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ORIGINAL ARTICLE

Adaptive neural control for a tilting quadcopter with finite-time convergence

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Abstract
This paper addresses the tracking control problem of the tilting quadcopter with unknown nonlinearities. A novel tilting quadcopter conception is proposed with a fully actuated version, which suggests that the translational and rotational movements can be controlled independently. Based on the Euler-Lagrange equations, the dynamics of tilting quadcopter is developed with uncertainties, where Neural Networks (NNs) are utilized to approximate the unknown nonlinearities in systems. We construct a novel auxiliary filter to obtain the estimation errors explicitly to achieve better approximation ability of NNs. By introducing new leakage terms in the adaptive scheme, the weights of identifier of NNs can converge to their optimal values. And a simple online verification is provided to test the parameter estimation convergence, which relaxes the requirement of persistent excitation condition. Moreover, we propose an Adaptive Finite-time Neural Control for the tilting quadcopter, where all the tracking errors can converge to a small neighborhood around zero in finite time as well as the estimation errors. Finally, comparative simulation results are presented to illustrate the effectiveness and superiority of our proposed control.

Keywords Adaptive control · Finite-time convergence · Neural networks · Parameter estimation · Tilting quadcopter

1 Introduction

Unmanned aerial vehicles (UAVs) have seen a boost in popularity and generated great interest for both military and civil applications [1–3]. Among various types of UAVs, quadcopters are adopted in many fields due to their maneuverability and simplicity [4–7]. However, the mobility and maneuverability of the quadcopter in standard configuration are limited since all propeller force vectors are limited in a single plane. Leading to only the cartesian position and the yaw angle can be controlled independently due to this under-actuated version [8]. With the increasing requirements in tasks, the abilities to interact with narrow and cluttered environments have attracted considerable attention, such as full-actuated, fault-tolerant and maneuverable version [9]. This indicates that exploring fully actuated quadcopter is preferred to greatly improve their performance in tasks [10].

Motivated by these requirements, some actuation strategies have constantly evolved in recent years. Considering an intuitive way by adding actuators in other directions, Hugo in [11] propose a new configuration with additional propellers in horizontal directions, where the translational and rotational movements can be controlled by two sets of rotors. Inspired by the tilt-wing in [12–14], Ryll first proposes a novel actuation concept in [15], where the propellers are allowed to tilt with respect to (w.r.t.) their axes. Due to the introduction of the tilting mechanism, the tilting quadcopter has the ability to generate arbitrary direction force or torque, which contributes to a fully-actuated version. And flight tests, although preliminary,

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clearly show its superior capability compared with the common quadcopter in [16]. A flight transition of the tilting quadcopter from 0 to 90° is then discussed since the maximum torque decreases as the pitch angle increasing [17]. To further improve the control bandwidth, a dual axis tilting quadcopter is proposed in [18] offering a wider range of control torques, which makes the vehicle be a more agile vision. Moreover, compared with the conventional quadcopter, the dual-axis version can still complete the task despite of half of actuators damaged [19, 20]. However, once a rotor failure of a common one, we will lose its control in yaw where the reliability and the survivability cannot be guaranteed [21]. As discussed, this tilting configuration can not only achieve full controllability with 6 DOF, but also fault-tolerant ability is dramatically improved [22].

Recently, the control design for the tilting quadcopter is receiving increased attention [23]. To address nonlinearities, an input-output linearization is applied to track arbitrary trajectories and desired orientations simultaneously in [24]. Considering external disturbances, a mixed sensitivity H_∞ optimal control is synthesized for the tilting quadcopter to compensate these effects [25]. The dynamics of tilting quadcopter with disturbances and uncertainties are studied in [26], where a second-order sliding mode is constructed to avoid the chattering phenomenon. In [27], the stability and PD control of tilting quadcopter are developed upon the failure of propeller during flight, where fully known dynamics are assumed. Moreover, a control scheme for the tilting quadcopter with H-configuration used for transporting unknown sling loads is proposed in [13]. Based on barrier Lyapunov functions, robust model reference adaptive control is employed to guarantee prior constraints on both tracking errors and control inputs despite of poor knowledge about vehicles and payloads. Various control designs have exploded over the years, such as backstepping control [28], sliding mode control [26], model predict control [29], robust control [30] and input output linearization [31]. Although the reliability and the flexibility are improved, the tilting mechanism makes the system into a complex nonlinear system. The additional unknown nonlinearities from the tilting mechanisms, aerodynamic damping and external environment should be fully considered in the control design to improve the robustness of the control scheme.

Adaptive control has been studied for decades with characterizing the ability of compensating uncertainties [32–34]. A well-known assumption in many existing literature is that uncertainties should follow the linear-in-parameter form or growth condition as supposed in [35–45]. However, this assumption is stringent in practice since many systems contain complex nonlinearities. To relax this condition and handle nonlinear uncertainties, function approximators are incorporated into the adaptive control, e.g., NNs [46–49] and fuzzy logic systems (FLSs) [50–52]. Gradient descent adaptive algorithm is developed in [32] to update the NNs weights, which may encounter a bursting phenomenon. To remedy this issue, several modified parameter estimations are proposed, e.g., e-modification [53] and σ-modification [54]. Due to additional damping terms in these algorithms, the closed-loop system can be guaranteed uniformly ultimately bounded [55]. However, most adaptive laws designed in [56–60] only concern about the overall system stability. Few considerations have been given to the estimated parameters convergence since the estimation errors is immeasurable or unknown generally. According to the certainty equivalence principle in [33], the control system performance can be greatly improved once the estimated parameters in adaptive laws converge to their optimal values. A well-recognized is noted that adaptive laws prefer containing some information on the parameter estimation error to guarantee its convergence. In [61], a novel composite-adaptation-based adaptive robust control method is proposed to achieve high tracking performance and accurate parameter estimation. This approach capturing the parameter error in the adaptation law can obtain better parameter estimation results than the traditional adaptive robust control. In [62], another parameter estimation scheme is proposed which features the ability to reconstruct the unknown parameters within finite time provided the PE condition being satisfied. And this idea is incorporated into the control design in [63] to obtain exponential convergence. Note that the above approaches need to online test the invertibility of a regressor-based matrix. And the designed auxiliary matrix and vector in [61–63] with unstable integrator inevitably increase all the time or even to infinite. Motivated by these requirements, to explore a control scheme, incorporated with a novel adaptive law with the ability of guaranteeing the system stability and the estimated parameters convergence, can pave the way for controlled systems with better tracking performance. Although these existing controllers are stable, the tracking errors converge to equilibrium points as time to infinity [64]. Then, an extension is paid to the fractal and fractional derivatives, where analysis and solution have been discussed for the linear and nonlinear fractional differential equations [65–71]. A finite-time stability theory has attracted considerable attention, which provides fast convergence, high-precision and better robustness to uncertainties [72]. Up to now, homogeneous [73], terminal sliding mode (TSM) [74] and nonsingular terminal sliding mode (NTSM) [75] are widely developed, where tracking errors converge to the equilibrium in finite-time. However, the existing TSM delivers a slower convergence when the system state is far away from the equilibrium, although its convergence rate lies in exponential one when near the equilibrium [76–78]. Thus, a fast finite-time theorem is proposed in [9, 79, 80], which can perform a fast convergence during the whole tracking process.

Motivated by the above discussion, we in this paper propose a novel tilting quadcopter conception with the translational and rotational movements controlled independently. To compensate for uncertainties in systems, an adaptive finite-time neural control is proposed, where a novel estimation error-based adaptive scheme is introduced. Compared to the existing work, the main contributions of this paper include:

(1) A novel fully actuated tilting quadcopter conception is proposed, where the push mechanism can achieve large tilting range. Compared with [13, 15–20, 24–31, 81–83], we establish the tilting quadcopter dynamics simultaneously considering uncertainties. The nonlinearities in systems do not need to follow the linear-in-parameter for or growth condition as assumed in [35–45], which can pave the way for the design of more general conditions.

(2) We propose a new auxiliary filter, where parameter estimation errors can be obtained explicitly and then be used as new leakage terms. By introducing these terms in the adaptive laws, compared with [56–60], high estimation performance can be achieved since weights of identifier NNs can converge to their optimal values in a bounded sense. Moreover, we provide a simple online approach to verify the parameters convergence, a relaxed alternative to the conventional PE-condition in [61–63].

(3) An adaptive finite-time neural control is proposed for the tilting quadcopter to compensate for the uncertainties in systems. By incorporating the new adaptive law into the control design, the tracking errors can fast converge to a small neighborhood around the equilibrium within finite time as well as the estimated errors.

The rest of this paper is organized as follows. Section 2 presents the design of a novel tilting quadcopter and develops its complete dynamics with uncertainties. The main results are given in Sect. 3, where an ANC with fast finite-time convergence is proposed for tilting quadcopter to achieve tracking and precise estimation simultaneously. In Sect. 4, simulation results are presented. Finally, we draw a conclusion in Sect. 5.

Notations: Throughout this paper, let $\mathbb{R}$ denote the set of real numbers and $\mathbb{R}^{n\times m}$ denote a $n\times m$-dimensional matrix. Let $K_i=\mathrm{diag}[k_{ij}]\in\mathbb{R}^{3\times 3}$ and $\bar{K_i}=\mathrm{diag}[\bar{k_{ij}}]\in\mathbb{R}^{3\times 3}$ denote positive-definite matrices with $i=1,2,\cdots,6,\ j=1,2,3.$ Let $z^k=[|z_1|^k\mathrm{sgn}(z_1),\cdots,|z_n|^k\mathrm{sgn}(z_n)]^{\mathrm{T}}$, where $z=[z_1,\cdots,z_n]^{\mathrm{T}}$ and $\mathrm{sgn}(\cdot)$ is the sign function.

2 Dynamic modeling

In this section, we propose a novel conception of the tilting quadcopter, which consists of four main rigid parts: the Body, Servo, Push and Propeller as shown in Fig. 1. Servo and Push mechanisms can be actuated to tilt about the corresponding axes w.r.t. the main body, which suggests a fully actuated vehicle can be obtained. Compared with [18] and [19], the tilting mechanisms of this novel conception can be indeed arbitrary within $[-\frac{\pi}{2},\frac{\pi}{2}]$ to provide more agile and reliable abilities.

2.1 Preliminary definitions

Let $R_W:\{o_w,x_w,y_w,z_w\}$ and $R_B:\{o_b,x_b,y_b,z_b\}$ denote the World reference frame and the Body frame. Let $R_{S_i}:\{o_{s_i},x_{s_i},y_{s_i},z_{s_i}\}$ and $R_{P_i}:\{o_{p_i},x_{p_i},y_{p_i},z_{p_i}\},i=1\cdots 4$ denote the Servo and Push frames associated with the i-th propeller as shown in Fig. 2.

