Neural Computing and Applications (2024) 36:16057–16070
https://doi.org/10.1007/s00521-024-09797-6

ORIGINAL ARTICLE

Prescribed performance adaptive fuzzy output feedback control for steer-by-wire vehicle system with intermittent actuator faults

Shifeng Zhou[1] · Yongming Li[1] · Shaocheng Tong[1]

Received: 28 October 2023 / Accepted: 25 March 2024 / Published online: 21 May 2024

Abstract
This paper investigates a finite-time adaptive fuzzy prescribed performance fault-tolerant control (FTC) issue for the steer-by-wire vehicle (SBWV) systems with intermittent actuator faults. Different from the steer-by-wire (SBW) system studied by the previous literatures, the SBWV system involved in this study consists of a vehicle dynamics model and an SBW system, including unmeasurable states and unknown nonlinear dynamics. Fuzzy logic systems (FLSs) are first used to identify the unknown model dynamics, and a fuzzy state observer is constructed to estimate the unmeasured states. Then, to compensate for the influence of intermittent actuator faults, a novel finite-time output-feedback prescribed performance adaptive FTC scheme is developed by using the adaptive backstepping control methodology and co-designing the last virtual controller. The presented control scheme not only guarantees that all signals of the closed-loop system are bounded in the presence of actuator faults, but also ensures that the tracking error converges to a small neighborhood of the zero within the prescribed performance bounded. The computer simulation and comparison results demonstrate the effectiveness of the proposed fuzzy control algorithm.

Keywords SBWV system · Fuzzy state observer · Adaptive finite-time control · Intermittent actuator faults · Prescribed performance

1 Introduction

In the past decade, intelligent manufacturing has made significant progress in various fields [1, 2], and automotive intelligent technology has also made significant progress, which has led to the widespread adoption of steer-by-wire (SBW) technology in automatic and semi-automatic intelligent vehicles. The SBW system transmits steering commands through electronic signals, replacing the traditional mechanical connection, which can control the vehicle steering behavior more accurately. The application of this technology improves driving safety and stability [3]. To achieve better steering effect and vehicle stability, some significant control results are reported [4–7]. In [4], a proportional-integral-derivative control method was applied to improve vehicle steering dynamics. In [5], a robust weighted gain-scheduling control method was applied to ensure vehicle yaw stability. By using the simplified linear model, a model predictive control method was developed in [6] to regulate steering of SBW systems. In [7], a super-twisting adaptive control method was designed for SBW systems. Note that in the practical engineering, since the considered steering systems and vehicle dynamic model are often complex and uncertainties, which makes it difficult to obtain precise models. Hence, the traditional control methods such as [4–7] can not obtain better control performance. To overcome this problem, some intelligent (fuzzy or neural network) control techniques have been investigated for the steering systems and vehicle model with unknown nonlinear dynamics [8–10]. In [8], the authors developed an adaptive neural network discrete-time control scheme. In [9], the authors proposed an adaptive hybrid learning neural network

✉ Shaocheng Tong
tongshaocheng@lnut.edu.cn

Shifeng Zhou
15265222661@163.com

Yongming Li
l_y_m_2004@163.com

[1] College of Electrical Engineering, Liaoning University of Technology, Shiying, Jinzhou 121001, Liaoning, China

control scheme and achieved the accurate tracking control performance. In addition, utilizing the Takagi-Sugeno fuzzy model, the authors [10] introduced a model predictive control method and realized the stability of the vehicle systems.

It is worth mentioning that actuator faults are common during the operation of the steering system, which may lead to a decrease in the required control performance and even instability of the vehicle system. Therefore, many scholars have investigated the FTC problem of SBW systems in the presence of actuator fault. In [11], a sliding mode predictive FTC strategy for SBW systems was proposed. In [12], a reinforcement-learning-based FTC scheme was proposed. However, references [11, 12] only addressed the compensation problem for one-time actuator faults, where the state of the actuator remains unchanged after the fault occurs. In practice, actuator often encounters various unpredictable intermittent faults. The state of the actuator often switches between normal operation and faults (or between various faults). For intermittent actuator faults. In [13], a nonlinear system adaptive compensation FTC method based on steering function technology was proposed. In [14], an adaptive neural network output feedback FTC method for nonlinear systems was proposed.

However, the above mentioned control schemes all assume that states of the controlled systems are completely measurable, and there are little research methods on the steer-by-wire systems with unmeasurable states. Note that when the states are unmeasurable, the state observer becomes an extremely effective technique for solving the problem of unmeasurable states. In [15–17], the observer-based control schemes were applied to control the steer-by-wire systems with unmeasurable states. However, the control schemes proposed in [15–17] all assume that the controlled steer-by-wire systems are free of actuator faults. In addition, these output feedback control schemes developed based on asymptotic stability theory, which only ensures the stability of the controlled system in infinite-time. For many practical systems, such as the steering system addressed in this study, it is usually more desirable for the state to converge to a stable equilibrium point within a finite-time, such as [18–20], and achieve tracking error convergence within a specified performance limit, such as [20, 21]. Note that the above control methods are all focus on the SBW systems or the vehicle dynamics systems, not the steer-by-wire vehicle (SBWV) systems addressed by this study, so they cannot reflect the changes of the steering wheel self-aligning torque and vehicle stability during the actual operation of the vehicle as what pointed out by [22]. At the same time, there are currently no research schmes on the adaptive intelligent output feedback finite-time prescribed performance control for the nonlinear SBW system with unmeasurable states and intermittent actuator faults, which inspires us to develop this study.

Based on the aforementioned research, this paper investigates the fuzzy finite-time output feedback control for the uncertain steer-by-wire vehicle (SBWV) systems with unmeasurable states and intermittent actuator faults. FLSs are used to identify the unknown dynamics and a fuzzy state observer is designed to estimate the immeasurable states. Prescribed performance function is introduced to ensure the transient performance. Subsequently, a novel finite-time prescribed performance adaptive fuzzy FTC scheme is proposed based on backstepping technique and finite-time control theory. The main contributions of this study are as follows

1. This paper first investigates the fuzzy adaptive control problem of uncertain SBWV system. The previous intelligent adaptive control methods [8, 9] were only applicable to the SBW system, which does not consider the vehicle dynamics model. However, this paper studies the control problem of the SBWV system composed of the SBW system and the vehicle dynamic model, which can accurately represent the dynamic response of the SBW system and changes in vehicle stability.
2. This paper first proposes a fuzzy finite-time adaptive output feedback FTC scheme for the SBWV system with intermittent actuator faults by designing a novel fuzzy state observer. Therefore, the proposed controller not only addresses the issues of unmeasurable states and actuator faults, but also guarantees that the SBWV system is stable within a finite-time interval and the tracking error does not exceed the prescribed performance bound. Although [23] studied intermittent faults in a SBW system, it depends on measurable states and asymptotic stability theory, and it can not guarantee the transient performance of the system.

2 Problem formulation and preliminaries

2.1 Steer-by-wire vehicle system model

This paper considers steer-by-wire vehicle (SBWV) system, which is composed of the SBW system (1) and the vehicle dynamics model (2). The SBW system mainly includes a steering motor, reducer, and steering actuator as shown in Fig. 1. The simplified vehicle dynamics model includes two degrees of freedom, namely the lateral motion and yaw motion of the vehicle as shown in Fig. 2. From [9] and [10], the SBW system dynamics model (1) and the vehicle dynamics model (2) can be represented as:

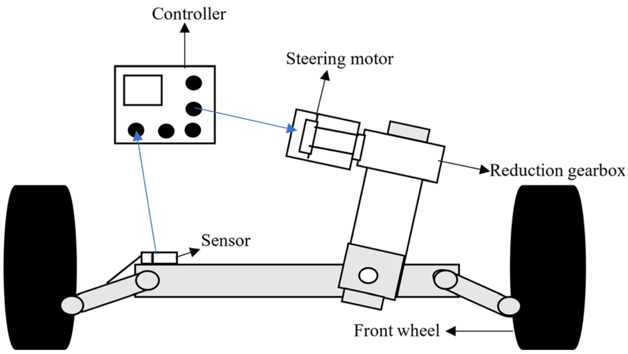

Fig. 1 Structure diagram of SBW system

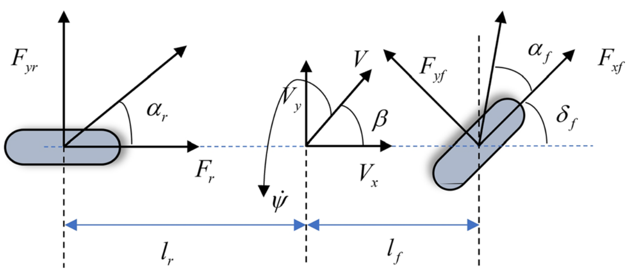

Fig. 2 Vehicle dynamic model

$$J_{eq}\ddot{\delta}_f + \tau_{f,fw} + \mu^2 B_{sm}\dot{\delta}_f + \tau_a - \mu\tau_d = \mu\tau_{sm} \tag{1}$$

$$\begin{cases} \dot{\beta} = \dfrac{C_f\cos\delta_f}{mV_x}\delta_f - \dfrac{C_f\cos\delta_f + C_r}{mV_x}\beta - \left(\dfrac{l_fC_f\cos\delta_f - l_rC_r}{mV_x^2} + 1\right)\dot{\psi} \\ \ddot{\psi} = \dfrac{l_fC_f\cos\delta_f}{I_z}\delta_f - \dfrac{l_fC_f\cos\delta_f - l_rC_r}{I_z}\beta - \dfrac{l_f^2C_f\cos\delta_f + l_r^2C_r}{I_zV_x}\dot{\psi} \end{cases} \tag{2}$$

where δ_f represents the front wheel steering angle. μ represents the ratio of motor output shaft angle to front wheels steering angle. $J_{eq} = J_{fw} + \mu^2 J_{sm}$ is the equivalent moment of inertia. J_{sm} is the rotational inertia of the motor. J_{fw} is the front wheel rotational inertia. B_{sm} is viscous friction coefficient. τ_{sm} is the motor output torque. $\tau_{f,fw}$ is the frictional torque. τ_d is the motor lumped torque perturbation. β and $\dot{\psi}$ are vehicle sideslip angle and the yaw rate. l_f and l_r are distances from the front and rear wheel axle to center of gravity (CG). C_f and C_r are front and rear wheels cornering stiffness. I_z is the vehicle around CG moment. V_x is the longitudinal vehicle velocity. m is the vehicle mass.

