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ORIGINAL ARTICLE

Observer-based adaptive control and faults estimation for T-S fuzzy singular fractional order systems

Yuqing Yan[1] · Huaguang Zhang[2] · Zhongyang Ming[1] · Yingchun Wang[1]

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Abstract

This paper handles the issue of adaptive control and faults estimation of a class of T-S singular fractional order systems(SFOSs) with H_∞ performance, where the fractional order belongs to (0, 1). Firstly, a novel observer for SFOSs is proposed, which estimate unmeasurable or partially measurable state and faults, simultaneously. Secondly, regarding to the information obtained by the above observer and the designed adaptive parameters, an adaptive controller is proposed to estimate actuator faults of the SFOSs. Further, it is indispensable to ensure the admissibility of the proposed fuzzy SFOSs with H_∞ performance, novel sufficient conditions are obtained by linear matrix inequalities (LMIs), Finally, to illustrate the method proposed above is available, simulation examples are presented.

Keywords T-S fuzzy singular systems · Fractional order systems · Adaptive fault-tolerant control · Robust H_∞

1 Introduction

In recent years, due to the description is more in line with real life, singular systems, that is, descriptor systems or generalised systems, have been widely conducted in various aspects of control field, such as electromagnetic systems, biological systems, flexible structures [1–3]. Generalized systems, which are more superior to complex phenomena than the normal system, so it has attracted the attention of many scholars [4–6]. Nevertheless, with the deepening of research, generalized system is far from meeting the needs of normal applications, so the fractional order system has slowly entered our sight. The industrial process has been refined under the fractional derivatives and calculus, which makes the research of fractional order systems(FOSs) very meaningful [7–9]. For an operating system, maintaining stable performance is the most valued actual issue. Lots of efficient stability conditions of FOSs have been established in [10–13]. According to LMIs, the stability analysis of fractional commensurate order systems with the cases of $0<\alpha<2$ was presented in [10]. As stated in [13], a novel conceptual unified framework for fractional neural networks was proposed because of fractional calculus has been classified as artificial neural networks. Since these two systems both play a vital role in control theory, the two systems are combined into a complex system, singular fractional order system, which is a new area worthy researching and have achieved abundant research results in recent years [14–16]. Guo et al. [14] focused on the stabilization problem for SFOSs by LMIs and a static output feedback controller was designed. In [15], a superior criteria have been put forward by strict LMI approach without equality constraints to obtain the stability of SFOS.

As we all know, in practice, most physical models are nonlinear. The control approach based on Takagi-Sugeno fuzzy systems is an efficient way to discuss complex nonlinear systems. Up to now, fuzzy systems have gained

✉ Huaguang Zhang
zhanghuaguang@mail.neu.edu.cn

Yuqing Yan
yanyuqing815@163.com

Zhongyang Ming
zhongyangming427@163.com

Yingchun Wang
drwangyc@gmail.com

1 College of Information Science and Engineering, Northeastern University, Box 134, 110819 Shenyang, People's Republic of China

2 State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, Liaoning 110004, China

widespread attention and significant results have been published for T-S fuzzy systems [17–20]. With a novel fuzzy observer and membership functions, Wang et al. [17] designed an algorithm and a novel controller to guarantee the asymptotically stable of the fuzzy H_∞ systems. Han et al. [18] addressed the application of multi-dimensional T-S fuzzy systems to eliminate the impact of failures. In [19], the novel descriptor observer and controller were designed to estimate faults, external noise and promoted the transformed closed-loop system to be asymptotically stable. Afterwards, an adaptive sliding mode controller strategy, whose sliding surface has already been constructed to process T-S fuzzy SFOSs with unknown constants in [21]. In order to be more realistic and achieve better performance, Asemani et al. [22] proposed sufficient conditions for stabilizing and designed a robust H_∞ observed-based controller for T-S fuzzy systems with uncertainty by LMI.

In fact, due to engineering and practical applications, it is inevitable that faults always exist, which requires many experts to spend a lot of energy to eliminate [23–26]. Compared with traditional feedback control, adaptive control design can better deal with the uncertainties in system dynamics and failures that may occur during system operation. In [26], a novel way is proposed to rape with the unknown singular systems by transforming it into the non-singular form with faults. Furthermore, an adaptive neural network approximation model for a nonlinear function is given. The key to the design task is to find a suitable adaptive law and a matched controller, so that under the condition of model matching, the adaptive controller still automatically adjust the remaining controller even though other actuators in the control system have unknown failures, which eventually achieve the desired control objective [27–29]. For the in-depth study, it is necessary to research the specific system, whose development is not completed, and there are still some aspects worth complementing.

Motivated by the above discussions, as a result of the increase in engineering accuracy requirements, it is inevitable to study the issue of adaptive observer for T-S fuzzy singular FOSs, which have not been studied completely. The contributions of the paper can be summarized as follows:

1. A new adaptive observer based on fault-tolerant control for SFOSs is proposed by designing sliding mode reaching law, which ensure that the stability of T-S fuzzy SFOSs and the regular condition towards the sliding surface has been improved.
2. A convex combination technique is developed, such that the proposed fault-tolerant control way is valid for the fuzzy systems with faults.
3. Due to the information obtained by the above observer and the designed adaptive parameters, according to the adaptive control law, an adaptive observer-based controller is proposed, which is more practical to estimate actuator faults of the SFOSs. The stability conditions in this paper reduce the conservatism and computational burden.
4. Further, novel sufficient conditions are obtained to ensure the stability ofthe proposed T-S fuzzy SFOSs with H_∞ performance in terms of LMI. Compared with the existed references, the H_∞ control method is more concise and more convenient for calculation with faults. Finally, to illustrate the proposed method is available, numerical examples are presented.

The rest of the paper is divided into the following sections. Section 2 presents some basic formula expressions. Section 3 gives an account of main results of the paper. In Sect. 4, numerical examples are given to display the validity of the theorem we proposed and Sect. 5 is the conclusion of the paper.

Notation: Here, it gives the definition of the symbols used in this paper. X^T expresses the transposition of the matrix X. sym$\{X\}$ means the form of $X+X^T$. $*$ notes a corresponding symmetric matrix. If there is no special description, for the dimensions of the matrix, which should be compatible.

