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ORIGINAL ARTICLE

Design of backtracking search heuristics for parameter estimation of power signals

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Abstract
This study presents a novel implementation of evolutionary heuristics through backtracking search optimization algorithm (BSA) for accurate, efficient and robust parameter estimation of power signal models. The mathematical formulation of fitness function is accomplished by exploiting the approximation theory in mean squared errors between actual and estimated responses, as well as, true and approximated decision variables. Variants of BSA-based meta-heuristics are applied for parameter estimation problem of power signals for identification of amplitude, frequency and phase parameters for different scenarios of noise variation. Analysis of performance evaluation for BSAs is conducted through exhaustive statistical observations in terms of mean weight deviation, root mean square error and Thiel inequality coefficient-based assessment metrics, as well as, ANOVA tests for statistical significance.

Keywords Parameter estimation · Power signals · Evolutionary algorithm · BSA

1 Introduction

In electrical power supply systems, frequency is a significant, as well as, fundamental parameter that specifies stability between power generation and power consumption [1]. Thus, the frequency component can be considered as a functional gauge to identify anomalous operating conditions [2]. In order to ensure regulated power supply to the clients and the utilities, it is mandatory requirement to scrutinize power quality of electrical grid through parameter estimation of power signal i.e., amplitudes, phases and frequencies [3, 4]. Research community has shown considerable interest in parameter estimation of power signal in power planning and distribution systems, for instance, Xu and Ding [5] presented the stochastic gradient (SG) and least squares (LS) procedures, Xu et al. [6] developed the hierarchical parameter estimation method, Li et al. [7] applied the reconstructing time sample techniques, Cao and Liu [8] gave the concept of the hierarchical identification approach, Phan et al. [3] described dedicated state space method, Chen et al.

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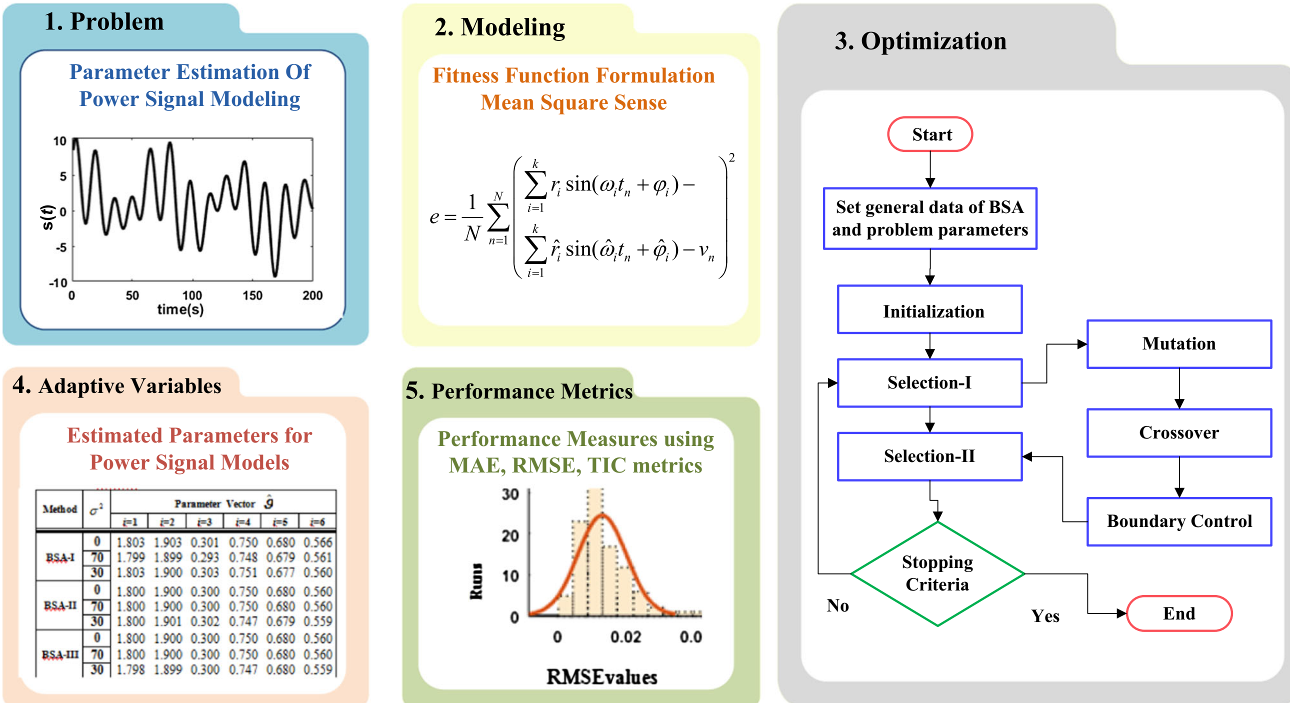


Fig. 1 Graphical flow diagram of parameter estimation of signal modeling problem

[9, 10] provided fast Fourier transform methods and Chaudhary et al. [11–13] provided the scheme of fractional adaptive filtering. The parameters of modeling power signals are identified by variety of the procedures introduced recently [14–20]. These are all deterministic procedures with their own perks, benefits and limitations while stochastic computing paradigm based on bioinspired heuristic looks promising to be explored exhaustively in the domain of power signal modeling.

