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ORIGINAL ARTICLE

Measuring congestion in sustainable supply chain based on data envelopment analysis

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Abstract
Sustainable Supply Chain Management (SSCM) involves the integrating of environmental, social and economic concerns into supply chain management activities with emphasis on the managers' efforts in the context of reducing the negative social and environmental impacts. Evaluating sustainable supply chain performance and efficiency is a significant topic for many researchers and scholars. Presence of input and intermediate product congestion is one of the key issues that results in lower efficiency and performance in a sustainable supply chain. Therefore, determination of congestion is of prime importance and removing it improves performance of the sustainable supply chain. One of the most appropriate methods for detecting congestion is Data Envelopment Analysis (DEA). Some studies have been conducted to detect the intermediate product congestion via solving Network DEA (NDEA) models without considering the role of intermediate products. In this study, a sustainable supply chain with two-stage structure was considered. Then, the congestion status according to the role of intermediate products was found out for the first time. Towards this aim, different scenarios which congestion can occur in intermediate products were identified. Then, in each scenario, the dominant cone definition was developed in network structure and NDEA models were proposed. Finally 20 Iranian sustainable supply chains of Resin manufacturing companies have been used to demonstrate applicability of the proposed models.

Keywords Sustainable Supply Chain Management (SSCM) · Network Data Envelopment Analysis (NDEA) · Congestion · Intermediate products role

1 Introduction

The current business environment has led organizations and institutions to seek cost savings and customer satisfactions in order to develop, leading them to Supply Chain Management (SCM). To overcome some problems, organizations have come up with a new management approach called the Sustainable Supply Chain Management (SSCM), which removes social, economic and environmental concerns and also has a high performance in the Supply Chain (SC). The benefits of a SSCM include customer satisfaction, quality, innovation and cost control, all of which are rooted in sustainability.

One of the most efficient methods for assessing the performance and the relative efficiency of the Sustainable Supply Chain (SSC) is Data Envelopment Analysis (DEA) [1].

DEA is a nonparametric method, to evaluate homogenous systems that consume multiple inputs to produce multiple outputs. In DEA method, any system is considered as a Decision Making Units (DMUs) and its relative efficiency is measured based on Linear Programming (LP). The two basic DEA models are CCR model, which was proposed by Charnes et al. [2], and BCC model, which was proposed by Banker et al. [3], rely on Constant Return to Scale(CRS) and Variable Return to Scale (VRS) assumption, respectively, to obtain relative efficiency of DMUs.

In conventional DEA models, the inner structure of a DMU is neglected and considered as a single process with

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primary inputs and final outputs. Awareness of internal structure can provide a broader view of SSC performance. In innovative DEA method, moreover the external inputs and final outputs, intermediate products have also been investigated in total performance. In general, three types of structures are considered for network structures which are called series structures, parallel structures and mixture of series and parallel structures. Recently, a lot of attention has been paid to measuring the efficiency of network systems.

Assessing the performance of a system in network structure has been discussed in many of studies in the field of DEA and different approaches have been developed.

DEA approach to assess the efficiency of systems (DMUs) by considering the internal structures of systems is called Network DEA (NDEA) [4]. Some of them are in the two envelopment and multiplier forms, for examples see, Seiford and Zhu [5], Zhu [6], Lewis and Sexton [7], Tsolas [8], Adler et al. [9]. A number of studies illustrate the multiplicative decomposition approach; for instance see; Kao and Hwang [10], Kao [11], Chen et al. [12]. Some approaches are based on Nash bargaining game model, for examples; Liang et al. [13], Zha and Liang [14], Du et al.[15] and Zhou et al. [16]. In this vein, some studies recommended additive approach, for instance, Chen and Zhu [17], Yu and Lin [18], Chiou et al. [19], Tone and Tsutsui [20] and Liu and Lu [21]. Identifying the role of intermediate products in the NDEA method is a significant and difficult issue [22].

In previous studies, the role of intermediate products has not been clarified accurately. According to the concept of intermediate products role (links control), six scenarios among different stages of supply chains have been introduced by Hassanzadeh and Mostafaee [23]. By their classification, intermediate products value control through (*i*) the prior stage, (*ii*) the latter stage, (*iii*) both first and second stages which have no cooperation (*iv*) both first and second stages which have cooperation, (*v*) no stages which have non-cooperative approach and (*vi*) no stages which have cooperative approach.

Existence of congestion in inputs, intermediate products or both is one of the factors that decrement efficiency of DMUs. Congestion is an event in the production processes in which the increment in one or more inputs does not increase outputs. Due to the definition, congestion can be considered as shortage in outputs. Eliminating the congestion leads to improved performance and efficiency of DMUs. The economic concept of congestion has been extensively studied within the context of DEA method. Many studies have been carried out to evaluate congestion in content DEA.

The presence of congestion in an SSC reduces its efficiency, so its determination is of great significance. So, eliminating the congestion leads to improved performance and efficiency which leads to the importance of congestion issue in SSC.

The study of congestion was started by Fare and Svensson in 1980 [24]. Afterwards, their study was developed by Fare and Grosskopf [25] and they presented a model based on content of DEA. Next, Fare et al. [26] suggested two DEA models in input-oriented and output-oriented (Called FGL approach). Their study only identified congested DMUs but could not compute its value. The detecting congestion issue was seriously investigated by the efforts of Cooper et al. They played a major role in determining DEA-based congestion: see for instance, [27–29]. Jahanshahloo et al., Khodabakhshi and Noura et al. each proposed various methods for detecting congestion within the framework of DEA: see [30–32]. Wei and Yan [33] investigated the necessary and sufficient conditions for occurrence of congestion according to the type of Returns To Scale (RTS) due to DEA efficiency. Also, Tone and Sahoo [34] defined a new Production Possibility Set (PPS) with the assumption of convexity and strong output disposability and omitted input possibility postulate which is named T_{convex}. Then, by proposing an algorithm, congested DMUs were identified and new notions of “strong congestion” and “weak congestion” were suggested by them. Some previous studies deal with problems while determining congested inefficient units. According to their definition, the congested inefficient unit is defined at its efficient projection which is determined via the congestion-based DEA model. If multiple projections occur, one of them is chosen arbitrary, whereas choosing any of the efficient projections lead to changes in their congestion status. To overcome this problem, further studies have been done on this crucial issue. For example, Sueyoshi and Sekitiani [35] proposed “wide congestion” definition which covering both strong and weak congestion and then suggested an approach which produce a unique optimal solution and a unique projection for inefficient units. Afterwards, two DEA models were proposed which uniquely determined the status of wide congestion. Next, Mehdiloozad et al. [36] proposed an LP model for identifying the Max projection of units (as reference unit) and demonstrated that the congestion status of DMUs is similar to the status congestion of their reference unit. Also, Shadab et al. [37] suggested a novel algorithm through making connection between S-shape form of production function and the geometric properties of the anchor points to identify weakly and strongly congested DMUs and congestion amount without determining the efficiency value and inefficient DMUs projection on the efficiency frontier.

None of the existing studies discussed congestion in a network system; until Khoveyni et al. [38] considered a system with two-stage structure and specified the effect of intermediate products variations on final outputs of second stage and suggested an algorithm to find out the congestion of extreme efficient systems (a system which is efficient in both stages) via solving NDEA models. In addition, in terms of theory of economics of scale, Shabanpour et al. [39] divided the congested inputs into two positive inputs (lead to increases efficiency) and negative inputs (lead to decrease efficiency). Then, they proposed a new NDEA model to evaluate and rank sustainable suppliers performance. It is noticeable that no studies have mentioned the role of intermediate products, and only considered one scenario for determining congestion in intermediate products.

In this study, we consider a two-stage supply chain as shown in Fig. 1. Then, congestion of intermediate products (links) in terms of links role is detected for the first time. Then "strongly" or "weakly" congested supply chains are introduced. To achieve this goal, among six scenarios which is introduced according to links role [23], four scenarios which congestion occurs in intermediate products are identified. Then for each scenario the dominant cone definition, which is introduced by Mehdiloozad et al. [36] in conventional DEA, used in network structure. Next, NDEA models are proposed in each mentioned scenarios to find out strongly or weakly congested supply chains. The rest of this paper organized as follows:

Section 2 provides some essential preliminaries and definitions. In Sect. 3, four scenarios where congestion occurs in intermediate products, according to the role of links, are identified and by using dominant cone in network structure and proposing NDEA models, the status of congestion is obtained. In Sect. 4, proposed model have been implemented on 20 Iranian Resin companies. Eventually, Sect. 5 includes some conclusions.

