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Finite-time lag synchronization for uncertain complex networks involving impulsive disturbances

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Abstract

This paper focuses on the finite-time lag synchronization (*FTLS*) of uncertain complex networks involving impulsive disturbance effects. By designing two different controllers, some Lyapunov-based conditions are established in terms of linear matrix inequalities to ensure the *FTLS* of impulsive systems, where the upper bound of the synchronizing times can be estimated via constructing Lyapunov functions. It is interesting to discover that the synchronizing time depends not only on the initial value but also on the impulse sequences, which implies that different impulses will lead to different synchronization times. Finally, a numerical example is given to illustrate the feasibility and effectiveness of the proposed *FTLS* criterion.

Keywords Finite-time lag synchronization · Lyapunov method · Uncertain complex networks · Impulsive disturbance · Linear matrix inequality (LMI)

1 Introduction

Since Huygens discovered that the pendulum oscillates synchronously, the synchronization phenomenon has been widely concerned and continuously studied. In 1990, Pecora and Carroll [1] of the US Navy laboratory put forward the drive-response synchronization method, which further triggered the research upsurge of synchronization and control for dynamic systems in various fields, such as signal processing, engineering, combinatorial optimization, modeling brain activity, and secure communication [2–5]. Up to now, many studies on several types of synchronization including complete, generalized, anticipated, lag, and phase synchronization have been proposed [6–10]. Among them, lag synchronization is very common in the implementation of electronic networks, which requires the current state of one node to be synchronized with the past state of another node; that is to say, there is a time shift on synchronization between these two nodes [11, 12]. In the telephone communication network, for example, the voice received by the receiver at time t^* is sent by the sender at time $t^* - \rho$, where ρ is the time shift. That is to say, the real-time transmission cannot be realized in many real models, i.e., complete synchronization cannot be effectively realized. In this case, it is natural to consider the lag synchronization. In addition, from the practical engineering application of parallel image processing and secure communication, this is a reasonable synchronization strategy. Therefore, the lag synchronization research has been applied to many practical systems, such as laser, neural networks, and electronic circuit [7, 13–16].

As we all know, synchronization performance is a key performance index in the synchronization of dynamical

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systems. However, in most closed-loop system control design methods, the fastest synchronization rate is in exponential form, which is why better synchronization performance cannot be achieved. The fundamental reason is that Lipschitz continuity of the closed-loop system needs to be satisfied. Therefore, these control analysis methods belong to the synchronization control problem over the infinite time. In addition, since the life span of human and machine is limited, people want to realize synchronization in finite time as much as possible. Especially in the field of engineering technology and economic management, if the goal of synchronization can be achieved within a certain period of time, it will greatly improve economic benefits. Based on above motivations, a synchronization called finite-time synchronization has attracted attention. It requires synchronization within a finite time. The improvement in this performance not only ensures the fastest convergence time of network synchronization, but also has better robustness against disturbance and uncertainty [17, 18], in which robustness refers to the ability to keep synchronization performance unchanged under uncertain interference. Therefore, network synchronization based on finite-time stability theory has been studied in the field of physics and engineering, and there have been a lot of researches on finite-time stability and chaos synchronization [18–27]. For example, by periodic intermittent control and impulsive control, Mei et al. [18] studied the finite-time synchronization (*FTS*) of complex networks (CNs) with delayed and non-delayed coupling; in [25], authors studied the *FTS* of hierarchical delayed neural networks by using Lyapunov–Krasovskii functional methods; Jing et al. [26] adjusted and designed a periodically intermittent strengths and feedback controller, respectively, to realize finite-time lag synchronization (*FTLS*) of delayed CNs. Therefore, it is necessary to study *FTS* for different types of network systems via different methods.

On the other hand, the key to realize *FTS* for CNs is to design a suitable controller. In recent years, there are various kinds of controllers to realize *FTS* for many kinds of systems, such as observer-based controller, sliding mode controller, adaptive controller, impulsive controller, and feedback controller [18, 28–34]. When designing the controller, we should not only consider whether the controller can achieve *FTS* successfully, but also consider the excellent performance of the designed controller. Hence, the design of controller should be simple in structure and easy to implement, and moreover, it should be continuous to avoid the chattering phenomenon. However, when designing *FTS* controllers, these two points are rarely fully considered in many cases. For example, the boundedness of controllers designed in [18] is hard to guarantee due to the special structure of the controllers; the proposed controllers in [33] are not precise and need to be further improved.

Based on above discussions, it is very necessary to design a proper controller to study the problem of lag synchronization in finite time. Moreover, in practical application, when the control input is transmitted, it is often affected by frequency change, switching phenomenon, or other burst noise in impulse form. Therefore, considering the impulse noise interference in the real control is a very normal thing in many cases. This paper considers the *FTLS* of uncertain drive-response systems involving impulsive disturbances. There are several main contributions as follows.

(1) Unlike previous papers [24–26, 35–37], impulse noise interferences for *FTLS* of CNs are fully considered. When system is involved in impulse disturbances before reaching the synchronization time, the obtained results show that the synchronizing time is related to the impulses and will be delayed. Furthermore, for impulse, we do not impose restriction on lower bound of two consecutive impulse disturbances and the size of impulse intervals does not affect *FTLS* for addressed system.

(2) Different from the designed controllers in [24, 33, 36], the boundedness problem of the controllers is overcome, and it is easier to implement through LMI toolbox.

The rest of the paper is organized as follows. Section 2 presents some preliminaries. By designing two different control laws, in Sect. 3, we establish some *FTLS* criterion for uncertain drive-response systems. A numerical example is given in Sect. 4. Section 5 shows the conclusion of this paper.

