performed.

One way to keep the number of particles low and still simulate more closely the properties of a macroscopic system is to use periodic boundary conditions. The examination of flow behaviour is restricted to a statistically representative flow section bounded upstream and downstream by periodic boundaries (Fig. 3). When a particle leaves the simulation cell through one of the periodic boundaries it instantaneously reenters the cell from the other side without change in direction, velocity and height over ground. This approach is known as the molecular dynamics technique (Metropolis et al. 1953; Alder and Wainwright 1960), first introduced by Cundall and Strack (1979) to solve problems in soil mechanics, and by Campbell (1982) to model rapid granular flows. Since then the molecular dynamics technique has been used widely to investigate mechanics of granular flows under various conditions (Campbell 1982, 1986, 1989b, 1990; Campbell and Brennen 1985a, 1985b; Campbell and Gong 1986; Hopkins and Shen 1986; Walton and Braun 1986a, 1986b; Zhang and Campbell 1992). The computer model presented herein has been developed to investigate geological problems that arise from the interpretation of highspeed, high-concentration flows (Straub 1994).

In the presented simulations a constant number of particles move under the force of gravity on their ballistic trajectories repeatedly leaving, reentering and crossing the simulation cell from left to right. Kinetic energy is entered only once by an initial velocity in flow direction and is dissipated by inelastic collisions until the particles come to rest. The particles are rigid spherical

Fig. 3 The simulation cell. *Small black arrows* depict individual velocity vectors. The *black particle* is an example of a particle leaving the simulation cell to one side and simultaneously reentering the cell from the other side

bodies interacting in instantaneous collisions. This ''instantaneous collision'' assumption allows to exclude the possibility of complex collisional events where three or more particles are involved. A function of restitution for the elastic properties and a coulomb-type coefficient of friction for the surface roughness are the input parameters to model dissipative particle collisions. Figure 4 shows a plot of a typical function of restitution ε that is defined as the ratio of the impact to the rebound momentums of the colliding particles. It depends on the relative particle velocity v_r normal to the particle surfa ces at the point of contact (see Fig. 5). The idealized function is defined by two points: one point represents full elasticity at zero relative velocity and the second point is defined by the input parameters of minimum restitution e *input* at a specific relative velocity *v input*. Between the two points a linear relation between restitution and relative velocity is assumed. Above *v input* restitution is kept on the constant value of e *input*. The result of $\varepsilon(v_r)$ is used to calculate the normal velocities after the event. Rubbing contacts are governed by a coulomb-type friction, i.e. the tangential force between the surfaces is limited by the normal force times a coefficient of friction μ_{sf} . Consequently, in addition to the coefficient of friction μ_{sf} , surface friction depends directly on the function of restitution ε and the treatment of the acting normal forces. Because of the instantaneous collision assumption, this coefficient of friction also includes assumptions on the duration of the rubbing contact.

Figure 6 shows a snapshot of a typical situation during a simulation run. Each particle of this mechanical system is followed exactly and therefore its location and momentum can be obtained at any time to charac-

Fig. 4 The function of restitution. ε_{input} and v_{input} are input parameters of the simulation and define the shape of the function

Fig. 5 Local reference system of a binary collision. The velocity vectors v_A and v_B of the particles can be divided into the velocities v_{NA} and v_{NB} in normal and v_{TA} and v_{TB} in tangential direction. The relative velocity in normal direction is defined by $v_r = | v_{NA} - v_{NB} |$. v_{TA} and v_{TB} in combination with the angular velocities ω_A and ω_B result in a relative motion in tangential direction

terize the state of the system. By dividing the simulation cell into horizontal strips and time averaging over the particles inside a strip vertical profiles of the flow properties are determined. The profiles plotted in Fig. 6 are the solid fraction, the flow velocity and the granular temperature. The solid fraction is the relative volume fraction of particles in a strip. It shows a low-density area at the base of the flow, an increasing density upward and a slight decrease near the open surface. This corresponds to the velocity profile that shows its lowest values near the base and an increasing upward to the main body of the flow. The granular temperature (Ogawa 1978) describes the particle motions relative to the mean flow velocity. Every collision results in particle motions that deviate from the mean flow field, more or less perpendicular to the mean motion. Its magnitude is proportional to the square of the local velocity gradient. The highest granular temperature is produced below the main body of the flow where the velocity gradient between ground and flow has its maximum. It decreases upwards into the main body where the velocity gradient gets lower. Granular temperature that is conducted upwards is quickly damped by inelastic collisions. This shows the fundamental difference between thermal temperature and granular temperature: molecular collisions are elastic and non-dissipative, whereas particle collisions are inelastic and dissipative. Therefore, granular temperature has to be continuously generated by shear work or it would vanish rapidly by inelastic damping.