In this paper, we select Euler angles to describe the attitude, and the rotation matrix R_{E2B} which represents the orientation of R_W w.r.t. R_B is given

$$R_{E2B}=\begin{bmatrix} \mathrm{c}\theta\mathrm{c}\psi & \mathrm{c}\theta\mathrm{s}\psi & -\mathrm{s}\theta \\ -\mathrm{c}\phi\mathrm{s}\psi+\mathrm{s}\phi\mathrm{s}\theta\mathrm{c}\psi & \mathrm{c}\phi\mathrm{c}\psi+\mathrm{s}\phi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{s}\phi\mathrm{c}\theta \\ \mathrm{s}\phi\mathrm{s}\psi+\mathrm{c}\phi\mathrm{s}\theta\mathrm{c}\psi & -\mathrm{s}\phi\mathrm{c}\psi+\mathrm{c}\phi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\theta \end{bmatrix}, \tag{1}$$

with the denotations $\mathrm{c}\cdot=\cos(\cdot)$ and $\mathrm{s}\cdot=\sin(\cdot)$ for clarity and conciseness. ϕ, θ and ψ are roll, pitch and yaw, respectively. The rotational equations take the form

Fig. 1 The tilting quadcopter vehicle

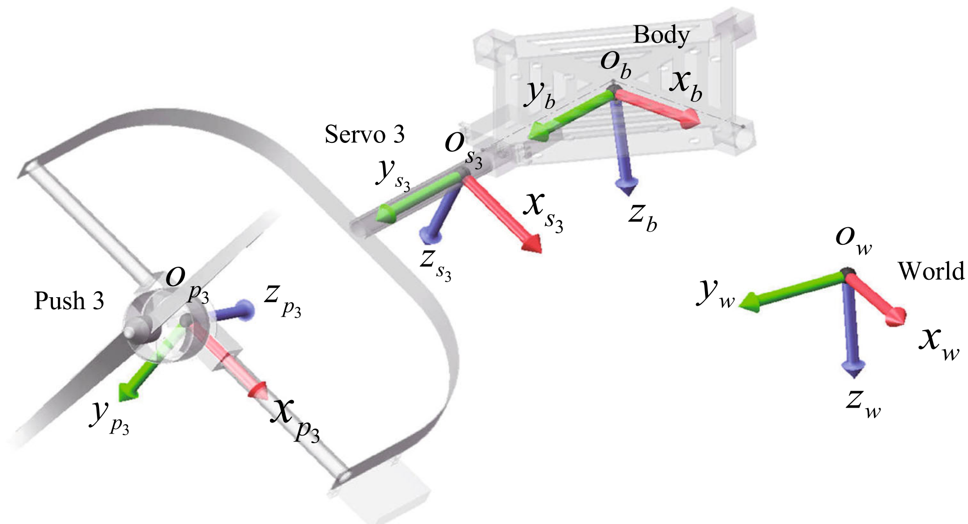

Fig. 2 The reference frame of 3-th propeller group

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{s}\phi \mathrm{t}\theta & \mathrm{c}\phi \mathrm{t}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \dfrac{\mathrm{s}\phi}{\mathrm{c}\theta} & \dfrac{\mathrm{c}\phi}{\mathrm{c}\theta} \end{bmatrix} \begin{bmatrix} p_v \\ q_v \\ r_v \end{bmatrix} = {R_1}^{-1} \omega_b, \tag{2}$$

where $\omega_b = [p_v, q_v, r_v]^T$ is the angle velocity projected into R_B. Let $\eta = [\phi, \theta, \psi]^T$ be the attitude vector. From (2), we have $R_1 \dot{\eta} = \omega_b$ where the inverse of matrix R_1 exists with the assumption of $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

2.2 Dynamic modeling of tilting quadcopter

According to the Euler–Lagrange equations, the dynamics of the tilting quadcopter can be described as

$$\begin{aligned} J(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} &= \tau + \tau_f \\ m_v \ddot{\xi} &= R_{E2B}^{\mathrm{T}} u + F_g + F_f, \end{aligned} \tag{3}$$

where $J(\eta) \in \mathbb{R}^{3\times 3}$ is the effective inertial matrix, $C(\eta, \dot{\eta}) \in \mathbb{R}^{3\times 3}$ is the Coriolis and Centrifugal force matrix, m_v is the mass of vehicle, $\xi = [x_v, y_v, z_v]^{\mathrm{T}}$ and $F_g = [0, 0, m_v g]^{\mathrm{T}}$ denote the position and the gravity vector in R_W; $\tau_f = [-g_1|p_v|p_v, -g_2|q_v|q_v, -g_3|r_v|r_v]^{\mathrm{T}}$ and $F_f = [-d_1|\dot{x}_v|\dot{x}_v, -d_2|\dot{y}_v|\dot{y}_v, -d_3|\dot{z}_v|\dot{z}_v]^{\mathrm{T}}$ are aerodynamic damping with positive coefficients g_i and d_i, $\tau \in \mathbb{R}^3$ and $u \in \mathbb{R}^3$ are control inputs expressed in R_B. From (2), we have $J = R_1^T I_v R_1$ where $I_v = \mathrm{diag}(I_{xx}, I_{yy}, I_{zz})$ is the moment inertia of the whole vehicle. If readers are interested in $J(\eta)$ and $C(\eta, \dot{\eta})$, more details can be found in Appendix A.

When outdoor flights, the aerodynamic damping may dramatically deteriorate the tracking performance. The coefficients like g_i and d_i in τ_f and F_f are always obtained based on the specific operation conditions. However, these coefficients may not cover all the realistic environment which becomes more complex than the specific conditions. It is impossible to capture these explicit parameters to achieve a perfect compensation. However, such a novel tilting conception makes the vehicle decouple the translational and rotational movements. Note that the skeleton of tilting quadcopter is varied while mechanisms tilting. This means that inherent parameters like the inertia of moment I_v would be different from the normal value but bounded by $[\underline{I}_v, \bar{I}_v]$ with positive constants $\underline{I}_v$ and $\bar{I}_v$. Motivated by the above discussions, the dynamics of tilting quadcopter is inevitably subjected to parametric uncertainties. For our control design, the following properties of the dynamics (3) are proposed.

Property 1 (see [36]): There exist positive constants $\underline{j}$ and $\bar{j}$ such that for any attitude vector η and vector $\vartheta = [\vartheta_1, \vartheta_2, \vartheta_3]^T$ with $\vartheta_i, (i = 1, 2, 3)$ being the real constants:

1. The effective inertial matrix $J(\eta)$ is symmetric, positive definite, and is bounded from below and above, i.e., $\underline{j} \leq \|J(\eta)\| \leq \bar{j}$.
2. The matrix $\dot{J}(\eta) - 2C(\eta, \dot{\eta})$ is skew-symmetric such that $\vartheta^{\mathrm{T}}(\dot{J}(\eta) - 2C(\eta, \dot{\eta}))\vartheta = 0$

3 Adaptive finite time neural control

3.1 Preliminaries

Lemma 1 (*see* [84]): *Suppose a continuous positive definite function* $V(\zeta)$, *and scalars* $a > 0$, $b > 0$, $0 < \beta < 1$ *and* $\varrho > 0$ *such that*

$$\dot{V}(\zeta) \leq -aV(\zeta) - bV^{\beta}(\zeta) + \varrho. \tag{4}$$

Then for any given time t_0, *the function* $V(\zeta)$ *is bounded by* *a small region* $\Omega_\zeta = \{\zeta | V^\beta(\zeta) \le \frac{\varrho}{b(1-c)}\}$ *with* $0<c<1$ *and the settling time* t_s

$$t_s \le t_0 + \frac{1}{a(1-\beta)} \ln\left(\frac{aV^{1-\beta}(\zeta_0) + bc}{bc}\right). \tag{5}$$

where ζ_0 *and* t_0 *are initial values of* ζ *and initial time, respectively.*

Lemma 2 (*see* [85]): *For* $x_i \in \mathbb{R}$, $i = 1, 2, \cdots, q$, $0<k\le 1$, *the following inequalities hold*

$$\left(\sum_{i=1}^{q} |x_i|\right)^k \le \sum_{i=1}^{q} |x_i|^k \le q^{1-k}\left(\sum_{i=1}^{q} |x_i|\right)^k. \tag{6}$$

Lemma 3 (*see* [86]): *The Radial Basis Function* (*RBF*) *networks are employed to emulate any continuous function* $g(X) : \mathbb{R}^r \to \mathbb{R}$:

$$g_{nn}(X) = W^{\mathrm{T}} S(X), \tag{7}$$

where the input vector $X \in \mathbb{R}^r$, *the weight vector* $W \in \mathbb{R}^m$, *the weight number* $m > 1$ *and the basis function vector* $S(X) = [s_1(X), s_2(X), \cdots, s_m(X)]^{\mathrm{T}}$ *with*

$$s_i(X) = \exp\left[\frac{-(X-\mu_i)^{\mathrm{T}}(X-\mu_i)}{\sigma_i^2}\right], \tag{8}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{ir}]^{\mathrm{T}}$ *is the center of the receptive field and* σ_i *is the width of the Gaussian function. RBF NNs can approximate any smooth function over a compact set with convergent errors*

$$g(X) = W^{*\mathrm{T}} S(X) + \epsilon, \tag{9}$$

where W^* *is the ideal weight vector, and* ϵ *is the convergent error, satisfying* $|\epsilon| \le \epsilon_n$ *with a positive constant* ϵ_n.

Lemma 4 (*see* [87]): *For any real variables* y_1, y_2, *and any positive constants* c *and* d, *the following inequality holds*

$$|y_1|^c |y_2|^d \le \frac{c}{c+d}|y_1|^{c+d} + \frac{d}{c+d}|y_2|^{c+d}. \tag{10}$$

3.2 Estimation error-based adaptive scheme design

Following the function approximation given in (9), we can always develop our system into a form as:

$$\dot{y} = W^{\mathrm{T}} S + \epsilon, \tag{11}$$

where $y \in \mathbb{R}^{3\times1}$ can be system states; $W = [\bar{W}_1, \bar{W}_2, \bar{W}_3] \in \mathbb{R}^{h\times3}$ and $S \in \mathbb{R}^{h\times1}$ are augmented weight matrix and regressor, respectively; $\epsilon = [\epsilon_1, \epsilon_2, \epsilon_3]^{\mathrm{T}} \in \mathbb{R}^{3\times1}$ is the residual error. Note that $\dot{y}$ always represents acceleration information which may be sensitive to noises. A stable filter is thus adopted

$$\begin{aligned} l\dot{y}_f &= -y_f + y, \quad y_f(0) = 0, \\ l\dot{S}_f &= -S_f + S, \quad S_f(0) = 0, \\ l\dot{\epsilon}_f &= -\epsilon_f + \epsilon, \quad \epsilon_f(0) = 0, \end{aligned} \tag{12}$$

where y_f, S_f and ϵ_f are the filtered variables, l is a positive constant. We then introduce the following auxiliary matrices P and vector Q

$$\dot{P} = -\delta P + S_f S_f^{\mathrm{T}}, \tag{13}$$

$$\dot{Q}_i = -\delta Q_i + S_f\left(\frac{y_i - y_{fi}}{l}\right)^{\mathrm{T}}, \ i = 1, 2, 3, \tag{14}$$

where δ is any positive constant. Based on matrices P and Q_i, a new leakage term L_i can be constructed as

$$L_i = P\hat{\bar{W}}_i - Q_i, \tag{15}$$

where $\hat{\bar{W}}_i$ is the estimated vector of $\bar{W}_i$. One can obtain a solution of the differential Eq. (13) as,

$$P = \int_0^t e^{-\delta(t-r)} S_f S_f^{\mathrm{T}} \mathrm{d}r \tag{16}$$

$$Q_i = \int_0^t e^{-\delta(t-r)} S_f \left(\frac{y_i - y_{fi}}{l}\right)^{\mathrm{T}} \mathrm{d}r, \tag{17}$$

From (15) and (16), L_i can be rewritten as

$$L_i = P\hat{\bar{W}}_i - Q_i = P\hat{\bar{W}}_i - P\bar{W}_i + C_i = -P\tilde{\bar{W}}_i + C_i, \tag{18}$$

where $C_i = -\int_0^t e^{-\delta(t-r)} S_f \epsilon_{fi}^{\mathrm{T}} \mathrm{d}r$ and $\tilde{\bar{W}}_i = \bar{W}_i - \hat{\bar{W}}_i$ is the estimation error. Since the NNs error ϵ and the regressor S are both bounded, then the filtered terms ϵ_{fi} and S_f are bounded. Thus, the term C_i is bounded by ϵ_{ci}.

