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ORIGINAL ARTICLE

A fuzzy computing approach to aggregate expert opinions using parabolic and exparabolic approximation procedures for solving multi-criteria group decision-making problems

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Abstract

Triangular fuzzy numbers (TFNs) are widely used for selection problems to determine expert opinions using linguistic expressions. Some aggregation procedures are developed to determine expert opinions more accurately. However, there is a need for a simple and more useful procedure to solve the selection problems more suitably. For this purpose, our study offers a triangular, exparabolic, and parabolic area calculation-based approximation approach for TFNs to aggregate the possible hedges (very and more or less) for TFNs. Hence, this aggregation procedure provides a tuning opportunity for classical TFN expressions to capture possible tuning processes to reflect the hesitancies of experts. The technique for order preferences by similarity to ideal solution (TOPSIS) method is applied in the two studies from extant literature, and suitable alternatives are determined as a result of the ranking process. Finally, a comparative analysis is presented to illustrate the efficiency of the proposed procedure. The conventional TOPSIS model's ranking scores are very close for exemplified examples (i.e., 0.5308, 0.4510, 0.4550 and 0.5304, 0.4626, 0.4940), but the proposed model's result has fluctuated for the same examples (i.e., 0.346, 0,669, 0,567 and 0.208, 0.991, 0.148). So, the main advantage of the proposed aggregation procedure is the alternative ranking scores separation capability analyzed with their linguistic diversification.

Keywords Exparabolic approximation · Selection problem · TOPSIS · Fuzzy decision making

List of symbols

TFNs	Triangular fuzzy numbers
TOPSIS	The technique for order preferences by similarity to ideal solution
MCDM	Multi-criteria decision-making
GRA	Grey relational analysis
VIKOR	VIseKriterijumska Optimizacija I Kompromisno Resenje: multi-criteria optimization and compromise solution
PROMETHEE	Preference ranking organization method for enrichment of evaluations
MOORA	Multi-objective optimization on the basis of ratio analysis
a_i	Alternative
C_i	Criteria
n	The number of criteria
k	The number of alternatives
a	Lover value of the parabolic or exparabolic shapes-based TFNs
b	Medium value of the parabolic or exparabolic shapes-based TFNs
c	Upper value of the parabolic or exparabolic shapes-based TFNs
l	Lover value of the TFN
m	Medium value of the TFN
u	Upper value of the TFN
A^*	Positive ideal solution
A^-	Negative ideal solution
D^+	Separation measure for positive ideal solution
D^-	Separation measure for negative ideal solution
I	Benefit type measure
I'	Cost type measure
C_i^*	Ranking score
W	Criteria Weight
w_j	The aggregate criteria weight
V_{ij}	Weighted normalized decision matrix

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$\widetilde{D}$	Fuzzy decision matrix
$\widetilde{A}, \widetilde{B}$	Fuzzy triangular numbers
x_{ij}	Aggregate value for the area
$\widetilde{x}_{ij}$	The TFN equivalent of the linguistic performance value of the alternative
$\mu(x)$	Membership function for the TFN
$\mu_{A_i^k}(x)$	Membership function for the exparabolic or parabolic TFN
x_N	The characteristics for the selection problem
A_N^k	The linguistic values used to discretize the continuous value of the criteria
r_s	Spearman's rank correlation coefficient
d_i	The difference between the two ranks of each observation
t	The number of observations
A	Total area for exparabolic shape TFN
B	Cross lined segment area for special segments
D	Total area for more or less segment
C	Semi-area for semi-parabolic area segment

1 Introduction

Nowadays, selection problems are becoming a more important problem type in multi-criteria decision-making (MCDM) environments due to the sharp competition in the different sectors. Selection problems are widely used in many areas, such as machine-tool selection, robot selection, manufacturing process selection, supplier selection, personnel selection, material selection, etc. Developments of new technologies impact the products and human capabilities to complete tasks more rapidly and efficiently. So, alternative capabilities are rapidly growing, and rating alternatives is a complex issue in the MCDM-based selection problems. We must assign criteria weights and determine alternatives' ratings to set MCDM-based selection models. In these stages, expert opinions are very crucial to select the most appropriate alternative. In the criteria weight assignment and alternative ranking process, linguistic expressions of experts are very critical to assigning more accurate criteria weights and alternative rating scores.