As a result of the particle fluctuations pressure develops that counteracts the load of the mass acting on the shear zone. By lifting the main body of the flow this "granular pressure" (Bagnold 1954) reduces the solid

Fig. 6 A simulation run. The *strip* between the *thin vertical lines* is the simulation cell. »*ertical strips* left and right of the cell are copies of the situation in the simulation cell. The vertical profiles depict the distribution of the solid fraction, the flow velocity and the granular temperature

 0.7

 $\mathbf 0$

4.0

 $\overline{0}$

fraction within the region of shear. With progress of the simulation generation of granular temperature consumes kinetic energy and the flow slows down. The lower shear rate between flow and ground results in a lower granular temperature. To equal the load of the main body by dispersive pressure from a lower granular temperature the thickness of the basal shear zone has to decrease until the balance between normal load and granular temperature is reached.

20

 10

 $\overline{0}$

 \bigcirc Ω

 $\mathbf 0$

 $t = 0.5115 s$

particle diameters Profile height

Self-organization

The sliding block model by Heim (1932) is a very simple model to characterize the geometry of the flow path of a landslide. Although physically motivated, the assumption to construct the energy line by connecting the outermost ends of the flow path is physically incorrect. A landslide is a many particle system and a physical description of such a system has to refer to its center of mass. Figure 7 depicts the correct model for landslide motion with the drop height $\langle H \rangle$ and the travel distance $\langle L \rangle$ of the center of mass.

Applying the energy line concept to the centre of mass of a granular flow simulation uncovers a linear relation between the travel distance $\langle L \rangle$ and the kinetic energy of the flow (Fig. 8). It has been demonstrated that every high-concentration flow in the rapid flow regime is forced to behave this way (Straub 1996). The inclination of the energy line depends only on the material properties that control the collisional energy dissipation. For another flow of the same material but of different thickness, different grain size, different drop height or different flow path, the inclination of the energy line is almost identical. Key to this bulk behaviour is a kind of synchronization of the particle motions. Every collision is a transmission of a portion of a particle's momentum to the surrounding flow, and

Fig. 7 The correct energy line concept refers to the centre of mass of a landslide (compare Fig. 1)

vice versa. Therefore, every particle has influence on the flow development in its neighbourhood and, simultaneously, its motion is controlled by the surrounding flow. This feedback results in an organized motion of the bulk flow. In dynamic systems theory this is called self-organization. Self-organization forces the flow into a state of minimum energy dissipation and the corresponding dissipation rate is responsible for the constant slope of its energy line. Even an external disturbance that changes the energy dissipation rate for a moment has no permanent effect. The system immediately returns to this equilibrium state of minimum dissipation, the attractor of the dynamic system. Straub (1996) named this the "rapid granular flow attractor''.

Increasing the number of particles in a simulation run stabilizes the attractor and makes it insensitive to external disturbance because it reduces the influence of single-particle fluctuations. Unfortunately, the increase in particle number simultaneously increases the necessary computation time.

The existence of an attractor is of eminent significance for the prediction of landslide motion. It gives a new meaning to the energy line concept that was not

 0.6

Granular

 (m^2/s^2)

Fig. 8 Energy line of the simulation in Fig. 6. The representing point of the situation shown in Fig. 6 is marked by an *arrow*

more than a geometrical description of landslide motion before.

Rheology

Self-organization in the rapid granular flow regime results in an almost straight energy line. Its constant slope is the representation of a constant coefficient of internal friction μ_{CM} . Regarding the stresses that act in a simple shear flow this coefficient of internal friction μ_{CM} can be expressed as ratio of shear stress τ_{xy} to normal stress τ_{yy} .

$$
\mu_{CM} = \tau_{xy}/\tau_{yy}.\tag{2}
$$

Normal stress arises from the load acting on the plane of shear. Because of the constant number of particles in a simulation the load acting on a shear plane stays constant. From a constant load, a constant coefficient of friction μ_{CM} and Eq. (2) follows a constant shear stress τ_{xy} during the whole simulation. This constant shear stress is obviously independent of the acting shear rate. Such behaviour corresponds to a plastic body rheology.