As the introduction of the leakage term L_i, the unknown estimation error $\tilde{\bar{W}}_i$ can be abstracted, which can be used to drive the adaptive law to improve the estimation convergence. And we first give the following lemma.

Lemma 1 *If the regressor vector* S *satisfies the persistently excited* (*PE*) *condition, then the matrix* P_f *in* (13) *is positive-definite, i.e., its minimum eigenvalue fulfills* $\lambda_{\min}(P_f) > \delta_{P_f} > 0$ *for a positive constant* δ_{P_f}. *Moreover, if the matrix* P *is positive-definite, then* S *is PE.*

Proof The first part is omitted here for clarity and conciseness, and we provide the proof for the second part in Appendix B. □

Remark 1 Lemma 1 indicates that the required excitation $\lambda_{\min}(P_f) > \delta_{P_f} > 0$ can be obtained under the standard PE condition. According to [33], PE condition is necessary for the parameter estimation convergence. However, it is difficult to test the PE condition on-line. Lemma 1 provides an intuitive way to validate the PE condition, i.e., whether the minimum eigenvalue of $\lambda_{\min}(P_f) > 0$ holds. Namely, we provide a simple online approach to verify the parameters convergence, a relaxed alternative to the conventional PE-condition in [61–63].

Remark 2 The gradient-based adaptive law in [88] has not fully considered the convergence of parameter estimation. Although the e-modified adaptive law in [50] can eliminate the potential bursting phenomenon incurred by the gradient scheme, it only contains the estimated parameter $\hat{\bar{W}}_i$. And this scheme cannot achieve the estimated parameters converging to their optimal values. It should be noted that if we adopt the leakage term L_i in the adaptive law, with the help of the estimation error $\tilde{\bar{W}}_i$, the estimated parameter $\hat{\bar{W}}_i$ characters the ability to converge to a small neighborhood around its optimal value. The performance of the overall control system can be dramatically improved. Please refer to the following control design for more details.

3.3 Rotational subsystem control

This section contributes to develop an estimation error-based AFTNC with for the rotational subsystem of (3). Let η_d be the desired attitude and we define $z_1 = \eta - \eta_d$ as attitude errors. Then, a virtual control α_1 is given as

$$\alpha_1 = \dot{\eta}_d - K_1 z_1 - K_2 z_1^{2\beta_1 - 1}, \tag{19}$$

with $0 < \beta_1 < 1$. We then introduce an integral term

$$\chi_1 = \int_0^t z_2(s)\mathrm{d}s, \tag{20}$$

where $z_2 = \dot{\eta} - \alpha_1$. This integral term can dramatically reduce the sensitivity to parametric uncertainties [89]. Differentiating z_1, we have

$$\dot{z}_1 = \dot{\eta} - \dot{\eta}_d = z_2 + \alpha_1 - \dot{\eta}_d. \tag{21}$$

Inspired by the backstepping idea, the convergence of z_1 is completely depended on the convergence of z_2. This recursive methodology provides systematic steps on designing the control input to guarantee z_2 to converge. Differentiating z_2 w.r.t. time leads to

$$\begin{aligned} J\dot{z}_2 + Cz_2 &= J\ddot{\eta} - J\dot{\alpha}_1 + Cz_2 \\ &= \tau + \tau_f - C\alpha_1 - J\dot{\alpha}_1, \end{aligned} \tag{22}$$

where J and C are the abbreviations of $J(\eta)$ and $C(\eta, \dot{\eta})$. Then the above equation can be rewritten as

$$\dot{F}_{11}(z) + F_{12}(z) = \tau + F_{13}(z), \tag{23}$$

where $F_{11} = Jz_2,\ F_{12} = Cz_2 - \dot{J}z_2,\ F_{13} = \tau_f - C\alpha_1 - J\dot{\alpha}_1$ and $z = \left[\eta^{\mathrm{T}}, \dot{\eta}^{\mathrm{T}}, \alpha_1^{\mathrm{T}}, \dot{\alpha}_1^{\mathrm{T}}\right]^{\mathrm{T}}$. Thus, the control input τ, which can be used to develop an estimation error-based adaptive scheme, can indirectly reflect all uncertainties in system. According to Lemma 3, we apply NNs to approximate the unknown dynamics functions F_{11}, F_{12} and F_{13}

$$\begin{aligned} F_{11}(z) &= W_{11}^{*\mathrm{T}} S_{11} + \epsilon_{11}, \\ F_{12}(z) &= W_{12}^{*\mathrm{T}} S_{12} + \epsilon_{12}, \\ F_{13}(z) &= W_{13}^{*\mathrm{T}} S_{13} + \epsilon_{13}, \end{aligned} \tag{24}$$

where $W_{1i}^* \in \mathbb{R}^{m_1 \times 3}$ is the optimal weight matrix; $S_{1i}(z) \in \mathbb{R}^{m_1 \times 1}$ is the corresponding regressor, $\epsilon_{1i} \in \mathbb{R}^{3\times 1}$ is the estimation error bounded by $\|\epsilon_{1i}\| \le \epsilon_{1in}$ with positive constants ϵ_{1in}, m_1 is the number of NNs node. We can rewrite (23) as

$$W_1^{*\mathrm{T}} S_1 + \bar{\epsilon}_1 = \tau, \tag{25}$$

where $W_1^* \in \mathbb{R}^{3m_1 \times 3}$, $S_1 = \left[\dot{S}_{11}^{\mathrm{T}}, S_{12}^{\mathrm{T}}, -S_{13}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{3m_1 \times 1}$ and $\bar{\epsilon}_1 = \dot{\epsilon}_{11} + \epsilon_{12} - \epsilon_{13}$ with $\|\bar{\epsilon}_1\| \le \bar{\epsilon}_{1n}$ where $\bar{\epsilon}_{1n}$ is a positive constant. We define the form W_1^* as

$$W_1^* = \begin{bmatrix} W_{11}^* \\ W_{12}^* \\ W_{13}^* \end{bmatrix} = \left[W_{c1}^*, W_{c2}^*, W_{c3}^*\right], \tag{26}$$

where $W_{ci}^* \in \mathbb{R}^{3m_1 \times 1}$. According to the estimation error-based adaptive scheme, we first construct the following stable filters

$$\begin{aligned} l_1\dot{\tau}_{fi} &= -\tau_{fi} + \tau_i, & \tau_{fi}(0) &= 0, \\ l_1\dot{S}_{1f} &= -S_{1f} + S_1, & S_{1f}(0) &= 0, \\ l_1\dot{\bar{\epsilon}}_{1fi} &= -\bar{\epsilon}_{1fi} + \bar{\epsilon}_{1i}, & \bar{\epsilon}_{1fi}(0) &= 0, \end{aligned} \tag{27}$$

where τ_{fi}, S_{1f} and $\bar{\epsilon}_{1fi}$ are the filtered states. Then the following auxiliary matrices are established

$$\begin{aligned} \dot{P}_1 &= -\delta_1 P_1 + S_{1f} S_{1f}^{\mathrm{T}}, \\ \dot{Q}_{1i} &= -\delta_1 Q_{1i} + S_{1f}\tau_{fi}, \\ \mathcal{L}_{1i} &= P_1 \hat{W}_{ci} - Q_{1i}, \end{aligned} \tag{28}$$

where δ_1 is a positive constant and $\hat{W}_{ci}$ are the estimated vector of W_{ci}. From (18), we have

$$\mathcal{L}_{1i} = -P_1\tilde{W}_{ci} + C_{1i}, \tag{29}$$

where $C_{1i} = -\int_0^t e^{-\delta_1(t-r)} S_{1f}\bar{\epsilon}_{1fi}\mathrm{d}r$ with $\|C_{1i}\| \le \bar{\epsilon}_{1ci}$, $\bar{\epsilon}_{1ci}$ is a positive constant and $\tilde{W}_{ci} = W_{ci}^* - \hat{W}_{ci}$ is the estimated error. Considering the dynamics (21) and (22), we propose the following estimation error-based AFTNC

$$\begin{aligned}
&\tau = \tau_{c1} + \tau_{c2} + \tau_{c3},\\
&\tau_{c1} = -K_3 z_2 - K_5 z_2 \chi_1^{\mathrm{T}} \chi_1,\\
&\tau_{c2} = -K_4 z_2^{2\beta_1 - 1} - K_6 z_2 \chi_1^{\mathrm{T}} \chi_1^{2\beta_1 - 1},\\
&\tau_{c3} = -z_1 - \chi_1 - \hat{W}_1^{\mathrm{T}} \bar{S}_1,\\
&\dot{\hat{W}}_{ci} = \Gamma_{1i}\left(\bar{S}_1 z_{2i} - \gamma_{1i}\frac{P_1^{\mathrm{T}}\mathcal{L}_i}{\|\mathcal{L}_i\|}\right),\ i = 1,2,3,
\end{aligned} \tag{30}$$

where $\bar{S}_1 = [0, 0, S_{13}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{3m_1 \times 1}$ in which $S_{13} \in \mathbb{R}^{m_1 \times 1}$, γ_{1i} is a positive constant and Γ_{1i} is positive-definite matrix.