The self-aligneding torque τ_a given by [22] as follows

$$\tau_a = (t_m + t_p)F_{yf} = C_f(t_m + t_p)\left(\delta_f - \beta - \frac{l_f\dot{\psi}}{V_x}\right) \tag{3}$$

where t_m is the mechanical trail, t_p is the pneumatic trail.

From (1) and (2), the SBWV system can be formulated as follows:

$$\begin{aligned} \dot{\beta} &= \frac{C_f\cos\delta_f}{mV_x}\delta_f - \frac{C_f\cos\delta_f + C_r}{mV_x}\beta \\ &\quad - \left(\frac{l_fC_f\cos\delta_f - l_rC_r}{mV_x^2} + 1\right)\dot{\psi} \\ \ddot{\psi} &= \frac{l_fC_f\cos\delta_f}{I_z}\delta_f - \frac{l_fC_f\cos\delta_f - l_rC_r}{I_z}\beta \\ &\quad - \frac{l_f^2C_f\cos\delta_f + l_r^2C_r}{I_zV_x}\dot{\psi} \\ \ddot{\delta}_f &= \frac{-C_f(t_m+t_p)}{J_{eq}}\delta_f + \frac{C_f(t_m+t_p)}{J_{eq}}\beta \\ &\quad + \frac{C_f(t_m+t_p)l_f}{V_xJ_{eq}}\dot{\psi} - \frac{\mu^2B_{sm}\dot{\delta}_f}{J_{eq}} \\ &\quad + \frac{\mu}{J_{eq}}\tau_{sm} + \frac{\mu\tau_d - \tau_{f,fw}}{J_{eq}} \end{aligned} \tag{4}$$

Remark 1 Note that the previous literatures [8, 9] only considered the adaptive control problem of SBW system (1), and its self-alignedment torque is based on a simplified hyperbolic tangent function, which cannot accurately reflect the changes in self-alignedment torque under complex driving environments in actual vehicle driving. While the literature [10] only focused on vehicle control for the vehicle dynamic system (2). Different from previous studies, this paper focuses on investigating the control problem of the SBWV system (4), which is composed of the systems (1) and (2). Therefore, it can accurately describe the dynamic response of the SBW system in complex driving environments and changes in vehicle stability.

Define $x = [x_1, x_2, x_3, x_4]^T = [\delta_f, \beta, \dot{\psi}, \dot{\delta}_f]^T \in R^4$, $f_1(x) = x_4 - x_2$, $f_2(x) = -\frac{C_f\cos x_1 + C_r}{mV_x}x_2 - \left(\frac{l_fC_f\cos x_1 - l_rC_r}{mV_x^2} + 1\right)x_3 + \frac{C_f\cos x_1}{mV_x}x_1 - x_3$, $f_3(x) = -\frac{l_fC_f\cos x_1 - l_rC_r}{I_z}x_2 - \frac{l_f^2C_f\cos x_1 + l_f^2C_r}{I_zV_x}x_3 + \frac{l_fC_f\cos x_1}{I_z}x_1 - x_4$, $f_4(x) = \frac{-C_f(t_m+t_p)}{J_{eq}}x_1 + \frac{C_f(t_m+t_p)}{J_{eq}}x_2 + \frac{C_f(t_m+t_p)l_f}{V_xJ_{eq}}x_3 - \frac{\mu^2B_{sm}x_4 + \tau_{f,fw}}{J_{eq}}$.

Then, the system (4) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x) \\ \dot{x}_2 &= x_3 + f_2(x) \\ \dot{x}_3 &= x_4 + f_3(x) \\ \dot{x}_4 &= f_4(x) + \bar{h}u(t) + d(t) \\ y &= x_1 \end{aligned} \tag{5}$$

In (5), $\bar{h} = \mu/J_{eq}$ represents the known control gain, $u(t) = \tau_{sm}$ represents the control input. $d(t) = \mu\tau_d/J_{eq}$ represents the disturbance. Note that in the actual running of the vehicle, since the system uncertainty caused by driving changes has a large fluctuation range, $f_i, i = 1, \ldots, 4$ are unknown nonlinear functions.

2.2 Intermittent actuator fault model

From [13] and [14], the intermittent actuator fault model is described as follows

$$u(t) = \eta(t)u_m(t) + u_k(t) \tag{6}$$

$$\eta(t) = \eta_k, t \in [t_k, t_{k+1}), k \in Z^+ \tag{7}$$

where $0 < \bar{\eta} \leq \eta_k \leq 1$ with $\bar{\eta}$ and η_k being unknown constants. $u_k(t)$ is the bounded signal. $t \in [t_k, t_{k+1})$ for $k \in Z^+$, t_k and t_{k+1} denote the time instants at which the actuator fault occurs and ends.

Remark 2 Due to frequent use and aging of actuator, the actuator usually suffers intermittent faults. When the actuator encounters intermittent faults, the steering motor may not be able to generate sufficient torque to drive the front wheels according to the expected control signal, which may cause the vehicle to deviate from the expected path. Therefore, effectively handling intermittent actuator faults has become crucial. Traditional fault diagnosis and prediction methods [11, 12] are not suitable for this situation because they do not take into account the random intermittency of faults. Therefore, this paper mainly studies the SBW system affected by intermittent actuator faults.

Assumption 1 There exists a positive constant u_k^* satisfying $|u_k(t)| \leq u_k^*$.

Assumption 2 There exists a positive constant d^* satisfying $|d(t)| \leq d^*$.

Control Objective: This study will develop a finite-time adaptive fuzzy FTC approach for the SBWV system (5) so that the following properties holds

1. The controlled steer-by-wire vehicle system is semi-global practical finite-time stable (SGPFS).
2. The tracking error converges to a small neighborhood of zero in a finite-time interval and does not exceed the prescribed performance bound.

2.3 Fuzzy logic systems

Since the considered SBWV system (5) contains the uncertain nonlinear dynamics, this paper will adopt the FLSs to model the SBWV system (5). In the following, we briefly review the notion and properties of the FLS.

Suppose IF-THEN fuzzy rules are as follows:

$\Re^l$: if s_1 is G_1^q and s_2 is G_2^q , ... and s_n is G_n^q, then ζ is H^q, $q = 1, 2, \cdots, D$.

where $s = (s_1, s_2, \ldots, s_n)^T$ and ζ are the FLS input and output, respectively. Fuzzy set G_i^q and H^q are fuzzy sets with the membership function $\mu_{G_i^q}(s_i)$ and $\mu_{H^q}(\zeta)$. D is the rule member.

Through singleton function, center average defuzzification, the FLS can be represented as

$$\zeta(s) = \frac{\sum_{q=1}^{D} \bar{\zeta}_l \prod_{i=1}^{n} \mu_{G_i^q}(s_i)}{\sum_{q=1}^{D} \left(\prod_{i=1}^{n} \mu_{G_i^q}(s_i)\right)} \tag{8}$$

where $\bar{\zeta}_q = \max_{\zeta \in R} \mu_{H^q}(\zeta)$.

Define the fuzzy basis function as

$$\Phi_q = \frac{\prod_{i=1}^{n} \mu_{G_i^q}(s_i)}{\sum_{q=1}^{D} \left(\prod_{i=1}^{n} \mu_{G_i^q}(s_i)\right)} \tag{9}$$

Denoting $\hat{W}^T = [\bar{\zeta}_1, \bar{\zeta}_2, \ldots, \bar{\zeta}_D] = [\hat{W}_1, \hat{W}_2, \ldots \hat{W}_D]$ and $\Phi(s) = [\Phi_1(s), \ldots, \Phi_D(s)]^T$, then FLS (9) can be rewritten as

$$\zeta(s) = \hat{W}^T \Phi(s) \tag{10}$$

Lemma 1 [24]: *let f(s) be a continuous positive function defined on a compact set* Ω. *Then, there exists a FLS* (10) *such as*

$$\sup_{s \in \Omega} |f(s) - \hat{W}^T \Phi(s)| < \varepsilon \tag{11}$$

where ε is the approximation error, which is usually assumed that there exists a constant $\bar{\varepsilon}$ such that $|\varepsilon| < \bar{\varepsilon}$.

Remark 3 Note that FLSs are introduced to address unknown nonlinear functions in the controlled systems (5) because they have the property of approximating unknown nonlinear functions in compact sets. Moreover, there are some other nonlinear approximators, such as NNs [14] and type-2 fuzzy logic systems, which can replace FLSs and achieve the same results.

2.4 Prescribed performance

In reference to [20, 21], the prescribed performance can be described by the following inequality:

$$-\rho_{\min}\mu(t) < \chi_1(t) < \rho_{\max}\mu(t), \forall t \geq 0 \tag{12}$$

where $\rho_{\min} > 0$ and $\rho_{\max} > 0$, $\mu(t) = (\mu_0 - \mu_\infty)e^{-rt} + \mu_\infty$ represents a bounded performance with $\lim_{t \to \infty} \mu(t) = \mu_\infty$ and r are positive design parameters, $\mu_0 = \mu(0)$, μ_0 is selected such that $\mu_0 > \mu_\infty$. $\chi_1(t) = y - y_d$ denotes the tracking error. It follows from (5) that $\chi(t)$ is guaranteed to be less than $\max\{\rho_{\min}\mu_0, \rho_{\max}\mu_0\}$.

In order to achieve the prescribed performance in (12), we can convert the constrained tracking error behavior into an equivalent unconstrained tracking error behavior. We define

$$\chi_1(t) = \mu(t)\phi(\xi(t)), t \geq 0 \tag{13}$$

where ξ is the converted error, $\phi(\xi) = (\rho_{\max}e^{\xi} - \rho_{\min}e^{-\xi})/(e^{\xi} - e^{-\xi})$ is smooth, strictly increasing function.