2 Problem formulation and preliminaries

In this part, some preliminary for T-S fuzzy singular fractional order systems are given.

Definition 1 [11] The Caputo derivative of $f(t)$ is expressed as follows , where α belongs (0, 1) :

$$ {}_cD_t^\alpha x(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(\varsigma-\alpha)} \int_0^t \frac{f^{(\varsigma)}(\tau)}{(t-\tau)^{\alpha+1-\varsigma}} d\tau, \tag{1} $$

where $\varsigma - 1 < \alpha \leq \varsigma, \varsigma \in Z^+$, and $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ is gamma function. For convenience, we simplify ${}_cD_t^\alpha x(t)$ to $D^\alpha x(t)$.

Immediately afterwards, the continuous-time fuzzy T-S singular fractional order systems is given as follows:

$R_i(t)$: IF $\mu_1(t)$ is D_{1i}, $\mu_2(t)$ is D_{12},..., $\mu_j(t)$ is D_{1j}, THEN

$$ \begin{cases} ED^\alpha x(t) = A_i x(t) + B_i(u(t) + f_a(t)), \\ \qquad y(t) = C_i x(t). \end{cases} \tag{2} $$

where i is the number of IF-THEN rules, $i = 1, 2, ..., r$. D_{ij} are the fuzzy sets, $x(t) \in \mathbb{R}^n$ stands state vector, $u(t) \in \mathbb{R}^m$ represents control input, $f_a(t)$ means actuator faults. A_i, B_i, C_i are constant matrices. $E \in \mathbb{R}^n$ and rank$(E) =$

$r<n$, which is singular. Afterwards, the overall T-S fuzzy SFOSs are expressed as follows:

$$\begin{cases} ED^{\alpha}x(t) = \sum_{i=1}^{r} h_i(\mu(t))\{A_i x(t) + B_i(u(t) + f_a(t))\}, \\ y(t) = \sum_{i=1}^{r} h_i(\mu(t))C_i x(t). \end{cases} \tag{3}$$

where $h_i(\mu(t)) = \frac{\varpi_i(\mu(t))}{\sum_{i=1}^{r}\varpi_i(\mu(t))}$, $\varpi_i(\mu(t)) = \prod_{j=1}^{p} D_{ij}(\mu_j(t))$, in which $D_{ij}(\mu_j(t))$ are the grade of membership of $\mu_j(t)$ in D_{ij}. If $\varpi_i(\mu(t)) > 0$, then, $\sum_{i=1}^{r}\varpi_i(\mu(t)) \geq 0$. Therefore, $h_i(\mu(t)) \geq 0$, $\sum_{i=1}^{r} h_i(\mu(t)) = 1$.

In addition, the fault-tolerant control problem is investigated. The next step is to devise an adaptive control law that forces the system state to reach the sliding surface under possible actuator fault. The detailed flow diagram of the proposed approach is shown in Fig. 1 to clarify the design procedure and structure.

Next, in order to achieve our objective, some Lemmas are needed in the sequel.

Definition 2 [30] For a generalized fractional order system $ED^{\alpha}(t) = Ax(t)$, which is said to be regular if $\det(s^{\alpha}E - A)$ is not identially zero. When $\deg(\det(sE - A)) = \operatorname{rank}(E)$, unforced SFOSs are impulse free, which is also stable as the generalized eigenvalues of $\det(\lambda E - A) = 0$ lying in $D_{\alpha} = \{\lambda : |arg(\lambda)| > \frac{\alpha\pi}{2}\}$. SFOSs are admissibile if the above three conditions are fulfilled.

Lemma 1 [15] *The singular FOS in Definition* 2 *with* $0<\alpha<1$ *is admissible iff there exist matrices X and Y, P satisfying*

$$\begin{bmatrix} EX & EY \\ -EY & EX \end{bmatrix} = \begin{bmatrix} X^T E^T & -Y^T E^T \\ Y^T E^T & X^T E^T \end{bmatrix} \geq 0, \tag{4}$$

and

$$\operatorname{sym}\{rA^T P\} < 0.$$

where $r = e^{j(1-\alpha)\frac{\pi}{2}}$.

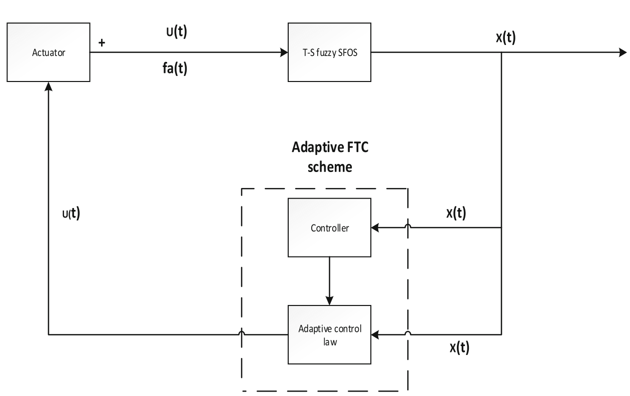

Fig. 1 Flow diagram in SFOS (3)

Lemma 2 [14] *Suppose SFOS in Definition* 2 *is regular, and M and N are invertible matrices such that*

$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & N_{n-m} \end{bmatrix}, MAN = \begin{bmatrix} \bar{A_1} & 0 \\ 0 & I_{n-m} \end{bmatrix}, \tag{5}$$

where $\lambda_{max}(\bar{A_1}) > 0$, N_{n-m} is nilpotent matrix.