Theorem 1 *Consider systems (21) and (22) with AFTNC and learning algorithm (30). If matrix P_1 is a positive-definite matrix, it can be derived that tracking errors z_1, $z_2$and estimated error $\tilde{W}_{ci}$ converge to a small neighborhood around the origin in finite-time. Moreover, the estimated weight $\hat{W}_{ci}$ converges to the region around its optimal values, simultaneously.*

Proof We first adopt the following Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^T z_1. \tag{31}$$

Taking its derivative, we have

$$\begin{aligned}
\dot{V}_1 &= z_1^T z_2 - z_1^T K_1 z_1 - z_1^T K_2 z_1^{2\beta_1 - 1}\\
&\le z_1^{\mathrm{T}} z_2 - \underline{k}_1 \sum_{i=1}^{3} z_{1i}^2 - \underline{k}_2 \sum_{i=1}^{3} \left(z_{1i}^2\right)^{\beta_1},
\end{aligned} \tag{32}$$

where $\underline{k}_1 = \min(k_{1j_1})$, $\underline{k}_2 = \min(k_{2j_1})$, $j_1 = 1,2,3$. From the above inequality, the tracking error z_1 can achieve finite-time stability once z_2 converges. Then we consider a Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\chi_1^{\mathrm{T}}\chi_1 + \frac{1}{2} z_2^{\mathrm{T}} J z_2 + \frac{1}{2}\sum_{i=1}^{3} \tilde{W}_{ci}^{\mathrm{T}} \Gamma_{1i}^{-1} \tilde{W}_{ci}. \tag{33}$$

Differentiating V_2 along (22) yields

$$\begin{aligned}
\dot{V}_2 &= z_1^{\mathrm{T}} z_2 - z_1^T K_1 z_1 - z_1^T K_2 z_1^{2\beta_1 - 1}\\
&\quad + \chi_1^{\mathrm{T}} z_2 + \frac{1}{2} z_2^{\mathrm{T}} \dot{J} z_2 + z_2^{\mathrm{T}}\Big(\tau\\
&\quad + \tau_f - C\dot{\eta} - J\dot{\alpha}_1\Big) - \sum_{i=1}^{3} \tilde{W}_{ci}^{\mathrm{T}} \Gamma_{1i}^{-1} \dot{\hat{W}}_{ci}\\
&= z_1^{\mathrm{T}} z_2 - z_1^T K_1 z_1 - z_1^T K_2 z_1^{2\beta_1 - 1}\\
&\quad + \chi_1^{\mathrm{T}} z_2 + \frac{1}{2} z_2^{\mathrm{T}} (\dot{J} - 2C) z_2\\
&\quad + z_2^{\mathrm{T}} (\tau + F_{13}) - \sum_{i=1}^{3} \tilde{W}_{ci}^{\mathrm{T}} \Gamma_{1i}^{-1} \dot{\hat{W}}_{ci}.
\end{aligned} \tag{34}$$

From the Property (2), the matrix $\dot{J} - 2C$ is skew-symmetric where $z_2^{\mathrm{T}}(\dot{J} - 2C) z_2 = 0$ holds. Substituting (24) and (30) into (34), we have

$$\begin{aligned}
\dot{V}_2 &= z_1^{\mathrm{T}} z_2 - z_1^T K_1 z_1 - z_1^T K_2 z_1^{2\beta_1 - 1}\\
&\quad + \chi_1^{\mathrm{T}} z_2 + z_2^{\mathrm{T}}\Big(\tau + W_{13}^{*\mathrm{T}} S_{13}\\
&\quad + \epsilon_{13}\Big) - \sum_{i=1}^{3} \tilde{W}_{ci}^{\mathrm{T}}\left(\bar{S}_1 z_{2i} - \gamma_{1i}\frac{P_1^{\mathrm{T}}\mathcal{L}_i}{\|\mathcal{L}_i\|}\right)\\
&= -z_1^T K_1 z_1 - z_2^{\mathrm{T}} K_3 z_2 - z_2^{\mathrm{T}} K_5 z_2 \chi_1^{\mathrm{T}} \chi_1\\
&\quad - z_1^T K_2 z_1^{2\beta_1 - 1} - z_2^{\mathrm{T}} K_4 z_2^{2\beta_1 - 1}\\
&\quad - z_2^{\mathrm{T}} K_6 z_2 \chi_1^{\mathrm{T}} \chi_1^{2\beta_1 - 1} + z_2^{\mathrm{T}}\left(\tilde{W}_{13}^{\mathrm{T}} S_{13} + \epsilon_{13}\right) - \sum_{i=1}^{3} \tilde{W}_{ci}^{\mathrm{T}} \bar{S}_1 z_{2i}\\
&\quad + \sum_{i=1}^{3} \gamma_{1i} \tilde{W}_{ci}^{\mathrm{T}} \frac{P_1^{\mathrm{T}}\mathcal{L}_i}{\|\mathcal{L}_i\|}\\
&= -z_1^T K_1 z_1 - z_2^{\mathrm{T}} K_3 z_2 - z_2^{\mathrm{T}} K_5 z_2 \chi_1^{\mathrm{T}} \chi_1\\
&\quad - z_1^T K_2 z_1^{2\beta_1 - 1} - z_2^{\mathrm{T}} K_4 z_2^{2\beta_1 - 1}\\
&\quad - z_2^{\mathrm{T}} K_6 z_2 \chi_1^{\mathrm{T}} \chi_1^{2\beta_1 - 1} + z_2^{\mathrm{T}} \epsilon_{13} - \sum_{i=1}^{3} \gamma_{1i} \frac{\tilde{W}_{ci}^{\mathrm{T}} P_1^{\mathrm{T}} P_1 \tilde{W}_{ci}}{\|\mathcal{L}_i\|}\\
&\quad + \sum_{i=1}^{3} \gamma_{1i} \frac{\tilde{W}_{ci}^{\mathrm{T}} P_1^{\mathrm{T}} C_{1i}}{\|\mathcal{L}_i\|}.
\end{aligned} \tag{35}$$

Note that P_1 is a symmetrical positive-definite matrix such that $\frac{\tilde{W}_{ci}^{\mathrm{T}} P_1^{\mathrm{T}} P_1 \tilde{W}_{ci}}{\|\mathcal{L}_i\|} \ge \lambda_1 \tilde{W}_{ci}^{\mathrm{T}} \tilde{W}_{ci}$ with $\lambda_1 > 0$ being the minimum eigenvalue of matrix $\frac{P_1^{\mathrm{T}} P_1}{\|\mathcal{L}_i\|}$. And $\sum_{i=1}^{3} \gamma_{1i} \frac{\tilde{W}_{ci}^{\mathrm{T}} P_1^{\mathrm{T}} C_{1i}}{\|\mathcal{L}_i\|}$ is bounded by a positive constant ϵ_{1c}. Since K_5 and K_6 are both positive-definite matrices, we can always find positive constants $\underline{k}_5$ and $\underline{k}_6$ such that $z_2^{\mathrm{T}} K_5 z_2 \ge \underline{k}_5$, and $z_2^{\mathrm{T}} K_6 z_2 \ge \underline{k}_6$, $\forall z_2 \ne 0$. Then, we have

$$\begin{aligned}
\dot{V}_2 \le &- \underline{k}_1 \sum_{i=1}^{3} z_{1i}^2 - \underline{k}_2 \sum_{i=1}^{3} \left(z_{1i}^2\right)^{\beta_1}\\
&- \left(\underline{k}_3 - \frac{1}{2}\right) \sum_{i=1}^{3} z_{2i}^2 - \underline{k}_4 \sum_{i=1}^{3} \left(z_{2i}^2\right)^{\beta_1}\\
&- \underline{k}_5 \sum_{i=1}^{3} \chi_{1i}^2 - \underline{k}_6 \sum_{i=1}^{3} \left(\chi_{1i}^2\right)^{\beta_1}\\
&- \lambda_1 \sum_{i=1}^{3} \sum_{j=1}^{3m_1} \gamma_{1i} \tilde{W}_{cij}^2 + \frac{1}{2}\epsilon_{13n}^2 + \epsilon_{1c},
\end{aligned} \tag{36}$$

where $\underline{k}_3 = \min(k_{3j_1})$, $\underline{k}_4 = \min(k_{4j_1})$, $j_1 = 1,2,3$, and a suitable value of $\underline{k}_3$ with $\underline{k}_3 > \frac{1}{2}$. For term $-\sum_{j=1}^{3m_1} \gamma_{1i} \tilde{W}_{cij}^2$ according to Lemma 4 yields

$$
\begin{aligned}
&-\gamma_{1i}\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2\\
&=-\gamma_{1i}\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2-\gamma_{1i}\sum_{j=1}^{3m_1}\left(\tilde{W}_{cij}^2\right)^{\beta_1}+\gamma_{1i}\sum_{j=1}^{3m_1}\left(\tilde{W}_{cij}^2\right)^{\beta_1}\\
&\leq-\gamma_{1i}\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2-\gamma_{1i}\sum_{j=1}^{3m_1}\left(\tilde{W}_{cij}^2\right)^{\beta_1}\gamma_{1i}\sum_{j=1}^{3m_1}\left(\beta_1\tilde{W}_{cij}^2+(1-\beta_1)\right)\\
&\leq-\gamma_{1i}\sum_{j=1}^{3m_1}(1-\beta_1)\tilde{W}_{cij}^2-\gamma_{1i}\sum_{j=1}^{3m_1}\left(\tilde{W}_{cij}^2\right)^{\beta_1}+\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1).
\end{aligned}
\tag{37}
$$

Substituting (37) into (36), we can get

$$
\begin{aligned}
\dot{V}_2\leq&-\underline{k}_1\sum_{i=1}^{3}z_{1i}^2-\left(\underline{k}_3-\frac{1}{2}\right)\sum_{i=1}^{3}z_{2i}^2-\underline{k}_5\sum_{i=1}^{3}\chi_{1i}^2-k_7\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2\\
&-\underline{k}_2\left(\sum_{i=1}^{3}z_{1i}^2\right)^{\beta_1}-\underline{k}_4\left(\sum_{i=1}^{3}z_{2i}^2\right)^{\beta_1}-\underline{k}_6\left(\sum_{i=1}^{3}\chi_{1i}^2\right)^{\beta_1}+\epsilon_{1c}\\
&-k_8\sum_{i=1}^{3}\left(\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2\right)^{\beta_1}+\lambda_1\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1)+\frac{1}{2}\epsilon_{13n}^2,
\end{aligned}
\tag{38}
$$

where $k_7=\min(\lambda_1\gamma_{1i}(1-\beta_1))$ and $k_8=\min(\lambda_1\gamma_{1i})$. The effective inertial matrix J satisfies Property (1), and thus we have,

$$
\begin{aligned}
&-\sum_{i=1}^{3}z_{2i}^2=-\frac{1}{\bar{j}}\sum_{i=1}^{3}\bar{j}z_{2i}^2\leq-\frac{1}{\bar{j}}z_2^{\mathrm{T}}Jz_2\\
&-k_7\sum_{j=1}^{3m_1}\tilde{W}_{cij}^2=-\frac{k_7}{\lambda_2}\sum_{j=1}^{3m_1}\lambda_2\tilde{W}_{cij}^2\leq\underline{k}_7\tilde{W}_{ci}^{\mathrm{T}}\Gamma_{1i}^{-1}\tilde{W}_{ci},
\end{aligned}
\tag{39}
$$

where λ_2 is the minimum eigenvalue of positive-definite matrices Γ_{1i}. Substituting (39) into (38) and applying Lemma 2 yields

