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ORIGINAL ARTICLE

Observer-based adaptive finite-time prescribed performance NN control for nonstrict-feedback nonlinear systems

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Abstract

This article focuses on an adaptive neural network (NN) finite-time prescribed performance control problem for nonstrict-feedback nonlinear systems subject to full-state constraints. Specifically, a finite-time performance function is employed, which can guarantee that the tracking error converges to a prescribed region within a finite-time. Neural networks (NNs) are used to approximate the unknown nonlinear function. The unmeasurable states are estimated via constructing a state observer. By using the dynamic surface control (DSC) technique, the complexity problem has been avoided in traditional backstepping control. In order to satisfy the state constraint condition, the barrier Lyapunov function (BLF) is incorporated in the process of backstepping. The developed adaptive finite-time NN backstepping control strategy can make that the closed-loop system is semiglobally practical finite-time stability (SGPFS). Meanwhile, all states can be guaranteed to remain in the constrained space. Simulation results demonstrate the validity of the control method.

Keywords Neural networks (NNs) · Adaptive control · Nonstrict-systems · Prescribed performance · Finite-time

1 Introduction

Over the last decades, adaptive backstepping control has attracted widespread attention due to its broad engineering applications, such as aircraft attitude systems [1], power systems [2] and so on. With the development of the Lyapunov stability theorem, the backstepping control technique and other tools, some remarkable results have been proposed (see, [3–6] and their references). It should be noticed that the control gains in considered nonlinear systems are assumed known in [7–9]. However, it is difficult for practical systems to satisfy this assumption [10]. In order to be more suitable for the practical situation in engineering applications, neural networks (NNs) or fuzzy logic systems (FLSs) are incorporated into the adaptive backstepping control technique. The authors in [11] and [12] utilized NNs or FLSs $f_i(\bar{\chi}_i|\Theta_i)$ to approximate the unknown function $f_i(\bar{\chi}_i)$. And then, Wang et al. [13] developed a NN controller for uncertain nonlinear systems subject to unknown control directions and time delay. The authors in [14] developed an adaptive backstepping control method for nonlinear systems with time-varying delay. In [15], an adaptive NN backstepping controller was developed for nonlinear systems in presence of unmodeled dynamics. Combining NNs with the backstepping technique, Li and Yang [16] developed an adaptive control strategy for nonlinear systems subject to unknown dead-zone inputs.

However, an inherent shortcoming of backstepping control is the complexity problem, which is caused by the repeated differentiation of the virtual controller in the process of recursive design. Fortunately, the dynamic surface control (DSC) was proposed to deal with the

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complexity problem. The main idea of the DSC is to introduce a first-order filter in the traditional backstepping control procedures. Thus, the proposed DSC-based controller not only overcomes the explosion of differential terms, but also makes the controller simpler. By combining the DSC with the backstepping technique, some remarkable adaptive controllers have been proposed in [17–20].

It should be pointed out that the aforementioned literature can only guarantee the infinite-time stabilization. Theoretically, the control systems are regulated to the desired performance when the time variable goes to infinity. However, from the view of practical applications, the control objective is expected to be achieved in the finite-time, which has drawn extensive practical interest as well as theoretical significance. Therefore, the adaptive finite-time (AFT) control design has attracted considerable attention. For example, Li et al. [21] developed an AFT controller for nontriangular nonlinear systems, and it can ensure that the closed-loop system is semiglobally practical finite-time stability (SGPFS). Based on the event-triggering strategy, Wang et al. [22] solve the AFT control problem for nontriangular nonlinear systems. The authors in [23] developed an AFT tracking control scheme for the spacecraft system with state constraints. Based on introducing a new barrier power integrator, Zhang et al. [24] developed a novel AFT control scheme for switched systems, which can deal with the finite-time control problem for some types of tracking and stabilization issues. By employing the approximate ability of the RBF-NNs, an AFT control strategy for MIMO nonlinear systems with the actuator hysteresis has been developed in [25]. In [26], an AFT recursive terminal sliding mode controller was proposed for a linear motor positioner. Based on an integral terminal sliding mode, Shao et al. [27] developed an adaptive control method for high-order uncertain nonlinear systems.

Even though the above results regarding the control theory of nonlinear systems have made abundant progress. The results proposed in [21–27] do not consider the prescribed performance control problem. Apparently, with the development of society, tracking control is demanded to realize tracking error in a precise range. The prescribed performance control (PPC) technique is a useful method to realize this demand. The PPC was first proposed in [28], which can guarantee the transient performance of the considered systems, and the tracking error can enter into a given bounded set. Wang et al. [29] presented a prescribed tracking performance control method for n-link manipulator, and they solved the problem caused by input saturation. For uncertain MIMO nonlinear systems with input quantization, Bikas et al. [30] constructed an effective adaptive NN controller to ensure the tracking error can remain in a prescribed range.

In summary, although the above results have made great progress in adaptive tracking control, there are still some problems to be solved in this paper: (1) how to address the algebraic loop problem caused by the structure of the nonstrict-feedback systems, (2) how to overcome the complexity problem in the process of backstepping procedure, (3) how to ensure that the closed-loop system is SGPFS and the tracking error converge to a prescribed range, (4) how to estimate unmeasurable states and make all states are not transgressed the constrained space. Based on the above discussion, this paper focuses on the AFT control for nonstrict-feedback nonlinear systems subject to unmeasurable states and full-state constraints. The main work consists of the following aspects:

(1) As more general nonlinear systems, nonstrict-feedback nonlinear systems proposed in this study can be used to control strict-feedback (or pure-feedback) nonlinear systems.
(2) By utilizing a finite-time performance function (FTPF), the presented AFT prescribed performance control method can not only guarantee that the considered system is stable, but also make the tracking error enters into the predefined range in a known time. It can be seen that the adaptive PPC schemes in [31–33] can only ensure the tracking error remains in the predefined bounded set without giving a precise time.
(3) By adopting the DSC, the computational explosion is avoided in the process of backstepping control. The log-type BLF is introduced in the process of backstepping design, and it can guarantee that all states do not transgress the constrained space.

2 Problem formulation

2.1 System description

Consider the following nonlinear systems

$$\begin{cases} \dot{\chi}_i(t) = \chi_{i+1}(t) + f_i(\chi(t)), \; i = 1,2,\ldots,n-1 \\ \dot{\chi}_n(t) = u(t) + f_n(\chi(t)), \\ y(t) = \chi_1(t), \end{cases} \tag{1}$$

where $\chi(t) \in \mathbb{R}^n$ is the state variable, $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ represents the input. $f_i(\cdot)$ is the unknown nonlinear function.

Remark 1 It should be noticed that the controller developed in [34] does not consider the issue of the full state constraints. However, it has a solid engineering background, and the investigation of nonlinear systems subject to full-state constraints is much more challenging. Meanwhile, as we all know that the full-state constraints are widely exist in engineering systems, for example, power systems [2] and missile systems [35]. If one does not consider the problem of full-state constraints, stability and good performance cannot be obtained.

Remark 2 If $f_i(\cdot) = f_i(\bar{\chi}_i)$ (or $f_i(\cdot) = f_i(\bar{\chi}_i, \chi_{i+1})$) with $\bar{\chi}_i = [\chi_1, \ldots, \chi_i]^{\mathrm{T}}$, system (1) is a class of strict (or pure)-feedback nonlinear systems. However, the unknown nonlinear function $f_i(\cdot)$ considered in this paper contains the whole state variables $\chi_i = [\chi_1, \ldots, \chi_n]^{\mathrm{T}}$. Thus, the nonlinear system (1) is a class of nonstrict-feedback systems. In the process of backstepping control, the considered systems are divided into n subsystems. The states variables χ_i (or χ_{i+1}) are viewed as state variables for the first ith subsystems in the strict-feedback (or pure-feedback) form. And then, the virtual control signal α_i should be constructed to ensure the stability for the first ith subsystems. However, $f_i(\cdot)$ contains the whole state variable in nonstrict-feedback systems, which means that α_i also contains the whole state variables for the first ith subsystems. Then, the designed controller for nonstrict-feedback systems has more challenges, and the algebraic loop problem is generated.

In this paper, the designed adaptive tracking controller satisfies the following objectives:

(1) the closed-loop system is SGPFS.
(2) all the states do not transgress their constrained sets.
(3) the output $y(t)$ can track the desired signal $y_d(t)$, that is to say, the tracking error $y(t) - y_d(t) = \omega(t)$ can converge to a predefined range in finite-time.

Assumption 1 [36] The reference signal $y_d(t)$ is a known bounded function. The time derivatives of $\dot{y}_d(t)$ and $\ddot{y}_d(t)$ are known and bounded. A_0, A_1 and A_2 such that $|y_d| \le A_0 \le k_{c1}$, $|\dot{y}_d| \le A_1$ and $|\ddot{y}_d| \le A_2$, $\forall t > 0$. And there exists a compact $\Omega_{y_d} = \{(y_d, \dot{y}_d, \ddot{y}_d)^{\mathrm{T}} : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le \delta_{y_d}\} \subset \mathbb{R}^3$ such that $(y_d, \dot{y}_d, \ddot{y}_d)^{\mathrm{T}} \in \Omega_{y_d}$, where δ_{y_d} is a positive constant.

Assumption 2 [37] For $\forall \aleph_1, \aleph_2$, there exists a known constant l_i, such that

$$|f_i(\aleph_1) - f_i(\aleph_2)| \le l_i |\aleph_1 - \aleph_2|.$$

Lemma 1 [38] *For $\forall(\ell_1, \ell_2) \in \mathbb{R}^2$, the following inequality can be obtained*

$$\ell_1 \ell_2 \le \frac{\hbar^p}{p} |\ell_1|^p + \frac{1}{q\hbar^q} |\ell_2|^q,$$

where $\hbar > 0$, $p > 1$, $q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Remark 3 Assumption 1 implies only the reference signal without its high-order times derivatives. In addition, for a given nonlinear function, the locally Lipschitz condition is easily satisfied. Assumption 2 allows us to consider unknown functions. Similar assumptions can be found in [16–18]. Without these assumptions, the proposed control scheme cannot be realized.

2.2 Prescribed performance function

Definition 1 [39] Suppose that there exists a function $\rho(t)$ satisfies the following properties:

(1) $\rho(t) > 0$;
(2) $\dot{\rho}(t) \le 0$;
(3) $\lim_{t \to T_r} \rho(t) = \rho_{T_r}$, and ρ_{T_r} is an arbitrary positive constant;
(4) when $t > T_r$, $\rho(t) = \rho_{T_r}$ with T_r being a designed time.

Then, $\rho(t)$ is a FTPF. It can be defined in this paper as

$$\begin{cases} \rho(t) = (\rho_0^l - l\varrho t)^{\frac{1}{l}} + \rho_{T_r}, & 0 \le t < T_r \\ \rho_{T_r}, & t \ge T_r \end{cases} \tag{2}$$

where l, ϱ and ρ_0 are positive parameters, $l = \frac{p}{q}$ with $q \ge p$, q and p are positive odd and even integers. And $\rho_0 + \rho_{T_r} = \rho(0)$ denotes the initial value, $T_r = \frac{\rho_0^l}{l\varrho}$ is the set time. And $\rho(t)$ is a smooth function, which has been verified in [40].

In order to realize the $\omega(t)$ enters into a prescribed range in a finite-time, the following error transformation function is defined

$$\psi(t) = \tan\left(\frac{\pi\omega(t)}{2\rho(t)}\right), \omega(0) < \rho(0). \tag{3}$$

According to (3), one can obtain

$$\omega(t) = \frac{2}{\pi}\rho(t)\arctan(\psi(t)). \tag{4}$$

From (4) and $y(t) - y_d(t) = \omega(t)$, we have

$$\dot{\omega}(t) = \frac{2}{\pi}\dot{\rho}(t)\arctan(\psi(t)) + \frac{2}{\pi}\rho(t)\frac{\dot{\psi}(t)}{1 + \psi^2(t)}, \tag{5}$$

which yields

$$\dot{\psi}(t) = \frac{\pi(1 + \psi^2(t))}{2\rho(t)}\left(\dot{\chi}_1 - \dot{y}_d - \frac{2}{\pi}\dot{\rho}(t)\arctan(\psi(t))\right).$$

Figure 1 exhibits the bound of the $\omega(t)$ under the FTPF. Thus, the proposed method can better satisfy the actual engineering application.