From (12), one obtains

$$\xi(t) = \phi^{-1}\left(\frac{\chi_1(t)}{\mu(t)}\right) = \frac{1}{2}\ln\frac{\phi + \rho_{\min}}{\rho_{\max} - \phi} \tag{14}$$

and

$$\dot{\xi}(t) = \psi\left(\dot{\chi}_1 - \frac{\dot{\mu}_1\chi_1}{\mu}\right) \tag{15}$$

where $\psi = \frac{1}{2\mu}\left(\frac{1}{\phi+\rho_{\min}} - \frac{1}{\rho_{\max}-\phi}\right)$.

Define the following state transformation

$$z_1 = \xi(t) - \frac{1}{2}\ln\frac{\rho_{\min}}{\rho_{\max}} \tag{16}$$

Then, we have

$$\dot{z}_1 = \psi\left(\dot{\chi}_1 - \frac{\chi_1\dot{\mu}}{\mu}\right) \tag{17}$$

3 Fuzzy state observer and controller design

This section first gives the design of a fuzzy observer for the steer-by-wire vehicle system (5). Then, a fuzzy adaptive output feedback controller is presented by the finite-time theory.

3.1 Fuzzy state observer design

In practice, due to space and hardware limitations, the steering angular velocity, vehicle sideslip angle, and yaw angle are difficult to measure by sensor technology directly. Therefore, it is necessary to design a state observer to get unavailable states based on FLSs.

To begin with, we firstly use a FLS $\hat{f}_i(x|\hat{W}_i) = \hat{W}_i^T\Phi_i(x)$ to identify $f_i(x)$, and let

$$f_i(x) = W_i^{*T}\Phi_i(x) + \varepsilon_i(x) \tag{18}$$

where W_i^* is the ideal weight. ε_i is fuzzy approximating error, which is such that $|\varepsilon_i| \leq \bar{\varepsilon}_i$, here $\bar{\varepsilon}_i$ is a constant.

Substituting (18) into (5) yields

$$\begin{aligned}
\dot{x}_1 &= x_2 + W_1^{*T}\Phi_1(x) + \varepsilon_1(x)\\
\dot{x}_2 &= x_3 + W_2^{*T}\Phi_2(x) + \varepsilon_2(x)\\
\dot{x}_3 &= x_4 + W_3^{*T}\Phi_3(x) + \varepsilon_3(x)\\
\dot{x}_4 &= W_4^{*T}\Phi_4(x) + \varepsilon_4(x) + \bar{h}u(t) + d(t)\\
y &= x_1
\end{aligned} \tag{19}$$

We design the fuzzy observer by

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \hat{W}_1^T\Phi_1(\hat{x}) + k_1(y - \hat{y}),\\
\dot{\hat{x}}_2 &= \hat{x}_3 + \hat{W}_2^T\Phi_2(\hat{x}) + k_2(y - \hat{y}),\\
\dot{\hat{x}}_3 &= \hat{x}_4 + \hat{W}_3^T\Phi_3(\hat{x}) + k_3(y - \hat{y})\\
\dot{\hat{x}}_4 &= \bar{h}u(t) + \hat{W}_4^T\Phi_4(\hat{x}) + k_4(y - \hat{y})\\
\hat{y} &= \hat{x}_1
\end{aligned} \tag{20}$$

In (20), $\hat{x}_i$ stands for the estimate of x_i, and $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_4]^T$, $\hat{W}_i$ expresses estimate of W_i^*.

Let $e = x - \hat{x}$ be observer error.

From (19) and (20), we have

$$\begin{aligned}
\dot{e} &= Ae + \sum_{i=1}^{4} B_iW_i^{*T}(\Phi_i(x) - \Phi_i(\hat{x}))\\
&\quad + \sum_{i=1}^{4} B_i\tilde{W}_i^T\Phi_i(\hat{x})) + \varepsilon + B_4d(t)
\end{aligned} \tag{21}$$

Here $\tilde{W}_i = W_i^* - \hat{W}_i$, stands for parameter estimating error.

$$A = \begin{bmatrix} -k_1 & 1 & 0 & 0\\ -k_2 & 0 & 1 & 0\\ -k_3 & 0 & 0 & 1\\ -k_4 & 0 & 0 & 0 \end{bmatrix}, \qquad K = [k_1, k_2, k_3, k_4]^T,$$

$\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4]^T$, $B_i = [\underbrace{0 \cdots 0,\ 1}_{i}, 0]^T$ and $B_4 = [0, 0, 0, 1]^T$.

Choose the observer gain vector K such that A is a stable matrix. That is, the following Lyapunov equation holds:

$$A^TP + PA = -Q \tag{22}$$

where $Q = Q^T > 0$ is a given matrix.

Consider the Lyapunov function $V_0 = e^TPe$. By (21)–(22), one has

$$\begin{aligned}
\dot{V}_0 &= -e^TQe + 2e^TP\Bigg(\sum_{i=1}^{4} B_iW_i^{*T}(\Phi_i(x) - \Phi_i(\hat{x}))\\
&\quad + \sum_{i=1}^{4} B_i\tilde{W}_i^T\Phi_i(\hat{x})) + \varepsilon + B_4d(t)\Bigg)
\end{aligned} \tag{23}$$

From [18, 19], we can obtain

$$\dot{V}_0 \leq -e^T Qe + \|P\|^2 \Big(\sum_{i=1}^{4} (\tilde{W}_i^T \tilde{W}_i + 2\|W_i^*\|^2) + \|P\|^2 \|\bar{\varepsilon}\|^2 + \|P\|^2 d^{*2} \qquad (24)$$
$$\leq -\lambda_0 \|e\|^2 + \|P\|^2 \sum_{i=1}^{4} \tilde{W}_i^T \tilde{W}_i + M_0$$

where $\lambda_0 = (\lambda_{\min}(Q) - 5) > 0$, $M_0 = \|P\|^2 \|\bar{\varepsilon}\|^2 + \|P\|^2 \sum_{i=1}^{4} \|W_i^*\|^2 + \|P\|^2 d^{*2}$.

Remark 4 Note that since a FLS has a good approximating ability of a nonlinear function, $\|\bar{\varepsilon}\|^2$ can be made smaller if the number of the IF-Then rules is chosen enough. In addition, if the considered SBWV (5) are free of the disturbance $d^{*2} = 0$. Therefore, when $\lambda_0 = (\lambda_{\min}(Q) - 5) > 0$ is selected large enough, we can conclude from (20) that the observer error vector can be smaller. Therefore, the designed fuzzy state observer (16) is reasonable.

3.2 Finite-time fuzzy adaptive controller design

This section will present the output feedback finite-time prescribed performance adaptive fuzzy FTC design based on the prescribed performance function and finite time concept. Then the stability proof of the controlled system is given.

Define the coordinate transformations as

$$\chi_1 = x_1 - y_d$$
$$z_i = \hat{x}_i - \omega_i \qquad (25)$$
$$s_i = \omega_i - \alpha_{i-1}, (i = 2, 3, 4)$$

where χ_1 is the tracking error, z_i is the error surface, s_i is the output error of one-order filter, α_{i-1} is the virtual controller, ω_i is newly introduced state variable obtained by the one-order filter as follows

$$\kappa_i \dot{\omega}_i + \omega_i = \alpha_{i-1}, \omega_i(0) = \alpha_{i-1}(0) \qquad (26)$$

where κ_i is a positive design parameter.

By using the above coordinate transformations (25) and the first-order filter (26), 4-step adaptive backstepping control design procedures are given for the SBWV system (5).

Step 1: By (5), (17) and $x_2 = \hat{x}_2 + e_2$, we have

$$\dot{z}_1 = \psi\Big(\dot{\chi}_1 - \frac{\chi_1 \dot{\mu}_1}{\mu_1}\Big)$$
$$= \psi\Big(z_2 + s_2 + \alpha_1 + \hat{W}_1^T \Phi_1(\hat{x}_1) - W_1^{*T} \Phi_1(\hat{x}_1) \qquad (27)$$
$$\textit{Consider the Lyapunov} + \tilde{W}_1^T \Phi_1(\hat{x}_1)$$
$$+ W_1^{*T} \Phi_1(\hat{x}) + e_2 + \varepsilon_1 - \dot{y}_d - \frac{\chi_1 \dot{\mu}}{\mu}\Big)$$

Consider Lyapunov function candidate as follows

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2\gamma_1} \tilde{W}_1^T \tilde{W}_1 \qquad (28)$$

where $\gamma_1 > 0$ is a given positive constant.

From (27), $\dot{V}_1$ is

$$\dot{V}_1 \leq z_1 \dot{z}_1 - \frac{1}{\gamma_1} \tilde{W}_1^T \dot{\hat{W}}_1$$
$$\leq + z_1 \psi\big(z_2 + s_2 + e_2 + \alpha_1 + W_1^{*T} \Phi_1(\hat{x}) - W_1^{*T} \Phi_1(\hat{x}_1)$$
$$+ \varepsilon_1 - \dot{y}_d + \tilde{W}_1^T \Phi_1(\hat{x}_1) + \hat{W}_1^T \Phi_1(\hat{x}_1) - \frac{\chi_1 \dot{\mu}}{\mu}\Big) - \frac{1}{\gamma_1} \tilde{W}_1^T \dot{\hat{W}}_1 \qquad (29)$$

Remark 5 Note that since $W_1^{*T} \Phi_1(\hat{x})$ in (27) includes the whole state variables of the system (5), if we apply traditional backstepping control design methods [15, 16] to design virtual control signal α_1, then α_1 will be a function of the entire state vector $x = [x_1, \ldots, x_4]$, which is not permitted by backstepping control design technique [18, 19]. In the following control design, we will use the property of fuzzy logic system to solve this problem.

By using Young's inequality and $0 < W_1^T(x) W_1(x) < 1$, we have

$$z_1 \psi(z_2 + s_2 + e_2 + \varepsilon_1) \leq 2 z_1^2 \psi^2 + \frac{1}{2} \|e\|^2 + \frac{1}{2} \bar{\varepsilon}_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} s_2^2 \qquad (30)$$

$$z_1 \psi(W_1^{*T} \Phi_1(\hat{x}) - W_1^{*T} \Phi_1(\hat{x}_1)) \leq \frac{\tau}{2} z_1^2 \psi^2 + \frac{2}{\tau} \|W_1^*\|^2 \qquad (31)$$

where τ is a given positive constant.