2 Preliminaries

Notations Let $\mathbb{Z}_+$ denote the set of positive integer numbers, $\mathbb{R}$ the set of real numbers and $\mathbb{R}^n$ the n-dimensional real spaces equipped with the Euclidean norm $|\cdot|$. $B>0$ $(B<0)$ denotes that B is symmetric and positive (negative) definite matrix. $\Theta=\{1,2,\ldots,N\}$ and I is an identity matrix. For $H\subseteq\mathbb{R}$, set $W\subseteq\mathbb{R}^m(1\leq m\leq n), C(H,W)=\{\omega:H\to W$ is a continuous function$\}$ and $PC(H,W)=\{\omega:H\to W$ is a continuous function everywhere except at finite number of instances t, at which $\omega(t^+)$, $\omega(t^-)$ exist, and moreover, $\omega(t^+)=\omega(t)\}$. $\mathcal{K}=\{b(\cdot)\in C(\mathbb{R}_+,\mathbb{R}_+)|\ b(0)=0$, $b(\delta)>0$ for $\delta>0$, and b is strictly increasing in $\delta\}$.

Consider the CNs composed of N coupled nodes and each of the nodes is an n-dimensional network. The dynamic of the ith networks is described by

$$\dot{x}_i(t) = \bar{A}x_i(t) + \bar{B}f(x_i(t)) + c\sum_{j=1}^{N} h_{ij}\Gamma x_j(t),\ i \in \Theta, \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ is the ith network state; $\bar{A} = A + \triangle A$, $\bar{B} = B + \triangle B$, in which $A,\ B \in \mathbb{R}^{n\times n}$ are the connection weight matrices, and $\triangle A,\ \triangle B \in \mathbb{R}^{n\times n}$ are parametric uncertainties; $c > 0$ is the coupling strength;

$$f(x_i(t)) = (f_1(x_{i1}(t)),\ f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))^T$$

denotes nonlinear function and satisfies $|f_j(\vartheta_1) - f_j(\vartheta_2)| \le l_j|\vartheta_1 - \vartheta_2|$ for any $\vartheta_1,\ \vartheta_2 \in \mathbb{R}$, where $l_j > 0$ are constants for $j = \{1, 2, \ldots, n\}$; the coupling configuration $H = (h_{ij})_{N\times N}$ is defined as: for element h_{ij}, if there is a connection from node j to node i, then $h_{ij} > 0$, or else $h_{ij} = 0$, and the diagonal elements $h_{ii} = -\sum_{i=1, j\neq i}^{N} h_{ij}$. The inner coupling matrix $\Gamma = \mathrm{diag}\{\gamma_1,\ \gamma_2,\ \ldots, \gamma_n\} > 0$ and $x_i(0) = x_{i0}$ denotes initial value of the network (1).

In order to study the lag synchronization, we refer to system (1) as the drive system without losing generality. In addition, it is assumed that during the transmission of the input signal, the state may suddenly jump at some discrete time, that is, the impulse phenomenon is generated [4, 38]. Hence, this paper considers the following CNs involving impulse as response system:

$$\begin{cases} \dot{y}_i(t) = \bar{A}y_i(t) + \bar{B}f(y_i(t)) \\ +c\sum_{j=1}^{N} h_{ij}\Gamma y_j(t) + u_i(t),\ t \in [t_{k-1}, t_k), \\ \Delta y_i(t_k) = D(y_i(t_k^-) - x_i(t_k^- - \sigma)),\quad k \in \mathbb{Z}_+, \end{cases} \quad (2)$$

with the initial value $y_i(\sigma) = y_{i\sigma},\ i \in \Theta$, where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t))^T \in \mathbb{R}^n$ and σ is a positive constant; the control input $u_i(t)$ will be designed later; $\Delta y_i(t_k) = y_i(t_k) - y_i(t_k^-)$; D represents the impulse matrix. The time sequence $\{t_k, k \in \mathbb{Z}_+\}$ is the set of impulse sequences which is strictly increasing on $\mathbb{R}_+$. We denote such sequence by set $\mathcal{F}$ for later use. The rest parameters in system (2) are the same as in system (1).

Define the lag synchronization error $\zeta_i(t) = y_i(t) - x_i(t - \sigma)$. Then, we can obtain the following error system between drive-response system (1) and (2)

$$\begin{cases} \dot{\zeta}_i(t) = \bar{A}\zeta_i(t) + \bar{B}g(\zeta_i(t)) \\ +c\sum_{j=1}^{N} h_{ij}\Gamma\zeta_j(t) + u_i(t),\ t \ge \sigma,\ t \ne t_k, \\ \Delta\zeta_i(t_k) = D\zeta_i(t_k^-),\quad k \in \mathbb{Z}_+, \end{cases} \quad (3)$$

where $g(\zeta_i(t)) = f(y_i(t)) - f(x_i(t - \sigma))$ and $\Delta\zeta_i(t_k) = \zeta_i(t_k) - \zeta_i(t_k^-)$ for $i \in \Theta$. For convenience, let $\zeta(t) = (\zeta_1^T(t), \zeta_2^T(t), \ldots, \zeta_N^T(t))^T$ and $G(\zeta(t)) = (g^T(\zeta_1), g^T(\zeta_2), \ldots, g^T(\zeta_N))^T$. Then, the Kronecker product form of the error system (3) can be transformed into

$$\begin{cases} \dot{\zeta}(t) = \big(I_N \otimes \bar{A} + c(H \otimes \Gamma)\big)\zeta(t) \\ +(I_N \otimes \bar{B})G(\zeta(t)) + U,\ t \ge \sigma,\ t \ne t_k, \\ \zeta(t_k) = \big(I_N \otimes (I + D)\big)\zeta(t_k^-),\quad k \in \mathbb{Z}_+, \end{cases} \quad (4)$$

where $U = (u_1^T, u_2^T, \ldots, u_N^T)^T$, and the initial value $\zeta(\sigma) = \zeta_\sigma = (\zeta_{1\sigma}^T, \zeta_{2\sigma}^T, \ldots, \zeta_{N\sigma}^T)^T$, $\zeta_{i\sigma} = y_{i\sigma} - x_{i0},\ i \in \Theta$.