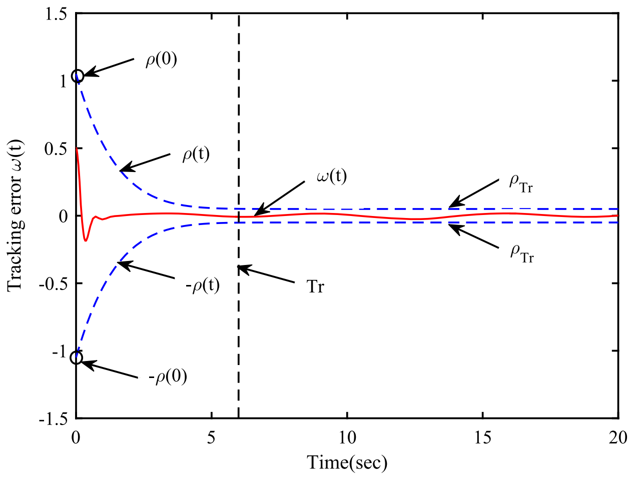

Fig. 1 The finite-time prescribed performance on the error

Remark 4 The main intention of using a state transition (3) is transform the tracking error $\omega(t)$ into another state $\psi(t)$. Then, we will design a controller to guarantee the boundedness for $\psi(t)$ instead of directly control the tracking error $\omega(t)$. From (3), we can see that the boundedness of $\psi(t)$ stands that $-\rho(t) < \omega(t) < \rho(t)$. Furthermore, $\psi(t) \to 0 \Rightarrow \omega(t) \to 0$. The readers can refer to [40] for more details about formula (5).

2.3 NNs approximation

RBF-NNs are utilized to approximate the nonlinear function. By [41], RBF-NNs can evaluate arbitrary continuous nonlinear function *f*(*X*) with the neural node number $q > 1$, and

$$f(X) = (\Gamma^*)^{\mathrm{T}}\phi(X) + \zeta(X), |\zeta(X)| \le \zeta_M$$

where the approximation error $\zeta(X)$ satisfies $|\zeta(X)| \le \zeta_M$ with ζ_M is an unknown constant, $\phi(X) = [\phi_1(X), \ldots, \phi_q(X)]^{\mathrm{T}}$ denotes the basis function vector, and Γ^* denotes the ideal weight vector,

$$\Gamma^* = \arg\min[\sup_{X\in\Omega_0} |f(X) - \Gamma^{\mathrm{T}}\phi(X)|] \tag{6}$$

with $\Gamma = [\jmath_1, \ldots, \jmath_q]^{\mathrm{T}}$ being the weight vector. And $\phi_i(X)$ is the Guassian function and it can be described as

$$\phi_i(X) = \exp\left(\frac{\| X - \wp_i\|^2}{\tau_i^2}\right), i = 1, 2, \ldots, q$$

where $\wp_i = [\wp_{i1}, \wp_{i2}, \cdots, \wp_{il}]^{\mathrm{T}}$ is the central vector of Gaussian function and τ_i represents the breadth of Gaussian function.

Remark 5 It should be noticed that the basis function of RBF-NNs satisfies $\phi_i^{\mathrm{T}}(X)\phi_i(X) \le \Im$, which will be greatly useful in controller design procedure.

According to the above NNs approximation results, it is known that $f_i(\chi(t))$ in system (1) is

$$f_i(\chi(t)) = \hat{f}_1(\chi(t)|\Gamma_1^*) + \zeta_1(\chi(t)), \tag{7}$$

where $\hat{f}_i(\chi(t)|\Gamma_1^*) = \Gamma_1^{*\mathrm{T}}$.

Further, according to (6) and (7), Γ_i^* is

$$\Gamma_i^* = \arg\min_{\Gamma_i\in\Omega_i}[\sup_{\hat{\chi}\in \mathrm{U}} |\hat{\mathrm{f}}_{\mathrm{i}}(\chi(\mathrm{t}))|\Gamma_{\mathrm{i}}) - \mathrm{f}_{\mathrm{i}}(\chi(\mathrm{t}))|], \tag{8}$$

where Ω_i and U are the compact set of Γ_i and $\chi(t)$, respectively.

By (7), system (1) is rewritten as

$$\begin{cases} \dot{\chi}_i(t) = \chi_{i+1}(t) + \Gamma_i^{*\mathrm{T}}\phi_i(\hat{\chi}(t)) + \Delta f_i + \zeta_i(\chi(t)), \\ \qquad i = 1, 2, \ldots, n-1 \\ \dot{\chi}_n(t) = u(t) + \Gamma_n^{*\mathrm{T}}\phi_n(\hat{\chi}(t)) + \Delta f_n + \zeta_n(\chi(t)), \\ y(t) = \chi_1(t), \end{cases} \tag{9}$$

where $\hat{\chi}(t)$ denotes the estimation value of $\chi(t)$ and $\Delta f_i = f_i(\chi(t)) - f_i(\hat{\chi}(t))$.

2.4 Finite-time

Definition 2 [42] Consider a simple nonlinear system $\dot{\eta} = f(\eta, u)$, where η represents the state, and u is the input. The nonlinear system $\dot{\eta} = f(\eta, u)$ is SGPFS, if for every initial condition $\eta(t_0) = \eta_0 \in \Omega_0$ with Ω_0 is the compact set, there exists $\jmath > 0$ and a setting time $T(\jmath, \eta_0) < \infty$ such that $\|\eta(t)\| < \jmath$, for all $t \ge t_0 + T$.

Lemma 2 [43] *For* $(\varphi_1, \varphi_2) \in \mathbb{R}^2$, *and there exists positive constants* ϵ_1, ϵ_2 *and* ϵ_3, *such that*

$$|\varphi_1|^{\epsilon_1}|\varphi_2|^{\epsilon_2} \le \frac{\epsilon_1}{\epsilon_1 + \epsilon_2}\epsilon_3|\varphi_1|^{\epsilon_1+\epsilon_2} + \frac{\epsilon_1}{\epsilon_1 + \epsilon_2}\epsilon_3^{-\frac{\epsilon_1}{\epsilon_2}}|\varphi_2|^{\epsilon_1+\epsilon_2}.$$

Lemma 3 [44] *Consider the nonlinear system* $\dot{\eta} = f(\eta)$. *If for all* $\eta_0 \in \Omega_0$, *there exists a smooth function* $V(\eta)$ *with constants* $c > 0$, $0 < \sigma < 1$, *and* $\beta > 0$, *such that*

$$\dot{V}(\eta) \le cV^{\sigma}(\eta) + \beta, t \ge 0$$

then $\dot{\eta} = f(\eta)$ is SGPFS.

Lemma 4 [45] *For any* $\chi_r \in \mathbb{R}, (r = 1, \ldots, n)$ *and* $0 < p \le 1$, *we have*

$$\left(\sum_{r=1}^{n}|\chi_r|\right)^p \le \sum_{r=1}^{n}|\chi_r|^p \le n^{1-p}\left(\sum_{r=1}^{n}|\chi_r|\right)^p.$$

3 Main results

3.1 Observer design

Since only the output $y(t)$ is measured, a NN state will be developed to obtain the unmeasurable states.

In order to design the state observer, we transform the nonlinear system (9) as the following matrix form

$$\begin{cases} \dot{\chi}(t) = A\chi(t) + Ky(t) + \sum_{i=1}^{n} B_i \Gamma_i^{*\mathrm{T}} \phi_i(\hat{\chi}(t)) \\ \qquad + B_n u(t) + \Delta f + \zeta, \\ y(t) = C\chi(t), \end{cases} \tag{10}$$

where

$$A = \begin{bmatrix} -k_1 & & & \\ -k_2 & & I_{n-1} & \\ \vdots & & & \\ -k_n & 0 & \ldots & 0 \end{bmatrix},$$

$K = [k_1, \ldots, k_n]^{\mathrm{T}}$, $B_i = [\underbrace{0, \ldots, 1}_{i}, \ldots, 0]^{\mathrm{T}}$, $C = [\underbrace{1, 0, \ldots, 0}_{n}]$, $\Delta f = [\Delta f_1, \cdots, \Delta f_n]^{\mathrm{T}}$, $\zeta = [\zeta_1(\hat{\chi}(t)), \ldots, \zeta_n(\hat{\chi}(t))]^{\mathrm{T}}$.

Vector K is chosen such that A is a Hurwitz matrix. And then, there exists a positive defined matrix $W > 0$, for any given matrix Q satisfies $Q = Q^{\mathrm{T}} > 0$ such that the following equality holds

$$A^{\mathrm{T}} W + WA = -Q. \tag{11}$$

An NN observer is established as

$$\begin{cases} \dot{\hat{\chi}}(t) = A\hat{\chi}(t) + Ky(t) + \sum_{i=1}^{n} B_i \Gamma_i^{\mathrm{T}} \phi_i(\hat{\chi}(t)) + Bu(t), \\ \hat{y}(t) = C\hat{\chi}(t). \end{cases} \tag{12}$$

Remark 6 Many works of literature require that all state variables are measured directly, for example, [46]. However, this requirement is almost impossible to achieve in practical engineering. Thus, we construct a state observer to estimate the unmeasurable states. Compared with [46], the system in this paper is more suitable for the actual systems.

According to (10) and (12), choose $e(t)$ as

$$e(t) = \chi(t) - \hat{\chi}(t) \tag{13}$$

satisfies

$$\dot{e}(t) = Ae(t) + \sum_{i=1}^{n} B_i \tilde{\Gamma}_i \phi_i(\hat{\chi}(t)) + \zeta + Bu(t) + \Delta f, \tag{14}$$

where $\tilde{\Gamma}_i = \Gamma_i^* - \Gamma_i$, and Γ_i is the optimal parameter of Γ_i^*.

Choose a Lyapunov function as

$$V_0(t) = e^{\mathrm{T}}(t) W e(t). \tag{15}$$

And then, the derivative of V_0 satisfies

$$\dot{V}_0 \le -q_0 \|e\|^2 + \Im \|W\|^2 \sum_{i=1}^{n} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i + M_0, \tag{16}$$

where $q_0 = \lambda_{\min}(Q) - \|W\|^2 \sum_{i=1}^{n} l_i^2 - 3$, $M_0 = \|W\|^2 \|\zeta^*\|^2$. Specifically, the calculation process can be found in Appendix 1.

Remark 7 For unmeasured state variables, a linear observer was proposed in [47] as the following form:

$$\begin{cases} \dot{\hat{\chi}}_i = \hat{\chi}_{i+1} - k_i(\chi_1 - \hat{\chi}_1), 1 \le i \le n-1 \\ \dot{\hat{\chi}}_n = u - k_n(\chi_1 - \hat{\chi}_1). \end{cases}$$

where the observer gain parameter k_i is chosen such that the polynomial $s(p) = p^n + k_1 p^{n-1} + \cdots + k_{n-1} p + k_n$ is Hurwizts. It will lead to the developed observer independent of considered systems. However, observer (12) utilizes RBF-NNs $\Gamma_i^{\mathrm{T}} \phi_i(\hat{\chi}(t))$ to estimate the unknown nonlinear functions $f_i(\chi_i(t))$, $(i = 1, \ldots, n)$. Via online estimation, a better estimation performance can be obtained.