Inserting (30)–(31) into (29) obtains

$$\dot{V}_1 \leq z_1 \psi\Big(\alpha_1 + 2\psi z_1 + \hat{W}_1^T \Phi_1(\hat{x}_1) + \frac{\tau}{2} \psi z_1 - \dot{y}_d - \frac{\chi_1 \dot{\mu}}{\mu}\Big)$$
$$+ \frac{1}{2} z_2^2 + \frac{1}{2} s_2^2 + M_1 + \frac{\tilde{W}_1^T}{\gamma_1} (\gamma_1 \psi z_1 \Phi_1(\hat{x}_1) - \dot{\hat{W}}_1) + \frac{1}{2} \|e\|^2 \qquad (32)$$

where $M_1 = \frac{2}{\tau}\left\|W_1^*\right\|^2 + \frac{1}{2}\bar{\varepsilon}_1^2$.

Construct the virtual controller α_1. the updating law of $\hat{W}_1$ as

$$\alpha_1 = -c_1\frac{z_1^{2p-1}}{\psi} - \hat{W}_1^T\Phi_1(\hat{x}_1) - \frac{4+\tau}{2}\psi z_1 + \dot{y}_d + \frac{\chi_1\dot{\mu}}{\mu} \tag{33}$$

$$\dot{\hat{W}}_1 = \gamma_1\psi z_1\Phi_1(\hat{x}_1) - \sigma_1\hat{W}_1 \tag{34}$$

where $p = \frac{2n-1}{2n+1}(n > 2, n \in N)$, $c_1 > 0$ and $\sigma_1 > 0$ are given constants.

Substituting (33)–(34) into (32) obtains

$$\dot{V}_1 \le -c_1 z_1^{2p} + \frac{1}{2}z_2^2 + \frac{1}{2}s_2^2 + \frac{\sigma_1}{\gamma_1}\tilde{W}_1\hat{W}_1 + M_1 + \frac{1}{2}\|e\|^2 \tag{35}$$

Step 2: According (20) and (25), $\dot{z}_2$ is

$$\begin{aligned}\dot{z}_2 &= \dot{\hat{x}}_2 - \dot{\omega}_2\\ &= \hat{x}_3 + k_2e_1 + W_2^{*T}\Phi_2(\hat{x}) - \tilde{W}_2^T\Phi_2(\hat{x}) - \dot{\omega}_2\\ &= z_3 + s_3 + \alpha_2 + k_2e_1 + \hat{W}_2^T\Phi_2(\hat{\bar{x}}_2) - W_2^{*T}\Phi_2(\hat{\bar{x}}_2)\\ &\quad + W_2^{*T}\Phi_2(\hat{x}) + \tilde{W}_2^T\Phi_2(\hat{x}_2) + \tilde{W}_2^T\Phi_2(\hat{\bar{x}}_2) - \dot{\omega}_2\end{aligned} \tag{36}$$

Construct the Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}s_2^2 + \frac{1}{2\gamma_2}\tilde{W}_2^T\tilde{W}_2 \tag{37}$$

where γ_2 is a given positive constant.

From (36), we have $\dot{V}_2$ as follows

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 + \omega_2\dot{\omega}_2 - \frac{1}{\gamma_2}\tilde{W}_2\dot{\hat{W}}_2\\ &\le -c_1z_1^{2p} + \frac{\sigma_1}{\gamma_1}\tilde{W}_1^T\hat{W}_1 + \frac{1}{2}z_2^2 + \frac{1}{2}s_2^2 + z_2(z_3 + s_3 + \alpha_2\\ &\quad + k_2e_1 + \hat{W}_2^T\Phi_2(\hat{\bar{x}}_2) - \dot{\omega}_2 + W_2^{*T}\Phi_2(\hat{x}) - W_2^{*T}\Phi_2(\hat{\bar{x}}_2)\\ &\quad - \tilde{W}_2^T\Phi_2(\hat{x}) + \tilde{W}_2^T\Phi_2(\hat{\bar{x}}_2) + s_2\dot{s}_2 - \frac{1}{\gamma_2}\tilde{W}_2^T\dot{\hat{W}}_2 + M_1 + \frac{1}{2}\|e\|^2\end{aligned} \tag{38}$$

By using the similar design procedures to step 1, the following inequalities hold

$$z_2(W_2^{*T}\Phi_2(\hat{x}) - W_2^{*T}\Phi_2(\hat{\bar{x}}_2)) \le \frac{\tau}{2}z_2^2 + \frac{2}{\tau}\left\|W_2^*\right\|^2 \tag{39}$$

$$z_2(z_3 + s_3 - \tilde{W}_2^T\Phi_2(\hat{x}) \le \frac{3}{2}z_2^2 + \frac{1}{2}\tilde{W}_2^T\tilde{W}_2 + \frac{1}{2}z_3^2 + \frac{1}{2}s_3^2 \tag{40}$$

Substituting (39)–(40) into (38) yields

$$\begin{aligned}\dot{V}_2 \le& -c_1z_1^{2p} + \frac{\sigma_1}{\gamma_1}\tilde{W}_1^T\hat{W}_1 + 2z_2^2 + \frac{1}{2}z_3^2 + \sum_{i=2}^{3}\frac{1}{2}s_i^2\\ &+ z_2\left(\alpha_2 + k_2e_1 + \hat{W}_2^T\Phi_2(\hat{\bar{x}}_2) - \dot{\omega}_2 + \frac{1+\tau}{2}z_2\right)\\ &+ s_2\left(-\frac{s_2}{\kappa_2} + B_2(\cdot)\right) + \frac{1}{2}\tilde{W}_2^T\tilde{W}_2 + M_2 + \frac{1}{2}\|e\|^2\\ &+ \frac{1}{\gamma_2}\tilde{W}_2^T(\gamma_2z_2\Phi_2(\hat{\bar{x}}_2) - \dot{\hat{W}}_2)\end{aligned} \tag{41}$$

where $M_2 = M_1 + \frac{2}{\tau}\left\|W_2^*\right\|^2$, and B_2 is a continuous function as follow

$B_2 = c_1\frac{\partial(z_1^{2p-1}/\psi)}{\partial t} + \dot{\hat{W}}_1^T\Phi_1 + \frac{\hat{W}_1^T\partial\Phi_1}{\partial\hat{x}_1}\dot{\hat{x}}_1 + \frac{4-\tau}{2}(\dot{z}_1\psi + z_1\dot{\psi}) - \ddot{y}_2 - c_1\frac{\partial(\chi_1\dot{\mu}/\mu)}{\partial t}$.

Construct the virtual controller α_2 , the updating law of $\hat{W}_2$ as

$$\alpha_2 = -c_2z_2^{2p-1} - k_2e_1 - \frac{5+\tau}{2}z_2 - \hat{W}_2\Phi_2(\hat{\bar{x}}_2) + \dot{\omega}_2 \tag{42}$$

$$\dot{\hat{W}}_2 = \gamma_2z_2\Phi_2(\hat{\bar{x}}_2) - \sigma_2\hat{W}_2 \tag{43}$$

where c_2 and σ_2 are given positive constants. Inserting (42)–(43) into (41) yields

$$\begin{aligned}\dot{V}_2 \le& -\sum_{i=1}^{2}c_iz_i^{2p} + \sum_{i=1}^{2}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \frac{1}{2}z_3^2 + \sum_{i=2}^{3}\frac{1}{2}s_i^2\\ &+ s_2\left(-\frac{s_2}{\kappa_2} + B_2\right) + \frac{1}{2}\tilde{W}_2^T\tilde{W}_2 + M_2 + \frac{1}{2}\|e\|^2\end{aligned} \tag{44}$$

Step 3: According to (25), the time derivative of z_3 is

$$\begin{aligned}\dot{z}_3 &= \dot{\hat{x}}_3 - \dot{\omega}_3\\ &= \hat{x}_4 + c_3e_1 + W_3^{*T}\Phi_3(\hat{x}) - \tilde{W}_3^T\Phi_3(\hat{x}) - \dot{\omega}_3\\ &= z_4 + s_4 + \alpha_3 + k_3e_1 + W_3^{*T}\Phi_3(\hat{x}_3) - \tilde{W}_3^T\Phi_3(\hat{x}_3)\\ &\quad + W_3^{*T}\Phi_3(\hat{\bar{x}}_3) + \hat{W}_3^T\Phi_3(\hat{\bar{x}}_3) + \tilde{W}_3^T\Phi_3(\hat{\bar{x}}_3) - \dot{\omega}_3\end{aligned} \tag{45}$$

Construct the Lyapunov function as follows:

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2}s_3^2 + \frac{1}{2\gamma_3}\tilde{W}_3^T\tilde{W}_3 \tag{46}$$

where $\gamma_3 > 0$ is a given constant.

From (45), $\dot{V}_3$ is

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 + s_3\dot{s}_3 - \frac{1}{\gamma_3}\tilde{W}_3^T\dot{\hat{W}}_3 \leq -\sum_{i=1}^{2}k_i z_i^{2p} + \sum_{i=1}^{2}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \sum_{i=2}^{3}\frac{1}{2}\tilde{W}_i^T\tilde{W}_i + 2z_3^2 + \frac{1}{2}z_4^2 + \sum_{i=2}^{4}\frac{1}{2}s_i^2 + \sum_{i=2}^{3}s_i\left(-\frac{s_i}{\kappa_i} + B_i(\cdot)\right) + z_3\left(\alpha_3 - \dot{\omega}_3 + \frac{1+\tau}{2}z_3 + k_3e_1 + \hat{W}_3^T\Phi_3(\hat{\bar{x}}_3)\right) + \frac{1}{\gamma_3}\tilde{W}_3^T(\gamma_3 z_3\Phi_3(\hat{\bar{x}}_3) - \dot{\hat{W}}_3) + \frac{1}{2}\|e\|^2 + M_3 \tag{47}$$

where $M_3 = M_2 + \frac{2}{\tau}\|W_3^*\|^2$;
$B_3 = (2p-1)c_2z_2^{2p-2} + k_2\dot{e}_1 + \frac{5+\tau}{2}\dot{z}_2 + \dot{\hat{W}}_2^T\Phi_2(\hat{\bar{x}}_2) + \frac{\hat{W}_2^T\partial\Phi_2}{\partial\hat{\bar{x}}_2}\dot{\hat{\bar{x}}}_2 + \frac{\dot{s}_3}{\kappa_3}$.