Definition 1 [35] For given impulse sequence $\{t_k\} \in \mathcal{F}$ and constant $\sigma > 0$, the system (1) and (2) is said to be *FTLS* if there exists a constant $T > 0$ such that

$$\lim_{t\to T+\sigma} |\zeta_i(t)| = \lim_{t\to T+\sigma} |y_i(t) - x_i(t - \sigma)| = 0$$

and

$$\lim_{t\to T+\sigma} |\zeta_i(t)| \equiv 0,\ \textit{if}\ \ t \ge T + \sigma,\ i \in \Theta,$$

where T called synchronizing time depends on the initial value $\zeta_{i\sigma}$ and impulse sequences $\mathcal{F}$.

Remark 1 From the description of lag synchronization error system, the response system does not receive the information from the driver system from time 0 to σ. Thus, in Definition 1, we describe this characteristic for *FTLS*, that is, synchronization time of the drive-response systems is delayed due to the constant σ which is named transmission delay. Due to the existence of transmission delay σ, the real synchronization time of the drive-response systems is $T + \sigma$, where T represents the synchronizing time starting from the time σ and depends not only on the initial value $\zeta_{i\sigma}$ but also on the impulse sequence $\mathcal{F}$. In particular, assume that a special case of $\sigma = 0$ is considered, and then, we can realize the complete synchronization of the systems (1) and (2) with synchronizing time T. Hence, there is a wide applicability in our results.

Definition 2 [32] For any vector $v = (v_1, v_2, \ldots, v_n)^T \in \mathbb{R}^n$ and constant μ, we define

$$S(v) = (\mathrm{sign}(v_1), \mathrm{sign}(v_2), \ldots, \mathrm{sign}(v_n))^T,$$
$$D(v) = \mathrm{diag}(|v_1|^\mu, |v_2|^\mu, \ldots, |v_n|^\mu).$$

Assumption 1 There exist constants $\varepsilon_1,\ \varepsilon_2 > 0$ such that uncertainties $\triangle A,\ \triangle B \in \mathbb{R}^{n\times n}$ satisfy the following conditions:

$$\triangle A^T\triangle A \le \varepsilon_1 I,\quad \triangle B^T\triangle B \le \varepsilon_2 I.$$

Before giving the main results, we first introduce the theoretical results of impulsive systems, which plays an important role in our proof.

Consider the following nonlinear impulsive system:

$$\begin{cases} \dot{x}(t) = f(x_t), \ t \neq t_k, \ t \geq t_0, \\ x(t) = g(x(t^-)), \quad t = t_k, \ k \in \mathbb{Z}_+, \\ x(0) = x_0. \end{cases} \tag{5}$$

Li et al. [22] provide the detailed descriptions for functions f, g and impulse sequence $\mathcal{S}$, which are omitted here. For system (5), the following lemma can be obtained.

Lemma 1 [22] *Let $U_\rho = \{x \in \mathbb{R}^n : |x| \leq \rho\}$ with $\rho > 0$. System (5) is FTS over the class $\mathcal{S}$ of impulse sequences if there exist constants $\beta \in [1, \infty)$, $\eta \in (0,1)$, $\alpha > 0$, functions $w_1, w_2 \in \mathcal{K}$, and locally Lipschitz continuous function $V(x) : \mathbb{R}^n \to \mathbb{R}_+$ such that*

(i) $w_1(|x|) \leq V(x) \leq w_2(|x|), \ \forall x \in \mathbb{R}^n$,

(ii) $V(g(x)) \leq \beta^{\frac{1}{1-\eta}} V(x), \ \forall x \in \mathbb{R}^n, t = t_k$,

(iii) $D^+V \leq -\alpha V^\eta(x), \ \forall x \in \mathbb{R}^n, \ t \neq t_k$, *and the impulse sequences $\{t_k\} \in \mathcal{S}$ satisfy*

$$\min\left\{ j \in \mathbb{Z}_+ : \ \frac{t_j}{\beta^{j-1}} \geq \frac{w_2^{1-\eta}(\rho)}{\alpha(1-\eta)} \right\} := N_0 < +\infty.$$

In addition, the settling time is bounded by

$$T(x_0, \{t_k\}) \leq \beta^{N_0 - 1} \frac{w_2^{1-\eta}(\rho)}{\alpha(1-\eta)}, \ \forall x_0 \in U_\rho, \ \forall \{t_k\} \in \mathcal{S},$$

where N_0 depends on $\{t_k\}$.

3 Main results

In this section, we focus on the design of controllers to ensure the *FTLS* of systems (1) and (2), where desynchronizing impulses are considered.

Theorem 1 *Under Assumption 1, if there exist constants $\alpha > 0$, $\rho > 0$, $\beta \in [1, \infty)$, $\mu \in (-1, 1)$, $n \times n$ matrices $P > 0$, $Q > 0$, $n \times n$ diagonal matrices $S > 0$, $R > 0$, and $n \times n$ real matrix W such that*

$$\begin{pmatrix} \Pi & I_N \otimes P & I_N \otimes (PB) & I_N \otimes P \\ * & -I_N \otimes Q & 0 & 0 \\ * & * & -I_N \otimes R & 0 \\ * & * & * & -I_N \otimes S \end{pmatrix} < 0, \tag{6}$$

$$(I + D)^T P (I + D) \leq \beta^{\frac{2}{1-\mu}} P, \tag{7}$$

where $\Pi = I_N \otimes (A^T P + PA + LRL) + 2cH \otimes (P\Gamma) - I_N \otimes W - I_N \otimes W^T, \ L = \mathrm{diag}(l_1, l_2, \ldots, l_n)$. *Then, the drive-response system (1) and (2) is FTLS over the class $\mathcal{F}$ under the controller given by*