Remark 8 In [13, 14], unmeasurable states make these methods no longer effective, and thus the NN state observer (12) is constructed. From (16), the convergence of the designed state observer cannot be ensured. It is thus necessary to develop a control scheme in the next section to guarantee the finite-time stability of the closed-loop system.

3.2 Backstepping control design

An AFT prescribed performance tracking controller is presented for considered nonstrict-feedback nonlinear systems in this subsection.

Choose the following coordinates change.

$$\begin{cases} z_1 = \psi, \\ z_i = \hat{\chi}_i - \gamma_i, \\ \varsigma_i = \gamma_i - \alpha_{i-1}, i = 2, \ldots, n \end{cases} \tag{17}$$

where γ_i represents the filter signal and ς_i denotes the error surface.

Design the virtual control signal α_1 and the adaptive function $\dot{\Gamma}_1$ as

$$\begin{cases} \alpha_1 = -\dfrac{2\rho(k_{b_1}^2 - z_1^2)^{1-\sigma}}{\pi(1+\psi^2)} c_1 z_1^{2\sigma-1} - \Gamma_1^{\mathrm{T}} \phi_1(\chi_1) \\ \qquad - \dfrac{(3+\tau)\pi(1+\psi^2)}{4\rho} z_1 + \dfrac{2}{\pi}\dot{\rho}\arctan(\psi) + \dot{y}_d, \\ \dot{\Gamma}_1 = \delta_1 \dfrac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)} z_1 \phi_1(\chi_1) - \mu_1 \Gamma_1, \end{cases} \tag{18}$$

where $0<\sigma<1$, $c_1 > 0$ and $\mu_1 > 0$ are design parameters.

Choose the virtual controller α_2 and the adaptive law $\dot{\Gamma}_2$ as

$$\begin{cases} \alpha_2 = -\dfrac{c_2 z_2^{2\sigma-1}}{(k_{b_2}^2 - z_2^2)^{\sigma-1}} - k_2 e_1 - \Gamma_2^{\mathrm{T}} \phi_2(\hat{\bar{\chi}}_2) + \dot{\gamma}_2 \\ \qquad - \dfrac{(\tau+2) z_2}{2(k_{b_i}^2 - z_i^2)} - \dfrac{\pi(1+v^2)(k_{b_2}^2 - z_2^2)}{k_{b_{i-1}}^2 - z_{i-1}^2} z_{i-1}, \\ \dot{\Gamma}_2 = \dfrac{\delta_2 \phi_2(\hat{\bar{\chi}}_2)}{k_{b_2}^2 - z_2^2} z_2 - \mu_2 \Gamma_2, \end{cases} \tag{19}$$

where c_2 and μ_2 are positive designed parameters.

Choose the virtual controller α_i and the adaptive function $\dot{\Gamma}_i$ as

$$\begin{cases} \alpha_i = -\dfrac{c_i z_i^{2\sigma-1}}{(k_{b_i}^2 - z_i^2)^{\sigma-1}} - k_i e_1 - \Gamma_i^{\mathrm{T}} \phi_i(\hat{\bar{\chi}}_i) \\ \qquad - \dfrac{\tau+2}{2(k_{b_i}^2 - z_i^2)} z_i - \dfrac{k_{b_i}^2 - z_i^2}{k_{b_{i-1}}^2 - z_{i-1}^2} z_{i-1} + \dot{\gamma}_i, \\ \dot{\Gamma}_i = \dfrac{\delta_i \phi_i(\hat{\bar{\chi}}_i)}{k_{b_i}^2 - z_i^2} z_i - \mu_i \Gamma_i, \end{cases} \tag{20}$$

where $c_i > 0$, $\mu_i > 0$ are designed parameters.

Choose the control input u and the adaptive function $\dot{\Gamma}_n$ as

$$\begin{cases} u = -\dfrac{c_n z_n^{2\sigma-1}}{(k_{b_n}^2 - z_n^2)^{\sigma-1}} - k_n e_1 - \dfrac{z_n}{2(k_{b_n}^2 - z_n^2)} \\ \qquad - \dfrac{(k_{b_n}^2 - z_n^2) z_{n-1}}{k_{b_{n-1}}^2 - z_{n-1}^2} - \Gamma_n^{\mathrm{T}} \phi_n(\hat{\bar{\chi}}) + \dot{\gamma}_n, \\ \dot{\Gamma}_n = \dfrac{\delta_n \phi_n(\hat{\chi})}{k_{b_n}^2 - z_n^2} z_n - \mu_n \Gamma_n, \end{cases} \tag{21}$$

where $c_n > 0$ and $\mu_n > 0$ are designed parameters.

The specific design process can be found in Appendix 2. Based on the above analysis, we summarize the main results in Theorem 1.

Theorem 1 *Choose virtual controllers (18), (19) and (20), the actual control input (21), adaptive laws (18), (19), (20) and (21), the NN state observer (12) for system (1). Then, the developed controller can be guaranteed the closed-loop system is SGPFS under the full-state constraints.*

Proof By Appendix 2 and Lemma 1, one has

$$\begin{aligned} \frac{\mu_i}{\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} \Gamma_i &= \frac{\mu_i}{\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} (\Gamma_i^* - \tilde{\Gamma}_i) \\ &\leq -\frac{\mu_i}{\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i + \frac{\mu_i}{\delta_i} \Gamma_i^{*\mathrm{T}} \Gamma_i^*, \end{aligned} \tag{22}$$

$$\varsigma_i Y_i(\cdot) \leq \frac{1}{2\tau} \varsigma_i^2 \vartheta_i^2 + \frac{\tau}{2}. \tag{23}$$

From (22) and (13), one gets

$$\begin{aligned} \dot{V}_n \leq & -\frac{q_1}{\lambda_{\max}(W)} e^{\mathrm{T}} W e - \sum_{i=1}^{n} 2^{\sigma} c_i \left(\frac{z_i^2}{2(k_{b_i}^2 - z_i^2)}\right)^{\sigma} \\ & - ((\mu_1 - \Im \|W\|^2 \delta_1) \frac{1}{2\delta_1} \tilde{\Gamma}_1^{\mathrm{T}} \tilde{\Gamma}_1 \\ & + \sum_{i=2}^{n} (\mu_i - \Im \delta_i - \Im \|W\|^2 \delta_i) \frac{1}{2\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i) \\ & + \sum_{i=2}^{n} \left(\frac{2}{\varpi_i} - \frac{\vartheta_i^2}{\tau} - 1\right) \frac{1}{2} \varsigma_i^2 + M, \end{aligned} \tag{24}$$

where $\lambda_{\max}(W)$ is the maximal eigenvalue of matrix W, and $M = M_n + \sum_{i=1}^{n} \left(\frac{\mu_i}{2\delta_i}\right) \Gamma_i^{*\mathrm{T}} \Gamma_i^* + \frac{n-1}{2}\tau$.

Defining

$$\begin{aligned} C = \min\Big\{ & 2^{\sigma} c_1, \ldots, 2^{\sigma} c_n, \mu_1 - \Im \|W\|^2 \delta_1, \\ & \mu_2 - \Im \|W\|^2 \delta_2 - \Im \delta_2, \ldots, \\ & \mu_n - \Im \|W\|^2 \delta_n - \Im \delta_n, \frac{2}{\varpi_2} - \frac{\vartheta_2^2}{\tau} - 1, \\ & \ldots, \frac{2}{\varpi_n} - \frac{\vartheta_n^2}{\tau} - 1 \Big\} \end{aligned} \tag{25}$$

and substituting (25) into (24), we can obtain

$$\begin{aligned} \dot{V}_n \leq & -\frac{q_1}{\lambda_{\max}(W)} e^{\mathrm{T}} W e - C \sum_{i=1}^{n} \left(\frac{z_i^2}{2\left(k_{b_i}^2 - z_i^2\right)} \right)^{\sigma} \\ & - \sum_{i=1}^{n} C \frac{1}{2\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i - C \sum_{i=2}^{n} \frac{1}{2} \varsigma_i^2 + M. \end{aligned} \tag{26}$$

According to Lemma 2, combining with $q_1 = 1$, $q_2 = \sum_{i=2}^{n} \varsigma_i^2$, $\varepsilon_1 = 1 - \sigma$, $\varepsilon_2 = \sigma$, and $\varepsilon_3 = \sigma^{\frac{\sigma}{1-\sigma}}$, we can obtain

$$\left(\sum_{i=2}^{n}\frac{1}{2}\varsigma_i^2\right)^{\sigma}\leq(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\sum_{i=2}^{n}\frac{1}{2}\varsigma_i^2. \tag{27}$$

Similarly, one has

$$\left(\sum_{i=1}^{n}\frac{1}{2\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i\right)^{\sigma}\leq(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\sum_{i=1}^{n}\frac{1}{2\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i, \tag{28}$$

$$\left(e^{\mathrm{T}}We\right)^{\sigma}\leq(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+e^{\mathrm{T}}We. \tag{29}$$

According to (27)-(29), it yields

$$\begin{aligned}\dot{V}_n\leq&-\frac{q_1}{\lambda_{\max}(W)}\left(e^{\mathrm{T}}We\right)^{\sigma}-C\sum_{i=1}^{n}\left(\frac{z_i^2}{2\left(k_{b_i}^2-z_i^2\right)}\right)^{\sigma}\\&-C\left(\sum_{i=1}^{n}\frac{1}{2\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i\right)^{\sigma}-C\left(\sum_{i=2}^{n}\frac{1}{2}\varsigma_i^2\right)^{\sigma}+M\\&+\left(2C+\frac{q_1}{\lambda_{\max}(W)}\right)(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}.\end{aligned} \tag{30}$$

Defining

$$\begin{aligned}c=&\min\left\{\frac{q_1}{\lambda_{\max}(W)},C\right\},\\\beta=&M+\left(2C+\frac{2q_1}{\lambda_{\max}(W)}\right)(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}},\end{aligned}$$

and utilizing Lemma 4, we can obtain

$$\dot{V}_n\leq-cV_n^{\sigma}+\beta. \tag{31}$$

According to (31), $\forall 0<\iota<1$, one has

$$\dot{V}_n(\theta)\leq-\iota cV_n^{\sigma}(\theta)-(1-\iota)cV_n^{\sigma}(\theta)+\beta, \tag{32}$$

where $\theta=[z_1,\ldots,z_n,\tilde{\Gamma}_1,\ldots,\tilde{\Gamma}_n,\varsigma_2,\ldots,\varsigma_n]^{\mathrm{T}}$.

Let

$$\begin{aligned}v_\theta=&\left\{\theta|V_n^{\sigma}(\theta)\leq\frac{\beta}{(1-\iota)c}\right\},\\\bar{v}_\theta=&\left\{\theta|V_n^{\sigma}(\theta)>\frac{\beta}{(1-\iota)c}\right\}.\end{aligned}$$

If $\theta\in\bar{v}_\theta$, then one has

$$\dot{V}_n(\theta)\leq-\iota cV_n^{\sigma}(\theta). \tag{33}$$

From (33), one has

$$\int_0^T\frac{\dot{V}_n(\theta)}{V_n^{\sigma}(\theta)}dt\leq-\int_0^T\iota cdt. \tag{34}$$

According to (34), one has

$$\frac{1}{1-\sigma}V_n^{1-\sigma}(\theta(T))-\frac{1}{1-\sigma}V_n^{1-\sigma}(\theta(0))\leq-\iota cT, \tag{35}$$

where $\theta(0)$ is the initial value of θ.

Let

$$T^*=\frac{1}{(1-\sigma)\iota c}[V_n^{1-\sigma}(\theta(0))-\left(\frac{\beta}{(1-\iota)c}\right)^{\frac{1-\sigma}{\sigma}}]. \tag{36}$$

It follows from (36) that $\theta\in v_\theta$ for $T\geq T^*$, the trajectory of θ does not outstrip the set v_θ.