The backtracking search optimization algorithm (BSA)-based evolutionary computing solvers have been explored to solving many problems arising in engineering and technological domains. A few latest applications of these schemes are in fluid dynamics [21], nanotechnology [22], wireless networks [23], system identification [24], power electronics [25], control [26], electric machines [27], economic load dispatch problems [28], nonlinear electric circuits [29], signal processing [30], biomedical [31], bioinformatics [32], finance [33]. All these contributions motivate authors to explore in meta-heuristic paradigm of BSA for accurate, reliable and robust system identification problems arising in power signal models. The aim of this research study is to exploit the well-known strength of BSA for parameter estimation of power signal systems. The prominent features of the proposed scheme are:

- A novel application of evolutionary computational heuristics through BSA is presented for effective, viable and reliable estimation of parameters in power signal modeling problems.
- Approximation theory is exploited for formulation of fitness function in terms of mean squared errors of actual and approximated parameters as well as responses for power signal models.
- Variants of BSA are implemented for parameter estimation of power signals with different degrees of freedom based on amplitude, frequency and phase for number of noise variances.
- Performance verification is ascertained through statistical results in terms of mean weight deviation, root mean square error and Thiel inequality coefficient-based evaluations metrics. Further, significance of the model is evaluated on the basis of ANOVA test.

Rest of the paper is organized as follows: Sect. 2 presents the necessary details of power signal modeling problem, designed methodology for parameter estimation is provided in Sect. 3, simulation of experimentation with interpretations is given in Sect. 4, while the conclusions and future recommendations are listed in Sect. 5.

2 System model: power signals

A distorted electric signal $s(t)$ from an AC power system can be expressed in the form of Fourier series [6, 34]:

Table 1 Pseudocode for BSA algorithm for optimization of periodic signal modeling

```
Part 1: BSA
       Input:
              Population Pop of k number of chromosomes Ch with dimension l

                        [ ϑ_1,1  ϑ_1,2  ···  ϑ_1,k ]
                        [ ϑ_2,1  ϑ_2,2  ···  ϑ_2,k ]
                  Pop = [   ⋮      ⋮     ⋮     ⋮   ] ,
                        [ ϑ_l,1  ϑ_l,2  ···  ϑ_l,k ]
       Output:
              The final Weights of BSA, i.e., Pop_BSA, with min e
       Begin BSA
              //Initialization of parameters
              Generations, i.e., Max_cycle = 200
              Description, i.e., Mix_rate = 1
              Scaling factor (F_s)= 0.5,
              population size (PS)= 60, i.e., n in P
              dimensionality (Dim), i.e., d in C
              Upper bound, ub = [30]_1*d
              Lower bound, lb = [-30] _1*d
              rnd is an operator for randomly generated real number between (0,1)
              rndi is an operator for randomly generated integers
              map is an operator for randomly generated binary matrix with PS and Dim.
              rndperturb is an operator for perturbation of parameters
              //Randomly generate population P
              for k= 1 to PS do
                     for j= 1 to Dim do
                            Pop_i,j = rnd x (ub_j + lb_j)+ lb_j
                            histPop_i,j= rnd x (ub_j + lb_j)+ lb_j
                     end for
                                   FitP_i= calculate e as in equation(14)
                                         globalminimum = min(FitP_i)
                                 globalminimizer= Pop_best weights of globalminimum
              end for
              //Selection-I
              for iteration from 1 to max_cycle do
                     if (rnd < rnd) then histPop := Pop end
                     histPop = rndperturb(histPop)
                     Trial-Population Creation
                            Mutant=Pop + F x rndi x (histPop-Pop)
                            T := Mutant
                            for i from 1 to PS do
                                   for j from 1 to Dim do
                                          if map_i,j then T_i,j = Pop_i,j end
                                   end for
                            end for
                            for i from 1 to PS do
                                   for j from 1 to Dim do
                                          if T_i,j < lb_j or T_i,j < ub_j then
                                          T_i,j = rnd*(lb_j - ub_j) + lb_j
                                          end
                                   end for
                            end for
              end for
              FitT = calculate e for all i=1,2,…,PS
              for i from 1 to PS do
                     if FitT_i < FitPop_i then
                            FitP_i =FitT_i
                            Pop_i = T_i
                     end
              end for
              FitP_BSA = min(FitPop_i) for all i =1,2, PS
              If FitP_BSA < globalminimum then
                     globalminimum = FitPop_BSA
                     globalminimizer= Pop_BSA weights of globalminimum
              end
       End BSA
       //Accumulation step
              Store P_BSA along with its e , time, generations executed for the current
       run of the BSA.
End Part 1
Part 2: Statistics
       For optimized parameter of periodic signal modeling problem repeat the procedure for
       multiple independent trials to obtain a dataset for effective statistical analysis on
       proposed scheme.
End Part 2
```

$$s(t) = \sum_{k=1}^{K} (c_k \cos k\omega t + d_k sin\, k\omega t) + v(t) \quad (1)$$

here, c and d are the Fourier coefficients, K is the harmonics index and ω is the fundamental frequency of the AC system.