$$\begin{aligned} U = &-(I_N \otimes P^{-1})\big[(I_N \otimes W)\zeta(t) + 0.5\big(\lambda_{\max}(Q)\varepsilon_1 I \\ &+ \lambda_{\max}(S)\varepsilon_2 LL\big)\zeta(t) + 0.5\alpha\lambda_{\max}^{\frac{1+\mu}{2}}(P) D(\zeta(t)) S(\zeta(t))\big], \end{aligned} \tag{8}$$

where impulse sequences $\{t_k\} \in \mathcal{F}$ satisfy

$$\min\left\{ j \in \mathbb{Z}_+ : \ \frac{t_j}{\beta^{j-1}} \geq \frac{2\lambda_{\max}^{\frac{1-\mu}{2}}(P)\rho^{1-\mu}}{\alpha(1-\mu)} \right\} := N_0 < +\infty. \tag{9}$$

In addition, the synchronizing time is bounded by

$$T(\zeta_\sigma, \{t_k\}) \leq \beta^{N_0 - 1} \frac{2\lambda_{\max}^{\frac{1-\mu}{2}}(P)\rho^{1-\mu}}{\alpha(1-\mu)}, \tag{10}$$

$\forall \zeta_\sigma \in U_\rho, \ \forall \{t_k\} \in \mathcal{F}$, where N_0 depends on $\{t_k\}$.

Proof Choosing Lyapunov function as

$$V(t) = \zeta^T(t)(I_N \otimes P)\zeta(t). \tag{11}$$

Taking the derivative of $V(t)$ over the time interval $t \in [t_{k-1}, t_k), \ t \geq \sigma$ along the solution of (4), we have

$$\begin{aligned} D^+V(t) &= 2\zeta^T(t)(I_N \otimes P)\dot{\zeta}(t) \\ &= 2\zeta^T(t)(I_N \otimes P)\big\{I_N \otimes [A + \triangle A]\zeta(t) \\ &\quad + I_N \otimes [B + \triangle B]G(\zeta(t)) + c(H \otimes \Gamma)\zeta(t) + U\big\} \\ &= \zeta^T(t)\big[I_N \otimes (A^T P + PA)\big]\zeta(t) \\ &\quad + 2\zeta^T(t)\big[(I_N \otimes P)(I_N \otimes \triangle A)\big]\zeta(t) \\ &\quad + 2\zeta^T(t)(I_N \otimes P)[I_N \otimes (B + \triangle B)]G(\zeta(t)) \\ &\quad + 2\zeta^T(t)[cH \otimes (P\Gamma)]\zeta(t) + 2\zeta^T(t)(I_N \otimes P)U. \end{aligned} \tag{12}$$

From Assumption 1, it is easy to derive

$$\begin{aligned} &2\zeta^T(t)\big[(I_N \otimes P)(I_N \otimes \triangle A)\big]\zeta(t) \\ &\leq \zeta^T(t)[I_N \otimes (PQ^{-1}P)]\zeta(t) \\ &\quad + \zeta^T(t)[I_N \otimes (\triangle A^T Q \triangle A)]\zeta(t) \\ &\leq \zeta^T(t)[I_N \otimes (PQ^{-1}P)]\zeta(t) \\ &\quad + \lambda_{\max}(Q)\zeta^T(t)[I_N \otimes (\triangle A^T \triangle A)]\zeta(t) \\ &\leq \zeta^T(t)\big[I_N \otimes ((PQ^{-1}P) + \lambda_{\max}(Q)\varepsilon_1 I)\big]\zeta(t) \end{aligned} \tag{13}$$

and

$$
\begin{aligned}
&2\zeta^T(t)(I_N\otimes P)[I_N\otimes(B+\triangle B)]G(\zeta(t))\\
&\quad=2\zeta^T(t)[I_N\otimes(PB)]G(\zeta(t))\\
&\qquad+2\zeta^T(t)[I_N\otimes(P\triangle B)]G(\zeta(t))\\
&\quad\le\zeta^T(t)[I_N\otimes(PBR^{-1}B^TP)]\zeta(t)\\
&\qquad+G^T(\zeta(t))(I_N\otimes R)G(\zeta(t))\\
&\qquad+\zeta^T(t)[I_N\otimes(PS^{-1}P)]\zeta(t)\\
&\qquad+G^T(\zeta(t))[I_N\otimes(\triangle B^TS\triangle B)]G(\zeta(t))\\
&\quad\le\zeta^T(t)[I_N\otimes(PBR^{-1}B^TP+PS^{-1}P)]\zeta(t)\\
&\qquad+\zeta^T(t)[I_N\otimes(LRL)]\zeta(t)\\
&\qquad+\zeta^T(t)[I_N\otimes(\lambda_{\max}(S)\varepsilon_2LL)]\zeta(t)\\
&\quad=\zeta^T(t)\big[I_N\otimes(PBR^{-1}B^TP+PS^{-1}P+LRL\\
&\qquad+\lambda_{\max}(S)\varepsilon_2LL)\big]\zeta(t).
\end{aligned}\tag{14}
$$

Substituting (13)–(14) into (12) and considering condition (6), it holds that

$$
\begin{aligned}
D^+V(t)\le&\ \zeta^T(t)\big[I_N\otimes(A^TP+PA+PQ^{-1}P\\
&+\lambda_{\max}(Q)\varepsilon_1I\\
&+PBR^{-1}B^TP+PS^{-1}P+LRL\\
&+\lambda_{\max}(S)\varepsilon_2LL)\\
&+2cH\otimes(P\Gamma)\big]\zeta(t)\\
&+2\zeta^T(t)(I_N\otimes P)U\\
\le&-\alpha\lambda_{\max}^{\frac{1+\mu}{2}}(P)\zeta^T(t)D(\zeta(t))S(\zeta(t))\\
\le&-\alpha\big(\zeta^T(t)(I_N\otimes P)\zeta(t)\big)^{\frac{1+\mu}{2}}\\
=&-\alpha V^{\frac{1+\mu}{2}}(t).
\end{aligned}\tag{15}
$$