From (33) and (36), $\dot{V}_n\leq 0$. Therefore, it can now be concluded form stability that the dynamic of the closed-loop system is SGPFS. On the other hand, by Lemma 4 in [48], it can be obtained that $z_i\in\Omega_{z_i}$, that is, $|z_i|<k_{b_i}$. As mentioned in Remark 4, $z_1=v(t)$ is transformed form $\omega(t)=y(t)-y_d(t)$. Then, one can obtain $|\chi_1|<|z_1|+|y_d|<k_{b_1}+A_0$. Choose $k_{b_1}=k_{c_1}-A_0$. We can obtain $|\chi_1|<k_{c_1}$. The state of χ_1 is constrained in Ω_{χ_1}. According to $z_2=\hat{\chi}_2-\gamma_2=\chi_2+e_2+\varsigma_2+\alpha_1$, $|\chi_2|\leq|z_2|+|e_2|+|\varsigma_2|+|\alpha_1|$ holds. Because α_1 is a smooth function of χ_1, z_1, y_d and $\dot{y}_d$, $|\chi_1|<k_{c_1}$, $|z_1|<k_{b_1}$, $y_d\leq A_0$, and $|\dot{y}_d|\leq A_1$. Then there exists a constant $\bar{\alpha}_1>0$ makes $|\alpha_1|\leq\bar{\alpha}_1$. $\forall t\geq T^*$, $V_n^{\sigma}>\frac{\beta}{1-\iota c}$, one has $|e_2|\leq\|e\|\leq s_2$ and $|\varsigma_2|\leq\|(\varsigma_2,\ldots,\varsigma_n)\|\leq\Lambda_2$, which means $|\chi_2|\leq k_{b_2}+s_2+\Lambda_2+\bar{\alpha}_1$. Let $k_{b_2}=k_{c_2}-s_2-\Lambda_2-\bar{\alpha}_1$, $|\chi_2|<k_{c_2}$ can be obtained, and χ_2 is constrained in the set Ω_{χ_2}. In this way, the state variable $\chi_i\in\Omega_{\chi_i}$ can be obtained. That is, the state systems do not violate constraint conditions. □

Remark 9 For any positive constants δ_{y_d} and Π, the set $\Omega_{y_d}=\left\{(y_d,\dot{y}_d,\ddot{y}_d)^{\mathrm{T}}:y_d^2+\dot{y}_d^2+\ddot{y}_d^2\leq\delta_{y_d}\right\}\in\mathbb{R}^3$ and $\Omega_i=\left\{e^{\mathrm{T}}We+\sum_{j=1}^{i}\log\left(\frac{k_{b_j}^2}{k_{b_j}^2-z_j^2}\right)+\sum_{j=1}^{i}\frac{1}{\delta_j}\widetilde{\Gamma}_j^{\mathrm{T}}\widetilde{\Gamma}_j+\sum_{j=1}^{i-1}\varsigma_{i+1}^2\leq 2\Pi\right\}\in\mathbb{R}^{3i}$. Furthermore, $\Omega_{y_d}\times\Omega_i$ is compact in $\mathbb{R}^{3+3i}$. Thus, $Y_i(\cdot)$ has a maximum $\vartheta_i>0$, such that $|Y_i(\cdot)|\leq\vartheta_i$.

Remark 10 It can be seen that the nonlinear function $f_i(\chi)$ in the nonlinear system (1) contains the whole states. If the traditional backstepping control method for strict-feedback systems is adopted, the algebraic loop problem will be caused. To avoid this problem, the equation

$$\begin{aligned}\phi_i^{\mathrm{T}}\phi_i(\hat{\chi})=&\phi_i^{*\mathrm{T}}\phi_i(\hat{\chi})-\widetilde{\phi}_i^{\mathrm{T}}\phi_i(\hat{\chi})-\phi_i^{*\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i)\\&+\phi_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i)+\widetilde{\phi}_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i)\end{aligned}$$

is adopted in (72). By employing this equation, NNs concluding state variables $\hat{\bar{\chi}}_i$ appear. Then, the property of RBF-NNs is utilized to design the controller.

Remark 11 According to the Lyapunov stability theorem, the above theorem presents a result on AFT prescribed performance control strategy. Obviously, the tracking error can converge to a prescribed range by designing reasonable FTPF. It theoretically ensures that the tracking error can converge to a prescribed range in the finite-time. In addition, it should be noticed that the BLF may generate the infinite control signal or even the actuator saturation, which has been discussed in [49].

Remark 12 The AFT control diagram is exhibited in Fig. 2. It can be seen that an observer (12) is designed for the considered nonlinear system (1), which can estimate the unmeasurable states. And then, utilizing the state estimation, we introduce the finite-time performance function $\rho(t)$ and the reference signal $y_d(t)$ into the backstepping design procedure to obtain the virtual controller, the adaptive law and the actual controller. We summarize the algorithm in this paper as follows

Step I: Choose the vector $K = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$, such that A is the Hurwitz matrix. Furthermore, by solving equation (11), we can get the positive defined matrix W.

Step II: Select the initial conditions such that the initial tracking error is smaller than the initial value of FTPF.

Step III: Specify suitable designed parameters, such as $c_1 > 0$, $\delta_1 > 0$, $\mu_1 > 0$ and $\varpi_2 > 0$. And decide the virtual control signal α_1 and the parameter update law $\dot{\Gamma}_1$.

Step IV: Specify suitable designed parameters, such as $c_i > 0$, $\delta_i > 0$, $\mu_i > 0$ and $\varpi_i > 0$. And decide the virtual control signal α_i and the parameter update law $\dot{\Gamma}_i (i = 2, \ldots, n-1)$.

Step V: Specify suitable designed parameters, such as $c_n > 0$, $\tau > 0$, $\delta_n > 0$, $\mu_n > 0$ and $0 < \sigma < 1$. And decide the control input u and the parameter update law $\dot{\Gamma}_n$.

4 Simulation examples

In order to verify the reliability of the designed controller, a numerical example and a practical example are given.

Example 1 Choose a nonstrict-feedback nonlinear systems

$$\begin{cases} \dot{\chi}_1(t) = \chi_2(t) + \sin \chi_1 \chi_2^2, \\ \dot{\chi}_2(t) = u + \left(1 + \chi_1^2\right)\chi_2^2, \end{cases} \tag{37}$$

where χ_1 and χ_2 represent the system state, u is the system input, and the boundary of constrained space is selected as $k_{c_1} = 1.5$ and $k_{c_2} = 1.65$. The ideal reference signal $y_d = 0.5 \sin(t)$. In this example, Gaussian functions contain

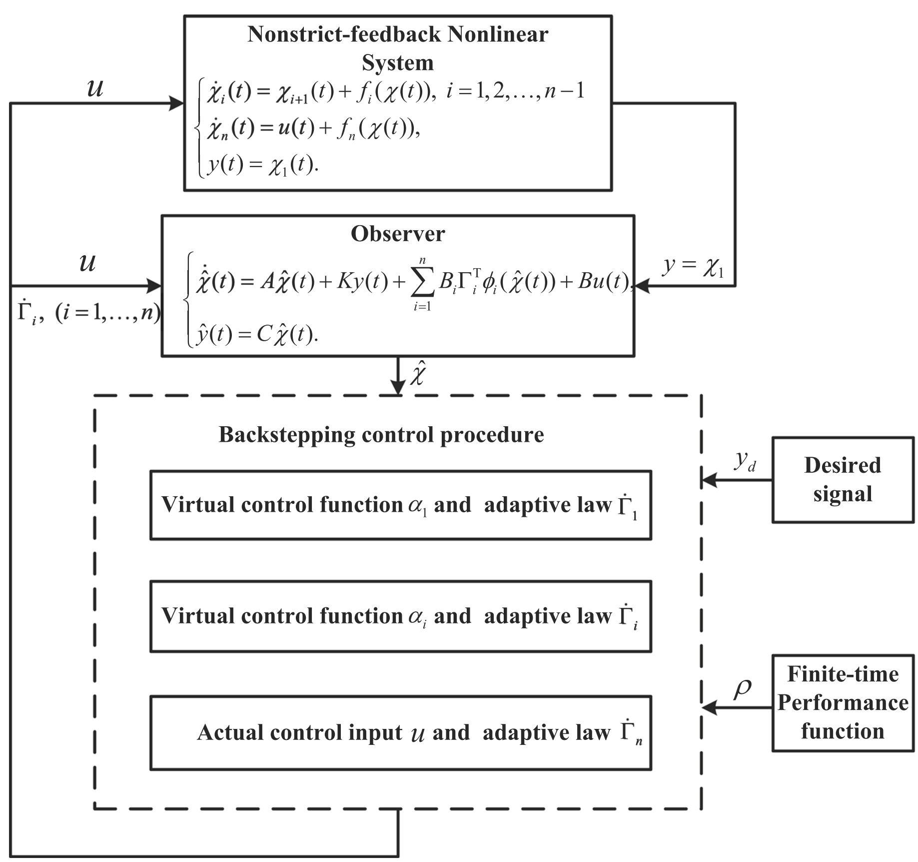

Fig. 2 Block diagram of the developed control strategy

eleven nodes with centers b_i evenly in $[-5, 5]$ and widths $b_i = 1$.

Define the FTPF $\rho(t)$ as

$$\begin{cases} \rho(t) = \left(\rho_0^l - l\varrho t\right)^{\frac{1}{l}} + \rho_{T_r}, & 0 \le t \le T_r \\ \rho_{T_r}, & t \ge T_r \end{cases} \tag{38}$$

with $l = \frac{2}{13}$, $\varrho = 0.7$, $\rho_0 = 0.6$ and $\rho_{T_r} = 0.025$. It should be noticed that the set time of finite-time performance function can be calculated as $T_r = \frac{\rho_0^l}{l\varrho} = 8.5839$ seconds.

Design α_1, u, $\dot{\Gamma}_1$ and $\dot{\Gamma}_2$ as

$$\begin{aligned} \alpha_1 = & -\frac{2\rho\left(k_{b_1}^2 - z_1^2\right)^{1-\sigma}}{\pi\left(1+\psi^2\right)} c_1 z_1^{2\sigma-1} + \dot{y}_d - \Gamma_1^{\mathrm{T}} \phi_1(\chi_1) \\ & - \frac{(3+\tau)\pi\left(1+\psi^2\right)}{4\rho} z_1 + \frac{2}{\pi}\dot{\rho}\arctan(v), \end{aligned} \tag{39}$$

$$\begin{aligned} u = & -\frac{c_2 z_2^{2\sigma-1}}{\left(k_{b_2}^2 - z_2^2\right)^{\sigma-1}} - k_2 e_1 + \dot{\gamma}_2 - \frac{z_2}{2\left(k_{b_2}^2 - z_2^2\right)} \\ & - \frac{\left(k_{b_2}^2 - z_2^2\right) z_1}{k_{b_1}^2 - z_1^2} - \Gamma_2^{\mathrm{T}} \phi_2(\hat{\bar{\chi}}), \end{aligned} \tag{40}$$

$$\dot{\Gamma}_1 = \frac{\delta_1 \pi\left(1+\psi^2\right)}{2\rho\left(k_{b_1}^2 - z_1^2\right)} z_1 \phi_1(\chi_1) - \mu_1 \Gamma_1, \tag{41}$$

$$\dot{\Gamma}_2 = \frac{\delta_2 \phi_2(\hat{\chi})}{k_{b_2}^2 - z_2^2} z_2 - \mu_2 \Gamma_2. \tag{42}$$