Construct the virtual controller α_3 , the updating law of $\hat{W}_3$ as

$$\alpha_3 = -c_3z_3^{2p-1} - k_3e_1 - \frac{5+\tau}{2}z_3 - \hat{W}_3\Phi_3(\hat{\bar{x}}_3) + \dot{\omega}_3 \tag{48}$$

$$\dot{\hat{W}}_3 = \gamma_3z_3\Phi_3(\hat{\bar{x}}_3) - \sigma_3\hat{W}_3 \tag{49}$$

where c_3 and σ_3 are given positive constants. Inserting (48)–(49) into (47) yields

$$\dot{V}_3 \leq -\sum_{i=1}^{3}k_iz_i^{2p} + \sum_{i=1}^{3}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \frac{1}{2}z_4^2 + \sum_{i=2}^{4}\frac{1}{2}s_i^2 + \sum_{k=2}^{3}s_k\left(-\frac{s_k}{\lambda_k} + B_k\right) + \sum_{k=2}^{3}\frac{1}{2}\tilde{W}_k^T\tilde{W}_k + M_3 + \frac{1}{2}\|e\|^2 \tag{50}$$

Step 4: According to (25), we can obtain

$$\dot{z}_4 = \dot{x}_4 - \dot{\omega}_4 = k_4e_1 + \bar{h}(\eta(t)u_m(t) + u_k(t)) + \hat{W}_4^T\Phi_4(\hat{x}) + \tilde{W}_4^T\Phi_4(\hat{x}) - \hat{W}_4^T\Phi_4(\hat{x}) - \dot{\omega}_4 \tag{51}$$

with $r^* = u_k^*$.

Construct the Lyapunov function as follows:

$$V_4 = V_3 + \frac{1}{2}z_4^2 + \frac{1}{2}s_4^2 + \frac{1}{2\gamma_4}\tilde{W}_4^T\tilde{W}_4 + \frac{\bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2 + \frac{1}{2\gamma_r}\tilde{r}^2 \tag{52}$$

where γ_4, γ_ϖ and γ_r are given positive constants. $\hat{\varpi}$ is estimate of $\varpi^* = 1/\bar{\eta}$, and $\tilde{\varpi} = \varpi^* - \hat{\varpi}$. $\hat{r}$ is estimate of r^*, and $\tilde{r} = r^* - \hat{r}$.

From (51), we have

$$\dot{V}_4 = \dot{V}_3 + z_4\dot{z}_4 + s_4\dot{s}_4 - \frac{1}{\gamma_4}\tilde{W}_4^T\dot{\hat{W}}_4 - \frac{\bar{\eta}}{\gamma_\varpi}\tilde{\varpi}\dot{\hat{\varpi}} - \frac{1}{\gamma_r}\tilde{r}\dot{\hat{r}} \leq -\sum_{i=1}^{3}c_iz_i^{2p} + M_3 + \sum_{i=1}^{3}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \frac{1}{2}z_4^2 + \sum_{i=2}^{4}\frac{1}{2}s_i^2 + \frac{1}{2}\|e\|^2 + z_4(\bar{h}\eta u_m + \bar{h}r^* + \alpha_4 - \alpha_4 + z_4 + k_4e_1 + \hat{W}_4^T\Phi_4(\hat{x}) - \dot{\omega}_4) + \sum_{i=2}^{4}\frac{1}{2}\tilde{W}_i^T\tilde{W}_i + \sum_{i=2}^{4}s_i\left(-\frac{s_i}{\kappa_i} + B_i\right) + \frac{\tilde{W}_4^T}{\gamma_4}(\gamma_4z_4\Phi_4(\hat{x}) - \dot{\hat{W}}_4) - \frac{\bar{\eta}}{\gamma_\varpi}\tilde{\varpi}\dot{\hat{\varpi}} - \frac{1}{\gamma_r}\tilde{r}\dot{\hat{r}} \tag{53}$$

where $B_4 = (2p-1)c_3z_3^{2p-2} + k_3\dot{e}_1 + \frac{5+\tau}{2}\dot{z}_3 + \dot{\hat{W}}_3^T\Phi_3(\hat{\bar{x}}_3) + \frac{\hat{W}_3^T\partial\Phi_3}{\partial\hat{\bar{x}}_3}\dot{\hat{\bar{x}}}_3 + \frac{\dot{s}_4}{\kappa_4}$.

Construct the virtual controller α_4, the updating law of $\hat{W}_4$ as follows

$$\alpha_4 = c_4z_4^{2p-1} + k_4e_1 + z_4 + \hat{W}_4^T\Phi_4(\hat{x}_4) - \dot{\omega}_4 - \bar{h}\hat{r} \tag{54}$$

$$\dot{\hat{W}}_4 = \gamma_4z_4\Phi_4(\hat{x}) - \sigma_4\hat{W}_4 \tag{55}$$

where c_4 and σ_4 are given positive constants.

Inserting (54)–(55) into (53) yields

$$\dot{V}_4 \leq -\sum_{i=1}^{4}c_iz_i^{2p} + \sum_{i=1}^{4}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \sum_{i=2}^{4}\frac{1}{2}\tilde{W}_i^T\tilde{W}_i + z_4(\bar{h}\eta u_m + \alpha_4) + z_4\bar{h}\tilde{r} + \sum_{i=2}^{4}s_i\left(-\frac{s_i}{\kappa_i} + B_i\right) + \sum_{i=2}^{4}s_i + \frac{1}{2}\|e\|^2 + M_3 - \frac{\bar{\eta}}{\gamma_\varpi}\tilde{\varpi}\dot{\hat{\varpi}} - \frac{1}{\gamma_r}\tilde{r}\dot{\hat{r}} \tag{56}$$

Design the controller u_m,, the updating laws of $\hat{\varpi}$ and $\hat{r}$ as

$$u_m = -\hat{\varpi}\alpha_4 \tag{57}$$

$$\dot{\hat{\varpi}} = \bar{h}z_4\gamma_\varpi\alpha_4 - \sigma_\varpi\hat{\varpi} \tag{58}$$

$$\dot{\hat{r}} = z_4\bar{h}\gamma_r - \sigma_r\hat{r} \tag{59}$$

where $\sigma_\varpi > 0$ and $\sigma_r > 0$ are design parameters.

By using (57), we obtain

$$z_4\alpha_4 - \bar{h}\eta z_4\hat{\varpi}\alpha_4 - \bar{h}\bar{\eta}z_4\tilde{\varpi}\alpha_4 \leq 0 \tag{60}$$

Then, one obtains

$$\dot{V}_4 \leq -\sum_{i=1}^{4}c_iz_i^{2p} + \sum_{i=2}^{4}s_i\left(-\frac{s_i}{\kappa_i} + B_i\right) + \sum_{i=1}^{4}\frac{\sigma_i}{\gamma_i}\tilde{W}_i^T\hat{W}_i + \sum_{i=2}^{4}\frac{1}{2}\tilde{W}_i^T\tilde{W}_i + \sum_{i=2}^{4}s_i + \frac{1}{2}\|e\|^2 + M_3 + \frac{\sigma_\varpi\bar{\eta}}{\gamma_\varpi}\tilde{\varpi}\hat{\varpi} + \frac{\sigma_r}{\gamma_r}\tilde{r}\hat{r} \tag{61}$$

The above control algorithm configuration is shown in Fig. 3.

4 Stability analysis

This section will analyze and prove the properties that the designed output feedback prescribed performance fuzzy FTC method has properties of the following theorem.

Theorem 1 *For the steer-by-wire vehicle system* (5), *with the Assumption* 1 *and* 2, *the fuzzy state observer* (20), *the virtual controllers* (33), (42), (48), (54), *the controller* (57), *and the parameter updating laws* (34), (43), (55), (58), (59), *the following properties are guaranteed.*

1. The controlled steer-by-wire vehicle system is SGPFS.
2. The tracking error converges to a small neighborhood of zero in a finite-time interval and does not exceed the prescribed performance bound.

Proof Take the whole Lyapunov function as

$$V = V_0 + V_4 \tag{62}$$

From (24) and (61), the time derivative of V is

$$\begin{aligned}\dot{V} \le\ & -\lambda_1\|e\|^2 - \sum_{i=1}^{4} c_i z_i^{2p} + \|P\|^2 \sum_{i=1}^{4} \tilde{W}_i^T \tilde{W}_i \\ & + \sum_{i=1}^{4} \frac{\sigma_i}{\gamma_i} \tilde{W}_i^T \hat{W}_i + \sum_{i=2}^{4} \frac{1}{2} \tilde{W}_i^T \tilde{W}_i + \sum_{i=2}^{4} s_i\left(-\frac{s_i}{\kappa_i} + B_i\right) \\ & + \sum_{i=2}^{4} s_i + M_4 + \frac{\sigma_\varpi \bar{\eta}}{\gamma_\varpi} \tilde{\varpi}\hat{\varpi} + \frac{\sigma_r}{\gamma_r} \tilde{r}\hat{r}\end{aligned} \tag{63}$$

where $\lambda_1 = \lambda_0 - \frac{1}{2}$ and $M_4 = M_0 + M_3$.

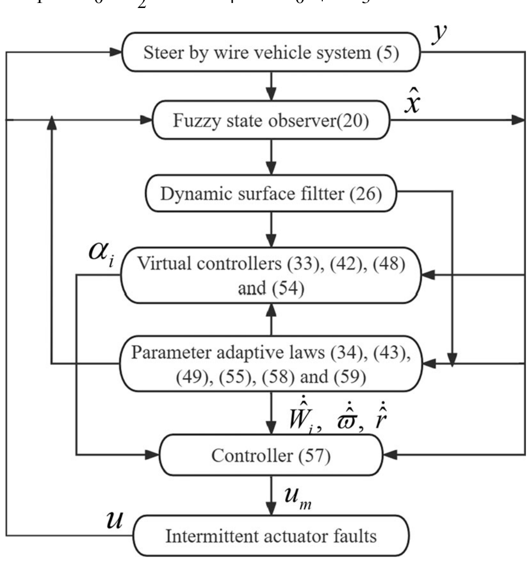

Fig. 3 Structure diagram of SBWV system

From [25], we define the set $\mathbb{Z} = \{e^T P e + \frac{1}{2}\sum_{i=1}^{4}(z_i^2 + \frac{1}{\gamma_i}\tilde{W}_i^T \tilde{W}_i) + \frac{1}{2}\sum_{i=2}^{4} s_i^2 + \frac{\bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2 + \frac{1}{2\gamma_r}\tilde{r}^2 \le \delta\}$, where $\delta > 0$ is a constant , which satisfies that $V(0) \le \delta$. Since $\mathbb{Z}$ is a compact in R^{17} with 17 denoting the dimension of $\mathbb{Z}$, function B_i is continuous on $\mathbb{Z}$, we have $\bar{B_i} \ge \|B_i\|$. where $\bar{B_i}$ is constant.