2 Preliminaries

2.1 DEA models

Suppose that we have a set of peer observed DMUs, (DMU_j, $j = 1,\ldots,n$) such that each DMU_j produces multiple nonnegative outputs y_{rj} $(r = 1,\ldots,s)$ utilizing multiple nonnegative inputs x_{ij} $(i = 1,\ldots,m)$. It is supposed that $\mathbf{x}_j = (x_{1j},\ldots,x_{mj})^T \neq \mathbf{0}_m$ and $\mathbf{y}_j = (y_{1j},\ldots,y_{sj})^T \neq \mathbf{0}_s$ for each j. Moreover, assume that $D_j = (\mathbf{x}_j, \mathbf{y}_j)^T$ expresses input and output vectors of each DMU_j, $j \in J = \{1,\ldots,n\}$. We assume that there is not any duplicate DMU.

The PPS is defined as the set of all possible input–output vectors as follow:

$$\mathrm{PPS} = \{(\mathbf{x},\mathbf{y}) : \mathbf{y} \text{ can be produced by } \mathbf{x}\}$$

After providing the CCR model [2], Banker et al. [3] introduced the following PPS under VRS assumption as follow:

$$T_v = \left\{(\mathbf{x},\mathbf{y})\,\middle|\,\mathbf{x}\lambda \le \mathbf{x}, \mathbf{y}\lambda \ge \mathbf{y}, \mathbf{1}^T\lambda = 1, \lambda \ge \mathbf{0}\right\}$$

T_v satisfies the postulates of inclusion of observations, convexity, input possibility postulate, output possibility postulate and minimum extrapolation. T_v is an unbounded convex polyhedral set.

According to the input possibility postulate, the following relationship is hold:

$$(\mathbf{x},\mathbf{y}) \in T_v, \bar{\mathbf{x}} \ge \mathbf{x} \Leftrightarrow (\bar{\mathbf{x}},\mathbf{y}) \in T_v$$

In the absence of input disposability postulates, Tone and Sahoo [35] introduced a new production technology with VRS assumption as following and named it T_{convex}.

$$T_{\text{convex}} = \left\{(\mathbf{x},\mathbf{y}) | \mathbf{x} = \mathbf{x}\lambda, \mathbf{y} \le \mathbf{y}\lambda, 1^T\lambda = 1, \lambda \ge 0\right\}$$

T_{convex} is a bounded convex set, which is satisfies the postulates of inclusion observations, convexity, output possibility postulate and minimum extrapolation.

Additive model with respect to T_v was proposed by Charnes et al. [40] to evaluate the efficiency of DMU_o and defined as follow:

$$\begin{aligned}
& Max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
& s.t \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \qquad \forall i \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad \forall r \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad s_i^-, s_r^+, \lambda_j \ge 0 \quad \forall i,r,j
\end{aligned} \tag{1}$$

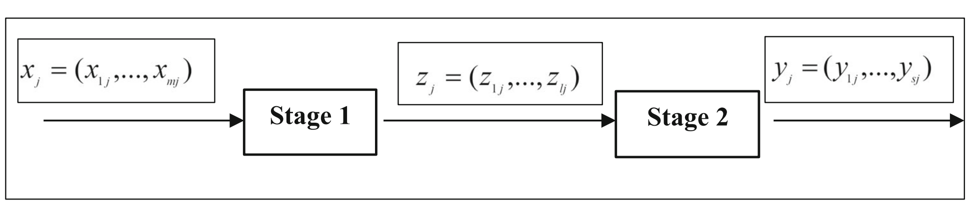

Fig. 1 The visual description of DMU_j

Definition 2.1 (Additive-efficiency) Let $\text{DMU}_o = (\mathbf{x}_o, \mathbf{y}_o)$ is a unit under evaluation. DMU_o is an additive efficient unit if and only if all output slack vectors $(\mathbf{s}^{+*})$ and all input slack vectors $(\mathbf{s}^{-*})$ are zero. $(\mathbf{s}^{+*} = \mathbf{0}_s$ and $\mathbf{s}^{-*} = \mathbf{0}_m)$. Else, it is called inefficient [40].

Through this paper "*" indicates optimality.

Definition 2.2 (Efficiency) Consider the following output-oriented model to evaluate efficiency of $\text{DMU}_o = (\mathbf{x}_o, \mathbf{y}_o)$ under set of T_{convex} and ε as a non-Archimedean number. DMU_o is called efficient if and only if $\beta^* = 1$ and $\mathbf{s}^{+*} = \mathbf{0}_s$ in model (2). Otherwise, DMU_o is called inefficient [34].

$$
\begin{aligned}
& Max\ \beta + \varepsilon\Big(\sum_{r=1}^{s} s_r^+\Big) \\
& s.t \\
& \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io} \qquad \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{ir} - s_r^+ = \beta\, y_{ro} \quad \forall r \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j, s_r^+ \geq 0 \qquad\qquad \forall j, r \\
& \beta : URS
\end{aligned}
\tag{2}
$$

Definition 2.3 (Projection Point) The projection point associated with an inefficient DMU_o in Model (2) is obtained as follow [34]:

$$
\begin{pmatrix} \mathbf{x}_o^{\text{congestion}} \\ y_o^{\text{congestion}} \end{pmatrix} = \begin{pmatrix} \mathbf{X}\lambda^* \\ \mathbf{Y}\lambda^* \end{pmatrix} = \begin{pmatrix} \mathbf{x}_o \\ \beta^* \mathbf{y}_o + \mathbf{s}^* \end{pmatrix}
$$

Note that the above projection is an efficient DMU of technology T_{convex}.

2.2 Congestion

Generally, a DMU is confronted with congestion if an increase in one or some inputs decreases one or some outputs with no worsening the rest of inputs or outputs.

Many methods have been used to investigate congestion by presenting the conventional DEA models. One of these studies was done by Tone and Sahoo [34]. They proposed an algorithm to detect congested DMUs and proposed the concept of "strong congestion" and "weak congestion" as follows:

Definition 2.4 (Strong congestion) DMU_o has strong congestion if it is efficient with respect to T_{convex} and there exists an activity $(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \in T_{convex}$ such that $\bar{\mathbf{x}} = \alpha\, \mathbf{x}_o (0 < \alpha < 1)$ and $\bar{\mathbf{y}} \geq \beta\, \mathbf{y}_o (\beta > 1)$. In other words, strong congestion occurs in DMU_o if proportional decrement in all inputs of DMU_o results an increment in all outputs.

Definition 2.5 (Weak congestion) DMU_o has weak congestion if there exists an activity $(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \in T_{convex}$ that consumes fewer resources in one or more inputs to obtain more products in one or more outputs, it means; $\bar{\mathbf{x}} \leq \mathbf{x}_o, \bar{\mathbf{y}} \geq \mathbf{y}_o$ and $\bar{\mathbf{y}} \neq \mathbf{y}_o$.

When multiple projections occur, some studies deal with problems for identifying congested inefficient DMUs from theoretical and applicable point of view. Some studies have been done to solve this problem. For instance, Mehdiloozad et al. [36] determined a dominant cone for DMU_o and stated theorem for identifying weak and strong congestion as follows:

Considering the dominant cone $C = \{(-\boldsymbol{\alpha}, \boldsymbol{\beta}) | (\boldsymbol{\alpha}, \boldsymbol{\beta}) \geq \mathbf{0}_{m+s}\}$ and defined D_o as follow:

$$D_o = \{(\mathbf{x}_o, \mathbf{y}_o) + (-\boldsymbol{\alpha}, \boldsymbol{\beta}) | (-\boldsymbol{\alpha}, \boldsymbol{\beta}) \in C\} \cap T_{\text{convex}}$$

Obviously, D_o is the set of all DMUs in T_{convex} that dominate DMU_o. Also, they defined:

$$
\begin{aligned}
S_o = \{ & (\boldsymbol{\alpha}, \boldsymbol{\beta}) |\ \mathbf{X}\lambda = \mathbf{x}_o - \boldsymbol{\alpha}, \mathbf{Y}\lambda \geq \mathbf{y}_o + \boldsymbol{\beta}, \\
& \mathbf{1}_n^T \lambda = 1,\ \lambda \geq \mathbf{0}_n,\ \boldsymbol{\alpha} \geq \mathbf{0}_m,\ \boldsymbol{\beta} \geq \mathbf{0}_s \}
\end{aligned}
$$

Also, they expressed the following two properties as a theorem.