Choose the NN state observer as

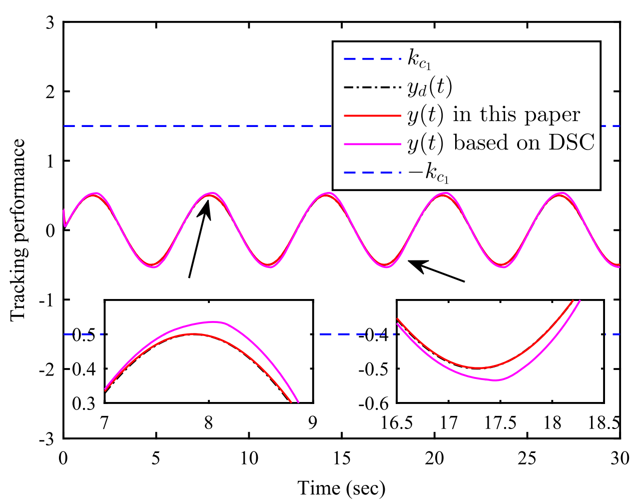

Fig. 3 Trajectories of y and y_d

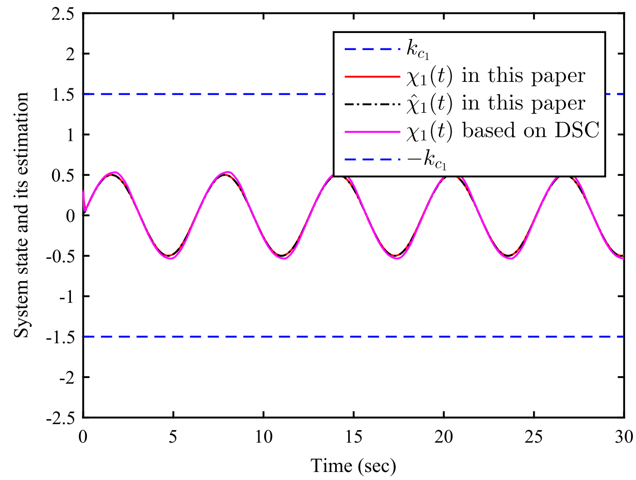

Fig. 4 Trajectories of χ_1 and $\hat{\chi}_1$

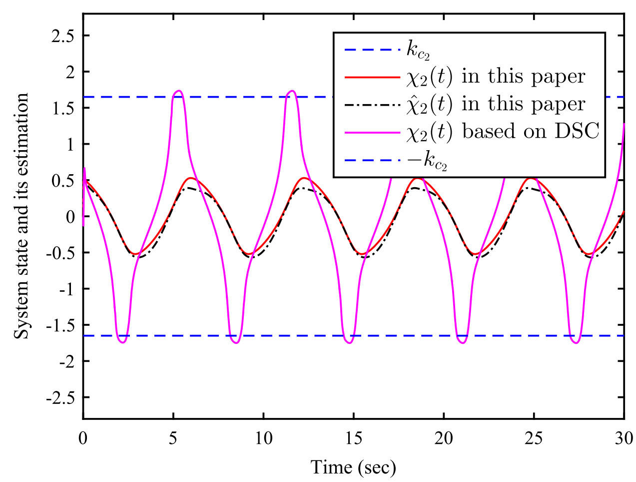

Fig. 5 Trajectories of χ_2 and $\hat{\chi}_2$

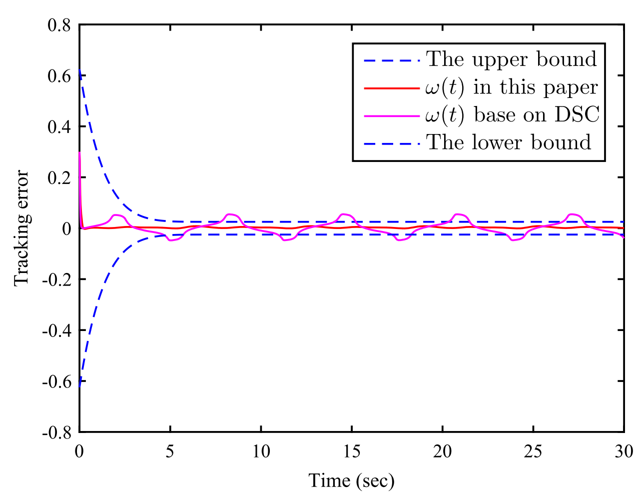

Fig. 6 Trajectories of ω

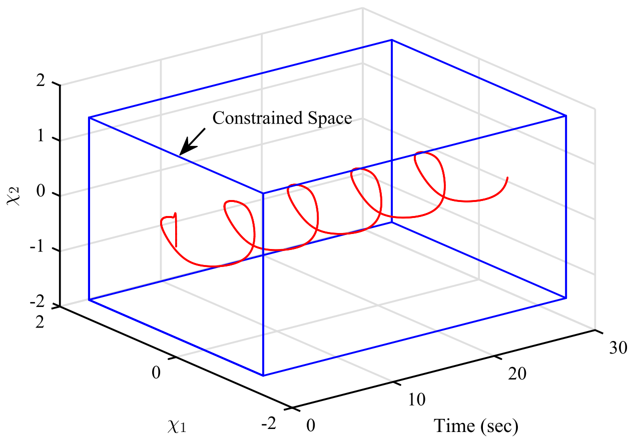

Fig. 7 State trajectories remain in the constrained space

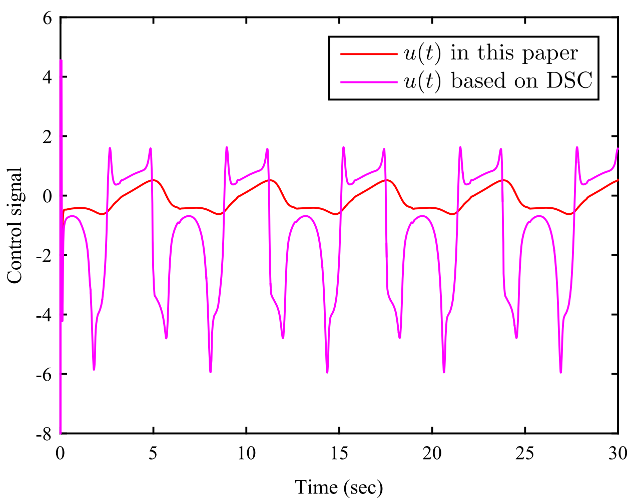

Fig. 8 Trajectories of u

$$\begin{cases} \dot{\hat{\chi}}_1 = \hat{\chi}_2 - k_1(\hat{\chi}_1 - \chi_1) + \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}), \\ \dot{\hat{\chi}}_2 = u - k_2(\hat{\chi}_1 - x_1) + \Gamma_2^{*\mathrm{T}}\phi_2(\hat{\chi}). \end{cases} \tag{43}$$

The parameters are selected as $\epsilon = 99/101$, $\tau = 1$, $\delta_1 = \delta_2 = 0.5$, $\mu_1 = \mu_2 = 0.5$, $c_1 = 15$, $c_2 = 18$, $k_1 = 20$, $k_2 = 50$. $[\chi_1(0), \chi_2(0)] = [0.3, 0]$, $[\hat{\chi}_1(0), \hat{\chi}_2(0)] = [0, 0]$, $\Gamma_1(0) = \Gamma_2(0) = [0,0,0,0,0,0,0,0,0,0,0]^{\mathrm{T}}$.

To verify the effectiveness of the developed controller, figures (Figs. 3, 4, 5, 6, 7 and 8) have been displayed for this example. Figure 3 exhibits the tracking performance of the considered system. Figure 4 plots the system state χ_1 and the estimation $\hat{\chi}_1$. Figure 5 depicts the system state χ_2 and its estimation $\hat{\chi}_2$. Figure 6 shows the tracking error, and it can be seen the tracking error remains in the prescribed range. Figure 7 depicts the constrained space and states trajectories remain in the constrained space. The system input u is exhibited in Fig. 8. It should be noticed that the designed parameters, the initial conditions, the centers of the receptive and the width of Gaussian functions based on DSC in [11] are selected the same with the controller proposed in this paper. Form Figs. 5 and6, we can see that the trajectory of the tracking error $\omega(t)$ based on DSC in [11] is out of the prescribed range and χ_2 based on DSC in [11] is out of constraint interval. Thus, the control scheme developed in this paper can ensure the full states and the tracking error do not violate constraint conditions and prescribed range, respectively.

Example 2 Based on the designed adaptive NN control scheme, the model of the electromechanical system exhibited in Fig. 9. Its dynamics can be modeled as the following form in [40]

$$\begin{aligned} M\ddot{q} + B\dot{q} + N\sin q + \Delta_1(\dot{q}, q, I) =& I, \\ V_e - RI - K_B\dot{q} + \Delta_2(\dot{q}, q, I) =& L\dot{I}, \end{aligned} \tag{44}$$

where

$$M = \frac{J}{K_\tau} + \frac{mL_0^2}{3K_\tau} + \frac{M_0L_0^2}{K_\tau} + \frac{2M_0R_0^2}{5K_\tau},$$
$$N = \frac{mL_0g}{2K_\tau} + \frac{M_0L_0g}{K_\tau}, B = \frac{B_0}{K_\tau},$$

and I represents the motor armature current, V_e is the input voltage, and q represents the angular motor position. To consider the influence of the model error on the real system, Δ_1 and Δ_2 denote the model error, respectively. The parameters of the electromechanical system are shown in Table 1.

Define $\chi_1 = q$, $\chi_2 = \dot{q}$, $\chi_3 = I$ and $u = V_e$. Equation (44) is transformed as

$$\begin{cases} \dot{\chi}_1(t) =& \chi_2(t), \\ \dot{\chi}_2(t) =& \frac{1}{M}\chi_3 - \frac{B}{M}\chi_2 - \frac{N}{M}\sin\chi_1 + \frac{B}{M}\cos\chi_2\sin\chi_3, \\ \dot{\chi}_3(t) =& \frac{1}{L}u - \frac{K}{L}\chi_2 - \frac{R}{L}\chi_3. \end{cases} \tag{45}$$

In this example, we give $y_d = 0.5\sin(t) + \sin(0.5t)$. The Gaussian functions in this example contain nine nodes with center b_i evenly in $[-4, 4]$ and widths $b_i = 1$. And the prescribed performance function keeps the same as Example 1. Furthermore, the design parameters are chosen as $\epsilon = 97/101$, $\tau = 1$, $\delta_1 = \delta_2 = \delta_3 = 1$, $c_1 = 8$, $c_2 = 10$, $c_3 = 12$, $k_1 = 20$, $k_2 = 40$, $k_3 = 50$, $l = \frac{2}{13}$, $\varrho = 0.7$, $\rho_0 = 0.6$, $\rho_{T_r} = 0.05$ and $T_r = 8.5839$.

Meanwhile, the initial values are selected as $\chi_1(0) = 0.3$, $\chi_2(0) = -1$, $\chi_3(0) = -1$, $\hat{\chi}_1(0) = 0.3$, $\hat{\chi}_2(0) = -1$, $\hat{\chi}_3(0) = 0.3$, $\Gamma_1(0) = \Gamma_2(0) = \Gamma_3(0) = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]^{\mathrm{T}}$.