$$
\begin{aligned}
\dot{V}_2\leq&-a_1\left(\frac{1}{2}\sum_{i=1}^{3}z_{1i}^2+\frac{1}{2}z_2^{\mathrm{T}}Jz_2\right.\\
&\left.+\frac{1}{2}\sum_{i=1}^{3}\chi_{1i}^2+\frac{1}{2}\sum_{i=1}^{3}\tilde{W}_{ci}^{\mathrm{T}}\Gamma_i^{-1}\tilde{W}_{1i}\right)\\
&-b_1\left(\frac{1}{2}\sum_{i=1}^{3}z_{1i}^2+\frac{1}{2}z_2^{\mathrm{T}}Jz_2+\frac{1}{2}\sum_{i=1}^{3}\chi_{1i}^2\right.\\
&\left.+\frac{1}{2}\sum_{i=1}^{3}\tilde{W}_{ci}^{\mathrm{T}}\Gamma_i^{-1}\tilde{W}_{ci}\right)^{\beta_1}\\
&+\frac{1}{2}\epsilon_{13n}^2+\epsilon_{1c}+\lambda_1\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1),
\end{aligned}
\tag{40}
$$

where $a_1=\min\left(2\underline{k}_1,\frac{2\underline{k}_3-1}{\bar{j}},2\underline{k}_5,2\underline{k}_7\right)$, $b_1=\min\left(2^{\beta_1}\underline{k}_2,2^{\beta_1}\underline{k}_4,2^{\beta_1}\underline{k}_6,2^{\beta_1}\underline{k}_8\right)$ where $\underline{k}_8=\frac{k_8}{\lambda_2^{\beta_1}}$. Referring to Lemma 1 yields,

$$
\dot{V}_2\leq-a_1V_2-b_1V_2^{\beta_1}+\varrho_1,
\tag{41}
$$

where $\varrho_1=\frac{1}{2}\epsilon_{13n}^2+\epsilon_{1c}+\lambda_1\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1)$. According to the Lemma 1, the function V_2 is bounded by a small region $\Omega=\left\{V_2^{\beta}\leq\frac{\varrho_1}{b_1(1-c_1)}\right\}$ with $0<c_1<1$. This suggests that z_1, z_2 and $\tilde{W}_{ci}$ converge to a small neighborhood around the equilibrium within finite-time and this completes the proof. Note that $\varrho_1=\frac{1}{2}\epsilon_{13n}^2+\epsilon_{1c}+\lambda_1\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1)$ where γ_{1i} and β_1 are control scheme parameters, ϵ_{13n} represents the boundary of residual error, and ϵ_{1c} is related to C_{1i} as described in (35). If the number of NN nodes is sufficiently large, the residual error $\epsilon_{1i},(i=1,2,3)$ can be zero in the ideal case, which means $\epsilon_{13n}=0$ holds. Due to the stable filters in (27) and the leakage term L_i in (29), we can also derive C_{1i} to be zero in this case and lead to $\epsilon_{1c}=0$. As the above condition, the item ϱ_1 related to the ultimate convergence region Ω becomes $\varrho_1=\lambda_1\sum_{i=1}^{3}\sum_{j=1}^{3m_1}\gamma_{1i}(1-\beta_1)$. If we appropriately select the control parameters, the final region can be arbitrary small. Moreover, although the number of NN nodes cannot be infinity, ϵ_{13n} and ϵ_{1c} are only small residual errors, and little effect on the final region Ω. □

Remark 3 For the existing gradient law or the modified adaptive law used in [88] and [50], the adaptive laws are established to guarantee the tracking errors convergence. However, the convergence of the estimated parameters to their optimal values may be difficult due to the uncertainties and damping effect in adaptive laws. This paper proposes a novel way to guarantee this convergence by introducing a new leakage item L_i in the adaptive law, where the estimation errors can be obtained explicitly to achieve the convergence of the parameter estimation.

Remark 4 The adaptive scheme (30) contains the estimated error $\tilde{\bar{W}}_i$ in the new leakage term L_i, which contributes to the convergence of the estimated parameter to its optimal value. In order to clearly illustrate the superiority of the adaptive law in (30), we adopt the e-modified adaptive law design scheme and let the adaptive law be $\dot{\hat{W}}_{ci}=\Gamma_{1i}(\bar{S}_1z_{2i}-\gamma_{1i}\hat{W}_{ci}),(i=1,2,3)$. Although finite-time stability can be achieved, due to the introduction of the damping term, the ultimate convergence region Ω depends not only on the residual error ϵ_{1i}, but also the optimal value W_{ci}^* of the unknown NN weights. For its ultimate convergence region proof, we omit here for clarity and conciseness. As discussed above, the ultimate region under e-modified adaptive law is much larger than Ω under the estimation error-based adaptive law (30). Thus the estimated parameters of e-modified cannot achieve converging to their optimal values. In other words, the estimation error-based adaptive scheme (30) can get better results in estimating, and the overall control system can be greatly improved.

3.4 Translational subsystem control

Incorporated with the estimation error-based adaptive scheme, an ANC is then developed for the translational subsystem. We first define $e_1 = \xi - \xi_d$ where ξ_d is the desired position. A virtual control input is developed as,

$$\alpha_2 = \dot{\xi}_d - \bar{K}_1 e_1 - \bar{K}_2 e_1^{2\beta_2 - 1}, \tag{42}$$

where $0 < \beta_2 < 1$. Similar to the rotational control, an integral term $\chi_2 = \int_0^t e_2(s)\mathrm{d}s$ is developed where $e_2 = \dot{\xi} - \alpha_2$. Differentiating e_1 and e_2 yields,

$$\begin{aligned} \dot{e}_1 &= e_2 + \alpha_2 - \dot{\xi}_d \\ \dot{e}_2 &= \ddot{\xi} - \dot{\alpha}_2 = f_2(\eta, u) + F_2(e), \end{aligned} \tag{43}$$

where $f_2(\eta, u) = \frac{1}{m_v}\left(R_{E2B}^{\mathrm{T}} u + F_g\right)$, $F_2(e) = \frac{1}{m_v} F_f - \dot{\alpha}_2$ with $e = \left[|\dot{\xi}|\dot{\xi}, \dot{\alpha}_2\right]^{\mathrm{T}}$. And we employ NNs to estimate the uncertain term $F_2(e)$ as,

$$F_2(e) = W_2^{*\mathrm{T}} S_2 + \bar{\epsilon}_2, \tag{44}$$

where $W_2^* = \left[W_{b1}^*, W_{b2}^*, W_{b3}^*\right] \in \mathbb{R}^{m_2 \times 3}$ is the optimal weight matrix, $\bar{\epsilon}_2 \in \mathbb{R}^{3\times 1}$ is bounded by a positive constant $\bar{\epsilon}_{2n}$ and $S_2 \in \mathbb{R}^{m_2 \times 1}$ is regressor. Then stable filters are given as,

$$\begin{aligned} l_2 \dot{e}_{2fi} &= -e_{2fi} + e_{2i}, \quad e_{2fi}(0) = 0, \\ l_2 \dot{f}_{2fi} &= -f_{2fi} + f_{2i}, \quad f_{2fi}(0) = 0, \\ l_2 \dot{S}_{2f} &= -S_{2f} + S_2, \quad S_{2f}(0) = 0, \\ l_2 \dot{\bar{\epsilon}}_{2fi} &= -\bar{\epsilon}_{2fi} + \bar{\epsilon}_{2i}, \quad \bar{\epsilon}_{2fi}(0) = 0, \end{aligned} \tag{45}$$

where l_2 is a positive constant. Employing the estimation error-based adaptive scheme, the following auxiliary matrices are developed,

$$\begin{aligned} \dot{P}_2 &= -\delta_2 P_2 + S_{2f} S_{2f}^{\mathrm{T}} \\ \dot{Q}_{2i} &= -\delta_2 Q_{2i} + S_{2f}\left(\frac{e_{2i} - e_{2fi}}{l_2} - f_{2fi}\right)^{\mathrm{T}} \\ \mathcal{L}_{2i} &= P_2 \hat{W}_{bi} - Q_{2i}, \end{aligned} \tag{46}$$

where δ_2 is a positive constant. According to (18), we have,

$$\mathcal{L}_{2i} = -P_2 \tilde{W}_{bi} + C_{2i}, \tag{47}$$

where $\tilde{W}_{bi} = W_{bi}^* - \hat{W}_{bi}$ are estimated errors and $C_{2i} = -\int_0^t e^{-\delta_2(t-r)} S_{2f} \bar{\epsilon}_{2fi} \mathrm{d}r$. Then, we propose an estimation error-based ANC with fast finite-time convergence as,

$$\begin{aligned} u &= u_{1c} + u_{2c} + u_{3c} \\ u_{1c} &= H_2\left(-\bar{K}_3 e_2 - \bar{K}_5 e_2 \chi_2^{\mathrm{T}} \chi_2\right) \\ u_{2c} &= H_2\left(-\bar{K}_4 e_2^{2\beta_2 - 1} - \bar{K}_6 e_2 \chi_2^{\mathrm{T}} \chi_2^{2\beta_2 - 1}\right) \\ u_{3c} &= H_2\left(-e_1 - \chi_2 - \frac{1}{m_v} F_g - \hat{W}_2^{\mathrm{T}} S_2\right) \\ \dot{\hat{W}}_{bi} &= \Gamma_{2i}\left(S_2 e_{2i} - \gamma_{2i} \frac{P_2^{\mathrm{T}} \mathcal{L}_{2i}}{\|\mathcal{L}_{2i}\|}\right), \ i = 1, 2, 3, \end{aligned} \tag{48}$$

where Γ_{2i} and γ_{2i} are positive-definite matrix and constant respectively; and $H_2 = m_v R_{E2B}$. Similar to the rotational subsystem, the following theorem can be obtained.

Theorem 2 *Consider the translational subsystem (43) with the estimation error-based AFTNC (48). If the matrix P_2 is positive-definite, tracking errors e_1, e_2 and estimated errors $\tilde{W}_{bi}$ converge to a small region around zero in finite-time. Moreover, estimated weights of RBF NNs can converge to the region around their optimal values simultaneously.*

The proof of Theorem 2 is similar to Theorem 1, we omitted its proof process for clarity and conciseness. If readers are interested in proof, please refer to the stability analysis in the rotational subsystem.

4 Simulation

In this section, we conduct comparative simulations to illustrate the effectiveness of our proposed control scheme for the tilting quadcopter with unknown nonlinearities. The vehicle is commanded to track the translational and rotational trajectories simultaneously. More details about the simulation designs are elaborated as follows.