Theorem 2.1

(i) DMU_o is weakly congested if and only if there exists some $(\boldsymbol{\alpha}, \boldsymbol{\beta}) \in S_o$ such that $\boldsymbol{\beta} \neq \mathbf{0}_s$
(ii) DMU_o is strongly congested if and only if there exists some $(\boldsymbol{\alpha}, \boldsymbol{\beta}) \in S_o$ such that $(\boldsymbol{\alpha}, \boldsymbol{\beta}) > \mathbf{0}_{m+s}$.

In previous studies, DMUs were considered as a single process (black box) which only used the primary inputs for producing the total outputs. Such an attitude ignored existence of congestion in intermediate products and measured congestion in primary inputs solely. Until, Khoveyni et al. [38] and Shabanpour et al. [39] considered DMUs with network structure and determined congested DMUs but the role of intermediate products in a network system was neglected and congestion was determined regardless of the role of intermediate products in studies. They introduced the concept of congestion in NDEA but the role of intermediate products was not mentioned in their definition and only one scenario was considered to obtain the congestion of intermediate products.

3 Identifying congestion in network DEA (NDEA)

In this study we consider a Supply Chain (SC) with two-stage structure with external inputs for first stage, final outputs for second stage and intermediate products(links) through stage one and stage two (see Fig. 1). A SC may confront with congestion in external inputs or intermediate products or both. Among six scenarios in NDEA, which are related to notion of intermediate products [23], congestion occurs in four scenarios which are called;

(i) intermediate products value control via the second stage, (ii) intermediate products value control via both first and second stages (Non-cooperative approach), (iii) intermediate products value control via both first and second stages (cooperative approach) and (iv) intermediate products value control via neither of two stages (cooperative approach).

In the two remaining scenarios congestion cannot occur. The scenario which is called intermediate products value control through the prior stage [23], the intermediate products play the role of outputs for the second stage. In this scenario, the second stage has no decision on receiving the amount of intermediate products and only the first stage decides on the products to be sent, and therefore congestion does not occur. Also, in the scenario called intermediate products value control through no stages which have non-cooperative approach [23], the intermediate products sent from first stage are equal to the amount of received by second stage.

In the following, the intermediate product congestion is explored in mentioned scenarios. To this aim, dominant cone in network structure is developed and NDEA models are proposed for the scenarios which congestion occurs in intermediate products. Finally, the status of congestion is found out.

Suppose there are a set of n homogenous DMUs, DMU_j; $j \in J = \{1, \ldots, n\}$, whose activity consists of k-stages $(k = 1, 2)$. The first stage consumes input vector $\mathbf{x}_j = (x_{1j}, \ldots, x_{mj})^T \neq \mathbf{0}_m$ and second stage produce final output vector $\mathbf{y}_j = (y_{1j}, \ldots, y_{sj})^T \neq \mathbf{0}_s$. In addition, the number of intermediate products from stage 1 to stage 2 is denoted by $\mathbf{z_j} = (z_{1j}, \ldots, z_{Lj})^T \neq \mathbf{0}_L$ which is depicted in Fig. 1. Moreover, assume that there is not duplicate DMU. Four scenarios which congestion occurs in intermediate products are introduced and proposed two NDEA models for each scenario to determine "strong" and "weak" congestion in a SC.

Scenario 1: Intermediate products value control via the second stage.

In this scenario, the optimal values of intermediate products are detecting by the second stage and the first stage has no power in controlling the intermediate products. Therefore, the intermediate products play the only role of the inputs under the full control of the second stage. So, congestion can happen in intermediate products of DMU_j;$j \in J = \{1, \ldots, n\}$. The technology production in this scenario can be expressed as follow:

$$PPS_1 = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geq y_{ro}, \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} \leq z_{lo}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geq 0, \forall i, r, l, j, k \right\}$$

where the superscript *1* stands for scenario 1.

In the absence of input disposability postulates, the PPS_1 is written as follow and named it $T_{1convex}$:

$$T_{1convex} = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geq y_{ro}, \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geq 0, \forall i, r, l, j, k \right\}$$

Definition 3.1 (Strong efficient) A $\text{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in T_{1convex}$ is a strong efficient DMU if the optimal objective value of Model (3) is zero, i.e., $\eta_1^* = 0$. The set of strong efficient DMUs are denoted by $SE_{1\ convex}$.

$$\begin{aligned}
&\eta_1^* = Max \sum_{r=1}^{s} \gamma_r \\
&s.t \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io} && \forall i \\
&\sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r && \forall r \\
&\sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} && \forall l \\
&\sum_{j=1}^{n} \lambda_j^k = 1 && \forall k \\
&\lambda_j^k, \gamma_r \geq 0 && \forall j, k, r
\end{aligned} \tag{3}$$

Definition 3.2: Let $\text{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{1convex}$. If there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}}) \in T_{1convex}$ and $(\boldsymbol{\beta}, \boldsymbol{\gamma}) \geq \mathbf{0}_{L+s}$ such that $\hat{\mathbf{x}} \leq \mathbf{x}_o$, $\hat{\mathbf{z}} = \mathbf{z}_o - \boldsymbol{\beta}$ and $\hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$ then the following statements are obtained:

(i) $\text{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o)$ evidences strong congestion in intermediate products if $(\boldsymbol{\beta}, \boldsymbol{\gamma}) > \mathbf{0}_{L+s}$.

(ii) $\text{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o)$ evidences weak congestion in intermediate products if $(\boldsymbol{\beta}, \boldsymbol{\gamma}) \geq \mathbf{0}_{L+s}$ and $\boldsymbol{\gamma} \neq \mathbf{0}_s$.

Now, we extent the definition of dominant cone presented by Mehdiloozad et al. [38] (DMU_o was considered as black box), to determine the congestion status of strongly efficient DMU_o respect to $T_{1convex}$. To this aim, consider the dominant cone C as bellow:

$$C = \{(-\boldsymbol{\omega}, -\boldsymbol{\beta}, \boldsymbol{\gamma}) \,|\, (\boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \geq \mathbf{0}_{m+l+s}\}$$

Also, D_o is defined as follow:

$$D_o = \{(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) + (-\boldsymbol{\omega}, -\boldsymbol{\beta}, \boldsymbol{\gamma}) | (-\boldsymbol{\omega}, -\boldsymbol{\beta}, \boldsymbol{\gamma}) \in C\} \cap T_{1\mathrm{convex}}$$

D_o is the intersection of dominant cone and $T_{1convex}$.

We define

$$S_o = \{(\boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\gamma}) | (\mathbf{x}_o - \boldsymbol{\omega}, \mathbf{z}_o - \boldsymbol{\beta}, \mathbf{y}_o + \boldsymbol{\gamma}) \in D_o, (\boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \geq \mathbf{0}_{m+l+s}\}$$

. Obviously, the convex set $S_o \neq \varnothing$ is a subset of nonnegative orthant and can be expressed as follow:

$$\begin{aligned} S_o = \big\{ & (\boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \big| \lambda^1 \mathbf{x} = \mathbf{x}_o - \boldsymbol{\omega}, \lambda^2 \mathbf{y} \geq \mathbf{y}_o + \boldsymbol{\gamma}, \\ & \lambda^1 \mathbf{z} = \lambda^2 \mathbf{z} = \mathbf{z}_o - \boldsymbol{\beta}, \mathbf{1}\lambda^k = 1, \lambda^k \geq \mathbf{0}_n, \\ & \boldsymbol{\gamma} \geq \mathbf{0}_r, \boldsymbol{\beta} \geq \mathbf{0}_l, \boldsymbol{\omega} \geq \mathbf{0}_m, \forall k\} \end{aligned}$$

According to the role of intermediate products in this scenario, we express the following definition to determine the congestion status of strong efficient units of $T_{1convex}$.

Definition 3.3 The following statements are true:

(i) DMU_o is weakly congested in intermediate products if there exists $(\boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ in S_o such that $\boldsymbol{\gamma} \neq \mathbf{0}_s$.

(ii) DMU_o is strongly congested in intermediate products if there exists $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ in S_o such that $(\boldsymbol{\beta}, \boldsymbol{\gamma}) > \mathbf{0}_{l+s}$.

The next model can be applied to identify the congestion status of intermediate products of two-stage DMUs.

$$\begin{aligned} & \upsilon_1^* = Max \ \underset{\forall l,r}{Min}\{\beta_l, \gamma_r\} \\ & s.t \\ & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_{io} && \forall i \\ & \sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r && \forall r \\ & \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^1 z_{lj} = z_{lo} - \beta_l && \forall l \\ & \sum_{j=1}^{n} \lambda_j^k = 1 && \forall k \\ & \lambda_j^k, \gamma_r, \beta_l \geq 0 && \forall j, k, r, l \end{aligned} \tag{4}$$

Model (4) is a bi-level problem that the inner problem has the objective function in minimizing form but the outer problem is in the maximizing form. Since, the inner and outer problems have the opposite sides for optimization; the model with some variable substitutions can be easily transferred into a single objective function problem.