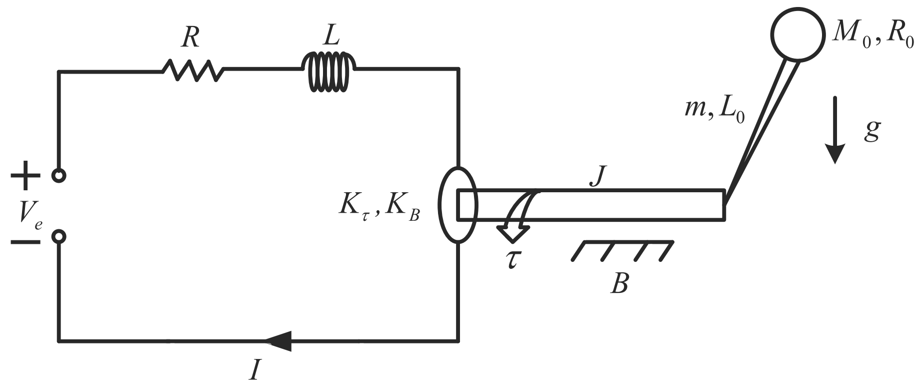

Fig. 9 Schematic of electromechanical system

Table 1 Parameters of the electromechanical system

Symbol	Physical meaning	Value
J	Rotor inertia	1.625×10^{-3} Kg·m^2
m	Link mass	0.506 Kg
R_0	Load radius	0.023 m
M_0	Load mass	0.434 Kg
L_0	Link length	0.305 m
K_B	Back-emf coefficient	0.9 N·m/A
L	Armature Inductance	25×10^{-3} H
B_0	Friction coefficient	0.01625 N·m·s/rad
R	Armature resistance	5 Ω
K_τ	Conversion value	0.9 N·m/A
g	Gravity coefficient	9.8 N/ kg

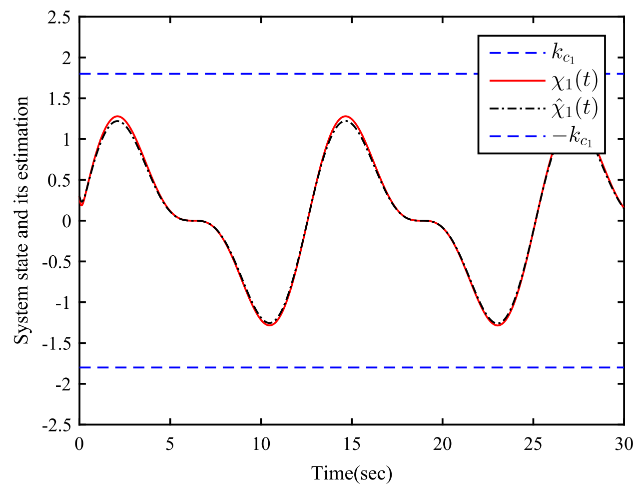

Fig. 11 Trajectories of χ_1 and $\hat{\chi}_1$

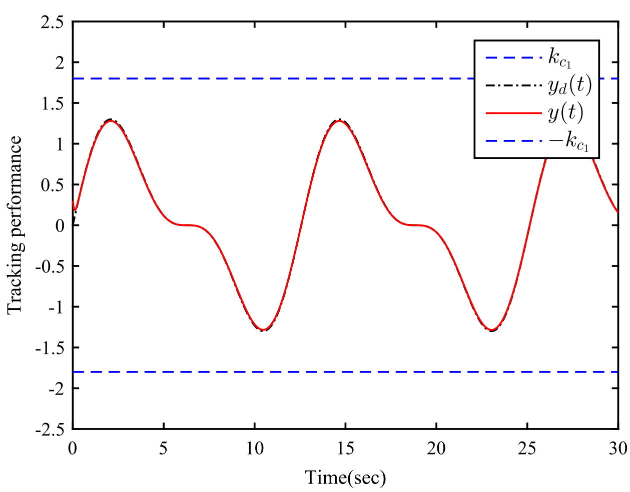

Fig. 10 Trajectories of y and y_d

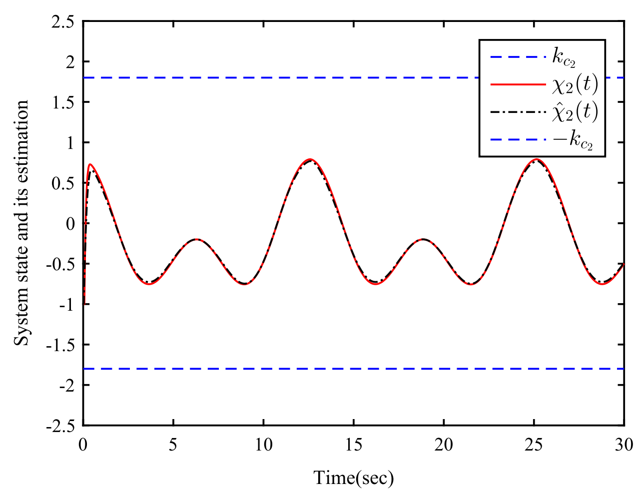

Fig. 12 Trajectories of χ_2 and $\hat{\chi}_2$

Figures 10, 11, 12, 13, 14 and 15 give the simulation results of the designed AFT control scheme. Figure 10 is described to show the tracking performance of the considered systems. The system state χ_1 and the estimation $\hat{\chi}_1$ are shown in Fig. 11. Figure 12 shows the system state χ_2 and its estimation $\hat{\chi}_2$. And the system state χ_3 and the estimation $\hat{\chi}_3$ are exhibited in Fig. 13. Figure 14 depicts the tracking performance, and we can see that the tracking error remains in the described range. The control input u is depicted in Fig. 15.

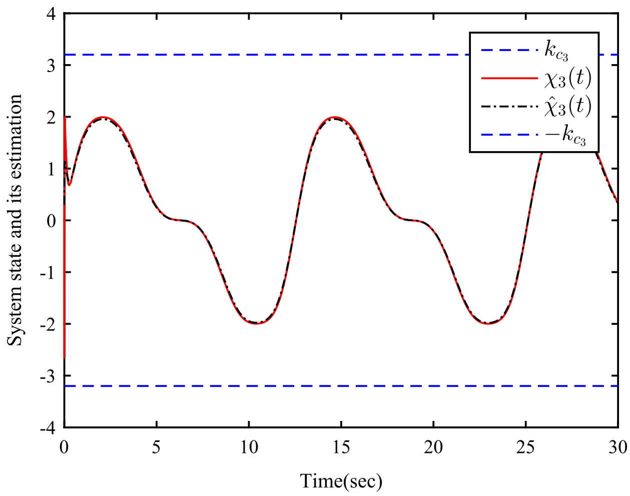

Fig. 13 Trajectories of χ_3 and $\hat{\chi}_3$

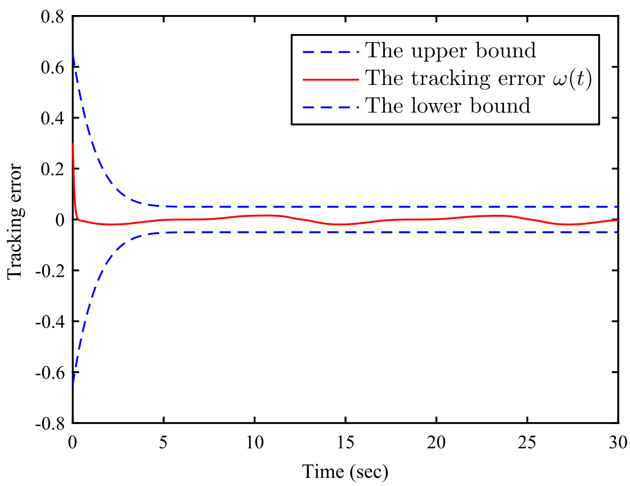

Fig. 14 Trajectories of ω

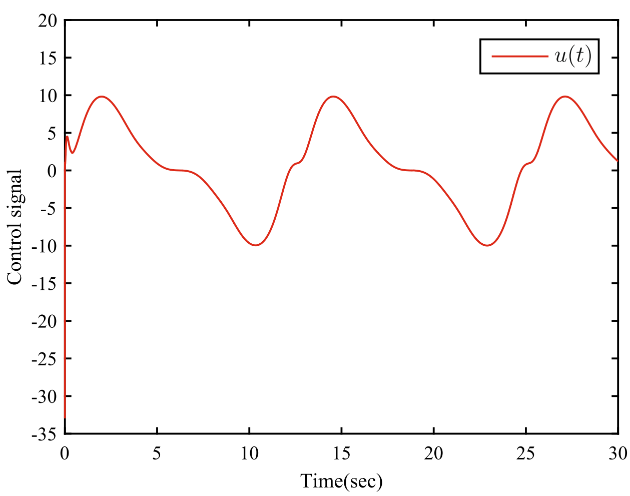

Fig. 15 Trajectories of u

5 Conclusion

This article has investigated the AFT prescribed performance control for nonstrict-feedback nonlinear systems subject to unmeasurable states and full-state constraints. The unmeasurable states are estimated via designing an NN observer. Meanwhile, the BLF method has been introduced in the process of the backstepping design, and the constraint condition is satisfied. It has been proven that the developed NN control strategy can ensure that the controlled system is SGPFS and the tracking error can converge to a predefined interval. Moreover, the DSC technique is utilized to address the complexity problem. Therefore, it has enriched the adaptive NN control theories of the nonstrict-feedback nonlinear systems.

Appendix 1

By (15), $\dot{V}_0$ is calculated as

$$\begin{aligned}\dot{V}_0(t) = & -e^{\mathrm{T}}(t)Qe(t) + 2e^{\mathrm{T}}(t)W\sum_{i=1}^{n}\tilde{\Gamma}_i\phi_i(\hat{\chi}(t)) \\ & + 2e^{\mathrm{T}}(t)W\big(\zeta + \Delta f\big).\end{aligned} \tag{46}$$

According to Lemma 1, Assumption 2 and $\phi_i^{\mathrm{T}}(X)\phi_i(X) \leq \Im$, the following inequalities hold

$$2e(t)^{\mathrm{T}}W\sum_{i=1}^{n}\tilde{\Gamma}_i\phi_i(\hat{\chi}) \leq \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + \|e\|^2, \tag{47}$$

$$2e(t)^{\mathrm{T}}W\zeta \leq \|W\|^2\|\zeta^*\|^2 + \|e\|^2, \tag{48}$$

$$2e(t)^{\mathrm{T}}W\Delta f \leq \left(\|W\|^2\sum_{i=1}^{n}l_i^2 + 1\right)\|e\|^2, \tag{49}$$

where $\zeta^* = [\zeta_1^*, \ldots, \zeta_n^*]^{\mathrm{T}}$.

According to (47, 48 and 49), it yields

$$\begin{aligned}\dot{V}_0 \leq & -e(t)^{\mathrm{T}}Qe(t) + \left(\|W\|^2\sum_{i=1}^{n}l_i^2 + 3\right)\|e\|^2 \\ & + \|W\|^2\|\zeta^*\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i \\ \leq & -q_0\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + M_0.\end{aligned} \tag{50}$$

Appendix 2

Step 1: From (1), (5) and (17), $\dot{z}_1$ can be given as

$$\begin{aligned}\dot{z}_1 =& \frac{\pi(1+\psi^2)}{2\rho}(\dot{\chi}_1 - \dot{y}_d - \frac{2}{\pi}\dot{\rho}\arctan(\psi))\\ =& \frac{\pi(1+\psi^2)}{2\rho}(z_2 + \alpha_1 + \varsigma_2 + \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}) - \dot{y}_d\\ &+ e_2 + \zeta_1 - \frac{2}{\pi}\dot{\rho}\arctan(\psi))\\ =& \frac{\pi(1+\psi^2)}{2\rho}(z_2 + \alpha_1 + \varsigma_2 + \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi})\\ &- \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}_1) + \Gamma_1^{\mathrm{T}}\phi_1(\hat{\chi}_1) + \tilde{\Gamma}_1^{\mathrm{T}}\phi_1(\hat{\chi}_1) + \zeta_1\\ &+ e_2 - \dot{y}_d - \frac{2}{\pi}\dot{\rho}\arctan(\psi)).\end{aligned} \tag{51}$$

Choose a Lyapunov function as

$$V_1 = V_0 + \frac{1}{2}\log\frac{k_{b1}^2}{k_{b_1}^2 - z_1^2} + \frac{1}{2\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}\tilde{\Gamma}_1, \tag{52}$$

where $k_{b_1} = k_{c_1} - A_0$, and δ_1 is the design parameter. Obviously, $V_0 \geq 0$ and $\tilde{\Gamma}_1^{\mathrm{T}}\tilde{\Gamma}_1 \geq 0$. In addition, $k_{b1}^2/(k_{b1}^2 - z_1^2) \geq 1$, that is , $\frac{1}{2}\log\frac{k_{b1}^2}{k_{b_1}^2 - z_1^2} \geq 0$. Then, $V_1 \geq 0$ can be obtained.