When the regularity of SFOS in definition 2 *is unknown, and M and N are invertible matrices such that*

$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{6}$$

Lemma 3 [31] A, B *are known matrices and* $\Theta \in \mathrm{H}_n, \Phi \in \mathrm{H}_2, \Omega \in \mathrm{H}_2$. *Define* Λ *as follows,*

$$\Lambda(\Phi, \Omega) = \left\{ \lambda \in C | \begin{bmatrix} \lambda \\ 1 \end{bmatrix}^* \Phi \begin{bmatrix} \lambda \\ 1 \end{bmatrix} = 0, \begin{bmatrix} \lambda \\ 1 \end{bmatrix}^* \Psi \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \geq 0 \right\}.$$

Then, there exist $P, Q > 0$ such that

$$\begin{bmatrix} A & B \\ E & 0 \end{bmatrix}^* (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} A & B \\ E & 0 \end{bmatrix} + \Theta < 0$$

Lemma 4 [30] *Suppose* S_1 *and* S_3 *are symmetric matrices and* S_2 *is constants matrix, then* $S_1 + S_2 S_3^{-1} S_2^T < 0$ *iff*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} < 0.$$

Lemma 5 [9] *Suppose D and E are constant matrices and S is a symmetric matrix, which satisfy the following inequality* $S + DFE + (DFE)^T < 0$ *with* F *satisfying* $F^T F \leq I$, *if and only if for some* $\varepsilon > 0$,

$$S + \begin{bmatrix} E^T & D \end{bmatrix} \begin{bmatrix} \varepsilon^{-1} I & 0 \\ 0 & \varepsilon I \end{bmatrix} \begin{bmatrix} E \\ D^T \end{bmatrix} < 0$$

3 Main results

3.1 Adaptive observer design

First, based on the adaptive control strategy, an observer is designed.

$$\begin{cases} ED^{\alpha}\hat{x}(t) = \sum_{i=1}^{r} h_i(\mu(t))\{A_i \hat{x}(t) + L_i(y(t) - \hat{y}(t))\}, \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(\mu(t))C_i \hat{x}(t). \end{cases} \tag{7}$$

Subsequently, $e(t) = x(t) - \hat{x}(t)$ means the error of state variable and $e_y(t) = y(t) - \hat{y}(t)$ represents the error of output variable. Therefore, the above-mentioned equivalent adaptive fuzzy controller can be rephrased into the following form:

$$u_a(t) = \sum_{j=1}^{r} h_j(\mu(t))\{\mathcal{K}_i\hat{x}(t) - (H_jB_j)^{-1}H_jA_je(t) - f_a(t)\}, \tag{8}$$

where $H_i, \mathcal{K}_i$ are constant matrices under the constraint condition $\det(H_iB_i) \neq 0$. Next, the error system is given as follows:

$$\begin{cases} ED^\alpha e(t) = \sum_{i=1}^{r} h_i(\mu(t))\{(A_i - L_iC_i)e(t) \\ \quad +B_i\mathcal{K}_i\hat{x} - B_i(H_iB_i)^{-1}H_iA_i)e(t)\}, \\ \quad e_y(t) = \sum_{i=1}^{r} h_i(\mu(t))C_ie(t). \end{cases} \tag{9}$$

Substituting the equivalent control law (8) into (3) and defining $\xi(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$, the error dynamic can be acquired as:

$$ED^\alpha\xi(t) = \sum_{i=1}^{r} h_i(\mu(t))\bar{A}_i\xi(t) \tag{10}$$

where $\bar{A}_i = \begin{bmatrix} A_i + B_i\mathcal{K}_i & -B_i\mathcal{K}_i - G_iA_i \\ A_i - L_iC_i - B_i\mathcal{K}_i - G_iA_i & B_i\mathcal{K}_i, \end{bmatrix}$, $G_i = B_i(H_iB_i)^{-1}H_i$.

3.2 Admissibility analysis

In this section, we consider the issue of the admissibility of closed-loop error dynamic system (10). In order to simplify, we make the following equivalent transformation of the symbol. $h_\mu = \sum \quad i = 1^r h_i(\mu(t)), \bar{A}_h = h_\mu\bar{A}_i, C_h = h_\mu C_i, G_h = h_\mu G_i, L_h = h_\mu L_i, \mathcal{K}_h = h_\mu\mathcal{K}_i$.

Theorem 1 *T-S singular FOS (3) is asymptotically stable with adaptive controller (8) and the fractional order belongs to* $0 < \alpha < 1$ *if* X *is symmetric matrix and* $P_{1i} > 0, P_{2i} > 0, Y$ *are constant matrices and a scalar* $\beta > 0$ *satisfying the following LMIs:*

$$\begin{bmatrix} EX & EY \\ -EY & EX \end{bmatrix} = \begin{bmatrix} X^TE^T & -Y^TE^T \\ Y^TE^T & X^TE^T \end{bmatrix} \geq 0, \tag{11}$$

$$\mathrm{sym}(r(\bar{A}_h^TP) < 0. \tag{12}$$

where

$$P = \begin{bmatrix} P_{1i} & 0 \\ 0 & \beta P_{2i} \end{bmatrix}, r = e^{j\frac{\pi}{2}\alpha}\omega^\alpha.$$

Proof Substituting $\bar{A}_h$, and P into (12), we can obtain the inequality (13), then, the following results are given:

$$\mathrm{sym}\begin{bmatrix} r(A_i + B_i\mathcal{K}_i)^TP_{1i} & r(A_i - L_iC_i - B_i\mathcal{K}_i - \bar{G}_iA_i)^TP_{2i} \\ -r(B_i\mathcal{K}_i + \bar{G}_i\bar{A}_i)^TP_{1i} - r\beta(B_i\mathcal{K}_i)^TP_{2i} & r(B_i\mathcal{K}_i)^TP_{2i} \end{bmatrix} < 0, \tag{13}$$

$$r(A_h + B_h\mathcal{K}_h)^TP_{1i} + r^*P_{1i}^T(A_h + B_h\mathcal{K}_h) < 0. \tag{14}$$

As a result of lemma 1, T-S singular FOS (10) is asymptotically stable.

Remark 1 It is obvious that the inequality (11) in Theorem 1, is not strict LMIs, which contains constraints condition and make calculation difficult. To solve the problem, matrix S is given, satisfying $ES = 0$ and we give the following Theorem.

Theorem 2 *T-S singular FOS (3) is asymptotically stable with adaptive controller (8) and the fractional order belongs to* $0 < \alpha < 1$ *if there exists real symmetric matrices* X, Y, P, Q_i *satisfying the following LMIs*:

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} > 0, \tag{15}$$

$$\mathrm{sym}\{r\bar{A}_h^T(PE^T + SQ_h) + r^*\bar{A}_hYE^T\} < 0. \tag{16}$$

After that, since the proof method is similar to Theorem 1, it will not be explained in detail herein.

3.3 Adaptive laws design

In this part, an adaptive law is designed to guarantee the error system reach stability state in a limited time. In order to be necessary, we pull in the following assumption.