Definition 3.4 Let $\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{1convex}$. If the optimal objective value of Model (4) is positive, i.e., $\upsilon_1^* > 0$, then DMU_o is said strongly congested.

To identify the weakly congested DMU_o in $SE_{1convex}$ the following model is suggested:

$$\begin{aligned} & \sigma_1^* = \underset{\forall r}{Max} \ \gamma_r \\ & s.t \\ & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_{io} && \forall i \\ & \sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r && \forall r \\ & \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} - \beta_l && \forall l \\ & \sum_{j=1}^{n} \lambda_j^k = 1 && \forall k \\ & \lambda_j^k, \gamma_r, \beta_l \geq 0 && \forall j, k, r, l \end{aligned} \tag{5}$$

Definition 3.5 Let $\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{1convex}$. DMU_o is weakly congested if the optimal objective value of Model (5) is positive, i.e., $\sigma_1^* > 0$.

Now, the next theorems show that Definition 3.2 is equivalent to Definitions 3.4 and 3.5.

Theorem 3.1 *$\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{1convex}$ evidences strong congestion in intermediate products according to definition* 3.2 *if and only if the optimal value of objective function of Model* (4) is positive.

Proof: Suppose that DMU_o has strong congestion in intermediate products. So, according to definition 3.2, there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}})$ and $(\boldsymbol{\beta}, \boldsymbol{\gamma}) > \mathbf{0}_{l+s}$ such that $\hat{\mathbf{x}} \leq \mathbf{x}_o$, $\hat{\mathbf{z}} = \mathbf{z}_o - \boldsymbol{\beta}$ and $\hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$. Consider $\underset{r,l}{Min}\{\beta_l, \gamma_r\} = \rho$. Obviously, $\rho > 0$ therefore $Max\,\rho$ is positive, i.e., the value of objective function of Model (4) is positive.

Assume that the optimal value of objective function of Model (4) is positive and $(\boldsymbol{\beta}, \boldsymbol{\gamma}, \lambda^1, \lambda^2)$ is optimal solution of Model (4) which is feasible solution for this model. Consider $\underset{r,l}{Min}\{\boldsymbol{\beta}_l, \gamma_r\} = \rho$ then $\rho \leq \boldsymbol{\gamma}$ and $\rho \leq \boldsymbol{\beta}$. According to assumption; $Max\,\rho > 0$, so $0 < \rho \leq \boldsymbol{\beta}$ and $0 < \rho \leq \boldsymbol{\gamma}$. According to the constraints of Model (4):

$$\begin{aligned}\hat{\mathbf{x}} &= \lambda^1\mathbf{x} \le \mathbf{x}_o\\ \hat{\mathbf{y}} &= \lambda^2\mathbf{y} = \mathbf{y}_o + \boldsymbol{\gamma}\\ \hat{\mathbf{z}} &= \lambda^1\mathbf{z} = \lambda^2\mathbf{z} = \mathbf{z}_o - \boldsymbol{\beta}\\ \mathbf{1}\lambda^k &= 1\ \forall k\end{aligned}$$

Therefore, there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}}) \in T_{1convex}$ such that $\hat{\mathbf{x}} \le \mathbf{x}_o$, $\hat{\mathbf{z}} < \mathbf{z}_o$ and $\hat{\mathbf{y}} > \mathbf{y}_o$. It indicates that DMU$_o$ has strong congestion in intermediate products.

Now we express theorem 3.2 for the evidence of weak congestion in intermediate products.

Theorem 3.2 *$DMU_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{1convex}$ evidences weak congestion in intermediate products according to definition* 3.2 *if and only if the optimal value of objective function of Model* (5) is positive.

Proof: The proof is similar to the proof of Theorem 3.1. □

Scenario 2: Intermediate products value control via both first and second stages (Non-cooperative approach).

The intermediate products are found out through both first and second stages simultaneously and independently. So, the intermediate products play the dual role of the output under control of the first and input of the second stage.

The PPS respect to the intermediate products role can be stated as bellow:

$$PPS_2 = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^n \lambda_j^1 x_{ij} \le x_{io}, \sum_{j=1}^n \lambda_j^2 y_{rj} \ge y_{ro}, \sum_{j=1}^n \lambda_j^1 z_{lj} \ge z_{lo}, \sum_{j=1}^n \lambda_j^2 z_{lj} \le z_{lo}, \sum_{j=1}^n \lambda_j^k = 1, \lambda_j^k \ge 0, \forall i, r, l, j, k \right\}$$

In the absence of input disposability postulates,PPS_2 is written as the following PPS and named it $T_{2convex}$:

$$T_{2convex} = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^n \lambda_j^1 x_{ij} = x_{io}, \sum_{j=1}^n \lambda_j^2 y_{rj} \ge y_{ro}, \sum_{j=1}^n \lambda_j^1 z_{lj} \ge z_{lo}, \sum_{j=1}^n \lambda_j^2 z_{lj} = z_{lo}, \sum_{j=1}^n \lambda_j^k = 1, \lambda_j^k \ge 0, \forall l, j, k \right\}$$

Definition 3.6 DMU$_o$ = $(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in T_{2convex}$ is said strong efficient DMU if the optimal objective value of Model (6) is zero, i.e., $\eta_2^* = 0$. The set of strong efficient DMUs belong to $T_{2convex}$ is named by $SE_{2convex}$.

$$\begin{aligned}\eta_2^* = Max(\sum_{r=1}^s \gamma_r &+ \sum_{l=1}^L \varphi_l)\\ &s.t\\ \sum_{j=1}^n \lambda_j^1 x_{ij} &= x_{io} && \forall i\\ \sum_{j=1}^n \lambda_j^2 y_{rj} &= y_{ro} + \gamma_r && \forall r\\ \sum_{j=1}^n \lambda_j^1 z_{lj} &= z_{lo} + \varphi_l && \forall l\\ \sum_{j=1}^n \lambda_j^2 z_{lj} &= z_{lo} && \forall l\\ \sum_{j=1}^n \lambda_j^k &= 1 && \forall k\\ \lambda_j^k, \gamma_r, \varphi_l &\ge 0 && \forall j, k, r, l\end{aligned} \tag{6}$$

Since intermediate products ($\mathbf{z}$) play the input and output role of stage 2 and stage 1, respectively, we consider $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2)$ in this scenario and propose the congestion definition in NDEA as bellow:

Definition 3.7 Let DMU$_o$ = $(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{2convex}$ such that $\mathbf{z}_o = (\mathbf{z}_{1o}, \mathbf{z}_{2o})$. If there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}}) \in T_{2convex}$ and $(\boldsymbol{\beta}^2, \boldsymbol{\gamma}) \ge \mathbf{0}_{l+s}$ that $\hat{\mathbf{x}} \le \mathbf{x}_o$,$\hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$,$\hat{z}_1 \ge z_{1o}$ and $\hat{\mathbf{z}}_2 = \mathbf{z}_{2o} - \boldsymbol{\beta}^2$ then the following definition can be obtained:

(i) DMU$_o$ is strongly congested in intermediate products if $(\boldsymbol{\beta}^2, \boldsymbol{\gamma}) > \mathbf{0}_{l+s}$.
(ii) DMU$_o$ is weakly congested in intermediate products if $\boldsymbol{\gamma} \ne \mathbf{0}_s$.