The fuzzy set theory-based applications are incorporated into the MCDM models to cope with this complex issue. In the literature, TFN-based aggregation methods are widely used to assign experts' linguistic expressions. For example, Memari et al. [18] proposed an intuitionistic fuzzy TOPSIS model to select the right sustainable supplier for automotive spare parts producer firms. Their presented model determined sustainable ranking scores of suppliers through a case study. Rahimdel and Karamoozian [21] used the technique for order preferences by similarity to the ideal solution (TOPSIS) model with fuzzy set theory to select the best primary crusher for the iron mine. Chu and Lin [8] presented a fuzzy TOPSIS approach to robot selection problems using TFN-based linguistic expressions. The membership function of each weight was determined by interval procedure for TFNs. Li et al. [14] presented an indicator system and a method for data integration by evaluating the specifications and role of third-party logistics, for 3PL provider selection. They established a comprehensive analysis approach for 3PL suppliers based on fuzzy sets. Lam et al. [13] proposed a fuzzy principal component analysis approach for solving the material supplier selection problem. They used TFNs to quantify the experts' expressions. Then, they employed principal component analysis to compress the criteria data and eliminate the multi-collinearity among them. Li et al. [15] developed a fuzzy portfolio selection model with background risk based on the definitions of the possibilistic return and possibilistic risk. They used LR-type possibility distribution for the returns of assets. Amindoust et al. [1] determined the sustainable supplier selection criteria and sub-criteria and presented an approach to rank suppliers. They used the fuzzy inference system-based ranking model to handle the subjectivity of experts' expressions for the selection problem. Liu [16] integrated fuzzy quality function deployment and the prototype product selection model to develop a product design and selection. They adopted the α-cut operation in the fuzzy quality function deployment model to determine the fuzzy set of each alternative. Keršulienė and Turskis [11] proposed a fuzzy MCDM model using the fuzzy information fusion principles, additive ratio assessment, and step-wise weight assessment ratio analysis models to select an architect. Their aggregation process was based on the unification of information using fuzzy sets on a basic linguistic term set. Mougouei and Powers [19] proposed a cost–value approach that considers the impacts of value-related requirement dependencies on the value of selected optimal requirements. They exploited the algebraic structure of fuzzy graphs for modeling value-related requirement dependencies and their strengths. Chan and Prakash [5] proposed a maintenance policy selection model at the level of the firm rather than the equipment level. Some selection criteria were crisp values, whereas others were obtained in linguistic terms. They presented a distance-based fuzzy MCDM model to select the appropriate maintenance policy. Their MCDM model was suitable for integrating data, in the form of linguistic variables, TFNs, and crisp numbers, into the analyzing study of maintenance policy alternatives. Chen and Lin [7] proposed a fuzzy geometric mean decomposition-based fuzzy MCDM method to enhance the flexibility of the fuzzy decision matrix. They used fuzzy subjudgment matrices to diversify the original fuzzy judgment matrix. Their presented approach was used to select the smart technology applicant for supporting mobile health care during and after the COVID-19 pandemic. Chai et al. [4] developed a fuzzy MCDM model based on the cumulative prospect theory, interval-valued fuzzy sets, and a combination of intuitionistic fuzzy sets for supplier selection problems. Huang et al. [9] introduced the patent value evaluation model. They considered the fuzziness of decision makers' expressions and the uncertainty of patent indicators and proposed a TFN-TOPSIS model based on the possibility degree relationship model. Chisale and Lee [3] prioritized barriers to renewable energy acceleration in Malawi using the analytic hierarchy process (AHP) and fuzzy TOPSIS combined model. They used TFNs to represent the expert's subjective judgments. Zhang et al. [26] used the fuzzy-TOPSIS model to obtain the hexagonal close-packed metallic crystal best structure among all structure alternatives when investigated under more than one criterion evaluating the TFNs.

We can see from the literature that the aggregation procedures in the selection problems using MCDM models depend on the linguistic expressions of the expert opinions. The fuzzy extensions of the TOPSIS model are developed in the literature to overcome model uncertainty since human judgments in real-life studies. They used aggregation procedures to aggregate different expert opinions in a useful way. However, the presented procedures have some disadvantages. They used fuzzy number characteristics-based aggregation procedures and ignored the prefixed membership function extensions or modifiers that were "very" and "more or less". In this work, we described an exparabolic and parabolic shape area calculation based on the TFN linguistic expression aggregating procedure for fuzzy TOPSIS models to fill this gap. Furthermore, this procedure provides the best alternative using extended linguistic hedges for the linguistic expressions, hence maintaining the descriptive capabilities of the fuzzy TFNs. The proposed aggregating procedure is very simple, reflects the classical TFN values in more extendable judgments, and provides a more tolerable way. Also, the provided aggregation procedure is capable of differentiating ranking results of alternatives that have more adjacent scores among them in the classical TFNs-based fuzzy TOPSIS models.

2 Methodology

2.1 Fuzzy TFN

In this study, triangular fuzzy numbers are used for linguistic expressions (see Fig. 1) [6, 22]. The fuzzy numbers are defined using the membership function ($\mu(x)$) ranging from 0 to 1. A TFN is illustrated in Fig. 1, represented by (l, m, u), where l, m, and u are the smallest, medium, and largest possible values, respectively. The linear TFN can be defined with a membership function as follows:

$$\mu(x) = \begin{cases} 1, & x = m \\ \dfrac{x-l}{m-l}, & l \leq x \leq m \\ \dfrac{u-x}{u-m}, & m \leq x \leq u \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

Fuzzy arithmetic for TFNs are as follows:

(i) TFN operations [6]:

Let $\tilde{A} = (l_1, m_1, u_1)$, and $\tilde{B} = (l_2, m_2, u_2)$ are positive TFN numbers, the arithmetic operations are as follows:

$$\tilde{A} \oplus \tilde{B} = (l_1 + l_2,\ m_1 + m_2,\ u_1 + u_2) \tag{2}$$

$$\tilde{A} \otimes \tilde{B} = (l_1 \times l_2,\ m_1 \times m_2,\ u_1 \times u_2) \tag{3}$$

$$\tilde{A} \oslash \tilde{B} = (l_1/u_2,\ m_1/m_2,\ u_1/l_2) \tag{4}$$

2.2 The new exparabolic and parabolic area calculation-based aggregation approach for TFNs

Reflecting the optimal meaning and appropriate system behavior for a given linguistic expression is difficult, even for problem-trained experts. This is a decisive factor in multi-criteria decision-making processes. The semantic representation of linguistic expressions creates a numerical value about the relevance of the concept that the expression represents. The well-known prefixed membership function modifiers are "very" and "more or less". The first modifier

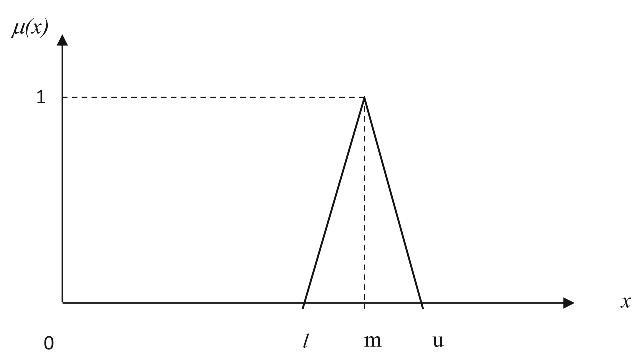


Fig. 1 Triangular fuzzy number (TFN)

causes a decrease in the membership degree of a value in the fuzzy set. The second modifier is a "more or less" fuzzy expansion operator because it increases the degree of membership [25]. These functions are

$$\mu_{\text{very}}A_i^k(x) = \left(\mu_{A_i^k}(x)\right)^2 \tag{5}$$

$$\mu_{\text{moreorless}}A_i^k(x) = \sqrt{\mu_{A_i^k}(x)} \tag{6}$$

where $x,\ldots,x_N$ are the characteristics for the selection problem, $A_1^k,\ldots,A_N^k$ are linguistic values used to discretize the continuous value of the criteria. The illustration of their effects on a normalized fuzzy set with TFN is shown in Fig. 2.

We can offer a new expression for the aggregation procedure for "more or less" or "very" hedges using some geometric properties of area elements (Fig. 3). These properties use aggregation operations to convert linguistic terms as crisp equivalents. Exparabolic and parabolic area calculations are a suitable way to convert linguistic terms to a crisp value.

Let $\widetilde{A}$=(a, b, c), a fuzzy triangular number, the aggregated equivalent of exparabolic shape (Fig. 4) can be expressed as follows:

Total area for exparabolic is

$$A = \frac{b-a}{3} + \frac{c-b}{3} = \frac{c-a}{3} \tag{7}$$

To aggregation process, we can divide the medium number of the TFN to this area:

$$\begin{aligned}\text{Aggregate value for exparabolic area} &= b/A \\ &= b/\left(\frac{c-a}{3}\right)\end{aligned} \tag{8}$$

To calculate cross lined segment area (Fig. 5), we express following equations:

$$B = [(b-a)\times(1)] - \left[\frac{b-a}{3} + \frac{b-a}{3}\right] \tag{9}$$

$$B = [(b-a)\times(1)] - \left(\frac{2b-2a}{3}\right) = \frac{5}{3}(b-a) \tag{10}$$

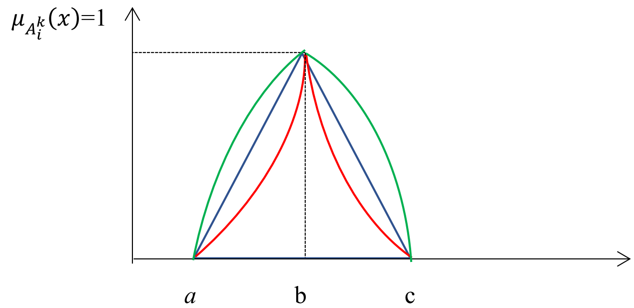


Fig. 3 Triangular (blue line) parabolic (green line) and exparabolic (red line) shapes

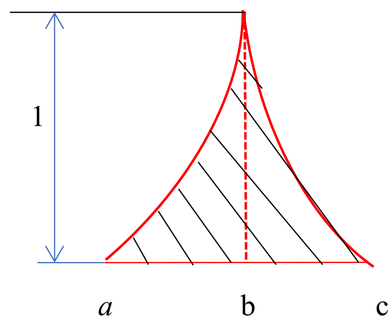


Fig. 4 Exparabolic shape area

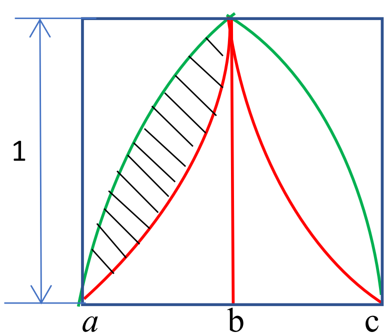
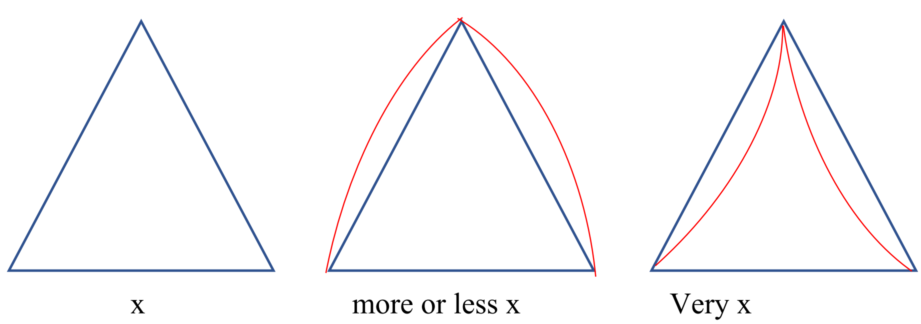