Assumption 1 The subsystem of the error dynamic system (9) is bounded and satisfying

$$\sup_{0 \leq t < \infty} \|e(t)\| \leq \delta_i, i = 1, 2, ..., r$$

where δ_i are known positive constant.

Subsequently, we give the adaptive control laws, whose parameters are given below. The adaptive parameters $\hat{\delta}_i, \hat{\gamma}_{i1}, \hat{\gamma}_{i2}$ to estimate $\delta_i, \gamma_{i1}, \gamma_{i2}$, respectively. The estimation errors are expressed as follows:

$$\tilde{\delta}_i = \hat{\delta}_i - \delta_i, \tilde{\gamma}_{i1} = \hat{\gamma}_{i1} - \gamma_{i1}, \tilde{\gamma}_{i2} = \hat{\gamma}_{i2} - \gamma_{i2}.$$

Then, we propose the adaptive controller laws such that the reachability condition is obtained.

$$u_{i_c}(t) = \mathcal{K}_i x(t) - (H_iB_i)^{-1}\left\{-H_iT_iC_ie(t) + \frac{v_i(t)}{\|v_i(t)\|}(\|H_iA_i + T_iC_i\|\hat{\delta}_i + \|H_iB_i\|\hat{\gamma}_{i1} + \|H_iB_i\|\|y(t)\|\hat{\gamma}_{i2} + \varepsilon_0)\right\} \tag{17}$$

$$\begin{cases} E_1D^\alpha\hat{\delta}_i = \sigma_{1i}\|v_i(t)\|\|H_iA_i + T_iC_i\|, \\ E_1D^\alpha\hat{\gamma}_{i1} = \sigma_{2i}\|v_i(t)\|\|H_iB_i\|, \\ E_1D^\alpha\hat{\gamma}_{i2} = \sigma_{3i}\|v_i(t)\|\|H_iB_i\|\|y(t)\|. \end{cases} \tag{18}$$

where $\sigma_{1i}, \sigma_{2i}, \sigma_{3i}, \varepsilon_0$ are positive scalar, $E_1 = MEN = \begin{bmatrix} I_m & 0 \\ 0 & N_{n-m} \end{bmatrix}$ is defined in lemma 2, $v_i(t)$ is continuous differentiable function, whose expression is given in (19) below and T_i is constant known matrix.

Afterwards, in order to prove that the designed controller make the system stable in a finite time, we propose a Lyapunov functional candidate.

$$V_i(t) = \frac{1}{2}v_i^T(t)v_i(t) + \frac{1}{2\sigma_{1i}}\tilde{\delta}_i^2 + \frac{1}{2\sigma_{2i}}\tilde{\gamma}_{i1}^2 + \frac{1}{2\sigma_{3i}}\tilde{\gamma}_{i2}^2 \tag{19}$$

Calculating the fractional derivative of $T_i(t)$, then

$$\begin{cases} E_1D^\alpha V_i(t) = E_1D^\alpha \\ \left\{v_i^T(t)v_i(t) + \frac{1}{\sigma_{1i}}\tilde{\delta}_i\hat{\delta}_i + \frac{1}{\sigma_{2i}}\tilde{\gamma}_{i1}\hat{\gamma}_{i1} + \frac{1}{\sigma_{3i}}\tilde{\gamma}_{i2}\hat{\gamma}_{i2}\right\}, \\ v_i(t) = h_\mu\{H_iE_1D^{\alpha-1}e(t) - H_i\int_0^t L_iC_ie(s)ds\}. \end{cases} \tag{20}$$

Substituting (17) into (19), the following inequality is given.

$$\begin{cases} E_1D^\alpha V_i(t) \leq \| v_i(t) \|^T E_1D^\alpha \| v_i(t) \| \\ +E_1D^\alpha\left\{\frac{1}{\sigma_{1i}}\tilde{\delta}_i\hat{\delta}_i + \frac{1}{\sigma_{2i}}\tilde{\gamma}_{i1}\hat{\gamma}_{i1} + \frac{1}{\sigma_{3i}}\tilde{\gamma}_{i2}\hat{\gamma}_{i2}\right\} \\ = \| v_i(t) \| \left\{\|H_iA_i + T_iC_i\|\tilde{\delta}_i\right. \\ \left. +\|H_iB_i\|\tilde{\gamma}_{i1} + \|H_iB_i\|\|y(t)\|\tilde{\gamma}_{i2} + \varepsilon_0\right\} \\ +\frac{1}{\sigma_{1i}}\dot{\tilde{\delta}}_i\dot{\hat{\delta}}_i + \frac{1}{\sigma_{2i}}\dot{\tilde{\gamma}}_{i1}\dot{\hat{\gamma}}_{i1} + \frac{1}{\sigma_{3i}}\dot{\tilde{\gamma}}_{i2}\dot{\hat{\gamma}}_{i2} \end{cases} \tag{21}$$

Then, to exchange (18) into (21), it obtains,

$$E_1D^\alpha V_i(t) \leq -\varepsilon_0 \| v_i(t) \| < 0. \tag{22}$$

After that, it prove that the designed controller make the error system stable in a finite time.

Remark 2 In practical applications, Under normal circumstances, $e(t)$ is unknown or difficult to obtain directly, so we usually find $e_y(t)$ to obtain $e(t)$ according to the equation $e_y(t) = Ce(t)$.

3.4 H_∞ performance control of T-S singular FOS

The control problem of continuous time T-S fuzzy SFOSs with H_∞ performance is considered in this section.