By using Youngs inequality, we have

$$\frac{\sigma_i}{\gamma_i}\tilde{W}_i \hat{W}_i \le \frac{\sigma_i}{\gamma_i} W_i^{*T} W_i^{*T} - \frac{\sigma_i}{\gamma_i}\tilde{W}_i^T \tilde{W}_i^T \tag{64}$$

$$\tilde{\varpi}\hat{\varpi} \le \frac{1}{2}\varpi^{*2} - \frac{1}{2}\tilde{\varpi}^2 \tag{65}$$

$$s_i B_i(\cdot) \le \frac{1}{2} s_i^2 + \frac{1}{2}\bar{B}_i^2 \tag{66}$$

$$\tilde{r}\hat{r} \le \frac{1}{2} r^{*2} - \frac{1}{2}\tilde{r}^2 \tag{67}$$

Substituting (64)–(67) into (63) yields

$$\begin{aligned}\dot{V} \le\ & -\lambda_1(Q)\|e\|^2 - \sum_{i=1}^{4} c_i z_i^{2p} - \sum_{i=2}^{4}\left(\frac{1}{\kappa_i} - 1\right) s_i^2 + M_4 \\ & - \frac{\sigma_\varpi \bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2 - \frac{\sigma_r}{2\gamma_r}\tilde{r}^2 - \frac{1}{2}\left(\frac{\sigma_1}{\gamma_1} - 2\|P\|^2\right)\tilde{W}_1^T \tilde{W}_1 \\ & - \frac{1}{2}\sum_{i=2}^{4}\left(\frac{\sigma_i}{\gamma_i} - 2\|P\|^2 - 1\right)\tilde{W}_i^T \tilde{W}_i\end{aligned} \tag{68}$$

where $M = \sum_{i=1}^{4}\frac{\sigma_i}{2\gamma_i} W_i^{*T} W_i^* + \sum_{i=2}^{4}\frac{1}{2}\bar{B}_i^2 + \frac{\sigma_\varpi \bar{\eta}}{2\gamma_\varpi}\tilde{\varpi} + \frac{\sigma_r}{2\gamma_r}\tilde{r}^2 + M_4$.

Choose $\bar{c_j} = \min\{\sigma_1 - 2\|P\|^2\gamma_1, \sigma_i - \gamma_i(2\|P\|^2 + 1), \frac{1}{\kappa_i} - 1, \sigma_\varpi, \sigma_r\}$, $j = 1, \ldots, 4$. Then, (68) can be rewritten as

$$\begin{aligned}\dot{V} \le\ & -\frac{\lambda_1}{\lambda_{\min}(Q)}\left(\lambda_{\min}(Q)\|e\|^2\right)^p - \lambda_1\|e\|^2 \\ & + \frac{\lambda_1}{\lambda_{\min}(Q)}\left(\lambda_{\min}(Q)\|e\|^2\right)^p \\ & - \sum_{i=1}^{4} c_i z_i^{2p} - \bar{c_j}\left(\sum_{i=1}^{4}\frac{1}{2\gamma_i}\tilde{W}_i^T \tilde{W}_i\right)^p \\ & + \bar{c_j}\left(\sum_{i=1}^{4}\frac{1}{2\gamma_i}\tilde{W}_i^T \tilde{W}_i\right)^p - \bar{c_j}\sum_{i=1}^{4}\frac{1}{2\gamma_i}\tilde{W}_i^T \tilde{W}_i \\ & - \bar{c_j}\sum_{i=2}^{4} s_i^{2p} + \bar{c_j}\sum_{i=2}^{4} s_i^{2p} - \bar{c_j}\sum_{i=2}^{4} s_i^2 - \bar{c_j}\left(\frac{\bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2\right)^p \\ & + \bar{c_j}\left(\frac{\bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2\right)^p + \frac{\bar{c_j}\bar{\eta}}{2\gamma_\varpi}\tilde{\varpi}^2 - \bar{c_j}\left(\frac{1}{2\gamma_r}\tilde{r}^2\right)^p \\ & + \bar{c_j}\left(\frac{1}{2\gamma_r}\tilde{r}^2\right)^p + \frac{\bar{c_j}}{2\gamma_r}\tilde{r}^2 + M_4\end{aligned} \tag{69}$$

By using the following inequality

Table 1 Parameters of the system simulation

Physical parameters	Values
m (kg)	2000
V_x (km/h)	30
B_{sm}	0.018
I_z (kg m^2)	1560
l_f, l_r (m)	1.0151, 1.8951
t_m, t_p (m)	0.016, 0.028
C_f, C_r (N/m)	30,000
J_{fw} (kg m^2)	3.8
J_{sm} (kg m^2)	0.0045
μ	20
F_s	5

$$|x|^{\imath}|y|^{r} \leq \frac{\imath}{\imath+r}\varsigma|x|^{\imath+r} + \frac{r}{\imath+r}\varsigma^{-\frac{\imath}{r}}|y|^{\imath+r} \tag{70}$$

and selecting $x = 1$, $\imath = 1-p$, and $\varsigma = p^{p/(1-p)}$, we can obtain the following inequalities

$$\sum_{k=2}^{4} s_k^{2p} \leq (1-p)\varsigma + \sum_{k=2}^{4} s_k^2 \tag{71}$$

$$\left(\sum_{k=1}^{4}\frac{1}{2\gamma_k}\tilde{W}_k^T\tilde{W}_k\right)^p \leq (1-p)\varsigma + \sum_{k=1}^{4}\frac{1}{2\gamma_k}\tilde{W}_k^T\tilde{W}_k \tag{72}$$

$$\left(\frac{\bar{\eta}}{2\gamma_{\varpi}}\tilde{\varpi}^2\right)^p \leq (1-p)\varsigma + \frac{\bar{\eta}}{2\gamma_{\varpi}}\tilde{\varpi}^2 \tag{73}$$

$$\left(\lambda_{\min}(Q)\|e\|^2\right)^P \leq (1-p)\varsigma + \lambda_{\min}(Q)\|e\|^2 \tag{74}$$

$$\left(\frac{1}{2\gamma_r}\tilde{r}^2\right)^p \leq (1-p)\varsigma + \frac{1}{2\gamma_r}\tilde{r}^2 \tag{75}$$

Substituting (71)–(75) into (70), one has

$$\begin{aligned}\dot{V} \leq & -\frac{\lambda_1}{\lambda_{\min}(Q)}\left(\lambda_{\min}(Q)\|e\|^2\right)^p - \sum_{i=1}^{4} c_i z_i^{2p} \\ & - \bar{c}_j\left(\sum_{i=1}^{4}\frac{1}{2\gamma_i}\tilde{W}_i^T\tilde{W}_i\right)^p - \bar{c}_j\sum_{i=2}^{4} s_i^{2p} \\ & + \bar{c}_j\tilde{\varpi}^{2p} + \bar{c}_j\tilde{r}^{2p} + D\end{aligned} \tag{76}$$

where $D = M_4 + 5(1-p)\varsigma$.

Then, choose $C = \min\left\{\frac{\lambda_1}{\lambda_{\min}(Q)}, 2^p c_i, \bar{c}_j, 2^p\bar{c}_j\right\}$, (76) can be rewritten as

$$\dot{V} \leq -CV^p + D \tag{77}$$

From the proof in [20], for $\forall 0 < \Upsilon < 1$, we obtain the reaching time as follows:

$$T_r = \frac{1}{\mu C(1-p)} \left\{V^{1-p}(z(0), \tilde{W}(0), \tilde{\varpi}(0), \tilde{r}(0)) - \left(\frac{D}{C(1-p)}\right)^{(1-p)/p}\right\} > 0 \tag{78}$$

In (78), $z(0) = [z_1(0), \ldots, z_4(0)]^T$, and $\tilde{W}(0) = \left[\tilde{W}_1(0), \ldots, \tilde{W}_4(0)\right]^T$. By the definition of $V(\aleph) \leq (\aleph = [z, \tilde{W}, \tilde{\varpi}])$, we can obtain $\|\aleph\| \leq \sqrt{2(\frac{D}{(1-\Upsilon)C})^{1/p}}$, then we have the closed-loop system is SGPFS. Combining $\mu(t) = (\mu_0 - \mu_\infty)e^{-ct} + \mu_\infty$ with $-\rho_{\min}\mu(t) < \chi(t) < \rho_{\max}\mu(t)$, we can obtain $\chi(t) \leq \max\{\rho_{\min}\mu_0, \rho_{\max}\mu_0\}$.

Therefore, from the previous discussion, the following guideline for the controller design and parameter selection is given as

Step 1: Define the fuzzy IF-THEN rules and membership functions [24], and determine the fuzzy basis functions, and then the FLSs.

Step 2: Specify vector $K = [k_1, k_2, k_3, k_4]^T$, such that A is a strict Hurwitz matrix.

Step 3: Specify a positive definite matrix Q and by solving the Lyapunov Eq. (22), a positive definite matrix P is obtained.

Step 4: The design parameter κ_i in nonlinear filter (26) is chosen as $\kappa_i > 0$.

Step 5: Select the design parameters $0 < p < 1$ and design virtual controllers (33), (42), (48) (54) and actual controller (57).

Step 6: Select the design parameters γ_i and design adaptive laws (34), (43), (49), (55), (58), (59).