According to the properties of $T_{2convex}$, we consider the dominant cone as follow:

$$C = \left\{ (-\boldsymbol{\omega}, -\boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) \middle| (\boldsymbol{\omega}, \boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) \ge \mathbf{0}_{m+2l+s} \right\}$$

Also, we define $D_o = \{(\mathbf{x}_o, \mathbf{z}_{2o}, \mathbf{z}_{1o}, \mathbf{y}_o) + (-\boldsymbol{\omega}, -\boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) | (-\boldsymbol{\omega}, -\boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) \in C\} \cap T_{2convex}$. D$_o$ is the intersection of dominant cone and $T_{2convex}$. We define:

$$S_o = \left\{ (\boldsymbol{\omega}, \boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) \geqslant \mathbf{0}_{m+2l+s} | (\mathbf{x}_o - \boldsymbol{\omega}, \mathbf{z}_{2o} - \boldsymbol{\beta}^2, \mathbf{z}_{1o} + \boldsymbol{\beta}^1, \mathbf{y_o} + \boldsymbol{\gamma}) \in \mathbf{D_o} \right\}$$

Obviously; S_o is a non-empty convex subset of non-negative orthant that can be expressed equivalently as below:

$$S_o = \left\{ (\boldsymbol{\omega}, \boldsymbol{\beta}^2, \boldsymbol{\beta}^1, \boldsymbol{\gamma}) | \lambda^1\mathbf{x} = \mathbf{x}_o - \boldsymbol{\omega}, \lambda^2\mathbf{y} \geqslant \mathbf{y}_o + \gamma, \lambda^1\mathbf{z}_{1o} \geqslant \mathbf{z}_{1o} + \boldsymbol{\beta}^1, \lambda^1\mathbf{z}_{2o} = \mathbf{z}_{2o} - \boldsymbol{\beta}^2, \boldsymbol{\beta}^1 \geqslant \mathbf{0}_l, \boldsymbol{\beta}^1 \geqslant \mathbf{0}_l, \boldsymbol{\gamma} \geqslant \mathbf{0}_r, \boldsymbol{\omega} \geqslant \mathbf{0}_m \right\}$$

In this scenario, based on definition S_o accompany with intermediate products role, the congestion status of DMU$_o$ in $SE_{2\mathrm{convex}}$ can be obtained via the following definition:

Definition 3.8

(i) (i) DMU_o is weakly congested in intermediate products if $\gamma \in \mathbf{S_o}$ such that $\gamma \neq \mathbf{0_s}$.

(ii) DMU_o is strongly congested in intermediate products if $(\boldsymbol{\beta}^2, \boldsymbol{\gamma}) \in S_o$ such that $(\boldsymbol{\beta}^2, \boldsymbol{\gamma}) > \mathbf{0}_{l+s}$.

Model (7) is used to detect the congestion status of intermediate products of DMU_o in.

$$\begin{aligned}
&\upsilon_2^* = Max \ \underset{l,r}{Min} \left\{\gamma_r, \beta_l^2\right\} \\
&s.t \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io} && \forall i \\
&\sum_{j=1}^{n} \lambda_j^2 y_{ir} = y_{ro} + \gamma_r && \forall r \\
&\sum_{j=1}^{n} \lambda_j^1 z_{lj} \geqslant z_{lo} && \forall l \\
&\sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} - \beta_l^2 && \forall l \\
&\sum_{j=1}^{n} \lambda_j^k = 1 && \forall j, k \\
&\lambda_j^k, \gamma_r, \beta_l^2 \geqslant 0 && \forall j, k, r, l
\end{aligned} \tag{7}$$

Definition 3.9 Let $\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{2\text{convex}}$. DMU_o is strongly congested if $\upsilon_2^* > 0$ in Model (7).

Model (7) is a bi-level program that can be converted to single objective function problem easily.

Also, by proposing the following model; weakly congested DMU belongs to $SE_{2\text{convex}}$ can be detected:

$$\begin{aligned}
&\sigma_2^* = \underset{r}{\mathrm{Max}} \ \gamma_r \\
&s.t \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io} \quad \forall i \\
&\sum_{j=1}^{n} \lambda_j^2 y_{ir} = y_{ro} + \gamma_r \quad \forall r \\
&\sum_{j=1}^{n} \lambda_j^1 z_{lj} \geqslant z_{lo} \quad \forall l \\
&\sum_{j=1}^{n} \lambda_j^2 z_{lj} \leqslant z_{lo} \quad \forall l \\
&\sum_{j=1}^{n} \lambda_j^k = 1 \quad \forall j, k \\
&\lambda_j^k, \gamma_r \geqslant 0 \quad \forall j, k, r
\end{aligned} \tag{8}$$

Definition 3.10 Let $\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{2\text{convex}}$. DMU_o evidences weak congestion in intermediate products if σ_2^*.

The following theorem shows that Definition 3.7 is equivalent to Definitions 3.8 and 3.9.

Theorem 3.3 *$\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{2\text{convex}}$ is strongly congested in intermediate products according to Definition 3.7 if and only if the optimal value of objective function of Model (7) is positive.*

Proof: Assume that DMU_o is strongly congested according to Definition 3.7. So, there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}})$ and $(\boldsymbol{\beta}^2, \boldsymbol{\gamma}) > \mathbf{0}_{l+s}$ such that $\hat{\mathbf{x}} \leqslant \mathbf{x}_o$, $\hat{\mathbf{z}}_2 = \mathbf{z}_o - \boldsymbol{\beta}^2$, $\hat{\mathbf{z}}_1 \geqslant \mathbf{z}_o$ and $\hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$. Consider $Min \left\{\boldsymbol{\beta}^2, \boldsymbol{\gamma}\right\} = \rho$ such that $\rho > 0$. Therefore, Max Min $\left\{\boldsymbol{\beta}^2, \boldsymbol{\gamma}\right\}$ is positive, i.e., the value of objective functions of Model (7) is positive.

Suppose that the optimal value of objective function Model (7) is positive and $(\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \boldsymbol{\beta}^2, \boldsymbol{\gamma})$ is an optimal solution which is feasible solution for this model too. Consider $Min \left\{\boldsymbol{\beta}^2, \boldsymbol{\gamma}\right\} = \rho$ then $\rho \leqslant \boldsymbol{\beta}^2$ and $\rho \leqslant \boldsymbol{\gamma}$. Regarding to the assumption, $0 < \rho \leqslant \boldsymbol{\beta}$ and $0 < \rho \leqslant \boldsymbol{\gamma}$. From constraints of Model (7):

$$\begin{aligned}
&\hat{\mathbf{x}} = \lambda^1 \mathbf{x} \leqslant \mathbf{x}_o \\
&\hat{\mathbf{y}} = \lambda^2 \mathbf{y} = \mathbf{y}_o + \gamma > \mathbf{y}_o \\
&\hat{\mathbf{z}}_1 = \lambda^1 \mathbf{z}_{1o} \geqslant \mathbf{z}_{1o} \\
&\hat{\mathbf{z}}_2 = \lambda^1 \mathbf{z}_{2o} = \mathbf{z}_{2o} - \beta^2 < \mathbf{z}_{2o} \\
&\mathbf{1}\lambda^k = 1 \qquad \forall k
\end{aligned}$$

Therefore, there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}}) \in T_{2\text{convex}}$ such that $\hat{\mathbf{x}} \leqslant \mathbf{x}_o$, $\hat{\mathbf{y}} > \mathbf{y}_o$, $\hat{\mathbf{z}}_2 < \mathbf{z}_{2o}$, and $\hat{\mathbf{z}}_1 \geqslant \mathbf{z}_{1o}$. It deduced that DMU_o has strong congestion in intermediate products. Therefore, Definition 3.7 is satisfied. □

Theorem 3.4 *$\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{2\text{convex}}$ is weakly congested in intermediate products according to definition 3.7 if and only if the optimal value of objective function problem of Model* (8) is positive.

Proof: The proof of Theorem 3.4 is the same as theorem 3.3. □

Scenario 3: Intermediate products value control via both stages (cooperative approach).

The intermediate products have a dual role of the input and output simultaneously, which are controlled by the both stages (the first and second stages). Therefore, congestion occurs in the intermediate products of DMU_j. In this scenario, PPS is defined as below:

$$PPS_3 = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \left| \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geqslant y_{ro}, \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} + s_{lz}^{1+} - s_{lz}^{1-}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geqslant 0 \, \forall j,k,l \right. \right\}$$

In the absence of input disposability postulates, the PPS_3 can be written as follow and named it $T_{3\text{convex}}$:

$$T_{3convex} = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \left| \sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geqslant y_{ro}, \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} + s_{lz}^{1+} - s_{lz}^{1-}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geqslant 0 \forall j,k,l \right. \right\}$$

Definition 3.10 DMU$_o$=$(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in T_{3convex}$ is called a strong efficient DMU if the optimal value function of Model (9) is zero, i.e.,$\eta_3^* = 0$. The set of strong efficient DMUs respect to $T_{3\text{convex}}$ is named$SE_{3\text{convex}}$

$$\begin{aligned}
\eta_3^* = \max \, & (\sum_{r=1}^{s} \gamma_r + \sum_{l=1}^{L} s_l^{1-} + \sum_{l=1}^{L} s_l^{1+}) \\
s.t \quad & \\
& \sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io} \qquad \forall i \\
& \sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r \qquad \forall r \\
& \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} + s_l^{1+} - s_l^{1-} \qquad \forall l \\
& \lambda_j^k, \gamma_r, s_l^{1+}, s_l^{1-} \geqslant 0 \qquad \forall j,k,l
\end{aligned} \tag{9}$$

.