Definition 3 [31] The T_{wz} is defined as the transfer function of the system (3) in the following form:

$$T_{wz} = C_i(s^\alpha E - A_i)^{-1}B_i + D_i, \tag{23}$$

Then, in frequency domain, the H_∞ norm is defined as follows:

$$\|T_{wz}\|_\infty = \sup_\omega \bar{\sigma}(T_{wz}(j\omega)), \omega \geq 0. \tag{24}$$

Theorem 3 *Continuous time T-S singular FOS (10) is asymptotically stable with fractional order belonging $0 < \alpha < 1$ and $\|T_{wz}\|_\infty < \gamma$, if there exists matrices $P_{1i} > 0, P_{2i} > 0, Z_{1i} > 0, Z_{2i} > 0$ and a scalar $\beta > 0$ such that (15) and the follow LMIs satisfied:*

$$\begin{bmatrix} \mathrm{sym}\{r\bar{A}_h^T P\} + \Lambda & r^*(Z^T - P^T) & \bar{C}^T \\ * & -\gamma I & \bar{D}^T \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{25}$$

where $\bar{A}_h$ is consistent with Theorem 1, and

$$Z = \begin{bmatrix} Z_{1i} & Z_{2i} \\ -\beta P_{2i} & \beta P_{2i} \end{bmatrix}, P = \begin{bmatrix} P_{1i} & 0 \\ 0 & \beta P_{2i} \end{bmatrix}, \bar{C}_h = \begin{bmatrix} C_i & 0 \\ 0 & C_i \end{bmatrix}, \bar{D} = \begin{bmatrix} D_i & 0 \\ 0 & D_i \end{bmatrix}, \Lambda = \begin{bmatrix} 0 & 0 \\ 0 & -\gamma I \end{bmatrix}. \tag{26}$$

Proof Substituting $\bar{A}_h, \bar{C}_h, Z, \bar{D}, P, \Lambda$ into (25), it obtains that:

$$\begin{bmatrix} \bigvee & r^*(Z^T - P^T) \\ * & -\gamma I \end{bmatrix} < 0. \tag{27}$$

where

$$\bigvee = \mathrm{sym}\{r\bar{A}_i^T P\} + \Lambda + \gamma^{-1}\bar{D}^T\bar{D} + \gamma^{-1}\bar{C}^T\bar{C}. \tag{28}$$

due to the inequality (27), it obtains that: $\bigvee < 0$, which is expressed in (29), $\bigvee =$

$$\mathrm{sym}\left\{ r\begin{bmatrix} (A_i + B_i\mathcal{K}_i)^T P_{1i} & & \\ -(B_i\mathcal{K}_i + \bar{G}_i\bar{A}_i)^T P_{1i} - \beta(B_i\mathcal{K}_i)^T P_{2i} & (A_i - L_iC_i - B_i\mathcal{K}_i - \bar{G}_iA_i)^T P_{2i} & Z_{1i} - P_{1i} \\ * & (B_i\mathcal{K}_i)^T P_{2i} & -\beta P_{2i} \\ & * & -\gamma I \end{bmatrix} \right\} < 0, \tag{29}$$

$$\Phi = \begin{bmatrix} 0 & e^{j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}, \Psi = \begin{bmatrix} 0 & e^{-j\theta} \\ e^{j\theta} & 0 \end{bmatrix}.$$

Further, $\Upsilon > 0$, it means that $\|T_{wz}\|_\infty < \gamma$. The proof of the theorem has been completed.

Theorem 4 *Continuous time T-S singular fractional order system (10) is output-feedback stabilizable if there exists matrices* $P_{1i} > 0, P_{2i} > 0$, *and* $Z_{ji}(j = 1, 2, ..., 6), Q_{ji}(j = 1, 2, ..., 8), J, M, N, \mathcal{K}$ *are constant matrices and a scalar* $\beta > 0$ *such that (15) and the LMI (34), which is at the top of next page, satisfied*:

$$\begin{bmatrix} \Omega_{11} & * & * & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * & * & * \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * & * \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & * & * & * & * \\ r^*Q_{2i}^T\bar{A}_h + rL & r^*Q_{2i}^T\bar{B}_h - rP_{2i} & r^*Q_{2i}^T\bar{B}_h & -r^*Q_{2i}^T & -rP_{2i} - r^*P_{2i} & * & * & * \\ \Omega_{61} & \Omega_{62} & \Omega_{63} & \Omega_{64} & -rQ_{6i} & -rQ_{7i} - r^*Q_{7i}^T & * & * \\ r^*Q_{4i}^T\bar{A}_h & r^*Q_{4i}^T\bar{B}_h & r^*Q_{4i}^T\bar{B}_h & -r^*Q_{4i}^T & 0 & -r^*Q_{8i}^T & -rP_{2i} - r^*P_{2i} & * \\ r\bar{C}_h & r\bar{D} & r\bar{D} & 0 & 0 & 0 & 0 & -\gamma I \end{bmatrix} < 0, \tag{34}$$

$$\Theta = \mathrm{sym}\{r\gamma(\bar{A}_i + B_i\mathcal{K}_i)^T P_{1i}\} + \bar{D}^T\bar{D}, \tag{30}$$

then,

$$-\Theta - \bar{C}^T\bar{C} > 0. \tag{31}$$

Since the continuous time system (10) is admissibile, pre- and post-multiplying (31) by $B_i^T(s^\alpha E - \bar{A}_i)^{-T}$ and its transform, respectively, we have that,

$$\Upsilon = \gamma^2 I - B_i^T(s^\alpha E - \bar{A}_i)^{-T}\bar{C}^T\bar{C}(s^\alpha E - \bar{A}_i)B_i^T, \tag{32}$$

where $\gamma^2 I > 0$. Then, the above inequality can be converted into

$$\Upsilon - B_i^T(s^\alpha E - \bar{A}_i)^{-T}\Theta(s^\alpha E - \bar{A}_i)B_i^T > 0,$$

From lemma 3 and substitute Ψ, Ω, we have that:

$$\begin{bmatrix} \bar{A}_i & B_i \\ E & 0 \end{bmatrix}^T (\Phi \otimes Z + \Psi \otimes P) \begin{bmatrix} \bar{A}_i & B_i \\ E & 0 \end{bmatrix} + \begin{bmatrix} \Theta & 0 \\ 0 & -\Upsilon \end{bmatrix} < 0, \tag{33}$$

where

where

$$\Omega_{11} = \mathrm{sym}\{r(\bar{A}_h^T Z_{1i} - \mathcal{K}^T Z_{1i} + \beta M^T P_{2i} M - \beta M^T J - \beta J^T M)\},$$

$$\Omega_{21} = r^* Z_{2i}^T \bar{A}_h + r\bar{B}_h^T Z_{1i}^T + r^*\beta J + r\beta J,$$

$$\Omega_{22} = r\bar{B}_h^T Z_{2i} + r^* Z_{2i}^T \bar{B}_h - r\beta P_{2i} - r^*\beta P_{2i},$$

$$\Omega_{31} = r\bar{A}_h^T Z_{3i} + r^* Z_{1i}^T \bar{B}_h - r\beta N^T J - r^*\beta N^T J + r\beta N^T P_{2i} M + r^*\beta N^T P_{2i} M,$$