4.1 Rotational simulation results

The initial attitude states of the tilting quadcopter are set to zero. The value selections of the inherent parameters, adaptive laws and control parameters are given in Table 1. To verify the robustness to the uncertainties, note that the moment inertia and damping parameters exist unknown nonlinearities as shown in Table 1. Moreover, the vehicle is commanded to track the desired rotational trajectories as $\eta_d(t) = [\cos(t-1), \cos t, \cos t + 0.5]^{\mathrm{T}}$. To illustrate the superiority of our scheme, we adopt the following Adaptive Neural Control (ANC) without finite-time convergence and estimation error-based adaptive law as the comparative method

Table 1 Parameters for Rotational Simulations

Parameters	Values	Parameters	Values
γ_{1i}	0.1	Γ_{1i}	0.4
m_1	512	β_1	$\frac{99}{101}$
l_1	0.01	δ_1	100
I_{xx}	$0.04463 + 0.01\sin 2t$	g_2	$0.5 + 0.2\sin(2t-1)$
I_{yy}	$0.04463 + 0.01\sin(4t+1)$	g_3	0.6
I_{zz}	$0.08844 + 0.02\sin(3t-2)$	K_1	$\mathrm{diag}(0.6, 0.6, 0.6)$
g_1	$0.5 + 0.1\sin(5t+2)$	K_2	$\mathrm{diag}(1.6, 1.6, 1.6)$
K_3	$\mathrm{diag}(0.4, 0.4, 0.4)$	K_4	$\mathrm{diag}(1.6, 1.6, 1.6)$
K_5	$\mathrm{diag}(0.4, 0.4, 0.4)$	K_6	$\mathrm{diag}(2.0, 2.0, 2.0)$

$$\begin{aligned} \alpha_1 &= \dot{\eta}_d - K_1 z_1 \\ \tau &= -K_3 z_2 - z_1 - \hat{W}_1^{\mathrm{T}} \bar{S}_1 \\ \dot{\hat{W}}_{ci} &= \Gamma_{1i}\left(\bar{S}_1 z_{2i} - \gamma_{1i} \hat{W}_{ci}\right), i = 1, 2, 3, \end{aligned} \tag{49}$$

where the above control parameters are the same to Table 1 to guarantee the reasonable results. The comparative simulation results are shown as follows.

The tracking performance is exhibited in Figs. 3, 4, 5 and 6. We use the abbreviation AFTNC as the proposed control scheme (30) and ANC as the results of (49). Figure 3 depicts the rotational tracking trajectories. AFTNC can not only enjoy better transient performance with fast convergence to the desired trajectories within finite-time, but also robustness rejection ability to the uncertainties. ANC can only achieve infinite-time stabilization, which leads to a long transient response. The estimation performance of neural networks with estimation error-based adaptive laws is shown in Fig. 4. Compared with ANC results, AFTNC can fast and well estimate the unknown nonlinearities F_{13}, where RBF NNs can converge to a small range around their real values. Let $\hat{W}_c = \left[\hat{W}_{c1}^{\mathrm{T}}, \hat{W}_{c2}^{\mathrm{T}}, \hat{W}_{c3}^{\mathrm{T}}\right]^{\mathrm{T}}$, and the norm value of $\hat{W}_c$ is shown in Fig. 5. The estimated weights of AFTNC can converge to a region around their real values in finite-time. Additionally, according to the estimation error-based adaptive scheme (29), the term $\mathcal{L}_{1i}$ contains the