Definition 3.11 (Congestion in NDEA) Let DMU$_o$ $= (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in T_{3convex}$. If there exists an activity $(\mathbf{x}, \hat{\mathbf{z}}, \mathbf{y}) \in T_{3convex}$ and $(\boldsymbol{\gamma}, \mathbf{s}^{1-}) \geqslant \mathbf{0}_{s+L}$ such that $\hat{\mathbf{x}} \leqslant \mathbf{x}_o, \hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$, and $\hat{\mathbf{z}} = \mathbf{z}_o - \mathbf{s}^{1-}$, then DMU$_o$ is facing strong congestion in intermediate products if $(\boldsymbol{\gamma}, \mathbf{s}^{1-}) > \mathbf{0}_{s+L}$ and weak congestion if $\boldsymbol{\gamma} \neq \mathbf{0}_s$.

Now, to detect the congestion status of strong efficient DMUs respect to $T_{3\text{convex}}$, the dominant cone is defined like $C = \{(-\boldsymbol{\omega}, -\mathbf{s}^{1-}, \boldsymbol{\gamma}) | (\boldsymbol{\omega}, \mathbf{s}^{1-}, \boldsymbol{\gamma}) \geqslant \mathbf{0}_{m+L+s}\}$.

Also, we define $D_o = \{(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) + (-\boldsymbol{\omega}, -\mathbf{s}^{1-}, \boldsymbol{\gamma}) | (-\boldsymbol{\omega}, -\mathbf{s}^{1-}, \boldsymbol{\gamma}) \in C\} \cap T_{3convex}$ and $S_o = \{(-\boldsymbol{\omega}, -\mathbf{s}^{1-}, \boldsymbol{\gamma}) \geqslant \mathbf{0}_{m+L+s} | (\mathbf{x}_o - \boldsymbol{\omega}, \mathbf{z}_o - \mathbf{s}^{1-}, \mathbf{y}_o + \boldsymbol{\gamma}) \in D_o\}$.

Obviously, $S_o \neq \varnothing$ is nonnegative convex subset of orthant. Respect to above definition, S_o can be presented equivalently as follow:

$$S_o = \{(-\boldsymbol{\omega}, -\mathbf{s}^{1-}, \boldsymbol{\gamma}) | \lambda^1 \mathbf{x} = \mathbf{x}_o - \boldsymbol{\omega}, \lambda^2 \mathbf{y} \geqslant \mathbf{y}_o + \boldsymbol{\gamma}, \lambda^1 \mathbf{z} = \lambda^2 \mathbf{z} = \mathbf{z}_o, \boldsymbol{\gamma} \geqslant \mathbf{0}_r, \mathbf{s}^{1-} \geqslant \mathbf{0}_L, \boldsymbol{\omega} \geqslant \mathbf{0}_m\}$$

According to above definition, the following statements can be expressed to identify the status congestion of intermediate products of strong efficient DMUs.

Definition 3.12

(i) DMU$_o$ evidences weak congestion in the intermediate products if there exists $\boldsymbol{\gamma} \neq \mathbf{0}_s$.
(ii) DMU$_o$ evidences strong congestion in the intermediate products if there exists $(\boldsymbol{\gamma}, \mathbf{s}^{1-}) > \mathbf{0}_{s+L}$.

By Definition 3.13 and 3.14, strongly and weakly congested DMUs can be obtained through solving Model (10) and Model (11), respectively.

$$\begin{aligned}
\upsilon_3^* = Max \, & \underset{r,l}{Min} \{\gamma_r, s^{1-}\} \\
s.t \quad & \\
& \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io} \qquad \forall i \\
& \sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r \qquad \forall r \\
& \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} + s_{lz}^{1+} - s_{lz}^{1-} \qquad \forall l \\
& \sum_{j=1}^{n} \lambda_j^k = 1 \qquad \forall k \\
& \lambda_j^k, \gamma_r, s_{lz}^{1-}, s_{lz}^{1+} \geqslant 0 \qquad \forall k,j,r,l
\end{aligned} \tag{10}$$

$$\begin{aligned}
\sigma_3^* = \, & \underset{r}{Max} \, \gamma_r \\
s.t \quad & \\
& \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io} \qquad \forall i \\
& \sum_{j=}^{n} \lambda_j^2 y_{rj} = y_{ro} + \gamma_r \qquad \forall r \\
& \sum_{j=1}^{n} \lambda_j^1 z_{lj} = \sum_{j=1}^{n} \lambda_j^2 z_{lj} = z_{lo} + s_{lz}^{1+} - s_{lz}^{1-} \qquad \forall l \\
& \sum_{j=1}^{n} \lambda_j^k = 1 \qquad \forall k \\
& \lambda_j^k, \gamma_r, s_{lz}^{1-}, s_{lz}^{1+} \geqslant 0 \qquad \forall k,j,r,l
\end{aligned} \tag{11}$$

Definition 3.13 Let DMU$_o$ = $(\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{3convex}$. It is strongly congested in intermediate products if $\upsilon_3^* > 0$.

Definition 3.14 Let $\mathrm{DMU}_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{3convex}$. DMU_o is weakly congested if $\sigma_3^* > 0$. Similar to the previous scenarios, we can express the following theorem.

Theorem 3.5 *Let $DMU_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{3convex}$. According to definition* 3.11, *DMU_o evidences strong congestion in intermediate products if and only if the optimal value of objective function Model* (10) is positive.

Proof: Suppose that DMU_o evidence congestion according to Definition 3.11. So, there exists an activity $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathbf{y}})$ and $(\boldsymbol{\gamma}, \mathbf{s}^{1-}) > \mathbf{0}_{s+L}$ such that $\hat{\mathbf{x}} \leqslant \mathbf{x}_o,\ \hat{\mathbf{z}} = \mathbf{z}_o - \mathbf{s}^{1-}$ and $\hat{\mathbf{y}} = \mathbf{y}_o + \boldsymbol{\gamma}$. Consider $Min\left\{\mathbf{s}^{1-}, \gamma\right\} = \rho$ such that $\rho > 0$. It results Max Min $\left\{\mathbf{s}^{1-}, \gamma\right\}$ is positive, i.e., the value of objective function of Model (10) is positive.

Assume that the optimal value of objective function Model (10) is positive. Let $(\lambda^1, \lambda^2, \mathbf{s}^{1-}, \mathbf{s}^{1+}, \boldsymbol{\gamma})$ is optimal solution of Model (10) which is feasible solution for Model (10). Therefore, we have:

$$\hat{\mathbf{x}} = \lambda^1 \mathbf{x} \leqslant \mathbf{x}_o$$
$$\hat{\mathbf{y}} = \lambda^2 \mathbf{y} = \mathbf{y}_o + \gamma$$
$$\hat{\mathbf{z}} = \lambda^1 \mathbf{z} = \lambda^2 \mathbf{z} = \mathbf{z}_o + \mathbf{s}^{1+} - \mathbf{s}^{1-}$$
$$\mathbf{1}\lambda^k = 1 \qquad \forall k$$

Let $\mathrm{Min}\left\{\mathbf{s}^{1-}, \boldsymbol{\gamma}\right\} = \rho$ then $\rho \leqslant \mathbf{s}^{1-}$, $\rho \leqslant \gamma$ and $\mathrm{Max}\,\rho > 0$. It results that $0 < \rho \leqslant \beta$ and $0 < \rho \leqslant \gamma$. Therefore, there exists an activity $(\hat{\mathbf{x}}], \hat{\mathbf{z}}, \hat{\mathbf{y}}) \in T_{3\mathrm{convex}}$ such that $\hat{\mathbf{x}} \leqslant \mathbf{x}_o, \hat{\mathbf{y}} \leqslant \mathbf{y}_o$ and$\hat{\mathbf{z}} < \mathbf{z}_o$. It indicates that DMU_o has strong congestion in intermediate products.

Now we prove above theorem for the status of weak congestion. □

Theorem 3.6 *Let $DMU_o = (\mathbf{x}_o, \mathbf{z}_o, \mathbf{y}_o) \in SE_{3convex}$. According to definition* 3.11 *DMU_o evidences strong congestion in intermediate products if and only if the optimal value of objective function of Model* (10) is positive.

Proof: The proof is similar to the proof of theorem 3.5.□

Scenario 4: Intermediate products value control via neither of the two stages (cooperative approach).