In addition, when $t=t_k,\ k\in\mathbb{Z}_+$, one obtains that

$$
\begin{aligned}
V(t_k)&=\zeta^T(t_k)(I_N\otimes P)\zeta(t_k)\\
&=\zeta^T(t_k^-)\{I_N\otimes[(I+D)^TP(I+D)]\}\zeta(t_k^-)\\
&\le\beta^{\frac{2}{1-\mu}}\zeta^T(t_k^-)(I_N\otimes P)\zeta(t_k^-)\\
&\le\beta^{\frac{2}{1-\mu}}V(t_k^-).
\end{aligned}\tag{16}
$$

Then, we see that it is easy for inequalities (15) and (16) to satisfy Lemma 1. Thus, the *FTLS* of systems (1) and (2) under the controller (8) over the class $\mathcal{F}$ of impulse sequences given in (9) is achieved. Moreover, the synchronizing time (10) is derived. This completes the proof. □

Remark 2 In Theorem 1, some sufficient conditions for synchronization control of systems (1) and (2) are presented. Note that it is necessary to ensure LMIs (6)–(7) hold simultaneously, that is to say, some decision matrices P, Q, W, S, R are solved to ensure LMIs (6)–(7) feasible for some given parameters at the same time. Moreover, in implementation, it can be seen from the derivation of inequality (16) that for given μ, it is desirable to find smallest constants β to ensure the term $(I+D)^TP(I+D)$ is as close as possible to $\beta^{\frac{2}{1-\mu}}P$. Therefore, when solving (6)–(7), the MATLAB LMI toolbox is used to find the smallest β so that the above inequalities hold.

In what follows, another *FTLS* result for the drive-response system (1) and (2) is derived based on a new Lyapunov function, in which a special case that the impulse matrix $D=\mathrm{diag}\{d_1,d_2,\ldots,d_n\}>0$ is considered.

Theorem 2 *Under Assumption 1, if there exist constants $\alpha>0,\ \rho>0,\ \beta\in[1,\infty),\ \mu\in(-1,1),\ k_1,\ k_2,\ k_3>0$, $n\times n$ diagonal matrix $P>0$, $n\times n$ real matrix W such that*

$(H_1) I_N\otimes\left(A^TP+PA+\frac{\varepsilon_1}{k_1}I+\frac{1}{k_2}L+\frac{\varepsilon_2}{k_3}L\right)+2cH\otimes(P\Gamma)-I_N\otimes W-I_N\otimes W^T\le 0,$

$(H_2) \lambda_{\max}(I+D)\le\beta^{\frac{2}{1-\mu}}I,$

where $L=\mathrm{diag}(l_1,l_2,\ldots,l_n)$. Then, the drive-response system (1) and (2) is FTLS over the class $\mathcal{F}$ under the controller given by

$$
U=U_1+U_2\tag{17}
$$

with

$$
\begin{aligned}
U_1=&-0.5\big\{[I_N\otimes(P^{-1}W)]\zeta(t)+[I_N\otimes(P^{-1}W^T)]\zeta(t)\\
&+(I_N\otimes P)S(\zeta(t))\cdot k_1\zeta^T(t)S(\zeta(t))\\
&+[I_N\otimes(BB^TP)]S(\zeta(t))\cdot k_2\zeta^T(t)(I_N\otimes L)S(\zeta(t))\\
&+(I_N\otimes P)S(\zeta(t))\cdot k_3\zeta^T(t)(I_N\otimes L)S(\zeta(t))\big\},\\
U_2=&-0.5\alpha(I_N\otimes P^{-1})S(\zeta(t))[2\zeta^T(t)(I_N\otimes P)S(\zeta(t)]^{\frac{1+\mu}{2}},
\end{aligned}
$$

where impulse sequences $\{t_k\}\in\mathcal{F}$ satisfy

$$
\min\left\{j\in\mathbb{Z}_+:\frac{t_j}{\beta^{j-1}}\ge\frac{2^{\frac{3-\mu}{2}}\lambda_{\max}^{\frac{1-\mu}{2}}(P)\rho^{\frac{1-\mu}{2}}}{\alpha(1-\mu)}\right\}:=N_0<+\infty.\tag{18}
$$

In addition, the synchronizing time is bounded by

$$
T(\zeta_\sigma,\{t_k\})\le\beta^{N_0-1}\frac{2^{\frac{3-\mu}{2}}\lambda_{\max}^{\frac{1-\mu}{2}}(P)\rho^{\frac{1-\mu}{2}}}{\alpha(1-\mu)},\tag{19}
$$

$\forall\zeta_\sigma\in U_\rho,\ \forall\{t_k\}\in\mathcal{F}$, where N_0 depends on $\{t_k\}$.