Figure 9 depicts the linear shear stress against shear rate relation in the rapid granular flow regime. In slow flows of granular material and a low viscous interstitial fluid it is bounded by the quasi-static regime. In case of an interstitial fluid of higher viscosity slow flow motion is governed by the macroviscous regime where the particle suspension adopts the viscous behaviour of the fluid with a modification for the frictional contacts of the solids (Bagnold 1954). The upper boundary of the rapid granular flow regime is represented by the transition to the saltation regime. For example, a thin granular flow accelerating downhill breaks up into individually saltating particles. This kind of transition

Fig. 9 In the rapid granular flow regime of open flows the shear stress τ_{xy} is independent of the shear rate γ . The flow adopts the behaviour of a plastic body. To lower shear rates the rapid granular flow regime is bounded by the quasi-static or the macroviscous regime. Higher shear rates result in a saltation regime

occurs where the dispersive pressure exceeds the normal load of the overburden.

Between these boundaries the rapid granular flow regime adopts a behaviour that can be characterized by

$$
\tau = c + \nu \gamma,\tag{3}
$$

with viscosity v and shear rate $\gamma = dv/dy$. The constant *c* stands for an unknown contribution of stress and can eventually be zero. The equation $\tau = v\gamma$ originally characterizes the rheology of a linear-viscous fluid. In contrast to a linear-viscous fluid we define τ to be constant and v to be a function of the shear rate γ .

This characterization agrees with the observation: the granular temperature is proportional to the square of the local shear rate and controls the solid fraction via the equilibrium between normal load and dispersive pressure. A higher shear rate results in a volumetric expansion of the sheared region while the shear stress is kept constant. Therefore, the particles have to move a longer vertical distance *dy* to carry the same amount of stress between the upper and the lower part of the shear zone. The viscosity is usually defined as a measure for the shear forces acting between two neighbouring layers moving parallel in a fixed distance with different speed, but this applies only to an incompressible fluid. In contrast to this, the expanding shear zone in a granular flow has a variable thickness *dy*. With an increasing shear rate a constant shear stress is transported over an increasing vertical distance *dy*. Thus, this must result in a reduction of the corresponding viscosity.

Boundary conditions

The above description of the discrete granular flow model used herein lacks a discussion of the way in which bottom collisions are treated. It seems that

Fig. 10 An example of a shear zone within a granular flow. The internal shear zone exhibits the same behaviour as the basal one. This observation is used to argue that the basal shear zone in the present model can be regarded as any shear zone within a flow

bottom collisions should have great influence in the development of granular flows. Particularly in flows approximately 12*—*15 particles thick as in the shown simulations one might expect a strong dependence on how bottom collisions are modelled. For a long time during the development of this granular flow model it was unclear how this should be handled. The solution was just as surprising as it was simple. Bottom collisions are assumed to be collisions with particles in a lower layer of the flow. The basal shear zone that evolves from the bottom collisions is regarded to be equal to a shear zone within a granular flow. This assumption has been validated by shear experiments, i.e. running simulations with a higher initial velocity in the upper half of the flow. Between the lower and the upper part an additional shear zone that is equivalent to the basal one is evolving (Fig. 10). Therefore, the simulated flows can be regarded as the upper part of a larger flow moving over a lower part with negligible velocity.

This solution, which seems surprising at first, is in accordance with the dynamic systems approach of rapid granular flows. Along every local shear zone the coefficient of internal friction μ_{CM} as the ratio between local shear stress and local normal stress keeps the same constant value. Every part of the flow is controlled by the same rapid granular flow attractor (Straub 1994). Therefore, energy dissipation is independent of the location of the main shear motion, whether it occurs along a basal shear zone or over the whole depth of the flow. The center of mass always covers the same travel distance $\langle L \rangle$.