Fig. 5 Special segment area

Semi-area (Fig. 6) for aggregation can be calculated for more or less is

$$C = \frac{5}{3}(b-a) + \frac{b-a}{3} = 2(b-a) \tag{11}$$

Fig. 2 Linguistic hedges



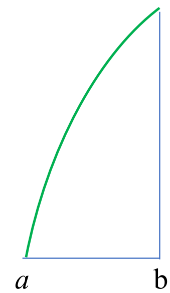


Fig. 6 Semi-parabolic area segment

Finally, we can calculate total area for more or less total area (Fig. 7) is as follows:

$$D = 2(b - a) + 2(c - b) = 2(c - a). \tag{12}$$

To aggregation process, we can divide the medium number of the TFN to this area:

$$\text{Aggregate value for exparabolic area} = b/[2(c - a)]. \tag{13}$$

The same procedure can be developed for triangular shape:

$$\text{Aggregate value for triangular area} = b/[(c - a)/2]. \tag{14}$$

Now, we can develop the new fuzzy TOPSIS application steps:

Step 1. Determine fuzzy decision matrix ($\tilde{D}$)

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \tilde{x}_{k1} & \tilde{x}_{k2} & \cdots & \tilde{x}_{kn} \end{bmatrix} \tag{15}$$

where the element $\tilde{x}_{ij}$ represents the triangular fuzzy number equivalent of the linguistic performance value of the alternative. Here, m is expressed as (i = 1,2,…,k alternatives) and j = 1,2,…., n, criteria [2, 6, 10, 23, 24].

Step 2. Obtain aggregate TFN.

For triangular area:

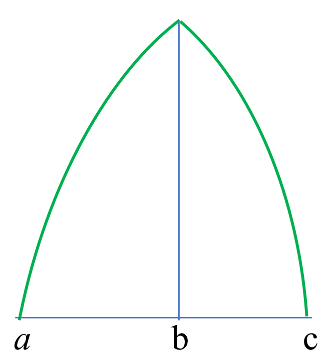


Fig. 7 More or less segment total area

$$\text{Aggregate value for triangular area} = x_{ij} = b/A = b/\left(\frac{c - a}{2}\right). \tag{16}$$

For exparabolic area:

$$\text{Aggregate value for exparabolic area} = x_{ij} = b/A = b/\left(\frac{c - a}{3}\right). \tag{17}$$

For parabolic area:

$$\text{Aggregate value for exparabolic area} = x_{ij} = b/[2(c - a)] = b/[2(c - a)]. \tag{18}$$

These aggregation operations provide better identification of TFN-based expert opinions. Dividing the medium value of the TFN (b) to the geometric shape area indicates not only the importance of the criteria or alternative rating but also the uncertainty degree of the linguistic expression. The larger value is the better type of evaluation suitable for assigning the criteria weights and alternative rantings.

Step 3. Set the normalized decision matrix using vector normalization method [23]:

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{k} x_{ij}} \tag{19}$$

Step 4. Set the weighted normalized decision matrix (v_{ij}):

$$V_{ij} = w_j \times r_{ij} \tag{20}$$

where w_j is the aggregate criteria weight calculated from triangular, exparabolic, or parabolic area calculation procedures in Eqs. (15–17).

Step 5. Calculate positive and negative ideal solutions:

$$A^* = \left\{ (\max_i v_{ij} \middle| j \in I), (\min_i v_{ij} | j \in I' \right\} \tag{21}$$

$$A^- = \left\{ (\min_i v_{ij} \middle| j \in I), \left(\max_{ij} v_{ij} | j \in I' \right\} \tag{22}$$

where I is a benefit type measure and I' is the cost type measure.

Step 6. Calculate the separation measures:

$$D^+ = \sqrt{\sum_{j=1}^{n} (v_i - v_i^*)^2} \tag{23}$$

$$D^- = \sqrt{\sum_{j=1}^{n} (v_i - v_i^-)^2} \tag{24}$$

Step 7. Calculate the ranking scores:

$$C_i^* = \frac{D_i^-}{D_i^- + D_i^+} \quad i = 1, \ldots, k \tag{25}$$

Table 1 Triangular area-based fuzzy TOPSIS model's result for example 1

	C1			C2			C3			C4			C5			C6			Total
A = (c − a)/2		*0.200*			0.213			0.275			0.288			0.175			0.200		
	0.6	*0.85*	1	0.525	0.8	0.95	0.1	0.4	0.65	0.15	0.45	0.725	0.45	0.65	1	0.6	0.85	1	
b/*A* = *W*		*4.250*			3.765			1.455			1.565			3.714			4.250		18.999
Normalized weight		0.224			0.198			0.077			0.082			0.196			0.224		1.00
Alternatives	Fuzzy decision matrix																		
a1	0.5	0.7	0.85	0.375	0.575	0.775	0.375	0.575	0.775	72.5	73	74	48.5	50	52	0.11	0.12	0.14	
a2	0.45	0.65	0.85	0.625	0.825	0.925	0.575	0.775	0.925	69	70	72	43	45	46.5	0.15	0.16	0.18	
a3	0.575	0.775	0.925	0.575	0.775	0.925	0.65	0.85	1	67.5	68	70	43.5	45	47.5	0.16	0.17	0.19	
		A = (c − a)/2	b/A																
a1		0.175	4.000		0.200	2.875		0.200	2.875		0.750	97.333		1.750	28.571		0.015	8.000	
a2		0.200	3.250		0.150	5.500		0.175	4.429		1.500	46.667		1.750	25.714		0.015	10.667	
a3		0.175	4.429		0.175	4.429		0.175	4.857		1.250	54.400		2.000	22.500		0.015	11.333	
	$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$		6.795			7.624			7.174			120.876			44.540			17.499	
	Normalized matrix																		
a1			0.589			0.377			0.401			0.805			0.641			0.457	
a2			0.478			0.721			0.617			0.386			0.577			0.610	
a3			0.652			0.581			0.677			0.450			0.505			0.648	
	Weighted normalized matrix																		
a1			0.132			0.075			0.031			0.066			0.125			0.102	
a2			0.107			0.143			0.047			0.032			0.113			0.136	
a3			0.146			0.115			0.052			0.037			0.099			0.145	
A^*			0.146			0.143			0.052			0.066			0.125			0.1449	
A^-			0.107			0.075			0.031			0.032			0.099			0.1023	
	D^+	D^-	C_i^*	Rank															
a1	0.084	0.050	0.3727	3															
a2	0.054	0.079	0.5936	2															
a3	0.048	0.074	0.6035	1															

Table 2 Exparabolic area-based fuzzy TOPSIS model's result for example 1

	C1			C2			C3			C4			C5			C6		
$\boldsymbol{A = (c-a)/3}$		0.133			0.213			0.275			0.288			0.175			0.200	
	0.6	0.85	1	0.525	0.8	0.95	0.1	0.4	0.65	0.15	0.45	0.725	0.45	0.65	1	0.6	0.85	1
$b/A = W$		6.375			3.765			1.455			1.565			3.714			4.250	
Normalized weight			0.302			0.178			0.069			0.074			0.176			0.201
Alternatives	Fuzzy decision matrix																	
a1	0.5	0.7	0.85	0.375	0.575	0.775	0.375	0.575	0.775	72.5	73	74	48.5	50	52	0.11	0.12	0.14
a2	0.45	0.65	0.85	0.625	0.825	0.925	0.575	0.775	0.925	69	70	72	43	45	46.5	0.15	0.16	0.18
a3	0.575	0.775	0.925	0.575	0.775	0.925	0.65	0.85	1	67.5	68	70	43.5	45	47.5	0.16	0.17	0.19
a1		0.117	6.000		0.133	4.313		0.133	4.313		0.500	146.000		1.167	42.857		0.010	12.000
a2		0.133	4.875		0.100	8.250		0.117	6.643		1.000	70.000		1.167	38.571		0.010	16.000
a3		0.117	6.643		0.117	6.643		0.117	7.286		0.833	81.600		1.333	33.750		0.010	17.000
			10.193			11.436			10.761			181.313			66.810			26.249
	Normalized matrix																	
a1			0.589			0.377			0.401			0.805			0.641			0.457
a2			0.478			0.721			0.617			0.386			0.577			0.610
a3			0.652			0.581			0.677			0.450			0.505			0.648
	Weighted normalized matrix																	
a1			0.178			0.067			0.028			0.060			0.113			0.092
a2			0.144			0.129			0.043			0.029			0.102			0.123
a3			0.197			0.104			0.047			0.033			0.089			0.130
$\boldsymbol{A^*}$			0.197			0.129			0.047			0.060			0.113			0.1303
$\boldsymbol{A^-}$			0.144			0.067			0.028			0.029			0.089			0.0920
	D^+	D^-	C_i^*	Rank														
a1	0.077	0.051	0.4000	3														
a2	0.063	0.071	0.5330	2														
a3	0.044	0.077	0.6385	1														