According to (51), the derivative of V_1 yields

$$\begin{aligned}\dot{V}_1 =& \dot{V}_0 + \frac{z_1\dot{z}_1}{k_{b_1}^2 - z_1^2} - \frac{1}{\gamma_1}\tilde{\Gamma}_1^{\mathrm{T}}\dot{\Gamma}_1\\ =& \dot{V}_0 + \frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1(\alpha_1 - \dot{y}_d + z_2 + \varsigma_2\\ &+ e_2 + \zeta_1 + \Gamma_1^{\mathrm{T}}\phi_1(\hat{\chi}_1) - \frac{2}{\pi}\dot{\rho}\arctan(\psi))\\ &+ \frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1(\Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}) - \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}_1))\\ &- \frac{1}{\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}(\dot{\Gamma}_1 - \delta_1\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1\phi_1(\hat{\chi}_1)).\end{aligned} \tag{53}$$

According to (53), Lemma 1 and $\phi_i^{\mathrm{T}}(X)\phi_i(X) \leq \Im$, we can obtain

$$\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1\zeta_1 \leq \frac{1}{2}(\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)})^2 z_1^2 + \frac{1}{2}\zeta_1^{*2}, \tag{54}$$

$$\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1\varsigma_2 \leq \frac{1}{2}(\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)})^2 z_1^2 + \frac{1}{2}\varsigma_2^2, \tag{55}$$

$$\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1 e_2 \leq \frac{1}{2}(\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)})^2 z_1^2 + \frac{1}{2}\|e\|^2, \tag{56}$$

$$\begin{aligned}&\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1(\Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}) - \Gamma_1^{*\mathrm{T}}\phi_1(\hat{\chi}_1))\\ &\quad \leq \frac{\tau}{2}(\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)})^2 z_1^2 + \frac{2\Im}{\tau}\|\Gamma_1^*\|^2,\end{aligned} \tag{57}$$

From (54)–(57), (53) can be written as

$$\begin{aligned}\dot{V}_1 \leq& \dot{V}_0 + \frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1(\alpha_1 - \dot{y}_d + \Gamma_1^{\mathrm{T}}\phi_1(\hat{\chi}_1)\\ &+ \frac{(3+\tau)\pi(1+\psi^2)}{4\rho(k_{b_1}^2 - z_1^2)}z_1 - \frac{2}{\pi}\dot{\rho}\arctan(\psi)) + \frac{1}{2}\|e\|^2\\ &+ \frac{2\Im}{\tau}\|\Gamma_1^*\|^2 + \frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1 z_2 + \frac{1}{2}\zeta_1^{*2} + \frac{1}{2}\varsigma_2^2\\ &- \frac{1}{\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}(\dot{\Gamma}_1 - \delta_1\frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1\phi_1(\hat{\chi}_1)).\end{aligned} \tag{58}$$

Considering (18), it follows that

$$\begin{aligned}\dot{V}_1 \leq& -q_1\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + M_1\\ &- \frac{c_1 z_1^{2\sigma}}{(k_{b_1}^2 - z_1^2)^{\sigma}} + \frac{\pi(1+\psi^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1 z_2\\ &+ \frac{\mu_1}{\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}\Gamma_1 + \frac{1}{2}\varsigma_2^2,\end{aligned} \tag{59}$$

where $q_1 = q_0 - \frac{1}{2}$ and $M_1 = M_0 + \frac{1}{2}\zeta_1^{*2} + \frac{2\Im}{\tau}\|\Gamma_1^*\|^2$.

The following first-order low-pass filter will be utilized to filter α_1 and to obtain γ_2

$$\varpi_2\dot{\gamma}_2 + \gamma_2 = \alpha_1, \gamma_2(0) = \alpha_1(0), \tag{60}$$

where ϖ_2 is a positive constant.

Define $\varsigma_2 = \gamma_2 - \alpha_1$. According to (60), it is easy to obtain $\dot{\gamma}_2 = -\frac{\varsigma_2}{\varpi_2}$, and then one gets

$$\dot{\varsigma}_2 = \dot{\gamma}_2 - \dot{\alpha}_1 = -\frac{\varsigma_2}{\varpi_2} + Y_2(\cdot), \tag{61}$$

where

$$Y_2(\cdot) = \frac{\partial\alpha_1}{\partial\hat{\chi}_1}\dot{\hat{\chi}}_1 - \frac{\partial\alpha_1}{\partial\Gamma_1}\dot{\Gamma}_1 - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d - \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d.$$

Step 2: According to (1), (5) and (17), $\dot{z}_2$ can be calculated as

$$\begin{aligned}\dot{z}_2 =& \dot{\hat{\chi}}_2 - \dot{\gamma}_2\\ =& \hat{\chi}_3 + k_2(y - \hat{\chi}_1) + \Gamma_2^{\mathrm{T}}\phi_2(\hat{\chi}) - \dot{\gamma}_2\\ =& z_3 + \varsigma_3 + \alpha_2 + k_2 e_1 + \Gamma_2^{*\mathrm{T}}\phi_2(\hat{\chi}) - \tilde{\Gamma}_2^{\mathrm{T}}\phi_2(\hat{\chi})\\ &- \Gamma_2^{*\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2) + \Gamma_2^{\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2) + \tilde{\Gamma}_2^{\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2) - \dot{\gamma}_2,\end{aligned} \tag{62}$$

where $\hat{\bar{\chi}}_2 = (\hat{\chi}_1, \hat{\chi}_2)^{\mathrm{T}}$.

Choose a Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\log\frac{k_{b_2}^2}{k_{b_2}^2 - z_2^2} + \frac{1}{2}\varsigma_2^2 + \frac{1}{2\delta_2}\tilde{\Gamma}_2^{\mathrm{T}}\tilde{\Gamma}_2, \tag{63}$$

where δ_2 is the positive designed parameter, and k_{b_2} will be given later. Similar to (52), $V_2 \geq 0$ can be obtained.

From (62) and (63), one has

$$\begin{aligned}\dot{V}_2 = \dot{V}_1 &+ \frac{z_2}{k_{b_2}^2 - z_2^2}(\alpha_2 + k_2 e_1 + z_3 + \varsigma_3 - \dot{\gamma}_2 \\ &+ \Gamma_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_2)) - \frac{z_2}{k_{b_2}^2 - z_2^2}\tilde{\Gamma}_2^{\mathrm{T}}\phi_2(\hat{\chi}) \\ &+ \frac{z_2}{k_{b_2}^2 - z_2^2}(\Gamma_2^{*\mathrm{T}}\phi_2(\hat{\chi}) - \Gamma_2^{*\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2)) \\ &+ \varsigma_2\dot{\varsigma}_2 - \frac{1}{\delta_2}\tilde{\Gamma}_2^{\mathrm{T}}(\dot{\Gamma}_2 - \frac{\delta_2}{k_{b_2}^2 - z_2^2}z_2\phi_2(\hat{\bar{\chi}}_2)).\end{aligned} \tag{64}$$

By Lemma 1, we can obtain the following inequalities

$$\begin{aligned}&\frac{z_2}{k_{b_2}^2 - z_2^2}(\Gamma_2^{*\mathrm{T}}\phi_2(\hat{\chi}) - \Gamma_2^{*\mathrm{T}}\phi(\hat{\bar{\chi}}_2)) \\ &\qquad \leq \frac{\tau z_2^2}{2(k_{b_2}^2 - z_2^2)} + \frac{2\Im}{\tau}\|\Gamma_2^*\|^2,\end{aligned} \tag{65}$$

$$\frac{z_2}{k_{b_2}^2 - z_2^2}\varsigma_3 \leq \frac{z_2^2}{2(k_{b_2}^2 - z_2^2)^2} + \frac{1}{2}\varsigma_3^2, \tag{66}$$

$$-\frac{z_2}{k_{b_2}^2 - z_2^2}\tilde{\Gamma}_2^{\mathrm{T}}\phi_2(\hat{\chi}) \leq \frac{z_2^2}{2(k_{b_2}^2 - z_2^2)^2} + \frac{\Im}{2}\tilde{\Gamma}_2^{\mathrm{T}}\tilde{\Gamma}_2, \tag{67}$$

According (65)-(67), one has

$$\begin{aligned}\dot{V}_2 \leq \dot{V}_1 &+ \frac{z_2}{k_{b_2}^2 - z_2^2}(\alpha_2 + k_2 e_1 - \dot{\gamma}_2 + \Gamma_2^{\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2) \\ &+ \frac{(2+\tau)z_2}{k_{b_2}^2 - z_2^2}) + \frac{1}{2}\varsigma_3^2 + \frac{\Im}{2}\tilde{\Gamma}_2^{\mathrm{T}}\tilde{\Gamma}_2 + \frac{2\Im}{\tau}\|\Gamma_2^*\|^2 \\ &+ \varsigma_2\dot{\varsigma}_2 - \frac{1}{\delta_2}\tilde{\Gamma}_2^{\mathrm{T}}(\dot{\Gamma}_2 - \frac{\delta_2}{k_{b_2}^2 - z_2^2}z_2\phi_2(\hat{\bar{\chi}}_2)) \\ &+ \frac{z_2 z_3}{k_{b_2}^2 - z_2^2},\end{aligned} \tag{68}$$

where $M_2 = M_1 + \frac{2\Im}{\tau}\|\Gamma_2^*\|^2$.

And then, we can obtain

$$\begin{aligned}\dot{V}_2 \leq &-q_1\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + M_2 \\ &+ \frac{z_2 z_3}{k_{b_2}^2 - z_2^2} + \frac{z_2}{k_{b_2}^2 - z_2^2}\Big(\alpha_2 + k_2 e_1 - \dot{\gamma}_2 \\ &+ \Gamma_i^{\mathrm{T}}\phi_2(\hat{\bar{\chi}}_2) + \frac{(2+\tau)z_2}{k_{b_2}^2 - z_2^2} \\ &+ \frac{\pi(1+v^2)(k_{b_2}^2 - z_2^2)}{2\rho(k_{b_1}^2 - z_1^2)}z_1\Big) + \frac{1}{2}\varsigma_3^2 \\ &+ \frac{\Im}{2}\tilde{\Gamma}_2^{\mathrm{T}}\tilde{\Gamma}_2 + \frac{\mu_1}{\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}\Gamma_1 + \frac{1}{2}\varsigma_2^2 - \frac{c_1 z_1^{2\sigma}}{(k_{b_1}^2 - z_1^2)^{\sigma}} \\ &+ \varsigma_2\dot{\varsigma}_2 - \frac{1}{\delta_2}\tilde{\Gamma}_2^{\mathrm{T}}(\dot{\Gamma}_2 - \frac{\delta_2}{k_{b_2}^2 - z_2^2}z_2\phi_2(\hat{\bar{\chi}}_2)).\end{aligned} \tag{69}$$

Substituting (19) into (69), one has

$$\begin{aligned}\dot{V}_2 \leq &-q_1\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + M_2 \\ &- \frac{c_1 z_1^{2\sigma}}{(k_{b_1}^2 - z_1^2)^{\sigma}} - \frac{c_2 z_2^{2\sigma}}{(k_{b_2}^2 - z_2^2)^{\sigma}} + \frac{\Im}{2}\tilde{\Gamma}_2^{\mathrm{T}}\tilde{\Gamma}_2 \\ &+ \frac{\mu_1}{\delta_1}\tilde{\Gamma}_1^{\mathrm{T}}\Gamma_1 + \frac{\mu_2}{\delta_2}\tilde{\Gamma}_2^{\mathrm{T}}\Gamma_2 + \frac{1}{2}\varsigma_2^2 + \frac{1}{2}\varsigma_3^2 \\ &+ \varsigma_2\left(-\frac{\varsigma_2}{\varpi_2} + Y_2(\cdot)\right) + \frac{z_2 z_3}{k_{b_2}^2 - z_2^2}.\end{aligned} \tag{70}$$