Influence of material properties

The relation between material properties and energy dissipation rate is not a linear one. In Fig. 11 the results

Fig. 11 Coefficients of friction μ_{CM} of the centers of mass of different simulations related to the material properties of the flowing particles. Every *dot* represents the result of one simulation. e *input* of the function of restitution is varied, whereas *v input* has a fixed value of 0.1 m/s. The coefficient of the surface friction μ_{sf} of the particles is varied between 0.0 (smooth) and 1.0 (rough). The axis of the surface friction μ_{sf} can serve only as a qualitative measure due to fluctuations in the small dynamic systems. It appears that even very inelastic and rough material has significant mobility. A distinct gap exists between the quasi-static and the rapid granular flow regime

of various simulations are plotted into a diagram of μ_{CM} as a function of the restitution function ε and the coefficient of friction μ_{sf} of the particle surfaces. To define the function of restitution ε the velocity v_{input} of

the defining point (see Fig. 4) has a fixed value of 0.1 m/s and the related minimum restitution ε_{input} varies from 0.95 (nearly elastic) to 0.05 (nearly unelastic) for different simulation runs. It appears that even for extremely inelastic particles with rough surfaces the coefficients of friction μ_{CM} of the simulated flows are significantly low. This causes a distinct gap between the data of the rapid granular flow regime and the quasi-static regime. The existence of this gap corresponds very well with the fundamental observation of the jump in mobility between soil creep and the flow of landslides that cannot be explained by simple frictional arguments.

For nearly inelastic rough particles the data seems to converge around a coefficient of friction μ_{CM} of 0.45 \pm 0.03. Because of the existence of the rapid granular flow attractor it is predictable that μ_{CM} of every additional performed flow simulation of inelastic and rough particles will end up within this small range. A similar behaviour can be expected for common rock materials that have to be regarded as relatively inelastic and rough. The obvious conclusion is that the coefficient of internal friction μ_{CM} of any high-speed, high-concentration granular flow of rock should be predictable.

From these results one can conclude that the behaviour of rapid granular flows is mainly controlled by the dynamic properties and that there is no linear dependence on the properties of the rock material. This has important implications for the prediction of landslide motion.

Spreading of granular flows

A frequently used approach to approximate the motion of landslides is to assume that the main body of a landslide behaves as a plastic body riding as a plug upon the basal shear zone (Savage 1993; Savage and Hutter 1989; Hutter et al. 1986a, 1986b; Campbell 1989a; Campbell and Cleary 1993). The loose granular material is governed by the quasi-static regime and internal shear is controlled by a large internal angle of friction. This is inconsistent with observations: indeed landslides preserve an existing initial stratification but they are also stretched along their flow path and folded onto themselves. Depending on the underlying topography the flow spreads perpendicular to its flow direction (Heim 1932; Eisbacher 1979; Campbell et al. 1995). Due to the short duration of a landslide event, this stretching, spreading and folding must be produced by rapid shear motion. Shear within the main body might be supported by the granular temperature that is conducted upwards from the basal shear zone. This granular temperature is in fact damped away by numerous particle collisions within the main body, but simultaneously it loosens the particle packing and reduces the resistance against shear.

Following these considerations to the extreme, such a flow can be regarded as a completely developed rapid granular flow that shears over its whole depth. The long-distance motion as well as internal deformation of this completely developed flow is governed by the same attractor. Consequently, flows of larger volume are more elongated along their flow path and their flow fronts reach greater distances.

Essentially, the idea of only one constant coefficient of friction μ that governs the entire development of a landslide has already been suggested by Davies (1982). He assumed the governing μ to be identical to the internal friction coefficient that controls the quasistatic flow regime of the same material. The present simulations show that the internal coefficient of friction μ_{CM} in the rapid granular flow regime is definitely smaller than the coefficient of quasi-static flow (Fig. 11).

Stretching and spreading of a landslide does not only depend on its volume. The flow needs kinetic energy to cover a long distance and, hence, to stretch along its path. Figure 12 shows the results of simulations with an equal number of particles and different initial flow velocities. The higher the initial velocity is, the farther are the distances \overline{L}_{max} that are covered by the respective flow front. Natural flows obtain their kinetic energy from the conversion of the potential energy of their drop height. Therefore, drop height, besides the flow volume, is another important factor to produce extreme travel distances L_{max} . On the other hand, small flow volumes and large drop heights do not result into long travel distances because these flows disintegrate into individually saltating particles. This must be considered as well if the flow depth drops below a minimum depth because of stretching of the flow.

Fig. 12 Maximum travel distance L_{max} as function of the initial velocity v_i . L_{max} is independent of the coefficient of internal friction μ_{CM} that varies here around 0.42. The data is obtained from simulations of rough and extremely inelastic particles $(\varepsilon_{input} = 0.05$, $\mu_{sf} = 0.5$). μ_{CM} increases a little with increasing initial velocity because of fluctuations of the small dynamic systems (75 particles) during the formation of the rapid granular flow. These fluctuations are larger with higher *v i*

Although the apparent coefficient of friction μ_{app} is a poor measure to characterize the geometry of landslide path and μ_{app} does not only depend on the landslide volume but also on the drop height, there is still a relatively good correlation between μ_{app} (= H_{max} / L_{max}) and landslide volume (Fig. 2).