Table 3 Parabolic area-based fuzzy TOPSIS model's result for example 1

	C1			C2			C3			C4			C5			C6		
A = 2*(c−a)		0.133			0.213			0.275			0.288			0.175		0.200		
	0.6	0.85	1	0.525	0.8	0.95	0.1	0.4	0.65	0.15	0.45	0.725	0.45	0.65	1	0.6	0.85	1
b/A = W		1.063			3.765			1.455			1.565			3.714			4.250	
Normalized weight		0.067			0.238			0.092			0.099			0.235			0.269	
Alternatives	Fuzzy decision matrix																	
a1	0.5	0.7	0.85	0.375	0.575	0.775	0.375	0.575	0.775	72.5	73	74	48.5	50	52	0.11	0.12	0.14
a2	0.45	0.65	0.85	0.625	0.825	0.925	0.575	0.775	0.925	69	70	72	43	45	46.5	0.15	0.16	0.18
a3	0.575	0.775	0.925	0.575	0.775	0.925	0.65	0.85	1	67.5	68	70	43.5	45	47.5	0.16	0.17	0.19
a1		0.700	1.000		0.800	0.719		0.800	0.719		3.000	24.333		7.000	7.143		0.060	2.000
a2		0.800	0.813		0.600	1.375		0.700	1.107		6.000	11.667		7.000	6.429		0.060	2.667
a3		0.700	1.107		0.700	1.107		0.700	1.214		5.000	13.600		8.000	5.625		0.060	2.833
			1.699			1.906			1.794			30.219			11.135			4.375
	Normalized matrix																	
a1			0.589			0.377			0.401			0.805			0.641			0.457
a2			0.478			0.721			0.617			0.386			0.577			0.610
a3			0.652			0.581			0.677			0.450			0.505			0.648
	Weighted normalized matrix																	
a1			0.040			0.090			0.037			0.080			0.151			0.123
a2			0.032			0.172			0.057			0.038			0.136			0.164
a3			0.044			0.138			0.062			0.045			0.119			0.174
A^*			0.044			0.172			0.062			0.080			0.151			0.1741
A^-			0.032			0.090			0.037			0.038			0.119			0.1229
	D^+	D^-	C_i^*	Rank														
a1	0.100	0.053	0.3461	3														
a2	0.047	0.095	0.6692	1														
a3	0.058	0.076	0.5670	2														

Table 4 Triangular area-based fuzzy TOPSIS model's result for example 2

	C1			C2			C3			C4			C5		
$A = (c - a)/2$		0.150			0.050			0.115			0.050			0.100	
	0.7	0.9	1	0.9	1	1	0.77	0.93	1	0.9	1	1	0.43	0.63	0.83
$b/A = W$		6.000			20.000			8.087			20.000			6.300	
Normalized weight		0.099			0.331			0.134			0.331			0.104	
Alternatives	Fuzzy decision matrix														
a1	5.7	7.7	9.3	5	7	9	5.7	7.7	9	8.33	9.67	10	3	5	7
a2	6.3	8.3	9.7	9	10	10	8.3	9.7	10	9	10	10	7	9	10
a3	6.3	8	9	7	9	10	7	9	10	7	9	10	6.3	8.3	9.7
a1		1.800	4.278		2.000	3.500		1.650	4.667		0.835	11.581		2.000	2.500
a2		1.700	4.882		0.500	20.000		0.850	11.412		0.500	20.000		1.500	6.000
a3		1.350	5.926		1.500	6.000		1.500	6.000		1.500	6.000		1.700	4.882
			8.789			21.172			13.712			23.877			8.129
	Normalized matrix														
a1			0.487			0.165			0.340			0.485			0.308
a2			0.555			0.945			0.832			0.838			0.738
a3			0.674			0.283			0.438			0.251			0.601
	Weighted normalized matrix														
a1			0.048			0.055			0.046			0.161			0.032
a2			0.055			0.313			0.111			0.277			0.077
a3			0.067			0.094			0.059			0.083			0.063
A^*			0.067			0.313			0.111			0.277			0.077
A^-			0.048			0.055			0.046			0.083			0.032
	D^+	D^-	C_i^*	Rank											
a1	0.295	0.077	0.2079	2											
a2	0.012	0.333	0.9658	1											
a3	0.298	0.055	0.1549	3											

3 Examples

We applied the proposed methodology to the two selection problems using the fuzzy TOPSIS method in the literature. The first example is related to robot selection [8], and the second example is related to software programmer selection [17]. Firstly, we applied the proposed methodology to these problems and discussed the suitability and advantages of the proposed methodology using a comparative analysis in the following sections.

3.1 Example 1: Robot selection problem

The robot selection problem from Chu and Lin [8] is applied to show the suitability of the proposed procedure. Chu and Lin [8] assumed that a manufacturing firm requires a robot to perform a material-handling task. Three candidates, a1, a2, and a3, are selected for the study. The four experts are set to evaluate the criteria weights and alternative rating scores. The triangular, exparabolic aggregation procedure-based solution and parabolic aggregation procedure-based solution results are presented in Tables 1, 2 and 3, respectively.

3.2 Example 2: Software programmer selection problem

Mahdavi et al. [17] proposed a TOPSIS approach using TFNs. They applied the measurement approach using fuzzy distance values with a lower bound of alternatives. They supposed that a software company desires to select a programmer. Three alternatives, a1, a2, and a3, are determined, and three experts are assigned for the evaluation process. Mahdavi et al. [17] used five benefit criteria that are considered emotional steadiness, oral communication skills, personality, past experience, and self-confidence in their study. The triangular, exparabolic aggregation