$$\Omega_{32} = r\bar{B}_h^T Z_{3i} + r^* Z_{2i}^T \bar{B}_h, \Omega_{42} = -rZ_{2i} + r^* Q_{1i}^T \bar{B}_h$$

$$\Omega_{33} = r\bar{B}_h^T Z_{3i} + r^* Z_{3i}^T \bar{B}_h + r\beta N^T P_{2i} N + r^*\beta N^T P_{2i} N - \gamma I,$$

$$\Omega_{41} = rP_{1i} - rZ_{1i} + r^* Q_{1i}^T A, \Omega_{43} = -rZ_{3i} + r^* Q_{1i}^T \bar{B}_h,$$

$$\Omega_{44} = -rQ_{1i} - r^* Q_{1i}^T, \Omega_{61} = rP_{1i} - rQ_{4i} + r^* Q_{3i}^T \bar{A}_h,$$

$$\Omega_{62} = -rZ_{5i} + r^* Q_{3i}^T \bar{B}_h, \Omega_{63} = -rZ_{6i} + r^* Q_{3i}^T \bar{B}_h, \Omega_{64} = -rQ_{5i} - r^* Q_{3i}^T.$$

Theorem 4 can be regarded as the extension of Theorem 3, so the proof is not expressed in detail.

4 Simulation results

Numerical examples are given to show the validity of the above approach in this section.

Procedure 1 The quantitative procedure of state, fault and adaptive parameters.

Step 1: Set the parameters and adaptive observer (7) with the aim of making the T-S SFOS asymptotically stable, and start the whole process.
Step 2: By considering internal actuator fault $f_a(t)$, this factor is taken into account in the design of the controller (8).
Step 3: According to theorems 1 and 2 of this paper, the LMI is used to solve (11)(12) or (15)(16) and determine whether the system is asymptotically stable under the action of this parameter and the control law (8) and thus whether it can reach the slide surface in finite time.
Step 4: Next, we found that the above steps are too computationally intensive in the process of adjusting parameters, so we designed the adaptive control law to automatically adjust the adaptive parameters $\hat{\delta}_i, \hat{\gamma}_{i1}, \hat{\gamma}_{i2}$ to find the optimal solution to make the system asymptotic stability.
*Step 5:*Finally, we present theorems 3 and 4, which are sufficient conditions for the H_∞ performance control of the system. Using the KYP lemma and the solution to LMI (25)(34), we can obtain that our proposed method is valid and H_∞ performance is satisfying $\|T_{wz}\|_\infty < \gamma$.

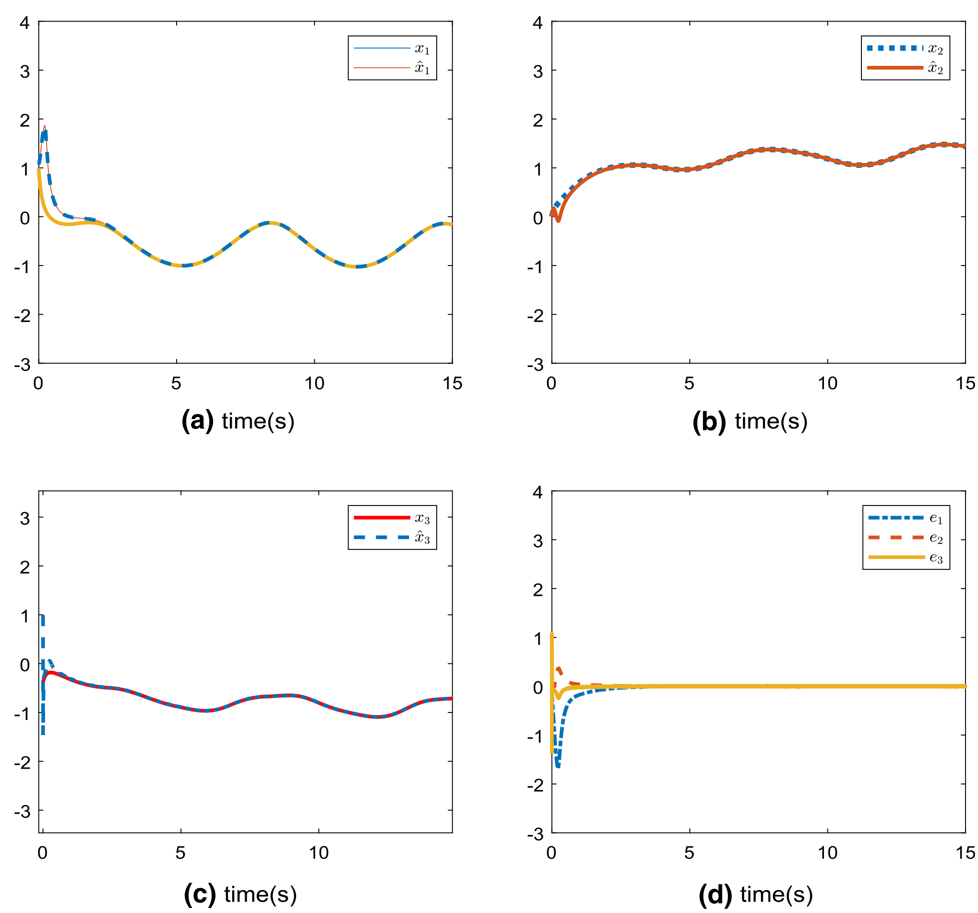

Fig. 2 (a) $x_1(t)$ and the error estimation of $x_1(t)$(b) $x_2(t)$ and the error estimation of $x_2(t)$(c) $x_3(t)$ and the error estimation of $x_3(t)$(d) State trajectories of error dynamic system (8)