A completely developed rapid granular flow can be regarded as a stack of shear zones, each of them controlled by the same μ_{CM} . Each of these shear zones again can be regarded as a stack of smaller shear zones as well. This approach to regard the shear motion in a rapid granular flow can be drawn down to the size of its particles. This is essentially the picture of a selfsimilar process. The behaviour along every shear zone is identical and independent of the regarded scale. This scale invariance can be used to scale simulation results to another size just by scaling the characteristic lengths $\langle H \rangle$, $\langle L \rangle$ and V by an appropriate factor.

Scale invariance of landslides can be proved by plotting log (μ_{max}) as a function of log (*Volume*). Using $\mu_{app} = H_{max}/L_{max}$ as an approximation for μ_{max} $\langle H \rangle / L_{max}$ the plot in Fig. 2 is appropriate to investigate the scale invariance of landslide motion. The regression line drawn in Fig. 2 reveals a log*—*log relation between μ_{app} and landslide volume. The scattering points around this regression line can be explained mainly by the different drop heights of the landslides. Other causes might be the approximation of μ_{max} by μ_{app} , the incorrect determination of landslide volumes, the individual shape of the initial mass and the influence of the topography on the flow path. Spreading over a flat plane instead of stretching along a narrow valley can reduce the minimum thickness of a flow and result in an earlier halt. Because there is still good correlation between μ_{app} and landslide volume, this log*—*log relation is assumed to be the result of a scaleinvariant, self-similar flow process.

Implications for landslide modelling

The present paper describes the results from a discrete particle model where particles move on their individual trajectories and interact only during particle collisions. This is essentially the picture of a thermodynamic system based solely on Newton's laws of motion. Thermodynamic terms, such as temperature and pressure, are used to describe its properties. These properties arise from particle collisions. The presented model does not rely on any assumptions for the bulk behaviour of the flow. This bulk behaviour is solely a result of the self-organization of the dynamic system.

On the contrary, the continuum treatment of granular flows is based on constitutive equations that are derived from experimental investigations, theoretical analysis or simple phenomenological postulations. The bulk behaviour is introduced as a given rheological model, e.g. a plastic or a non-newtonian viscous behaviour. Besides the given properties, there are no dynamic properties that evolve during the simulation of a continuum model.

The existence of the rapid granular flow attractor is a property that is not yet implemented in a rheological model. An important consequence of this attractor is the fact that the coefficient of friction μ_{CM} of a granular flow is highly reproducible and independent of flow path and volume. It seems that for nearly inelastic and rough materials, such as the common rock material in landslides, μ_{CM} can be easily predicted if the data from the simulations are applicable. However, this has to be checked against landslide data. Unfortunately, the location of the center of mass before and after an event has never been determined.

Furthermore, it is shown that the stretching of the landslide mass is controlled by the same μ_{CM} as the motion of its center of mass. The maximum flow distance of the flow front L_{max} , the most important information to be obtained from the prediction of landslide motion, does not only depend on the landslide volume as proposed by Davies (1982). The drop height $\langle H \rangle$ is another important factor in the stretching of a landslide mass. Motion and deformation is suggested to be a scale-invariant and self-similar process. This hypothesis can be tested against existing landslide data. In consideration of its quality the data shows good agreement. Furthermore, this is indirect confirmation of the hypothesis that μ_{CM} of landslides is a quasi-constant value. Following the regression line in Fig. 2, the corresponding volume of $\mu_{app} = 0.45$ is approximately $V = 0.001 \text{ km}^3$ (see the dashed lines in Fig. 2). This agrees well with the observation by Heim (1932) and Hsü (1975) that there is a minimum volume of 0.0005 km^3 for long runout landslides. Smaller mass movements, such as rockfalls, cannot develop rapid granular flow behaviour. However, further investigations have to prove this hypothesis.