In this scenario, there is no continuity in the amount of sending and receiving intermediate products among both stages. Therefore, congestion occurs in the intermediate products. The PPS of this scenario can be written as follows:

$$PPS_4 = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leqslant x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geqslant y_{ro}, \right.$$
$$\left. \sum_{j=1}^{n} \lambda_j^1 z_{lj} \geqslant \sum_{j=1}^{n} \lambda_j^2 z_{lj}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geqslant 0 \forall j, k, l \right\}$$

In the absence of input disposability postulates, the PPS_4 can be written as follow and named it$T_{4convex}$:

$$T_{4convex} = \left\{ (\mathbf{x}, \mathbf{z}, \mathbf{y}) \middle| \sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io}, \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geqslant y_{ro}, \right.$$
$$\left. \sum_{j=1}^{n} \lambda_j^1 z_{lj} \geqslant \sum_{j=1}^{n} \lambda_j^2 z_{lj}, \sum_{j=1}^{n} \lambda_j^k = 1, \lambda_j^k \geqslant 0 \forall j, k, l \right\}$$

PPS_4, can be converted to the PPS of the third scenario, which is named Intermediate products control via both stages (cooperative approach) [23]. Therefore, determining the strong efficient DMU and statuses of "weak" and "strong" congestion in intermediate products can be obtained similar to the third scenario.

4 An empirical study

In the Sustainable Supply Chain (SSC), members use environmental and social criteria to remain in the SC. One of these sustainable supply chains that play an important role in the Country's industry is Resin manufacturing companies. Resin is a synthetic or natural compound that is very sticky which has been known to human thousands of years ago. Advances in technology and understanding the conversion of Resin to polymer have led to the discovery of synthetic Resin. Most polymers are made with the synthetic Resin, because they are cheaper and easier to purification. The establishment of Resin manufactory associations in Iran dates back to the year 1971 when eight companies were formed and today their number has grown more than twenty companies.

Material flow management, information and capital with the integration of goals from all three dimensions of sustainability (economics, environment and social) based on the requirement of customers and stockholders are considered in Resin manufacturing companies. Today, million tons of chemical materials are emitted into the air. Resin manufacturing companies emit a considerable amount of chemicals. In this section, an empirical example about application of the proposed method into Iranian Resin companies is given. We are looking forward for identifying congested companies to help improve their performance. So, we consider 20 Iranian Resin companies (DMUs) as a two-stage sustainable supply chain depicted in Fig. 2. The first stage is supplier and the second stage is manufacturer. Inputs for the first stage are annual cost (economic sustainability), annual turnover of employees (social sustainability) and environmental costs (environmental sustainability). The annual cost of suppliers involves both the purchase price and all other costs associated with the acquisition during its life cycle such as investment cost, transportation cost, procurement and inventory. The annual turnover of employees is the loss of talent in the workforce overtime. Every supplier will experience some turnover. When a supplier has high employee turnover, it risks impacting the profitability of its organization, the culture and the productivity. Most supply chain managers spend a lot of their time looking for ways to reduce costs and employee turnover rate. So, the annual costs and annual turnover of employees are considered as inputs. According to a report from Organization for Economic Co-Operation and Development (OECD) published in 1997 the environmental costs are costs connected with the actual or potential deterioration of natural assets due to economic activities.

Such costs can be viewed from two different perspectives, namely as (a) costs caused, that costs associated with economic units actually or potentially causing environmental deterioration by their own activities or as (b) costs borne, that is, costs incurred by economic units independently of whether they have actually caused the environmental impacts. That is why the environmental costs have been considered as input. The intermediate products from the supplier to manufacturer are the number of trained personnel in the fields of work, safety and health, and the number of products from the supplier to the manufacturer. The outputs of stage 2 are the number of Resin products, the number of green products (environmental sustainability) and income (economic sustainability). Resin products are food packing, plastic trinket, toys and industrial products and so on. Green products have gained increasing attention because of their environmental friend lines in the green manufacturing process, low emissions in use, recyclability and so on. Table 1 reports used factors and notations. Inputs, outputs and links (intermediate products) value of each company is shown in Table 2. Manufacturer (stage 2) ordered supplier (stage 1) to train the specialists and prepare products needed to produce green products, reduce pollution and upgrade its technical knowledge. Therefore, intermediate products play the role of inputs for second stage. According to the different scenarios introduced based on the role of intermediate products, scenario1 which is called "Intermediate products value control via the second stage" should be applied in the link between Resin companies.

First, strong efficient and inefficient companies for determining the congested units are achieved through Model (3). The optimal value of objective function Model (3) is shown in Table 3 for all companies.

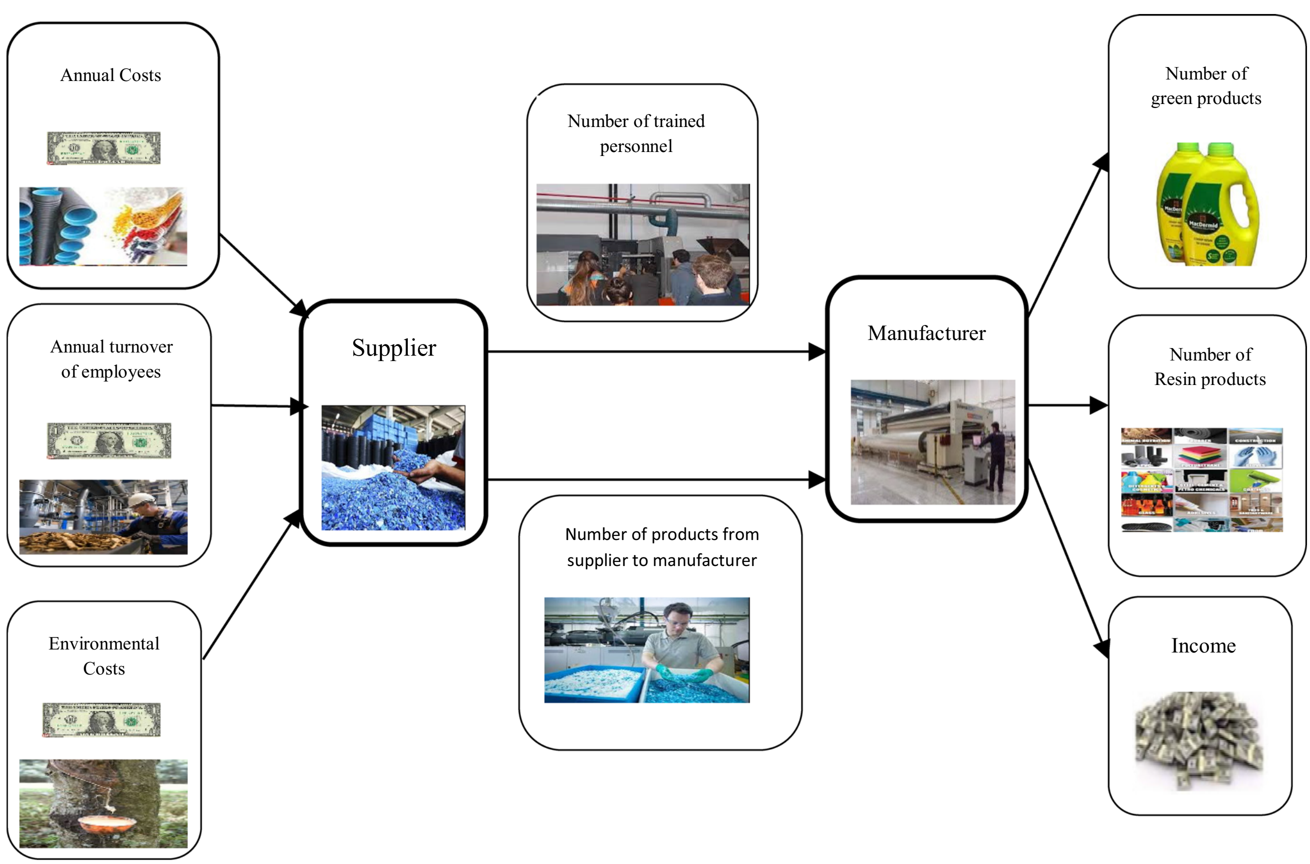

Fig. 2 The visual description of Resin sustainable supply chain

Table 1 Used factors and notations

Stages and links	Notations and Definitions
Supplier	x^1_{1j} Annual costs
	x^1_{2j} Annual turnover of employees
	x^1_{3j} Environmental costs
Manufacturer	y^2_{1j} The number of Resin products
	y^2_{2j} The number of green products
	y^2_{3j} Income
Intermediate inputs/outputs	z_{1j} The number of trained personnel in the fields of work, safety and health
	z_{2j} Number of products from the supplier to manufacturer