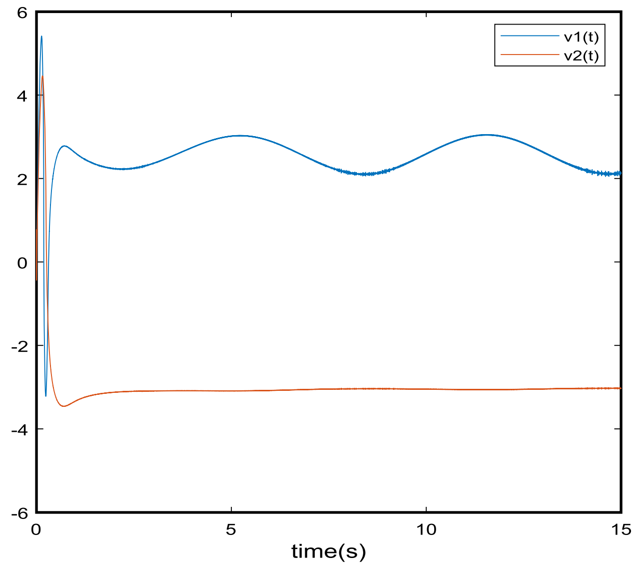

Fig. 3 The controllers $v_1(t)$ and $v_2(t)$ in Example 1

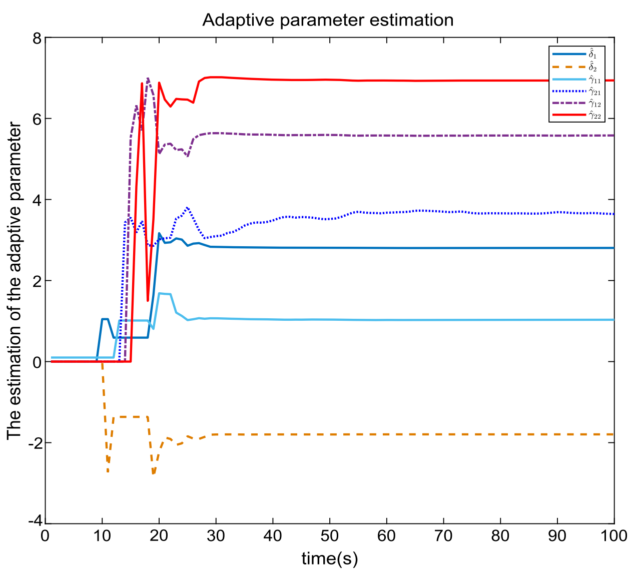

Fig. 4 The adaptive parameters in Example 1

Example 1 The following fuzzy singular fractional order systems are considered with two fuzzy rules and the parameter matrices are described as follows.

$$\begin{cases} ED^{\alpha}x(t) = \sum_{i=1}^{r} h_i(\mu(t))\{A_i x(t) + B_i(u(t) + f_a(t))\}, \\ y(t) = \sum_{i=1}^{r} h_i(\mu(t))C_i x(t). \end{cases} \tag{35}$$

where

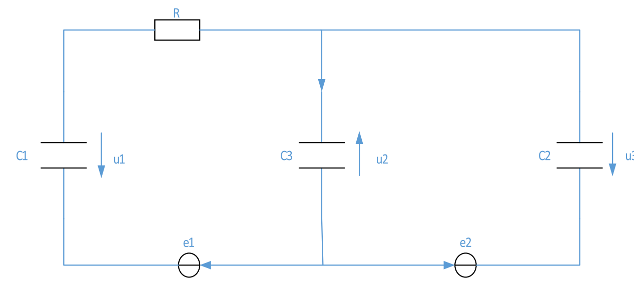

Fig. 5 Electrical circuit illustration in Example 2

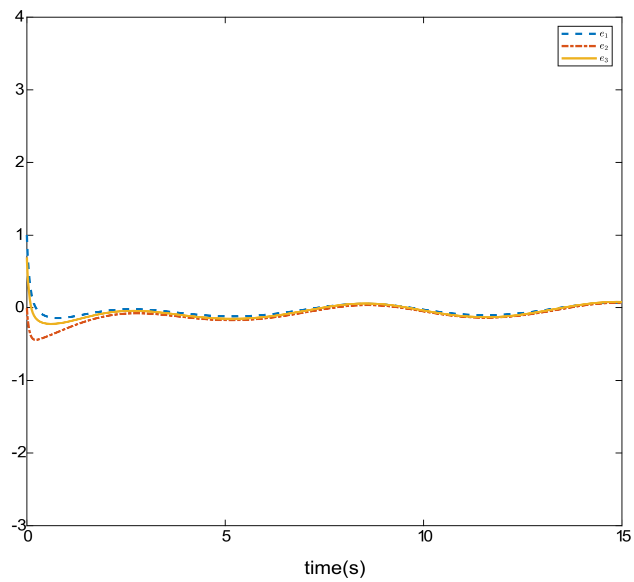

Fig. 6 The error estimation of the system (10) in Example 2

$$\alpha = \frac{3}{4}, E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 3 & 0 \\ 1 & 0 & -1 \\ -1 & -6 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & -6 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^T, C_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T,$$

$$h_1(\mu(t)) = \frac{1 + sin^2(x_1(t))}{3},$$

$$h_2(\mu(t)) = \frac{1 + cos^2(x_1(t))}{3}, f_a(t) = \begin{cases} 0, & t < 2 \\ \frac{1}{1 + 2^{-t}}, & t \geq 2 \end{cases}.$$

As a result of Theorem 3, to solve the inequalities (15) and (25), it obtains that the following solution:

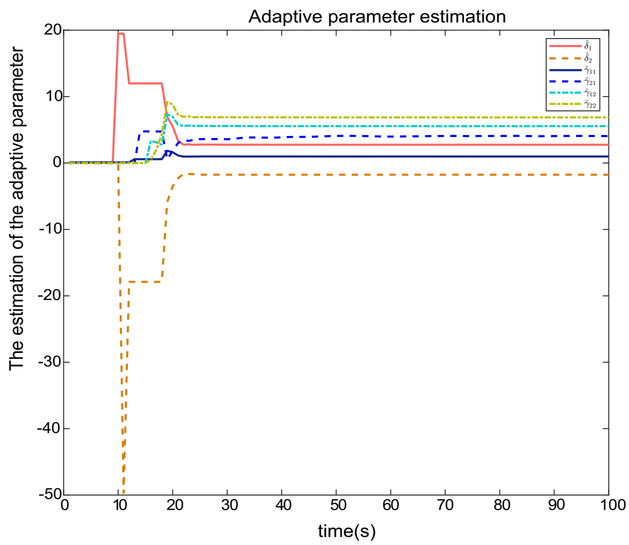

Fig. 7 The adaptive parameters in Example 2

$$P_1 = 10^8 * \begin{bmatrix} 0.8072 & -0.4548 & 0 \\ -0.4548 & 1.5022 & 1.7059 \\ -1.7059 & -1.7059 & 1.7059 \end{bmatrix},$$