Proof Consider the Lyapunov function

$$
V(t)=2\zeta^T(t)(I_N\otimes P)S(\zeta(t)).\tag{20}
$$

Taking the derivative of $V(t)$ over the time interval $t\in[t_{k-1},t_k),\ t\ge\sigma$ along the solution of (4), one has that

$$\begin{aligned}
D^{+}V(t) &= 2\zeta^{T}(t)(I_N\otimes P)\dot{S}(\zeta(t)) \\
&\quad + 2\dot{\zeta}^{T}(t)(I_N\otimes P)S(\zeta(t)) \\
&= 2S^{T}(\zeta(t))(I_N\otimes P)\dot{\zeta}(t) \\
&= S^{T}(\zeta(t))\{I_N\otimes[A^{T}P+PA] \\
&\quad + 2cH\otimes(P\Gamma)\}\zeta(t) \\
&\quad + 2S^{T}(\zeta(t))\big[(I_N\otimes P)(I_N\otimes\triangle A)\big]\zeta(t) \\
&\quad + 2S^{T}(\zeta(t))\big[I_N\otimes(PB+P\triangle B)\big]G(\zeta(t)) \\
&\quad + 2S^{T}(\zeta(t))(I_N\otimes P)U.
\end{aligned} \tag{21}$$

Note that

$$\zeta^{T}(t)(I_N\otimes(LL))\zeta(t)\le\big[\zeta^{T}(t)(I_N\otimes L)S(\zeta(t))\big]^{2}.$$

From Assumption 1, one obtains that when $|\zeta(t)|\neq 0$,

$$\begin{aligned}
&2S^{T}(\zeta(t))\big[(I_N\otimes P)(I_N\otimes\triangle A)\big]\zeta(t) \\
&\le S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t))\cdot k_1\zeta^{T}(t)S(\zeta(t)) \\
&\quad + \zeta^{T}(t)\big[I_N\otimes(\triangle A^{T}\triangle A)\big]\zeta(t)\cdot k_1^{-1}\frac{1}{\zeta^{T}(t)S(\zeta(t))} \\
&\le S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t))\cdot k_1\zeta^{T}(t)S(\zeta(t)) \\
&\quad + \frac{\varepsilon_1}{k_1}\zeta^{T}(t)S(\zeta(t))
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
&2S^{T}(\zeta(t))\big[I_N\otimes(PB+P\triangle B)\big]G(\zeta(t)) \\
&\le S^{T}(\zeta(t))\big[I_N\otimes(PBB^{T}P)\big]S(\zeta(t)) \\
&\quad \cdot k_2\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + G^{T}(\zeta(t))G(\zeta(t))\cdot k_2^{-1} \\
&\quad \frac{1}{\zeta^{T}(t)(I_N\otimes L)S(\zeta(t))} \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t)) \\
&\quad \cdot k_3\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + G^{T}(\zeta(t))\big[I_N\otimes(\triangle B^{T}\triangle B)\big]G(\zeta(t)) \\
&\quad \cdot k_3^{-1}\frac{1}{\zeta^{T}(t)(I_N\otimes L)S(\zeta(t))} \\
&\le S^{T}(\zeta(t))\big[I_N\otimes(PBB^{T}P)\big]S(\zeta(t)) \\
&\quad \cdot k_2\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + k_2^{-1}\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t)) \\
&\quad \cdot k_3\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + \frac{\varepsilon_2}{k_3}\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)).
\end{aligned} \tag{23}$$

When $|\zeta(t)|=0$,, it can be derived that

$$\begin{aligned}
&2S^{T}(\zeta(t))\big[(I_N\otimes P)(I_N\otimes\triangle A)\big]\zeta(t) \\
&= S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t)) \\
&\quad \cdot k_1\zeta^{T}(t)S(\zeta(t))+\frac{\varepsilon_1}{k_1}\zeta^{T}(t)S(\zeta(t)), \\
&2S^{T}(\zeta(t))\big[I_N\otimes(PB+P\triangle B)\big]G(\zeta(t)) \\
&= S^{T}(\zeta(t))\big[I_N\otimes(PBB^{T}P)\big]S(\zeta(t)) \\
&\quad \cdot k_2\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + k_2^{-1}\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t)) \\
&\quad \cdot k_3\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + \frac{\varepsilon_2}{k_3}\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)).
\end{aligned}$$

Thus, assertion Eqs. (22) and (23) hold for any $\zeta(t)\in\mathbb{R}^{Nn}$. Moreover, from the definition of $S(v)$, we have

$$S^{T}(\zeta(t))S(\zeta(t))=\begin{cases}0, & |\zeta(t)|=0,\\ M\in\{1,\ldots,Nn\}, & |\zeta(t)|\neq 0.\end{cases}$$

Combining this with (21)–(23) over the time interval $t\in[t_{k-1},t_k),\ t\ge\sigma,$, one has that

$$\begin{aligned}
&D^{+}V(t) \\
&\le S^{T}(\zeta(t))\{I_N\otimes[A^{T}P+PA]+2cH\otimes(P\Gamma)\}\zeta(t) \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t))\cdot k_1\zeta^{T}(t)S(\zeta(t)) \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PBB^{T}P)\big]S(\zeta(t)) \\
&\quad \cdot k_2\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + S^{T}(\zeta(t))\big[I_N\otimes(PP)\big]S(\zeta(t)) \\
&\quad \cdot k_3\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + \frac{\varepsilon_1}{k_1}\zeta^{T}(t)S(\zeta(t))+\bigg(\frac{1}{k_2} \\
&\quad + \frac{\varepsilon_2}{k_3}\bigg)\zeta^{T}(t)(I_N\otimes L)S(\zeta(t)) \\
&\quad + 2S^{T}(\zeta(t))(I_N\otimes P)U \\
&\le 2S^{T}(\zeta(t))(I_N\otimes P)U_2 \\
&= -\alpha S^{T}(\zeta(t))S(\zeta(t))[2\zeta^{T}(t)(I_N\otimes P)S(\zeta(t))]^{\frac{1+\mu}{2}} \\
&\le -\alpha V^{\frac{1+\mu}{2}}(t).
\end{aligned} \tag{24}$$

In addition, when $t=t_k,\ k\in\mathbb{Z}_{+}$,

$$\begin{aligned}
V(t_k) &= 2\zeta^{T}(t_k)(I_N\otimes P)S(\zeta(t_k)) \\
&= 2\zeta^{T}(t_k^{-})\big(I_N\otimes(I+D)\big)^{T}(I_N\otimes P) \\
&\quad S\big((I_N\otimes(I+D))\zeta(t_k^{-})\big) \\
&\le 2\lambda_{\max}(I+D)\zeta^{T}(t_k^{-})(I_N\otimes P)S(\zeta(t_k^{-})) \\
&\le 2\beta^{\frac{2}{1-\mu}}\zeta^{T}(t_k^{-})(I_N\otimes P)S(\zeta(t_k^{-})) \\
&= \beta^{\frac{2}{1-\mu}}V(t_k^{-}).
\end{aligned} \tag{25}$$