Table 5 Exparabolic area-based fuzzy TOPSIS model's result for example 2

	C1			C2			C3			C4			C5		
$A = (c - a)/3$		0.100			0.050			0.115			0.050			0.100	
	0.7	0.9	1	0.9	1	1	0.77	0.93	1	0.9	1	1	0.43	0.63	0.83
$b/A = W$		9.000			20.000			8.087			20.000			6.300	
Normalized weight		0.142			0.316			0.128			0.316			0.099	
Alternatives	Fuzzy decision matrix														
a1	5.7	7.7	9.3	5	7	9	5.7	7.7	9	8.33	9.67	10	3	5	7
a2	6.3	8.3	9.7	9	10	10	8.3	9.7	10	9	10	10	7	9	10
a3	6.3	8	9	7	9	10	7	9	10	7	9	10	6.3	8.3	9.7
a1		1.200	6.417		1.333	5.250		1.100	7.000		0.557	17.371		1.333	3.750
a2		1.133	7.324		0.333	30.000		0.567	17.118		0.333	30.000		1.000	9.000
a3		0.900	8.889		1.000	9.000		1.000	9.000		1.000	9.000		1.133	7.324
			13.184			31.758			20.567			35.816			12.194
	Normalized matrix														
a1			0.487			0.165			0.340			0.485			0.308
a2			0.555			0.945			0.832			0.838			0.738
a3			0.674			0.283			0.438			0.251			0.601
	Weighted normalized matrix														
a1			0.069			0.052			0.043			0.153			0.031
a2			0.079			0.298			0.106			0.264			0.073
a3			0.096			0.089			0.056			0.079			0.060
A^*			0.096			0.298			0.106			0.264			0.073
A^-			0.069			0.052			0.043			0.079			0.031
	D^+	D^-	C_i^*	Rank											
a1	0.282	0.074	0.2075	2											
a2	0.017	0.317	0.9495	1											
a3	0.284	0.056	0.1640	3											

procedure-based solution and parabolic aggregation procedure-based solution results are presented in Tables 4, 5, and 6, respectively.

3.3 Discussion of the examples' results

We list the results in Table 7 and apply Spearman's rank correlation test [24] to analyze ranking differentiations. The Spearman's rank correlation calculation equation is as follows:

$$r_s = 1 - \frac{6\sum d_i^2}{t(t^2 - 1)} \tag{26}$$

where d_i is the difference between the two ranks of each observation, t is the number of observations.

Spearman's correlation coefficient increases in impact as two rankings become closer to being perfectly monotone functions of each other. When two rankings are perfectly matched, Spearman's correlation coefficient will equal 1.

The ranking scores from the parabolic area calculation-based TOPSIS model are different from triangular and exparabolic area calculation-based TOPSIS results in Chu and Lin's [8] case problem. So, it is difficult to say which alternative is exactly more suitable than the other. However, a2 is suitable for Chu and Lin's [8] case problem due to ranking first or second for all approaches. If we can see the decision matrix of example 1, the expert evaluations are very close to each other. So, the proposed aggregation procedures provide two different results based on the "very" or "more or less" hedges for TFNs, and these new procedures are capable of the two-way perspectives for alternatives to select the more suitable one for the defined expectations.

On the other hand, we obtain similar results in example 2. Mahdavi et al. [17] ranking scores results are very close to each other (r_s = 0.976). The only differentiation is in a1 and a3 ranking scores. Mahdavi et al. [17] study depends on the small ranking score as a better type of methodology.

Table 6 Parabolic area-based fuzzy TOPSIS model's result for example 2

		C1			C2			C3			C4			C5	
$A = 2^*(c - a)$		0.600			0.050			0.115			0.050			0.100	
	0.7	0.9	1	0.9	1	1	0.77	0.93	1	0.9	1	1	0.43	0.63	0.83
$b/A = W$		1.500			20.000			8.087			20.000			6.300	
Normalized weight		0.027			0.358			0.145			0.358			0.113	
Alternatives	Fuzzy decision matrix														
a1	5.7	7.7	9.3	5	7	9	5.7	7.7	9	8.33	9.67	10	3	5	7
a2	6.3	8.3	9.7	9	10	10	8.3	9.7	10	9	10	10	7	9	10
a3	6.3	8	9	7	9	10	7	9	10	7	9	10	6.3	8.3	9.7
a1		7.200	1.069		8.000	0.875		6.600	1.167		3.340	2.895		8.000	0.625
a2		6.800	1.221		2.000	5.000		3.400	2.853		2.000	5.000		6.000	1.500
a3		5.400	1.481		6.000	1.500		6.000	1.500		6.000	1.500		6.800	1.221
			2.197			5.293			3.428			5.969			2.032
	Normalized matrix														
a1			0.487			0.165			0.340			0.485			0.308
a2			0.555			0.945			0.832			0.838			0.738
a3			0.674			0.283			0.438			0.251			0.601
	Weighted normalized matrix														
a1			0.013			0.059			0.049			0.174			0.035
a2			0.015			0.338			0.120			0.300			0.083
a3			0.018			0.101			0.063			0.090			0.068
A^*			0.018			0.338			0.120			0.300			0.083
A^-			0.013			0.059			0.049			0.090			0.035
	D^+	D^-	C_i^*	Rank											
a1	0.318	0.084	0.2082	2											
a2	0.003	0.359	0.9912	1											
a3	0.322	0.056	0.1475	3											

Also, their ranking scores are very close to alternatives. But the proposed aggregation procedures based on TOPSIS ranking scores are differentiated. The Mahdavi et al. [17] rankings present a smooth graph, but the proposed procedures-based result has a fluctuated shape (Fig. 8). The main advantage of the proposed aggregation procedure is the alternative separation capability analyzed with their linguistic diversification. They have modeling capability for the selection problems that alternative ratings are very closed. They used a geometrical area calculation procedure for the aggregation operator. So, the linguistic differences are much better modeled using developed procedures.