Remark 6 From (78), we know that the tracking error $|y - y_d|$ and the settling time T_r depend on the parameters

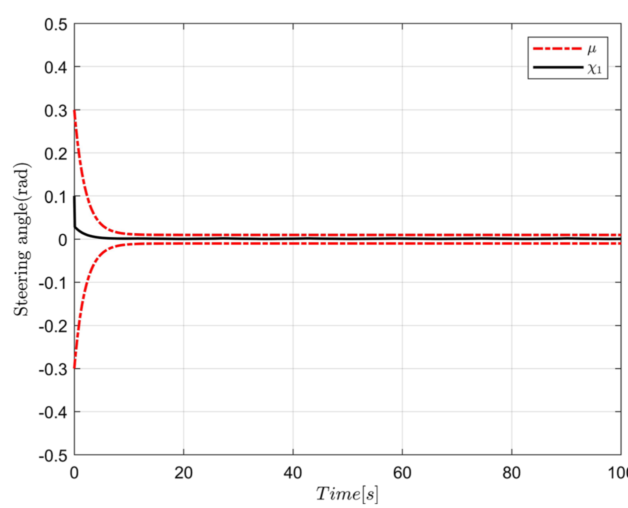

Fig. 4 The tracking error and performance bounds

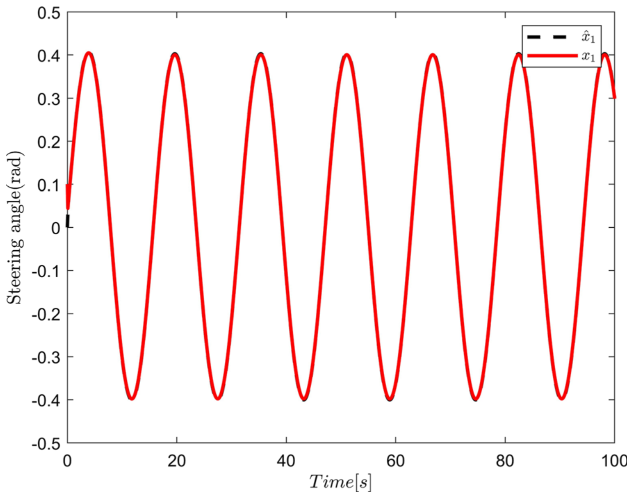

Fig. 5 The trajectories of the x_1

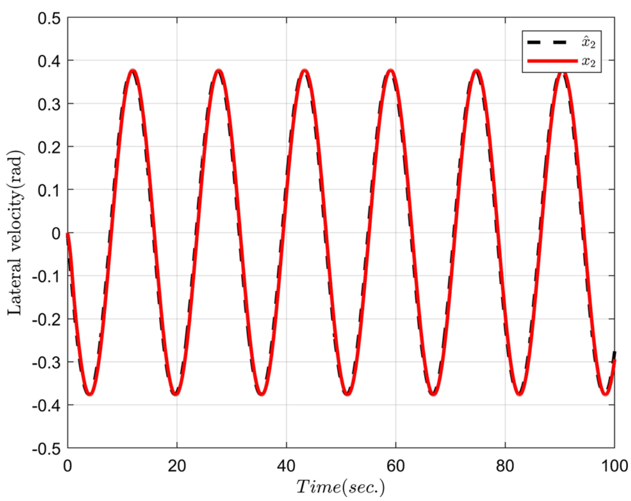

Fig. 6 The trajectories of the x_2

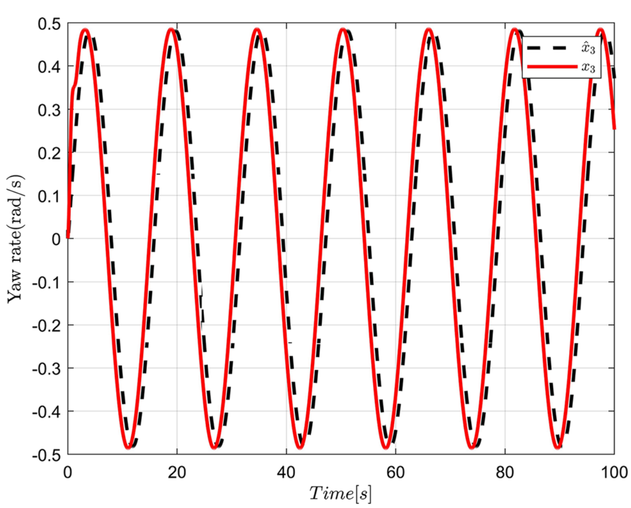

Fig. 7 The trajectories of the x_3

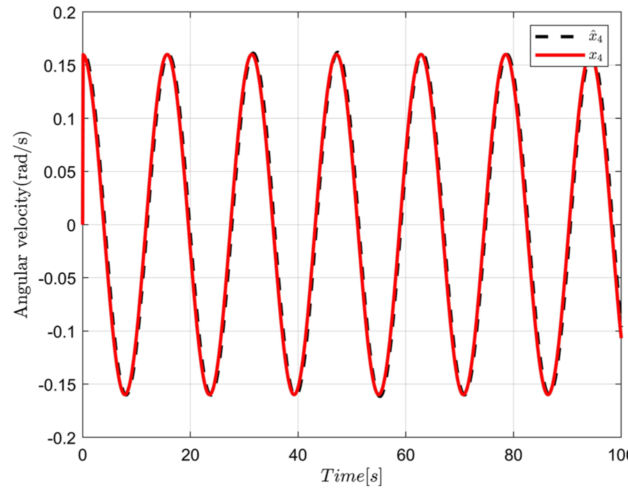

Fig. 8 The trajectories of the x_4

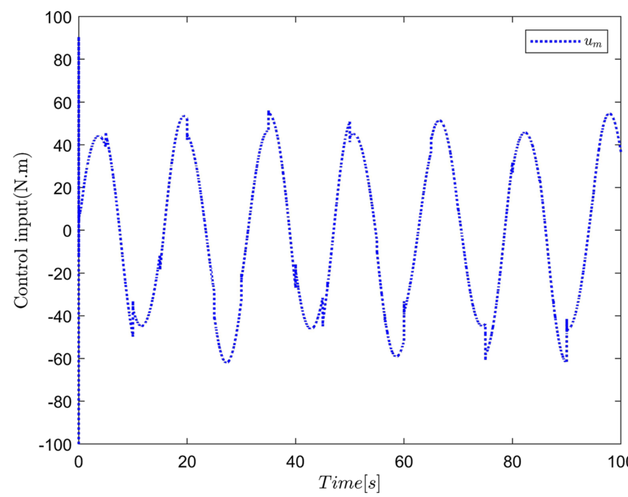

Fig. 9 The trajectory of controller u_m

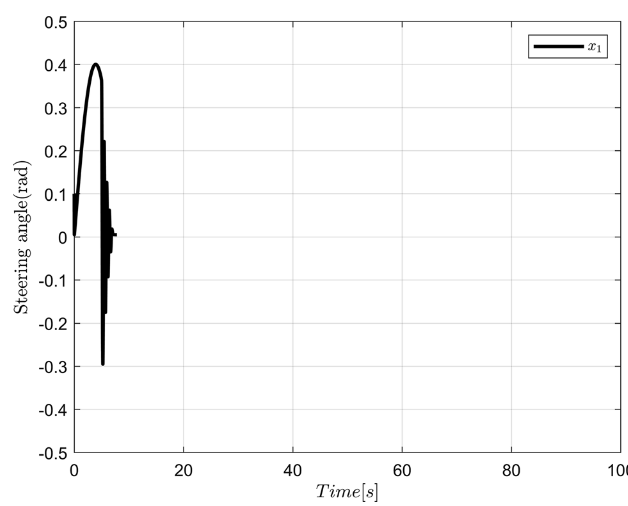

Fig. 10 The trajectory of steering angle

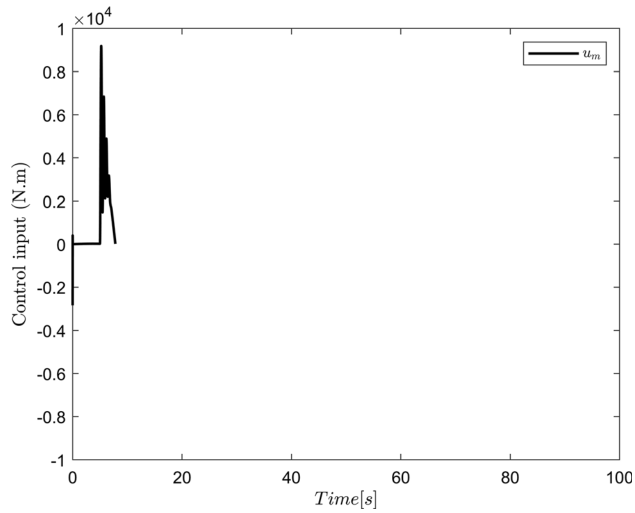

Fig. 11 The trajectory of controller u_m

Table 2 Time comparisons of two approaches

Simulation time (s)	Program execution time of the controller with DSC technology (s)	Program execution time of the controller without DSC technology (s)
30	1.78	2.54
50	2.56	4.76
100	4.98	7.23

p, C and$D$. However, from the definitions of C and D in (69), (76) and (77), we know that by increasing the design parameters γ_i, c_i, $\lambda_{\min}(Q)$ or decreasing the design parameters σ_i and p makes T_r, tracking error $|y - y_d|$ can be made to be smaller. Notice that smaller tracking errors would result in higher control energy. Therefore, in practice, there should be a tradeoff between improved tracking performance and control energy.

5 Simulation studies

In this section, we check the effectiveness of the presented fuzzy finite-time control method via computer simulation. The parameter selections of SBWV system (5) are shown by Table 1 [9, 12].

The reference function and the external disturbance are chosen as $y_d = 0.4\sin 0.4t$, $d(t) = 5\sin 0.5t$.

The prescribed performance function is defined as follows:

$\mu(t) = (0.3 - 0.01)e^{-0.5t} + 0.01$

The intermittent actuator fault model is defined as follows:

$u(t) = \eta(t)u_m(t) + u_k(t)$

where $\eta(t) = 0.8$, $u_k(t) = 5$ and $t \in [kT^*, (k+1)T^*)$, $k = 1, 3, 5, \ldots$, $T^* = 5\,s$.