According to Lemma 1, the *FTLS* problem of systems (1) and (2) under the controller (17) over the class $\mathcal{F}$ of impulse sequences given in (18) is achieved, and moreover, the synchronizing time (19) is derived. The proof is completed. □

Remark 3 In recent years, there are many results dealt with the problem for *FTLS* of CNs [24, 35–37]. However, note that the controllers in [24, 36] needed state feedback with special structure when the lag synchronization error is not zero, i.e., $\zeta_i/|\zeta_i|^2$ when $\zeta_i \neq 0$, which is named fractional state feedback. The obvious disadvantage of this kind of controllers is that although u_i is defined when $\zeta_i = 0$, when ζ_i is close to 0, it is difficult to determine whether u_i is bounded or not, which indicates that these controllers are difficult to apply in finite-time sense when considering the synchronization problem. In our results, the controllers (8) and (17) can effectively solve the problem of boundedness in [24, 36] and are easier to implement in applications. In addition, it should be noted that these researches are all based on continuous state dynamics. When considering a system with impulses, there are many difficulties and challenges, such as when the *FTLS* system is affected by impulse disturbance, the *FTLS* property may changed and the synchronizing time may increase or even tend to infinity. Hence, some *FTLS* conditions of drive-response systems with impulse effects are presented in our results, which generalizes the previous results.

Remark 4 In Theorems 1 and 2, by constructing different Lyapunov functions, the *FTLS* criteria of CNs involving impulsive disturbance are obtained, respectively. The proposed results show that different impulses will lead to different synchronization times. If the system is subjected to impulse disturbance with large disturbance strength or more frequent impulse sequence, the convergence rate will slow down, and then, the synchronization time will be delayed. Conversely, when smaller disturbance strength or less frequent impulse sequence is involved to the system, the convergence speed will speeded up and then the synchronization time will be shortened. This observation can be found from (10) and (19).

4 Numerical examples

In this section, a example is given to illustrate the *FTLS* of the drive-response system with impulse disturbance under the control design.

Example 1 CNs often arise in the modeling of practical systems, such as digital communication network, urban public transportation, epidemic spreading phenomena. In view of this, we consider the following 3D uncertain CNs as drive system

$$\dot{x}_i(t) = (A + \triangle A)x_i(t) + (B + \triangle B)f(x_i(t)) + c\sum_{j=1}^{N} h_{ij}\Gamma x_j(t), \; i = 1,2,3,4, \tag{26}$$

with

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -1 & 1 \\ 0 & -7 & 1 \end{pmatrix},$$

$$\triangle A = \begin{pmatrix} -0.1 & 0.2 & 0.1 \\ 0.1 & -0.1 & 0.1 \\ 0.1 & 0.1 & -0.1 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 0.2 & -0.1 & 1 \\ 7 & 0 & 0.1 \end{pmatrix},$$

$$\triangle B = \begin{pmatrix} 0.1 & 0.1 & 0.2 \\ 0.1 & 0.2 & -0.1 \\ 0.1 & -0.1 & -0.2 \end{pmatrix},$$

$$H = \begin{pmatrix} -2 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -2 \end{pmatrix},$$

$$\Gamma = \mathrm{diag}\{0.3, 0.4, 0.5\},$$

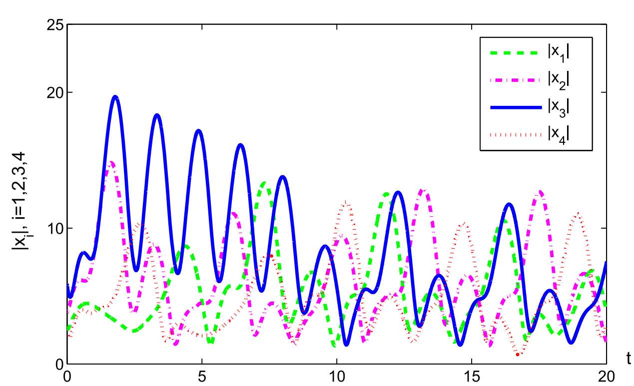

Fig. 1 Trajectories of drive system (26)

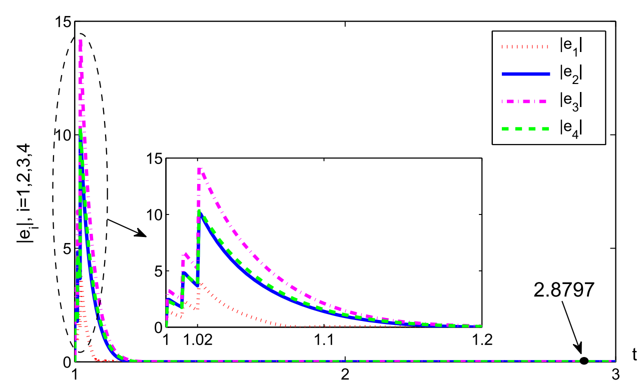

Fig. 2 Trajectories of error $\zeta_i = y_i(t) - x_i(t - \sigma)$ with $\sigma = 1$ under controller (17)