It was pointed out that the rapid granular flow attractor is the fundamental dynamic property that governs the bulk flow behaviour. It defines the requirements for a consistent theory of rapid granular flow. This must be completed by a treatment of the limits of the rapid granular flow regime to define the transitions where the attractor ceases to exist. These limits can be characterized by the ratio of shear to normal stress τ_{xy}/τ_{yy} . Within the field of rapid granular flow this ratio is forced to be at a constant value μ_{CM} . The relative increase in shear stress ($\tau_{xy} \gg \tau_{yy}$) marks the transition to the saltation regime. Due to the greater dispersive pressure, the flow expands and, if τ_{xy} is still larger than τ_{yy} , breaks up into individual particles.

A drop of the shear stress below the normal stress $(\tau_{xy} < \tau_{yy})$ causes the transition to the quasi-static flow regime. In a first approximation this situation can be regarded as a sudden stop of the flow because this regime does not contribute significantly to the flow motion. If the shear stress along the basal shear zone drops below the normal stress, the lower part of the flow stops and a new major shear zone develops higher in the flow where $\tau_{xy} = \tau_{yy}$.

Conclusion

Key to the bulk behaviour of large granular flows is the joint micromechanical interaction of its individual particles. One aspect of this behaviour is the way in which stresses are produced from particle interactions and how constitutive equations can be derived from this. Current continuum models of granular flow motion that are applied to landslide phenomena are based on recent developments in the formulation of constitutive equations of granular flow mechanics.

As shown herein this picture of granular flow mechanics must be enhanced. Granular flows are dynamic systems with dynamic properties that develop from particle interaction. The self-organization results in the development of an equilibrium state, the attractor of the flow. This attractor controls travel distance and bulk deformation of a granular flow, which are the significant factors that have to be determined for the prediction of landslide motion. Therefore, the rapid granular flow attractor has to be included in the constitutive equations of coming continuum mechanical approaches.