We design five If-Then fuzzy rules:

$\Re^l$: if x_1 is G_1^l and x_2 is G_2^l and x_3 is G_3^l and x_4 is G_4^l, then y is H^q, $q = 1, 2, \cdots, 5$.

where the fuzzy membership functions of G_i^q, $i = 1, 2, 3, 4$ are chosen as $\mu_{G_i^1}(x_i) = \mathrm{e}^{(-(x_i-2)^2/5)}$, $\mu_{G_i^2}(x_i) = \mathrm{e}^{(-(x_i-1)^2/5)}$, $\mu_{G_i^3}(x_i) = e^{(-(x_i)^2/5)}$, $\mu_{G_i^4}(x_i) = \mathrm{e}^{(-(x_i+1)^2/5)}$, $\mu_{G_i^5}(x_i) = \mathrm{e}^{(-(x_i+2)^2/5)}$.

According to [24], we construct FLSs $\hat{f}_i(x|\hat{W}_i) = \hat{W}_i^T\Phi_i(x)$, $(i = 1, \ldots, 4)$ to approximate $f_i(x)$ in system (5).

The observer gain vector (20) is designed as $K = [k_1, k_2, k_3, k_4]^T = [10, 160, 160, 500]^T$, then A is a Hurwitz matrix. For given a definite matrix $Q = 7I$, by solving Lyapunov equation $A^TP + PA = -7I$, we obtain

$$P = \begin{bmatrix} 0.4030 & 0.5302 & 1.6858 & 0.0070 \\ 0.5302 & 68.0998 & 81.3344 & 201.5808 \\ 1.6858 & 81.3344 & 36.1595 & 266.2275 \\ 0.0070 & 201.5808 & 266.2275 & 844.0186 \end{bmatrix}$$

In this simulation, the all design parameters in filters, virtual controller α_i, real controller u_m and parameter updating laws $\dot{\hat{W}}_i, \dot{\hat{\varpi}}, \dot{\hat{r}}$ are chosen as $\kappa_i = 0.1$, $p = 0.95$, $\tau = 1$, $c_1 = 100$, $c_2 = 50$, $c_3 = 60$, $c_4 = 70$, $\gamma_i = 1$, $\sigma_i = 1$.

The initial condition of variables and adaptive parameters of the controlled system are selected as $x_1(0) = 0.1$, $x_i(0) = 0$, $\hat{x}_i(0) = 0$ and $\hat{W}_i(0) = 0$. The simulation results are shown in Figs. 4, 5, 6, 7, 8, 9, 10. Figure 4 shows the responses of tracking error and performance bounds. Figures 5, 6, 7, 8 show the trajectories of steering angle, the vehicle sideslip angle, the yaw rate, and angular acceleration and their estimates. Figure 9 shows the trajectories of the control input u_m.

From Figs. 4, 5, 6, 7, 8, 9, it can be concluded that the proposed control scheme can ensure that the steer-by-wire vehicle system (5) is SGPFS, and the steering angle can track the expected trajectory within a finite-time interval and does not exceed the prescribed performance bound, while the vehicle always maintains a stable driving state.

Further, to demonstrate the robustness against actuator failures of the proposed control method, we make a simulation comparison with fuzzy control without fault-tolerant technology. In the simulation, $x_i(0)$, $\hat{x}_i(0)$ and $\hat{W}_i(0)$ are selected those in the above simulation. The simulation results are shown in Figs. 10 and 11 that in the presence of intermittent actuator faults, the stability of the controlled system cannot be guaranteed.

Remark 7 Furthermore, to illustrate this study can reduce the computational complexity problem, we compare with the adaptive fuzzy control method without using DSC technology in control design. We set the simulation time to 30, 50, 100 s, respectively. Then the corresponding program execution times are shown in Table 2. From Table 2, we see clearly that the proposed adaptive fuzzy DSC control approach in this paper uses much less program execution time to achieve stability than the adaptive control method without DSC technology.

6 Conclusion

This paper has investigated the problem of output-feedback fuzzy adaptive finite-time prescribed performance FTC design for the steer-by-wire vehicle system with immeasurable states and intermittent actuator faults. FLSs are used to model the uncertain SBWV system and a fuzzy state observer is formulated to estimate the immeasurable states. By adopting the adaptive backstepping control design technique, a finite-time adaptive fuzzy prescribed performance output feedback FTC approach has been developed. Based on Lyapunov theory of finite-time stability, the stability analysis has been provided. The main advantages of the presented control scheme can ensure that the controlled steer-by-wire vehicle system is SGPFS. Furthermore, it can make the tracking and observer errors converge to a small neighborhood of zero in a finite-time interval and does not violate the prescribed performance bounded. Finally, the computer simulation results and comparisons with the previous control method have demonstrated the effectiveness of the presented control approach.The further study direction will focus on the performance optimization for SBWV system based on this study.

Data availibility Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declared that they have no conflicts of interest to this work.

References

1. Gao HJ, Li ZK, Yu XH, Qiu JB (2022) Hierarchical multiobjective heuristic for PCB assembly optimization in a beam-head surface mounter. IEEE Trans Cybern 52(7):6911–6924
2. Zhuang SL, Li DX, Yu XH, Tong MS, Lin WY, Rodriguez-Andina JJ, Shi Y, Gao HJ (2023) Microinjection in biomedical applications: an effortless autonomous omnidirectional microinjection system. IEEE Ind Electron Mag. https://doi.org/10.1109/MIE.2023.3318831
3. Ji XW, He XK, Lv C, Liu YH, Wu J (2018) Adaptive-neural-network-based robust lateral motion control for autonomous vehicle at driving limits. Control Eng Pract 76:41–53
4. Marino R, Scalzi S, Netto M (2011) Nested PID steering control for lane keeping in autonomous vehicles. Control Eng Pract 19(12):1459–1467
5. Huang XY, Zhang H, Zhang GG, Wang JM (2014) Robust weighted gain-scheduling $H\infty$ vehicle lateral motion control with considerations of steering system backlash-type hysteresis. IEEE Trans Control Syst Technol 22(5):1740–1753
6. Huang C, Naghdy F, Du HP (2019) Sliding mode predictive tracking control for uncertain steer-by-wire system. Control Eng Pract 85:194–205
7. Sun Z, Zheng JC, Man ZH, Fu MY, Lu RQ (2019) Nested adaptive super-twisting sliding mode control design for a vehicle steer-by-wire system. Mech Syst Signal Process 122:658–672
8. Wang YL, Wang YF (2021) Discrete-time adaptive neural network control for steer-by-wire systems with disturbance observer. Expert Syst Appl 183:115395
9. Wang YL, Wang YF, Tie M (2021) Hybrid adaptive learning neural network control for steer-by-wire systems via sigmoid tracking differentiator and disturbance observer. Eng Appl Artif Intell 104:104393
10. Wei LT, Wang XY, Li L, Fan ZX, Dou RZ, Lin JG (2021) T-S fuzzy model predictive control for vehicle yaw stability in nonlinear region. IEEE Trans Veh Technol 70(8):7536–7546
11. Huang C, Naghdy F, Du HP (2017) Fault tolerant sliding mode predictive control for uncertain steer-by-wire system. IEEE Trans Cybern 49(1):261–272
12. Chen H, Tu YD, Wang H, Shi KB, He SP (2022) Fault-tolerant tracking control based on reinforcement learning with application to a steer-by-wire system. J Franklin Inst 359(3):1152–1171
13. Lai GY, Wen CY, Liu Z, Zhang Y, Chen CLP, Xie SL (2018) Adaptive compensation for infinite number of actuator failures based on tuning function approach. Automatica 87:365–374
14. Wu W, Li YM, Tong SC (2023) Neural network output-feedback consensus fault-tolerant control for nonlinear multiagent systems with intermittent actuator faults. IEEE Trans Neural Netw Learn Syst 34(8):4728–4740
15. Huang C, Naghdy F, Du HP (2018) Observer-based fault-tolerant controller for uncertain steer-by-wire systems using the delta operator. IEEE/ASME Trans Mechatron 23(6):2587–2598
16. Luo G, Wang ZZ, Ma BX, Wang YF, Xu JF (2021) Observer-based interval type-2 fuzzy friction modeling and compensation control for steer-by-wire system. Neural Comput Appl 33:10429–10448
17. Zhao J, Yang KH, Cao YC, Liang ZC, Li WF, Xie ZC, Wong PK (2023) Observer-based discrete-time cascaded control for lateral stabilization of steer-by-wire vehicles with uncertainties and disturbances. IEEE Trans Circ Syst I Regul Pap. https://doi.org/10.1109/TCSI.2023.3276945
18. Zhang J, Tong SC, Li YM (2022) Adaptive fuzzy finite-time output-feedback fault-tolerant control of nonstrict-feedback systems against actuator faults. IEEE Trans Syst Man Cybern Syst 52(2):1276–1287
19. Li YM, Li KW, Tong SC (2019) Finite-time adaptive fuzzy output feedback dynamic surface control for MIMO nonstrict feedback systems. IEEE Trans Fuzzy Syst 27(1):96–110
20. Zhou HD, Sui S, Tong CS (2023) Finite-time adaptive fuzzy prescribed performance formation control for high-order nonlinear multiagent systems based on event-triggered mechanism. IEEE Trans Fuzzy Syst 31(4):1229–1239
21. Li YM, Shao XF, Tong SC (2020) Adaptive fuzzy prescribed performance control of nontriangular structure nonlinear systems. IEEE Trans Fuzzy Syst 28(10):2416–2426

22. Iqbal J, Zuhaib KM, Han C, Khan AM, Ali MA (2017) Adaptive global fast sliding mode control for steer-by-wire system road vehicles. Appl Sci 7(7):738
23. Yu M, Wang ZC, Wang H, Jiang WH, Zhu RS (2023) Intermittent fault diagnosis and prognosis for steer-by-wire system using composite degradation model. IEEE J Emerg Select Top Circ Syst 13(2):557–571
24. Wang L-X (1993) Stable adaptive fuzzy control of nonlinear systems. IEEE Trans Fuzzy Syst 1(2):146–155
25. Swaroop D, Hedrick JK, Yip PP, Gerdes JC (2000) Dynamic surface control for a class of nonlinear systems. IEEE Trans Autom Control 45(10):1893–1899