According to the comparative analysis results, we reach the following outcomes about the advantages of the proposed methodology:

i. If the expert's linguistic evaluations are very close to each other, the proposed procedure uses two-way perspectives ("very" or "more or less" hedges) for alternatives to select the more suitable one for the defined expectations.
ii. The fuzzy TOPSIS model's ranking scores are very close for exemplified three alternatives (0.5308, 0.451, 0.455) in the Chu and Lin [8] approach. However, the proposed methodology's result has differentiated between each alternative for the same example (0.346, 0.669, 0.567). In Mahdavi et al.'s [17] ranking, scores are 0.5304, 0.4626, and 0.494, respectively, in example 2. The proposed methodology's results, for example 2, are 0.208, 0.991, and 0.148, respectively. So, the main advantage of the proposed aggregation procedure is the alternative ranking scores separation capability modeled with their linguistic diversification.
iii. The modeling capability of the proposed methodology has an advantage when alternative ratings are very close. The proposed methodology uses a simple geometrical area calculation procedure for the aggregation operation. So, it has much better

Table 7 Spearman's rank correlation test results

Exm 1	I	II	III	IV	Rankings				Ranking differences					
	Chu and Lin [8]	Triangular	Exparabolic	Parabolic	I	II	III	IV	I–II	I–III	I–IV	II–III	II–IV	III–IV
a1	0.5308	0.372706	0.400026	0.346	1	3	3	3	−2	−2	−2	0	0	0
a2	0.4510	0.593596	0.532969	0.669	3	2	2	1	1	1	2	0	1	1
a3	0.4550	0.603525	0.638537	0.567	2	1	1	2	1	1	0	0	−1	−1
								d_i	6	6	8	0	2	2
								r_s	−0.500	−0.500	−1.000	1.000	0.500	0.500
Exm 2	I	II	III	IV	Rankings				Ranking differences					
	Mahdavi et al. [17]	Triangular	Exparabolic	Parabolic	I	II	III	IV	I–II	I–III	I–IV	II–III	II–IV	III–IV
a1	0.5304	0.208	0.208	0.208	3	2	2	2	1	1	1	0	0	0
a2	0.4626	0.966	0.950	0.991	1	1	1	1	0	0	0	0	0	0
a3	0.4940	0.155	0.164	0.148	2	3	3	3	−1	−1	−1	0	0	0
								d_i	2	2	2	0	0	0
								r_s	0.976	0.976	0.976	1.000	1.000	1.000

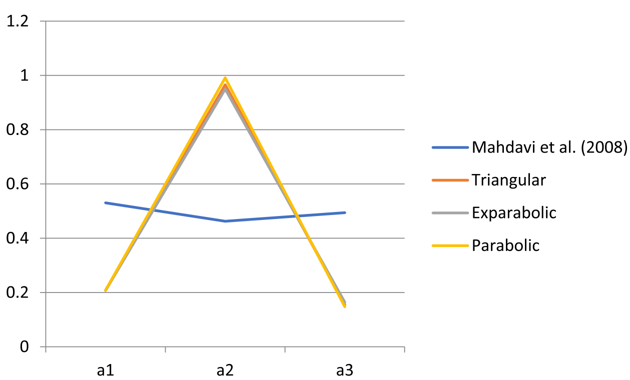


Fig. 8 Comparison graphics for Mahdavi et al. [17] study

modeling capability when the expert's linguistic expressions are very close.

4 Conclusions

In this work, we described an exparabolic and parabolic shape area calculation-based TFN linguistic expression aggregating procedure for fuzzy TOPSIS models. Furthermore, this procedure provides the best alternative using extended linguistic hedges for the linguistic expressions, hence maintaining the descriptive capabilities of the fuzzy TFNs.

We evaluated the results of different types of fuzzy TFNs-based TOPSIS model in selecting the best alternative. Results from the examples provide only the best alternative, so the presented new procedures consider the extended expressions as hedges and capture the information provided by overlapped expert opinions. The use of the proposed aggregating procedures that consider the information given by all the experts in the TFN expression process increases the generalization ability of the aggregating expressions. Nevertheless, it cannot determine a unique aggregate value as the best suitable for any type of problem, so it will be necessary to set aggregated TFNs for the new problems. This specification is an advantage for the modeling stage of the special TFNs for a specific selection problem. In future work, we intend to extend the application area to design a new kind of TFNs for the different MCDM methods such as GRA, VIKOR, PROMETHEE, and MOORA. On the other hand, the proposed model can be easily applied in different application areas requiring fuzzy multi-criteria decision-making processes, such as product or process quality improvement [12, 20], via evaluating the performance of artificial intelligence model-based predicted parameter values on the experimental or real quality characteristics' values.

Author contribution YTI was involved in conceptualization, supervision, methodology, validation, visualization, writing–review & editing.

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Data availability Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare no potential conflict of interest.

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