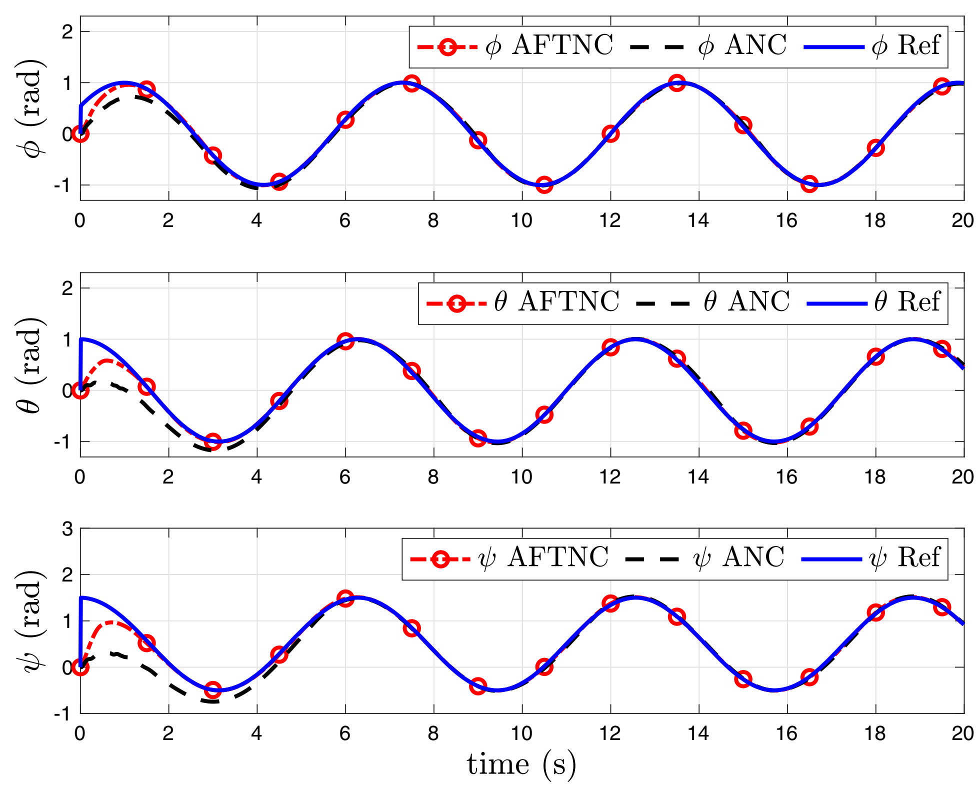

Fig. 3 The tracking performance of attitude

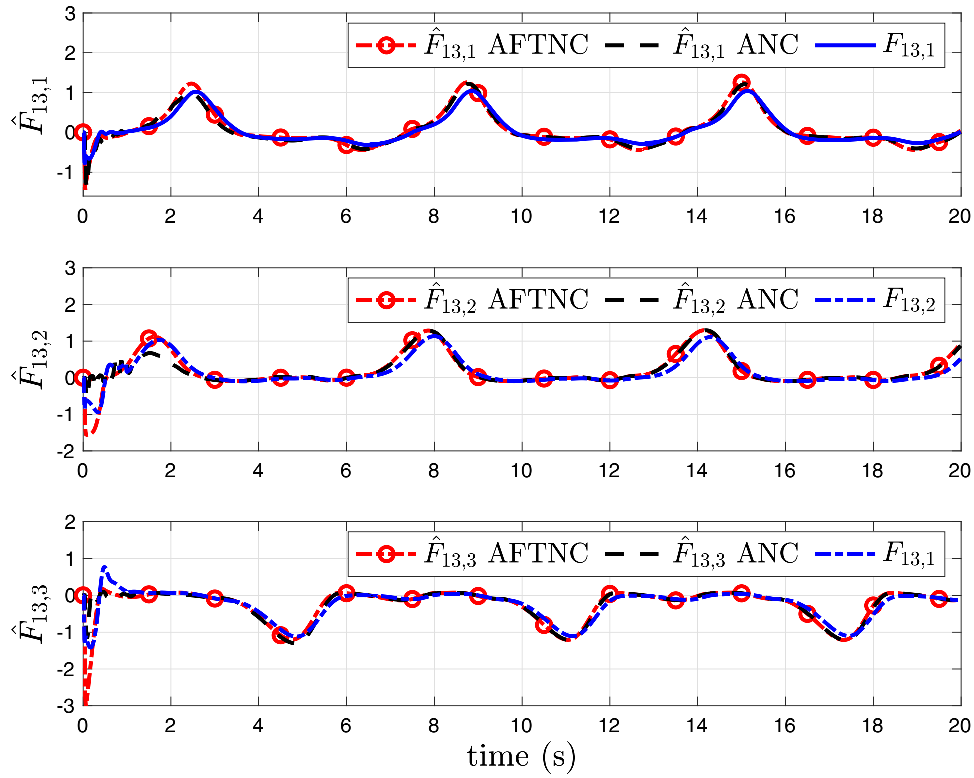

Fig. 4 The estimation performance of F_{13}

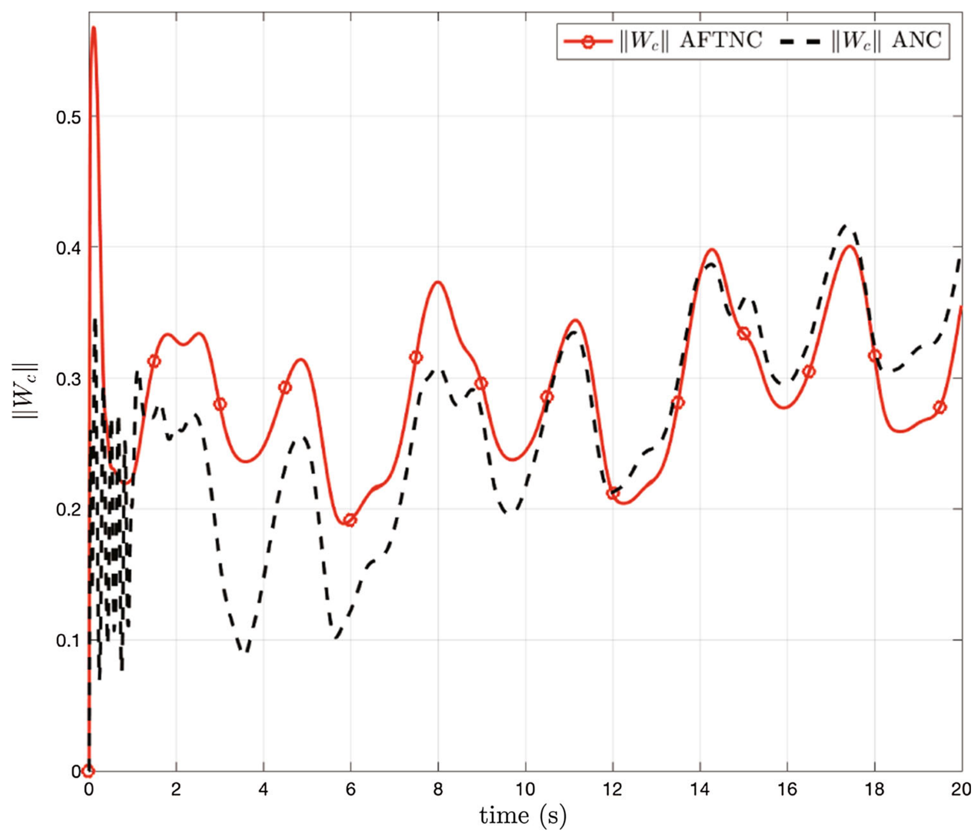

Fig. 5 The weight W_c with error-based adaptive scheme

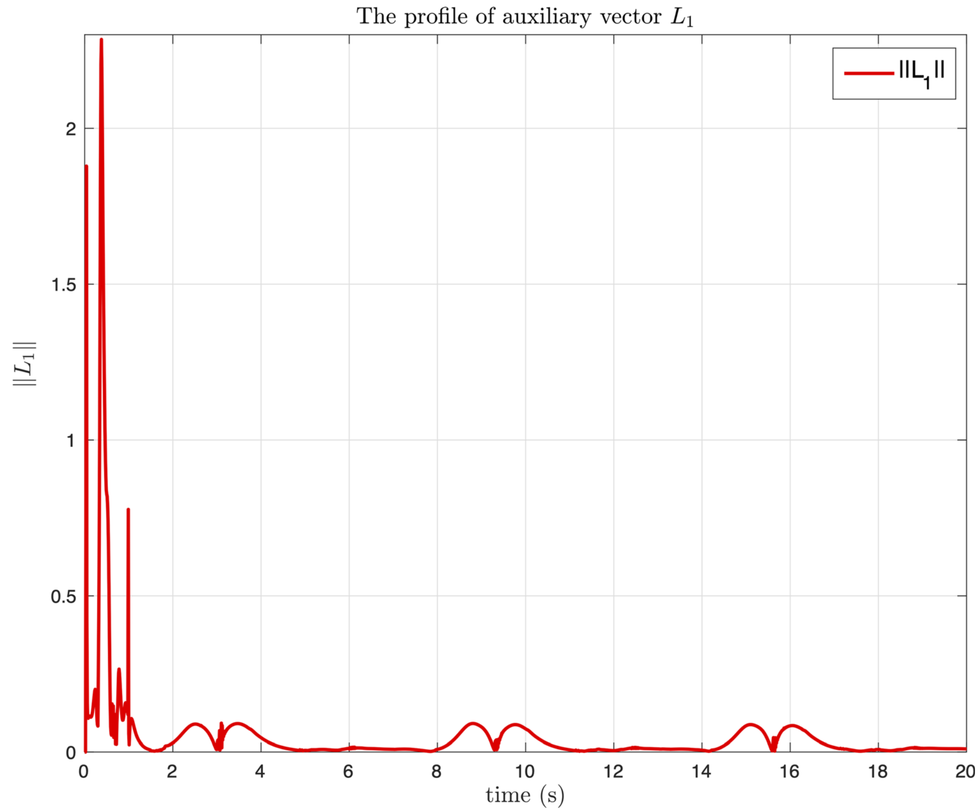

Fig. 6 The profile of auxiliary vector L_1

information of estimated error $\tilde{W}_{ci}$. Thus, $\mathcal{L}_{1i}$ should converge to a small neighborhood around zero, which depends on the estimation error $\bar{\epsilon}_1$. Let $L_1 = [\mathcal{L}_{11}^{\mathrm{T}}, \mathcal{L}_{12}^{\mathrm{T}}, \mathcal{L}_{13}^{\mathrm{T}}]^{\mathrm{T}}$ and the norm value of L_1 is shown in Fig. 6. Note that L_1 converges to a small range around zero which further represents the estimated weights can converge to their optimal values. All simulation results have illustrated the effectiveness and superior performance of AFTNC with estimation error-based adaptive law.

4.2 Translational simulation results

The initial position $\xi(0)$ is set to zero and the vehicle is commanded to track the trajectories $\xi_d = [\cos(t-0.8), \cos(t), \cos(t)+0.3]^{\mathrm{T}}$. All simulation parameters used in dynamics and control are listed in Table 2. As the rotational simulation design, we adopt the aerodynamic damping coefficient d_i with uncertainties as shown in Table 2 to demonstrate the robustness of the proposed control scheme. Moreover, the comparative ANC is designed as

$$\begin{aligned} &\alpha_2 = \dot{\xi}_d - \bar{K}_1 e_1, \\ &u = H_2 - \bar{K}_3 e_2 + H_2\left(-e_1 - \frac{1}{m_v} F_g - \hat{W}_2^{\mathrm{T}} S_2\right), \\ &\dot{\hat{W}}_{bi} = \Gamma_{2i}\left(S_2 e_{2i} - \gamma_{2i} \hat{W}_{bi}\right), i = 1,2,3, \end{aligned} \tag{50}$$

where all the above control parameters are set the same to Table 2 to guarantee reasonable simulation results. The translational tracking results are shown as follows.

The tracking performance of comparative results is exhibited in Figs. 7, 8, 9 and 10. We define AFTNC as the performance of the proposed control scheme and ANC as (50). Fig. 7 shows the translational tracking results. ANC can only achieve infinite-time convergence, and there exists obvious over-shoot leading to a long transient response. However, AFTNC can fast converge to the desired commands within finite-time and achieve well tracking performance to uncertainties. The estimations of unknown nonlinearities $F_{2,i}$ are shown in Fig. 8. With the help of estimation error-based adaptive law, it can be demonstrate that AFTNC can well approximate the smooth nonlinearities in high precision. Denote $W_b = [W_{b1}^{\mathrm{T}}, W_{b2}^{\mathrm{T}}, W_{b3}^{\mathrm{T}}]^{\mathrm{T}}$. And compared with ANC, NNs weights of AFTNC can fast converge to a smaller neighborhood around their optimal values as shown in Fig. 9, which can support the superior estimation results in Fig. 8. Moreover, let $L_2 = [\mathcal{L}_{2i}^{\mathrm{T}}, \mathcal{L}_{2i}^{\mathrm{T}}, \mathcal{L}_{2i}^{\mathrm{T}}]^{\mathrm{T}}$, which contains the estimated error information $\tilde{W}_{bi}$. According to the rigorous theoretical analysis, L_2 should converge around zero which suggests that $\hat{W}_{bi}$ can estimate their optimal value W_{bi}^*. And Fig. 10 can demonstrate the above convergence of $\hat{W}_{bi}$,

Table 2 Parameters for Translational Simulations

Parameters	Values	Parameters	Values
γ_{2i}	0.01	Γ_{2i}	0.1
m_2	125	β_1	$\frac{99}{101}$
m_v	2.878	l_2	0.01
δ_2	100	d_1	$0.7 + 0.2\cos 5t$
d_2	$0.7 + 0.3\sin(5t - 3)$	d_3	$0.7 + 0.2\sin(2t - 1)$
$\bar{K}_1$	diag(2.1, 2.1, 2.1)	$\bar{K}_2$	diag(3.0, 3.0, 3.0)
$\bar{K}_3$	diag(1.4, 1.4, 1.4)	$\bar{K}_4$	diag(3.0, 3.0, 3.0)
$\bar{K}_5$	diag(1.4, 1.4, 1.4)	$\bar{K}_6$	diag(3.0, 3.0, 3.0)

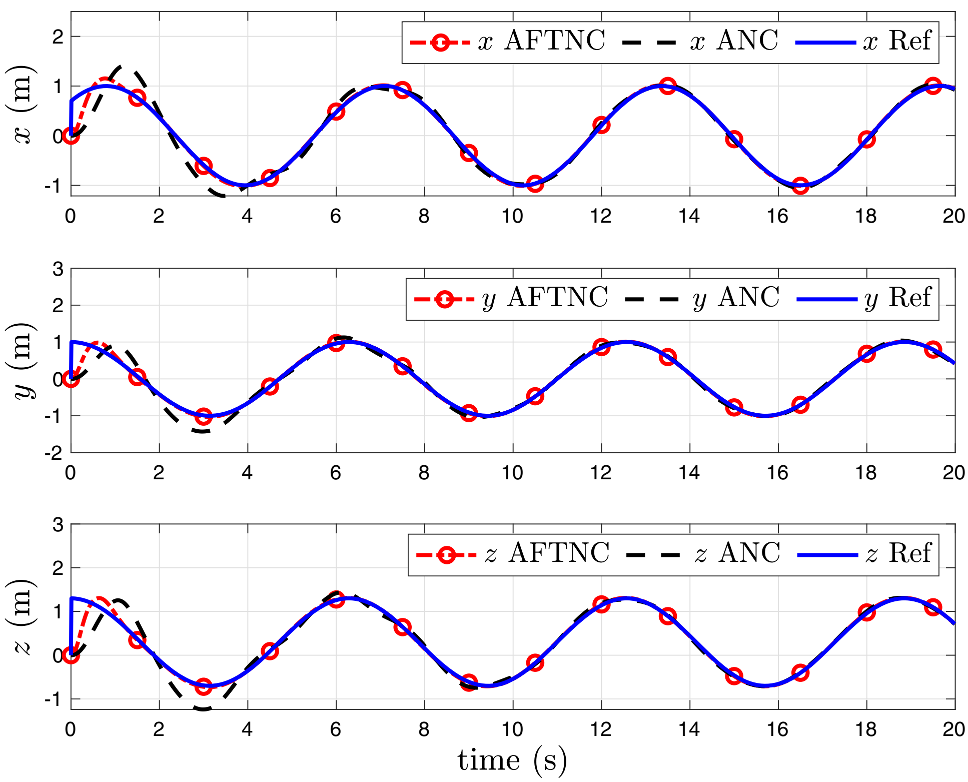

Fig. 7 The tracking performance of position

where L_2 converges to a small region around the equilibrium within finite-time.

According to the above simulation results, the tilting quadcopter can provide a fully actuation version with rotational and translational movements controlled independently. In comparison with the existing ANC, the proposed AFTNC can not only perform robustness ability to the uncertainties, but also guarantee finite-time convergence of tracking errors leading to a fast transient response. Furthermore, with the help of estimation error-based adaptive law, AFTNC enable NNs weights W_{ci} and W_{bi} converge to a smaller neighborhood around their optimal values in finite-time, which contributes to achieve better tracking performance.

5 Conclusion

In this paper, we first propose a novel conception of a tilting quadcopter with fully-actuated version, which suggests that the translational and rotational movements can be controlled independently. The dynamics of the tilting