References

- Ahn H, Brennen CE, Sabersky RH (1988) Experiments on chute flows of granular materials. In: Satake M, Jenkins JT (ed) Micromechanics of granular materials. Elsevier, Amsterdam, pp 339*—*348
- Alder BJ, Wainwright TE (1960) Studies in molecular dynamics. I. General method. J Chem Phys 31 : 459*—*480
- Bagnold RA (1954) Experiments on a gravity-free dispersion of large solid spheres in a Newton fluid under shear. Proc R Soc Lond, Ser A 225 : 49*—*63
- Campbell CS (1982) Shear flows of granular materials. PhD thesis, CalTech, Pasadena, California, pp 1*—*260
- Campbell CS (1986) Computer simulation of rapid granular flows. Proc 10th US National Congress of Applied Mechanics, Austin, Texas, June 1986, ASME, New York, pp 327*—*338
- Campbell CS (1989a) Self-lubrication for long runout landslides. J Geol 97 : 653*—*665
- Campbell CS (1989b) The stress tensor for simple shear flows of a granular material. J Fluid Mech 203 : 449*—*473
- Campbell CS (1990) Rapid granular flows. Annu Rev Fluid Mech 22 : 57*—*92
- Campbell CS, Brennen CE (1985a) Computer simulation of granular shear flows. J Fluid Mech 151 : 167*—*188
- Campbell CS, Brennen CE (1985b) Chute flows of granular material: some computer simulations. J Appl Mech 52 : 172*—*178
- Campbell CS, Cleary PW (1993) Self-lubrication for long runout landslides: examination by computer simulation. J Geophys Res 98 : 21911*—*21924
- Campbell CS, Gong A (1986) The stress tensor in a two-dimensional granular shear flow. J Fluid Mech 164 : 107*—*125
- Campbell CS, Cleary PW, Hopkins M (1995) Large-scale landslide simulations: global deformation, velocities and basal friction. J Geophys Res 100 : 8267*—*8283
- Crandell DR, Fahnestock RK (1964) Rock falls and avalanches, Little Tahoma Peak, Mt. Rainier, Washington. Bull US Geol Surv 1221A : 1*—*30
- Cundall PA, Strack ODL (1979) A discrete numerical model for granular assemblies. Geotechnique 29 : 47*—*65
- Davies TRH (1982) Spreading of rock avalanche debris by mechanical fluidization. Rock Mech 15 : 9*—*24
- Dent JD (1986) Flow properties of granular materials with large overburden loads. Acta Mech 64 : 111*—*122
- Eisbacher GH (1979) Cliff collapse and rock avalanches (sturtzstroms) in the MacKenzie Mountains, northwestern Canada. Can Geotech J 16 : 309*—*334
- Erismann TH (1979) Mechanisms of large landslides. Rock Mech 12 : 15*—*46
- Erismann TH (1986) Flowing, rolling, bouncing, sliding: synopsis of basic mechanisms. Acta Mech 64 : 101*—*110
- Erismann TH, Heuberger H, Preuss E (1977) Der Bimsstein von Köfels (Tirol), ein Bergsturz-"friktionit". Tschermaks Mineral Petrol Mitt 24 : 67*—*119
- Guest JE (1971) Geology of the farside crater Tsiolkovsky. In: Fielder G (ed) Geology and physics of the moon. Elsevier, Amsterdam, pp 93*—*103
- Harrison JV, Falcon NL (1938) An ancient landslip at Saidmarreh in southwestern Iran. J Geol 46 : 296*—*309
- Heim A (1882) Der Bergsturz von Elm. Z Dtsch Geol Ges 34 : 74*—*115
- Heim A (1932) Bergsturz und Menschenleben. Verlag Fretz und Wasmuth, Zurich, pp 1*—*218
- Hopkins MA, Shen HH (1986) Constitutive relations for a planar, simple shear flow of rough disks. Int J Eng Sci 11 : 1717*—*1730
- Howard K (1973) Avalanche mode of motion: implications from lunar examples. Science 180 : 1052*—*1055
- Hsü KJ (1975) Catastrophic debris streams (sturzstroms) generated by rockfalls. Geol Soc Am Bull 86 : 129*—*140
- Hsü KJ (1978) Albert Heim: observations on landslides and relevance to modern interpretations. In: Voight B, Pariseau WG (ed) Rockslides and avalanches. Elsevier, Amsterdam, pp 71*—*93
- Hsü KJ (1989) Physical principles of sedimentology. Springer, Berlin Heidelberg New York, pp 1*—*233
- Hutter K, Szidarovsky F, Yakowitz S (1986a) Plane steady shear flow of a cohesionless granular material down an inclined plane: a model for flow avalanches. Part I. Theory. Acta Mech 63 : 87*—*112
- Hutter K, Szidarovsky F, Yakowitz S (1986b) Plane steady shear flow of a cohesionless granular material down an inclined plane: a model for flow avalanches. Part II. Numerical results. Acta Mech 65 : 239*—*261
- Hutter K, Koch T (1991) Motion of a finite mass of granular material down a rough incline. Phil Trans R Soc Lond A 334 : 93*—*138
- Jenkins JT, Savage SB (1983) A theory for the rapid flow of identical, smooth, nearly elastic spherical particles. J Fluid Mech 139 : 187*—*202
- Johnson B (1978) Blackhawk landslide, California, U.S.A. In: Voight B, Pariseau WG (ed) Rockslides and avalanches. Elsevier, Amsterdam, pp 481*—*504
- Johnson PC, Jackson R (1987) Frictional*—*collisional constitutive relations for granular materials, with application to plane shearing. J Fluid Mech 176 : 67*—*93
- Johnson PC, Nott P, Jackson R (1990) Frictional*—*collisional equations of motion for particulate flows and their application to chutes. J Fluid Mech 210 : 501*—*535