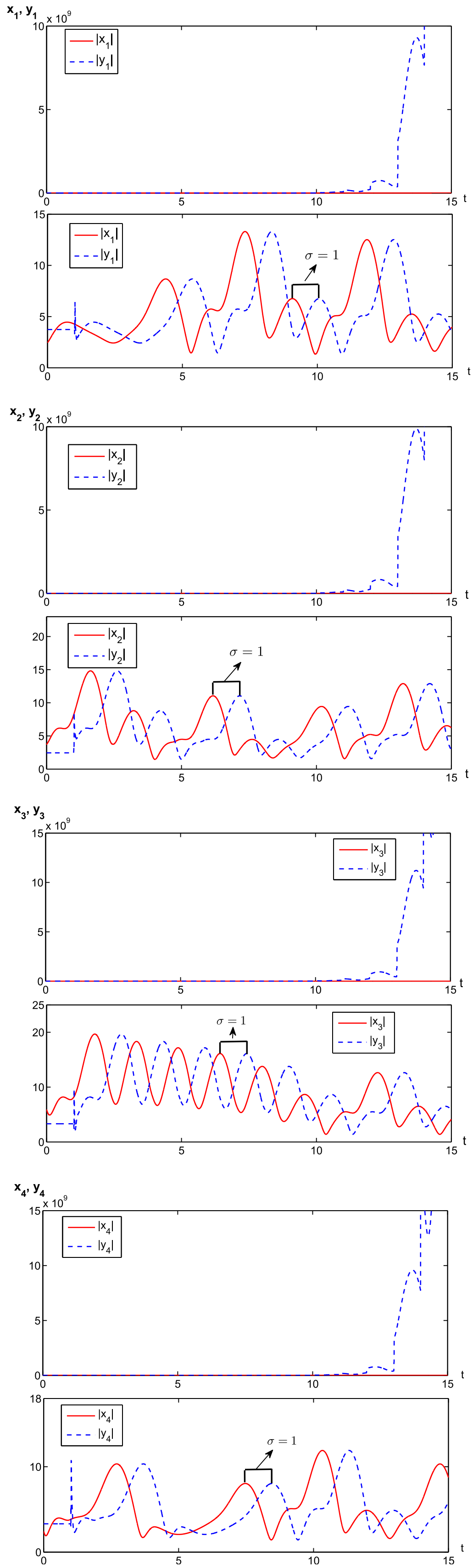

Fig. 3 Trajectories of systems (26) and (27) with $\sigma = 1$ without/with controller (17) in Example 1

$c = 0.2$ and $f(v) = (f_1(v), f_2(v), f_3(v))^T$, where $f_j(v) = 0.5(|v+1| - |v-1|)$, $j = 1, 2, 3$.

Consider the initial condition $x_{10} = (1, 2, -1)^T$, $x_{20} = (3, -1, 2)^T$, $x_{30} = (5, -3, -1)^T$, $x_{40} = (-1, 2, 1)^T$. Then, state trajectories of drive system (26) are shown in Fig. 1.

Considering the response system involving impulses disturbance in the form of

$$\begin{cases} \dot{y}_i(t) = (A + \triangle A) y_i(t) + (B + \triangle B) f(y_i(t)) \\ \quad + c \sum_{j=1}^{N} h_{ij} \Gamma y_j(t) + u_i(t), \ t \in [t_{k-1}, t_k), \\ \Delta y_i(t_k) = D(y_i(t_k^-) - x_i(t_k^- - \sigma)), \quad k \in \mathbb{Z}_+, \end{cases} \tag{27}$$

where impulses matrix

$$D = \begin{pmatrix} 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$$

and impulse sequences $t_{4k} = 3k$, $t_{4k-1} = 3k - 1$, $t_{4k-2} = 3k - 1.98$, $t_{4k-3} = 3k - 1.99$.

In simulation, the lag synchronization with $\sigma = 1$ is considered. In what follows, we choose $\alpha = 2$, $\beta = 1.3$, $\mu = 0.2$, $k_1 = k_2 = k_3 = 1$ and note that $L = I$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$. According to Theorem 2, the following feasible solution is derived by MATLAB LMI toolbox

$$P = \begin{pmatrix} 0.0388 & 0 & 0 \\ 0 & 0.0388 & 0 \\ 0 & 0 & 0.0388 \end{pmatrix},$$

$$W = \begin{pmatrix} 0.5254 & 0.0405 & 0 \\ 0.0525 & 0.5244 & -0.0981 \\ 0 & -0.0879 & 0.5855 \end{pmatrix}.$$

Then drive-response system (26) and (27) can achieve *FTLS* under the controller (17), where synchronizing time is bounded by $T(\zeta_\sigma, \{t_k\}) \leq 1.8797$. Under same conditions, when considering system without impulses, we can estimated the synchronizing time $T(\zeta_\sigma, \{t_k\}) \leq 0.8556$. It shows that due to the existence of impulse disturbances in response system, the synchronization time is delayed. The lag synchronization errors of CNs with impulse disturbance and the synchronizing time with $\sigma = 1$ are shown in Fig. 2. Correspondingly, the state trajectories without/with controller are shown in Fig. 3, where the initial condition is chosen as $y_{1\sigma} = [1, 3, -2]^T$, $y_{2\sigma} = [2, 1, 1]^T$, $y_{3\sigma} = [3, -1, 1]^T$, $y_{4\sigma} = [-3, 1, 1]^T$.

5 Conclusion

This paper studied the *FTLS* of uncertain CNs subjecting to impulsive disturbances, and some criteria on synchronization control were established by employing different Lyapunov function in two theorems. In particular, the synchronizing time for addressed impulsive system was estimated, which shows that different impulses will lead to different synchronization times. Finally, the effectiveness of the proposed results was verified by a numerical example. The development for systems involving delayed impulses with synchronizing-time estimation is an interesting topic in the future, and moreover, further interesting research topic is the case that synchronization time is independent of the initial value, i.e., the problem of fixed-time synchronization. In addition, inspired by [39, 40], our another future work will concern with controller design for finite-time problems of practical system.

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Declarations

Conflict of interest The authors declare that none of the authors have any competing interests in the manuscript.

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