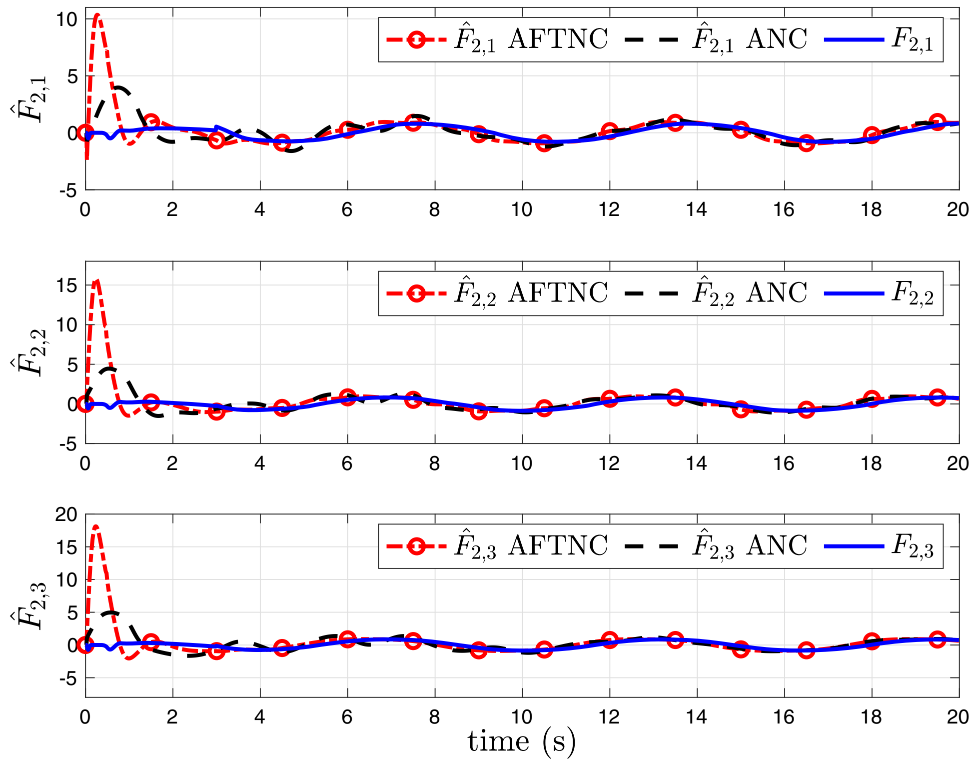

Fig. 8 The estimation of $F_{12}(e)$

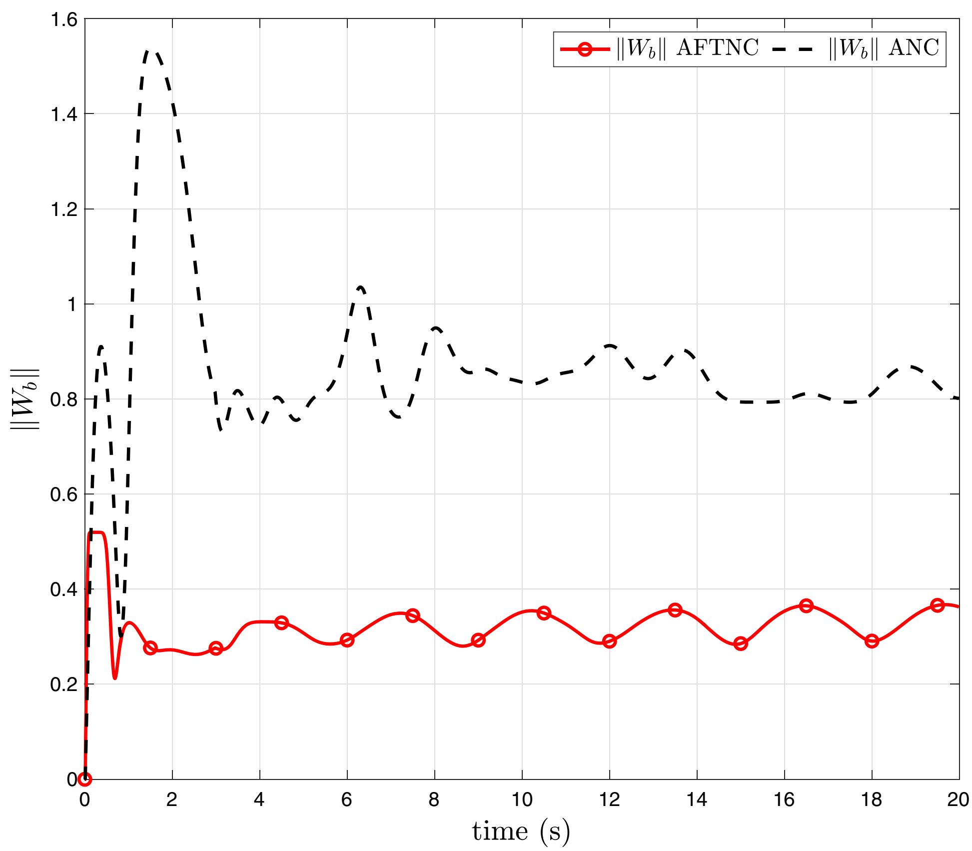

Fig. 9 The weight W_b with error-based adaptive scheme

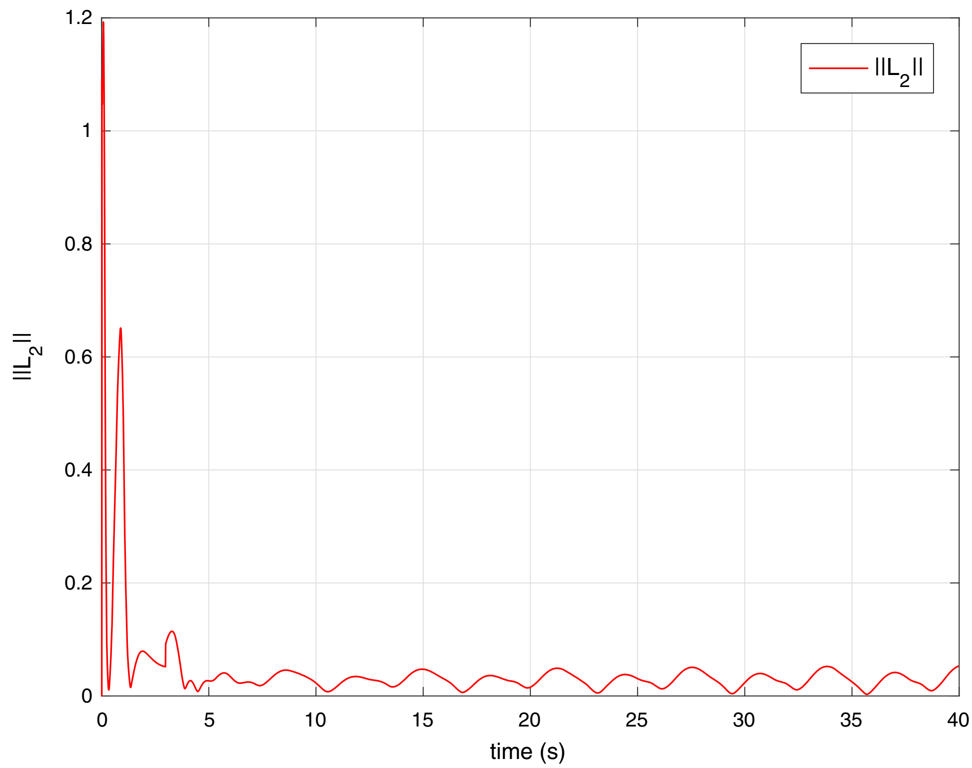

Fig. 10 The profile of auxiliary vector L_2

quadcopter is developed based on Euler-Lagrange equations and NNs are utilized to approximate the unknown nonlinearities in systems. An estimation error-based AFTNC is then proposed, where a new auxiliary filter is constructed to obtain the estimation errors explicitly as new leakage terms into the adaptive law. This indicates that not only the tracking and estimation errors converge to a small neighborhood around zeros, but also the estimated weights of NNs can converge to their optimal values simultaneously. Finally, the simulation results have been provided to demonstrate the effectiveness of our proposed ANC. In our future work, we will extend our approach to practical situations, such as states constrains, external disturbance and fault-tolerant control.

Appendix A

The matrices $J(\eta)$ and $C(\eta, \dot{\eta})$ are given as follows:

$$J(\eta) = [j_{ij}] \in \mathbb{R}^{3\times3}, \quad C(\eta, \dot{\eta}) = [c_{ij}] \in \mathbb{R}^{3\times3}, \tag{51}$$

with

$$\begin{aligned}
&j_{11} = I_{xx}; j_{12} = 0; j_{13} = -I_{xx}\mathrm{s}\theta; j_{21} = 0; j_{22} = I_{yy}\mathrm{c}^2\phi + I_{zz}\mathrm{s}^2\phi \\
&j_{23} = (I_{yy} - I_{zz})\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta; j_{31} = -I_{xx}\mathrm{s}\theta;\ j_{32} = (I_{yy} - I_{zz})\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta; \\
&j_{33} = I_{xx}\mathrm{s}^2\theta + I_{yy}\mathrm{s}^2\phi\mathrm{c}^2\theta + I_{zz}\mathrm{c}^2\phi\mathrm{c}^2\theta; \\
&c_{11} = 0;\ c_{12} = \frac{1}{2}(I_{yy} - I_{zz})(\dot{\theta}\mathrm{s}2\phi - \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta) \\
&c_{13} = -I_{xx}\dot{\theta}\mathrm{c}\theta - \frac{1}{2}(I_{yy} - I_{zz})(\dot{\theta}\mathrm{c}2\phi\mathrm{c}\theta + \dot{\psi}\mathrm{s}2\phi\mathrm{c}^2\theta); \\
&c_{21} = \frac{1}{2}I_{xx}\dot{\psi}\mathrm{c}\theta; c_{31} = -I_{xx}\dot{\theta}\mathrm{c}\theta; \\
&c_{22} = -(I_{yy} - I_{zz})\dot{\phi}\mathrm{s}2\phi + \frac{1}{2}(I_{yy} - I_{zz})\dot{\psi}\mathrm{c}\phi\mathrm{s}\phi\mathrm{s}\theta; \\
&c_{23} = (I_{yy} - I_{zz})(\dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta - \frac{1}{2}\dot{\theta}\mathrm{s}\phi\mathrm{c}\phi\mathrm{s}\theta) + \frac{1}{2}I_{xx}\dot{\phi}\mathrm{c}\theta - (I_{xx} \\
&\qquad - I_{yy}\mathrm{s}^2\phi - I_{zz}\mathrm{c}^2\phi)\dot{\psi}\mathrm{c}\theta\mathrm{s}\theta; \\
&c_{32} = (I_{yy} - I_{zz})(\dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta - \dot{\theta}\mathrm{s}\phi\mathrm{c}\phi\mathrm{s}\theta); \\
&c_{33} = I_{xx}\dot{\theta}\mathrm{s}2\theta + (I_{yy} - I_{zz})\dot{\phi}\mathrm{s}2\phi\mathrm{c}^2\theta - (I_{yy}\mathrm{s}^2\phi + I_{zz}\mathrm{c}^2\phi)\dot{\theta}\mathrm{s}2\theta.
\end{aligned} \tag{52}$$

Appendix B

The matrix P is positive-definite such that $P = \int_0^t e^{-\delta(t-r)} S_f S_f^{\mathrm{T}} \mathrm{d}r \geq \lambda_f I$ where $\lambda_f > 0$. Then we have,

$$\int_0^t e^{-\delta(t-r)} S_f S_f^{\mathrm{T}} \mathrm{d}r = \int_0^{t-T} e^{-\delta(t-r)} S_f S_f^{\mathrm{T}} \mathrm{d}r + \int_{t-T}^{t} e^{-\delta(t-r)} S_f S_f^{\mathrm{T}} \mathrm{d}r \leq \frac{e^{-\delta T}}{\delta} \|S_f\|_\infty^2 + \int_{t-T}^{t} S_f S_f^{\mathrm{T}} \mathrm{d}r. \tag{53}$$

We can further have,

$$\int_{t-T}^{t} S_f S_f^{\mathrm{T}} \mathrm{d}r \geq \left(\lambda_f - \frac{e^{-\delta T}}{\delta} \|S_f\|_\infty^2\right) I. \tag{54}$$

If we set T and δ in reason, $\lambda_f - \frac{e^{-\delta T}}{\delta} \|S_f\|_\infty^2$ can be positive. Since the filter in (13) is stable, then the regressor S is PE.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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