- Kent PE (1966) The transport mechanism in catastrophic rockfalls. J Geol 74 : 79*—*83
- Krumdieck MA (1984) On the mechanics of large landslides. IV Int Symposium on Landslides, Toronto, 1 : 539*—*544
- Lucchitta BK (1979) Landslides in Valles Marineris, Mars. J Geophys Res 84 : 8097*—*8113
- Lun CKK, Savage SB, Jeffrey DJ, Chepurniy N (1984) Kinetic theories for granular flow: inelastic particles in Couette flow and slightly inelastic particles in a general flow field. J Fluid Mech 140 : 223*—*256
- McEwen AS (1989) Mobility of large rock avalanches: evidence from Valles Marineris, Mars. J Geol 17 : 1111*—*1114
- Melosh HJ (1979) Acoustic fluidization: a new geological process? J Geophys Res 84 : 7513*—*7520
- Melosh HJ (1983) Acoustic fluidization. Am Sci 71 : 158*—*165
- Melosh HJ (1986) The physics of very large landslides. Acta Mech 64 : 89*—*99
- Metropolis N, Rosenbluth A, Rosenbluth M, Teller A, Teller E (1953) Equation of state calculations by fast computing machines. J Chem Phys 21 : 1087*—*1098
- Norem H, Irgens F, Schieldrop BA (1987) A continuum model for calculating snow avalanches. In: Sam B, Gubler H (ed) Avalanche formation, movement and effects. IAHS Publ 126: 363*—*379
- Ogawa S (1978) Multitemperature theory of granular materials. In: Cowin SC, Satake M (eds) Proc Continuum mechanical and statistical approaches in the mechanics of granular materials, Sendai, Japan, pp 208*—*217
- Plafker G, Ericksen GE (1978) Nevados Huascarán avalanches, Peru. In: Voight B, Pariseau WG (eds) Rockslides and avalanches. Elsevier, Amsterdam, pp 277*—*314
- Ridgeway K, Rupp R (1970) Flow of granular material down chutes. Chem Proc Eng 51 : 82*—*85
- Sanders BE, Hopkins MA, Ackermann NA (1988) Physical experiments and numerical simulations of two-dimensional chute flow. In: Satake M, Jenkins JT (eds) Micromechanics of granular materials, Elsevier, Amsterdam, pp 359*—*366
- Savage SB (1979) Gravity flow of cohesionless granular materials in chutes and channels. J Fluid Mech 92 : 53*—*96
- Savage SB (1993) Mechanics of granular flows. In : Hutter K (ed) CISM courses and lectures, no. 337. Springer, Berlin Heidelberg New York
- Savage SB, Hutter K (1989) The motion of a finite mass of granular material down a rough incline. J Fluid Mech 199 : 177*—*215
- Savage SB, Jeffrey DJ (1981) The stress tensor in a granular flow at high shear rates. J Fluid Mech 110 : 255*—*272
- Savage SB, Sayed M (1984) Stresses developed by dry cohesionless granular materials sheared in an annular shear cell. J Fluid Mech 142 : 391*—* 430
- Scheidegger AE (1973) On the prediction of the reach and velocity of catastrophic landslides. Reck Mech 5 : 231*—*236
- Shaller PJ (1991) Analysis and implications of large martian and terrestrial landslides. PhD thesis, CalTech, Pasadena, California, pp 1*—*586
- Shreve RL (1966) Sherman landslide, Alaska. Science 154 : 1639*—*1643
- Shreve RL (1968a) Leakage and fluidization in air*—*layer lubricated avalanches. Geol Soc Am Bull 79 : 653*—*658
- Shreve RL (1968b) The Blackhawk landslide. Geol Soc Am Spec Pap 108 : 1*—*47
- Siebert L (1984) Large volcanic debris avalanches : characteristics of source areas, deposits, and associated eruptions. J Volcanol Geotherm Res 22 : 163*—*197
- Siebert L, Glicken H, Ui T (1987) Volcanic hazards from Bezymiannyand Bandai-type eruptions. Bull Volcanol 49 : 435*—*459
- Straub S (1994) Rapid granular flow in subaerial pyroclastic flows. Dissertation, Univ Würzburg, pp 1-404 (in German)
- Straub S (1996) Self-organization in the rapid flow of granular material: evidence for a major flow mechanism. Geol Rundsch 85 : 85*—*91
- Ui T (1983) Volcanic dry avalanche deposits: identification and comparison with nonvolcanic debris stream deposits. J Volcanol Geotherm Res 18 : 135*—*150
- Ui T, Yamamoto H, Suzuki-Kamata K (1986) Characterization of debris avalanche deposits in Japan. J Volcanol Geotherm Res 29 : 231*—*243
- Voight B, Glicken H, Janda RJ, Douglas PM (1981) Catastrophic rockslide avalanche of May 18. In: Lipman PW, Mullineaux DR (eds) The 1980 eruptions of Mount St. Helens, Washington, US Geol Surv Prof Pap 1250 : 347*—*377
- Voight B, Janda RJ, Glicken H, Douglas PM (1983) Nature and mechanics of the Mount St. Helens rockslide-avalanche of 18 May 1980. Geotechnique 33 : 243*—*273
- Voight B, Pariseau WG (eds) (1978) Rockslides and avalanches: an introduction. Elsevier, Amsterdam, pp 1*—*67
- Walton OR, Braun RL (1986a) Viscosity and temperature calculations for shearing assemblies of inelastic, frictional discs. J Rheol 30 : 949*—*980
- Walton OR, Braun RL (1986b) Stress calculations for assemblies of inelastic spheres in uniform shear. Acta Mech 63 : 73*—*86
- Zhang Y, Campbell CS (1992) The interface between fluid-like and solid-like behaviour in two-dimensional granular flows. J Fluid Mech 237 : 541*—*568

.