$$P_2 = 10^8 * \begin{bmatrix} -0.8187 & -0.9555 & 0 \\ -0.9555 & 3.6438 & 1.5675 \\ -1.5675 & -1.5675 & 1.5675 \end{bmatrix},$$

$$Z_1 = 10^8 * \begin{bmatrix} 3.1138 & -1.6482 & 1.8387 \\ -1.6482 & 3.7259 & 3.2577 \\ 1.8387 & 3.2577 & 3.9148 \end{bmatrix},$$

$$Z_2 = 10^8 * \begin{bmatrix} -1.1041 & -3.6205 & 3.1062 \\ -3.6205 & 4.2988 & 0.0936 \\ 3.1062 & 0.0936 & 3.0042 \end{bmatrix},$$

$$\mathcal{K}_1 = \begin{bmatrix} 0.0305 & 0 & 0 \\ 0 & 0.0305 & 0 \\ 0 & 0 & 0.0305 \end{bmatrix},$$

$$\mathcal{K}_2 = \begin{bmatrix} 0.4100 & 0 & 0 \\ 0 & 0.4100 & 0 \\ 0 & 0 & 0.4100 \end{bmatrix},$$

$$L_1 = [0.0152 \quad 0.0152 \quad 0]^T,$$
$$L_2 = [-1.4322 \quad 0.1895 \quad 0.4176]^T.$$
$$\|T_{wz}\|_\infty \le \gamma_1 = 1.1991 * 10^8, \beta_1 = 3.2300 * 10^8,$$
$$\|T_{wz}\|_\infty \le \gamma_2 = 3.6749 * 10^8, \beta_2 = 3.6427 * 10^7.$$

Figures 1a–c shows $x_i(t)(i = 1,2,3)$ can be estimated and tracked accurately under observer-based adaptive fault-tolerant controller, whose curves are depicted in Fig. 2. After that, under the effect of the adaptive parameters, which is shown in Fig. 3, we get that the error system in Fig. 1d converges to zero, so that dynamic system (35) is asymptotically stable and Theorem 3 is valid.

Table 1 Application range comparison of existing methods and ours

References	C_1	C_2	C_3	C_4	C_5
The paper	Yes	Yes	Yes	Yes	No
9	No	Yes	Yes	No	Yes
15	No	Yes	No	Yes	Yes

Notations of Table 1: C_1 : less conservative for stabilization
C_2 : the stability interval of SFOS
C_3 : whether easy to solve or not
C_4 : convenient to be used in practice
C_5 : computational burden

Example 2 Considering electrical circuit shown in Fig. 5, which are widely used, including the following circuit element, a load resistance R, source voltage e_1, e_2, and capacitances C_1, C_2, C_3. Then, the circuit symbols are given , $u_1(t), u_2(t), u_3(t)$ represent the voltage of capacitances, respectively. Afterwards, the state equation is given as follows,

$$\begin{bmatrix} RC_1 & 0 & 0 \\ C_1 & C_2 & -C_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} D^\alpha u_1(t) \\ D^\alpha u_2(t) \\ D^\alpha u_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & -2 & 2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where $\alpha = 0.25, \quad C_1 = C_2 = C_3 = 1, \quad R = 1,$

$$f_a(t) = \begin{cases} 0, & t < 2 \\ \dfrac{1}{sin(0.5t - 5)}, & t \ge 2, \end{cases}$$

$$h_1(\mu(t)) = \frac{1 + sin(x_1(t))}{6}, h_2(\mu(t)) = 1 - h_1(\mu(t)).$$

Due to Theorem 2 and solving inequalities (15) and (16), it obtains a series of feasible solutions as follows:

$$X = \begin{bmatrix} 2.6613 & -0.8871 & 0.0534 \\ -0.8871 & 0.2957 & -0.0178 \\ 0.0534 & -0.0178 & 3.1853 \end{bmatrix},$$
$$Y = \begin{bmatrix} 1.1978 & 0.4285 & 0.5039 \\ -0.4285 & -0.3406 & 1.2105 \\ -0.5039 & -1.2105 & -3.0381 \end{bmatrix},$$
$$P = \begin{bmatrix} 2.7778 & 3.9200 & -0.8296 \\ 3.9200 & 1.1422 & 0.5348 \\ -0.8296 & 0.5348 & -2.7944 \end{bmatrix},$$

$$Q = [-0.3038 \quad 0.3467 \quad -1.2055],$$
$$L = [0.4854 \quad -0.6179 \quad -0.1441]^T.$$

Further, Fig. 6 shows that the error dynamic system (10) is asymptotically stable and the designed adaptive controller (8) with the adaptive parameters shown in Fig. 7, which means that the state variables of T-S fuzzy SFOSs are tracked precisely with actuator faults.

Remark 3 Through the above table, We extend the stability of the SFOS to fuzzy T-S systems. Compared with [15], our approach is easy to solve. We reduce the conservative for stabilization and own practical applications. [9] constructed a sliding surface with reduced dimension by the method of state transformation but increases the computational burden. In general, our result is better and stronger than the existing results.

5 Conclusion

This paper deals with the issue of adaptive control of T-S singular FOSs with faults and H_∞ performance, where the fractional order belongs to (0, 1). Firstly, according to adaptive laws and fuzzy approximation technology, an adaptive observer has been given to dedicate to guarantee the preset tracking performance, which keep the estimation of the unmeasurable or partially measurable state and faults, simultaneously. Secondly, a controller has been designed through constructing the sliding mode surface of fuzzy singular fractional order systems with adaptive sliding mode control strategy. Moreover, a convex combination technique has been developed, such that it is shown that the proposed approach is valid for the systems with faults and unknown disturbances. Further, in order to ensure the stability of the proposed system with H_∞ performance, novel sufficient conditions have been obtained by LMIs, Finally, to illustrate the availability of the presented method, numerical simulation and practical examples have been presented.

Declarations

Conflict of interest The authors declare that they have no conflict of interest. All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors. Informed consent was obtained from all individual.

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