The first-order filter is designed as

$$\varpi_3\dot{\gamma}_3 + \gamma_3 = \alpha_2, \gamma_3(0) = \alpha_2(0). \tag{71}$$

Define $\varsigma_3 = \gamma_3 - \alpha_2$. According to (71), we can obtain $\dot{\gamma}_3 = -\frac{\varsigma_3}{\varpi_3}$, which implies

$$\dot{\varsigma}_3 = \dot{\gamma}_3 - \dot{\alpha}_2 = -\frac{\varsigma_3}{\varpi_3} + Y_3(\cdot),$$

where

$$Y_3(\cdot) = -\frac{\partial\alpha_2}{\partial\hat{\chi}_1}\dot{\hat{\chi}}_1 - \frac{\partial\alpha_2}{\partial\hat{\chi}_2}\dot{\hat{\chi}}_2 - \frac{\partial\alpha_2}{\partial\Gamma_1}\dot{\Gamma}_1 - \frac{\partial\alpha_2}{\partial\Gamma_2}\dot{\Gamma}_2 - \frac{\partial\alpha_2}{\partial\gamma_2}\dot{\gamma}_2.$$

Step i ($3 \leq i \leq n-1$): By (17), $\dot{z}_i$ is

$$\begin{aligned}\dot{z}_i &= \dot{\hat{\chi}}_i - \dot{\gamma}_i \\ &= \hat{\chi}_{i+1} + k_i(y - \hat{\chi}_1) + \Gamma_i^{\mathrm{T}}\phi_i(\hat{\chi}) - \dot{\gamma}_i \\ &= z_{i+1} + \varsigma_{i+1} + \alpha_i + k_i e_1 + \Gamma_i^{*\mathrm{T}}\phi_i(\hat{\chi}) \\ &\quad - \tilde{\Gamma}_i^{\mathrm{T}}\phi_i(\hat{\chi}) - \Gamma_i^{*\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i) + \Gamma_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i) \\ &\quad + \tilde{\Gamma}_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i) - \dot{\gamma}_i.\end{aligned} \tag{72}$$

Choose a Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}\log\frac{k_{b_i}^2}{k_{b_i}^2 - z_i^2} + \frac{1}{2}\varsigma_i^2 + \frac{1}{2\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i, \tag{73}$$

where δ_i is the positive parameter, and k_{b_i} will be given later. Similar to (63), it can be seen $V_i \geq 0$.

From (72) and (73), one has

$$\begin{aligned}\dot{V}_i = &\dot{V}_{i-1} + \frac{z_i}{k_{b_i}^2 - z_i^2}(\alpha_i + k_2 e_1 + z_{i+1} + \varsigma_{i+1} \\ &- \dot{\gamma}_{i+1} + \Gamma_i^{\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i)) - \frac{z_i}{k_{b_i}^2 - z_i^2}\tilde{\Gamma}_i^{\mathrm{T}}\phi_i(\bar{\chi}) \\ &+ \frac{z_i}{k_{b_i}^2 - z_i^2}(\Gamma_i^{*\mathrm{T}}\phi_i(\bar{\chi}) - \Gamma_i^{*\mathrm{T}}\phi_i(\hat{\bar{\chi}}_i)) \\ &+ \varsigma_i\dot{\varsigma}_i - \frac{1}{\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\left(\dot{\Gamma}_i - \frac{\delta_i}{k_{b_i}^2 - z_i^2}z_i\phi_i(\hat{\bar{\chi}}_i)\right).\end{aligned} \tag{74}$$

Similarly, it yields

$$\begin{aligned}&\frac{z_i}{k_{b_i}^2 - z_i^2}(\Gamma_i^{*\mathrm{T}}\phi_i(\bar{\chi}) - \Gamma_i^{*\mathrm{T}}\phi(\hat{\bar{\chi}}_i)) \\ &\leq \frac{\tau z_i^2}{2(k_{b_i}^2 - z_i^2)} + \frac{2\Im}{\tau}\|\Gamma_i^*\|^2,\end{aligned} \tag{75}$$

$$\frac{z_i}{k_{b_i}^2 - z_i^2}\varsigma_{i+1} \leq \frac{z_i^2}{2(k_{b_i}^2 - z_i^2)^2} + \frac{1}{2}\varsigma_{i+1}, \tag{76}$$

$$-\frac{z_i}{k_{b_i}^2 - z_i^2}\tilde{\Gamma}_i^{\mathrm{T}}\phi_i(\bar{\chi}) \leq \frac{z_i^2}{2(k_{b_i}^2 - z_i^2)^2} + \frac{\Im}{2}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i. \tag{77}$$

The first-order filter is designed as

$$\varpi_{i+1}\dot{\gamma}_{i+1} + \gamma_{i+1} = \alpha_i, \gamma_{i+1}(0) = \alpha_i(0). \tag{78}$$

Define $\varsigma_{i+1} = \gamma_{i+1} - \alpha_i$. According to (78), we can obtain $\dot{\gamma}_{i+1} = -\frac{\varsigma_{i+1}}{\varpi_{i+1}}$, which implies

$$\dot{\varsigma}_{i+1} = \dot{\gamma}_{i+1} - \dot{\alpha}_i = -\frac{\varsigma_{i+1}}{\varpi_{i+1}} + Y_{i+1}(\cdot),$$

where

$$Y_{i+1}(\cdot) = -\sum_{j=1}^{i}\frac{\partial\alpha_i}{\partial\hat{\chi}_j}\dot{\hat{\chi}}_j - \sum_{j=1}^{i}\frac{\partial\alpha_i}{\partial\Gamma_j}\dot{\Gamma}_j - \sum_{j=1}^{i}\frac{\partial\alpha_i}{\partial\gamma_i}\dot{\gamma}_i.$$

Substituting (75)–(77) and (19) into (53), it yields

$$\begin{aligned}\dot{V}_i \leq &-q_1\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + \sum_{j=1}^{i}\frac{1}{2}\varsigma_{j+1}^2 \\ &- \sum_{j=1}^{i}\frac{c_j z_j^{2\sigma}}{(k_{b_j}^2 - z_j^2)^\sigma} - \sum_{j=1}^{i}\frac{\mu_j}{\delta_j}\tilde{\Gamma}_j^{\mathrm{T}}\Gamma_j + \frac{z_i z_{i+1}}{k_{b_i}^2 - z_i^2} \\ &+ \sum_{j=2}^{i}\frac{\Im}{2}\tilde{\Gamma}_j^{\mathrm{T}}\tilde{\Gamma}_j + \sum_{j=2}^{i}\varsigma_j\left(-\frac{\varsigma_j}{\varpi_j} + Y_j(\cdot)\right) + M_i,\end{aligned} \tag{79}$$

where $M_i = M_{i-1} + \frac{2\Im}{\tau}\|\Gamma_i^*\|^2$.

Step n: From (1) and (17), the derivation of z_n is given as

$$\begin{aligned}\dot{z}_n = &u + k_n e_1 + \Gamma_n^{\mathrm{T}}\phi_n(\hat{\chi}) - \tilde{\Gamma}_n^{\mathrm{T}}\phi_n(\bar{\chi}) \\ &+ \tilde{\Gamma}_n^{\mathrm{T}}\phi_n(\bar{\chi}) - \dot{\gamma}_i.\end{aligned} \tag{80}$$

The Lyapunov function is given as

$$V_n = V_{n-1} + \frac{1}{2}\log\frac{k_{b_n}^2}{k_{b_n}^2 - z_n^2} + \frac{1}{2}\varsigma_n^2 + \frac{1}{2\delta_n}\tilde{\Gamma}_n^{\mathrm{T}}\tilde{\Gamma}_n, \tag{81}$$

where $\delta_n > 0$ is a designed parameter. Similar to (73), $V_n \geq 0$ can be obtained.

According to (80) and (81), one has

$$\begin{aligned}\dot{V}_n = &\dot{V}_{n-1} + \frac{z_n}{k_{b_n}^2 - z_n^2}(u + k_n e_1 - \dot{\gamma}_n + \Gamma_n^{\mathrm{T}}\phi_n(\hat{\bar{\chi}})) \\ &- \frac{z_n}{k_{b_n}^2 - z_n^2}\tilde{\Gamma}_n^{\mathrm{T}}\phi_n(\bar{\chi}) + \varsigma_n\dot{\varsigma}_n \\ &- \frac{1}{\delta_n}\tilde{\Gamma}_n^{\mathrm{T}}(\dot{\Gamma}_n - \frac{\delta_n}{k_{b_n}^2 - z_n^2}z_n\phi_n(\hat{\bar{\chi}})).\end{aligned} \tag{82}$$

From Lemma 1, it yields

$$-\frac{\tilde{\Gamma}_n^{\mathrm{T}}\phi_n(\bar{\chi})}{k_{b_n}^2 - z_n^2}z_n \leq \frac{z_n^2}{2(k_{b_n}^2 - z_n^2)^2} + \frac{\Im}{2}\tilde{\Gamma}_n^{\mathrm{T}}\tilde{\Gamma}_n. \tag{83}$$

From (79), (82) and (83), one gets

$$\begin{aligned}\dot{V}_n \leq &-q_1\|e\|^2 + \Im\|W\|^2\sum_{i=1}^{n}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i + M_{n-1} \\ &- \sum_{i=1}^{n-1}\frac{c_i z_i^{2\sigma}}{(k_{b_i}^2 - z_i^2)^\sigma} + \frac{z_n}{k_{b_n}^2 - z_n^2}(u + k_n e_1 - \dot{\gamma}_n \\ &+ \frac{z_n}{2(k_{b_n}^2 - z_n^2)} + \frac{(k_{b_n}^2 - z_n^2)z_{n-1}}{k_{b_{n-1}}^2 - z_{n-1}^2} + \Gamma_n^{\mathrm{T}}\phi_n(\hat{\bar{\chi}})) \\ &- \sum_{i=1}^{n-1}\frac{\mu_i}{\delta_i}\tilde{\Gamma}_i^{\mathrm{T}}\Gamma_i + \sum_{i=1}^{n-1}\frac{1}{2}\varsigma_{i+1}^2 + \sum_{i=2}^{n}\frac{\Im}{2}\tilde{\Gamma}_i^{\mathrm{T}}\tilde{\Gamma}_i \\ &+ \sum_{i=2}^{n}\varsigma_i\left(-\frac{\varsigma_i}{\varpi_i} + Y_i(\cdot)\right) \\ &- \frac{1}{\delta_n}\tilde{\Gamma}_n^{\mathrm{T}}(\dot{\Gamma}_n - \frac{\delta_n}{k_{b_n}^2 - z_n^2}z_n\phi_n(\hat{\bar{\chi}})).\end{aligned} \tag{84}$$

Considering (21), it yields

$$\begin{aligned}\dot{V}_n \le & -q_1\|e\|^2 + \Im\|W\|^2 \sum_{i=1}^{n} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i + \sum_{i=1}^{n} \frac{1}{2}\varsigma_i^2 \\ & - \sum_{i=1}^{n} \frac{\mu_i}{\delta_i} \tilde{\Gamma}_i^{\mathrm{T}} \Gamma_i - \sum_{i=1}^{n} \frac{c_i z_i^{2\sigma}}{(k_{b_i}^2 - z_i^2)^{\sigma}} \\ & + \sum_{i=2}^{n} \varsigma_i \left(-\frac{\varsigma_i}{\varpi_i} + Y_i(\cdot) \right) + \sum_{i=2}^{n} \frac{\Im}{2} \tilde{\Gamma}_i^{\mathrm{T}} \tilde{\Gamma}_i + M_i. \end{aligned} \tag{85}$$

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Declarations

Conflict of interest The authors declare that there is no conflict of interest.

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