Table 2 Inputs, outputs and link value of 20 Resin sustainable supply chains

DMU_j	x^1_{1j}	x^1_{2j}	x^1_{3j}	z_{1j}	z_{2j}	y^1_{1j}	y^1_{2j}	y^1_{3j}
1	2981	0.2	117	8	145	158	5	4760
2	2683	0.5	101	6	135	191	5	3240
3	3753	0.15	84	11	213	217	9	4850
4	2961	0.1	121	9	152	295	13	4190
5	2789	0.35	116	5	139	337	7	4710
6	2951	0.6	135	14	91	263	8	4510
7	2856	0.2	174	8	153	338	13	4930
8	2654	0.45	132	11	175	194	11	4350
9	2921	0.2	110	7	97	172	4	4130
10	2723	0.7	98	10	64	387	3	3860
11	3975	0.5	164	11	142	419	6	5157
12	1855	0.65	135	7	118	476	9	4230
13	4186	0.3	139	13	164	117	10	5970
14	2774	0.2	112	7	143	218	6	3370
15	2654	0.45	176	9	115	176	5	4670
16	3852	0.5	161	12	178	197	12	5110
17	3758	0.1	95	8	126	423	9	4840
18	3984	0.3	153	15	114	259	12	5710
19	3656	0.55	76	11	89	110	9	4380
20	2814	0.6	241	7	135	73	6	20

According to Definition 3.1, companies whose optimal value of objective function Model (3) is equal to zero are strong efficient units else they are inefficient. Results show, except DMUs 1, 2, 8, 14, 15 and 20, which are inefficient, the rest of them are strong efficient. We solved Models (4) and (5) for strong efficient companies to find out congested DMUs. For each company, the optimal objective value and optimal solutions of Model (4) are given in Table 3. The optimal objective value of Model (4) for DMU6 and DMU17 is 0.21 and 0.77, respectively, and for other companies is zero. Therefore, the only company evidence strong congestion in their intermediate products are DMU_6 and DMU_{17}. Moreover, the optimal solution of model (4) for DMU_6 is $(\boldsymbol{\beta}, \boldsymbol{\gamma}) = (1.71, 0.21, 0.21, 0.21, 0.21) > \mathbf{0}_{2+3}$ which indicates there exists an observed activity belongs to the production technology with more outputs and less intermediate products compare with DMU_6. Therefore, according to definition 3.2, DMU_6 evidenced "strong congestion" in intermediate products. In addition, the optimal solution of model (4) for DMU_{17} is $(\boldsymbol{\beta}, \boldsymbol{\gamma}) = (2.38, 3.e+1, 1.e+2, 0.77, 0.77) > \mathbf{0}_{2+3}$. DMU_{17} is strongly congested too. To find weakly congested companies, Model (5) was solved for strong efficient units. The optimal objective value and the optimal solutions of Model (5) are given in Table 3. Results express that only DMU_3 has weak congestion in intermediate products among strong efficient companies. Its optimal objective function value and the optimal solutions are 7.1e + 2 and $\boldsymbol{\gamma} = (0, 0.48, 7.e+2) > \mathbf{0}_3$, respectively. It means, there exists an activity that produces more outputs by consuming less or equal intermediate products (inputs) compared with DMU_3.

Therefore, DMU_6, DMU_{17} and DMU_3 can improve their efficiency and performance by reducing their demands and orders to its supplier stage.

Briefly, out of 20 companies, only three of them are inefficient and the rest of them are strong efficient. Among strong efficient units, DMU_6 and DMU_{17} evidence strong congestion and DMU_3 is weakly congested. Eventually, in terms of obtained results, Resin sustainable supply chains are doing well.

Table 3 The results of Models (3), (4) and (5)

	Optimal objective value of model (4)	Optimal objective value of model (4)	Optimal solutions of model (4)					Optimal objective value of model (5)	Optimal solutions of model (5)		
	$\eta^{s_1^*}$	$\upsilon^{s_1^*}$	$\beta_2{}^*$	$\beta_1{}^*$	$\gamma_1{}^*$	$\gamma_1{}^*$	$\gamma_3{}^*$	$\sigma^{s_1^*}$	$\gamma_1{}^*$	$\gamma_2{}^*$	$\gamma_3{}^*$
1	5.0e + 2	–	–	–	–	–	–	–	–	–	–
2	1.7e + 3	–	–	–	–	–	–	–	–	–	–
3	0	0	0	0	0	0	0	7.1e + 2	0	0.48	7.e + 2
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0.21	1.71	0.21	0.21	0.21	2.e + 2	3.5e + 2	4.e + 1	0	3.e + 2
7	0	0	0	0	0	0	0	0	0	0	0
8	1.1e + 3	–	–	–	–	–	–	–	–	–	–
9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0
14	1.7e + 3	–	–	–	–	–	–	–	–	–	–
15	3.3e + 2	–	–	–	–	–	–	–	–	–	–
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0.77	2.38	3.e + 1	1.e + 2	0.77	0.77	4.2e + 2	7.e + 1	0	4.e + 2
18	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0
20	1.3e + 3	–	–	–	–	–	–	–	–	–	–

5 Conclusions

A Supply Chain (SC) is a network responsible for supplying materials, delivering materials to agents and delivering final products to customers. Sustainability in Supply Chain (SSC) as a new and influential sector attracts the attention of supply chain management professionals and environmental lovers. Therefore, SSC focuses on three aspects: social, economic and environmental aspects in a SC. One of the most suitable methods to assess the efficiency of sustainable supply chain is Data Envelopment Analysis (DEA).

In conventional DEA models, the internal structures of a supply chain are ignored and supply chain is considered as a single process (supply chain as a black box). DEA approach to evaluate the efficiency of supply chain with a network structure is called Network DEA (NDEA). Some NDEA approaches have been suggested to assess the performance of supply chains. Unfortunately, presence of congestion in sustainable supply chain reduces the efficiency and performance. Some studies have been done in this field to determine the congestion of intermediate products but none of them indicate the role of intermediate products (links) value in their methods and only consider one scenario. Now, in this research we considered a two-stage supply chain and identified four scenarios which congestion can occur in intermediate products: intermediate products value control via the second stage, intermediate products value control via both stages (Non-cooperative), intermediate products value control via both stages (cooperative) and intermediate products value control via no stages (Non-cooperative). In each mentioned scenarios we developed the dominant cone definition in network structure, and proposed NDEA models to find out the status of congestion in terms of the power of intermediate products, for the first time. Finally, 20 Iranian sustainable supply chains of Resin manufacturing companies have been used to demonstrate applicability of the proposed models. Each of companies is considered as a two-stage supplier chain. Stage 1 as a supplier and stage 2 as a manufacturer. In this applied example, second stage orders to stage 1 for training skilled personnel and providing essential products to improve its outcome and efficiency. Therefore, intermediate products role are considered as inputs of stage 2 that related to scenario 1 which was named link control by the second stage. We first determined the strong efficient companies by solving NDEA model and defined a dominant cone in network structure.

Then we proposed two bi-level models which were converted to LP models easily and the status of intermediate products congestion of strong efficient companies were obtained. The results show that out of 20 companies, companies 1,2,8,14 and 15 are inefficient and the rest of them are strong efficient. For strong efficient companies, we applied Models (3) and (4) presented in scenario 1 to determine the "strongly" or "weakly" congested units. The Results show that only companies 6 and 17 should reduce the amount of both intermediate products to achieve the maximum value in both outputs. The others by using professional experts and modern technology have acceptable performance and done well.

In this study, proposed models can persuade supply chain managers in planning sustainable policies. In addition, suggested models allow mangers, environmentalist and eco-fan society monitor the state of environment under the influence of industrial operations along with discovering congested sustainable supply chains according to the role of intermediate products.

To confirm the suggested models, a case study was discussed. The contributions of this study are as follows:

- To explore the intermediate product congestion of two-stage network structures, four scenarios are identified in terms of the role of intermediate products.
- By developing the dominant cone definition in network structure and proposing two novel NDEA models in each scenario, the congestion status of intermediate product is detected more accurately than previous studies.
- Improvement strategies are suggested for congested DMUs according to the role of intermediate product lead to improved performance and efficiency of DMUs.

Further researches can be done based on the results of this paper. Some of them can be mentioned as follows:

- The same models can be developed in the presence of fuzzy or interval data.
- Identifying the congested sustainable supply chain with multiple stages for future development.
- External inputs and outputs can be considered for each stage in a multi stages sustainable supply chain.

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