The generic description of alternating current electrical signal can be derived from Eq. (1) and given below in terms of amplitudes, frequencies and phases as [8]:

$$s(t) = \sum_{i=1}^{k} r_i \sin(x_i(t)) + v(t), \quad (2)$$

for $x_i(t) = \omega_i t + \varphi_i$.

The amplitudes, the frequencies and phases (in radians) are represented as $\boldsymbol{r} = [r_1, r_2, \ldots, r_k]$, $\boldsymbol{\omega} = [\omega_1, \omega_2, \ldots, \omega_k]$, and $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \ldots, \varphi_k],$, respectively. For all non-zero magnitudes of φ, the whole signal waveform appears to be shifted in time scale by φ/ω seconds. A positive magnitude of φ indicates an advance while a negative value is for a delay in the electric signal. Eqs. (1)–(2) have been reported from many studies of electrical and electronic engineering [35–38] and reference therein.

In this experimental procedure, $t_n = nh$ is the sampling whose sampling period is h. The observed data are $\{t_k, s(t_k)\}$. Let $r_n = s(t_n)$ for inference, then, the discretized electrical signal on the basis of sinusoidal function is formalized as:

$$s_n = \sum_{k=1}^{K} (c_k \cos k\omega t_n + d_k sin\, k\omega t_n) + v_n \quad \text{for } n = 1, 2, \ldots N. \quad (3)$$

The expression for the power signal (2) in discrete form as:

$$s_n = \sum_{i=1}^{k} r_i \sin(\omega_i t_n + \varphi_i) + v_n \quad \text{for } n = 1, 2, \ldots N. \quad (4)$$

Amplitudes, frequencies and phases are components of the electrical signals described in Eqs. (1)–(4). In addition to individual any arbitrary combination of these parameters based on can be formulized for the parameter estimation problems. The unknown fundamental components of power signal to be estimated are given as:

$$\boldsymbol{\vartheta}_{cd} = \boldsymbol{cd} = [c_1, c_2, \ldots, c_k, d_1, d_2, \ldots, d_k]^{\mathrm{T}} \in \mathbb{R}^k \quad (5)$$

$$\boldsymbol{\vartheta}_{r} = \boldsymbol{r} = [r_1, r_2, \ldots, r_k]^{\mathrm{T}} \in \mathbb{R}^k \quad (6)$$

$$\boldsymbol{\vartheta}_{\omega} = \omega = [\omega_1, \omega_2, \ldots, \omega_k]^{\mathrm{T}} \in \mathbb{R}^k \quad (7)$$

$$\boldsymbol{\vartheta}_{\varphi} = \varphi = [\varphi_1, \varphi_2, \ldots, \varphi_k]^{\mathrm{T}} \in \mathbb{R}^k \quad (8)$$

$$\boldsymbol{\vartheta}_{r,\omega} = [\boldsymbol{r}, \boldsymbol{\omega}] = [r_1, r_2, \ldots, r_k,\ \omega_1, \omega_2, \ldots, \omega_k]^{\mathrm{T}} \in \mathbb{R}^{2k} \quad (9)$$

$$\boldsymbol{\vartheta}_{r,\varphi} = [\boldsymbol{r}, \boldsymbol{\varphi}] = [r_1, r_2, \ldots, r_k,\ \varphi_1, \varphi_2, \ldots, \varphi_k]^{\mathrm{T}} \in \mathbb{R}^{2k} \quad (10)$$

$$\boldsymbol{\theta}_{\omega,\varphi} = [\boldsymbol{\omega}, \boldsymbol{\varphi}] = [\omega_1, \omega_2, \ldots, \omega_k,\ \varphi_1, \varphi_2, \ldots, \varphi_k]^{\mathrm{T}} \in \mathbb{R}^{2k} \quad (11)$$

$$\boldsymbol{\vartheta}_{r,\omega,\varphi} = [\boldsymbol{r}, \boldsymbol{\omega}, \boldsymbol{\varphi}] = [r_1, r_2, \ldots, r_k,\ \omega_1, \omega_2, \ldots, \omega_k,\ \varphi_1, \varphi_2, \ldots, \varphi_k]^{\mathrm{T}} \in \mathbb{R}^{3k}. \quad (12)$$

The individual parameter estimation problems are given in Eqs. (6)–(8), while parametric Eqs. (9)–(12) indicate the integrated parameter estimation systems.

3 Proposed methodology

In this section, proposed methodology for parameter estimation of signal modeling problem is presented in two steps; fitness function formulation and learning procedure by exploitation of meta-heuristics of BSA. The framework of the proposed methodology is presented graphically in Fig. 1.

3.1 Construction of fitness function

In the first step, the fitness/merit function e is constructed in mean square sense as:

$$e = e_1 + e_2, \quad (13)$$

where e_1 is the difference actual s and estimated $\hat{s}$ response and is expressed as:

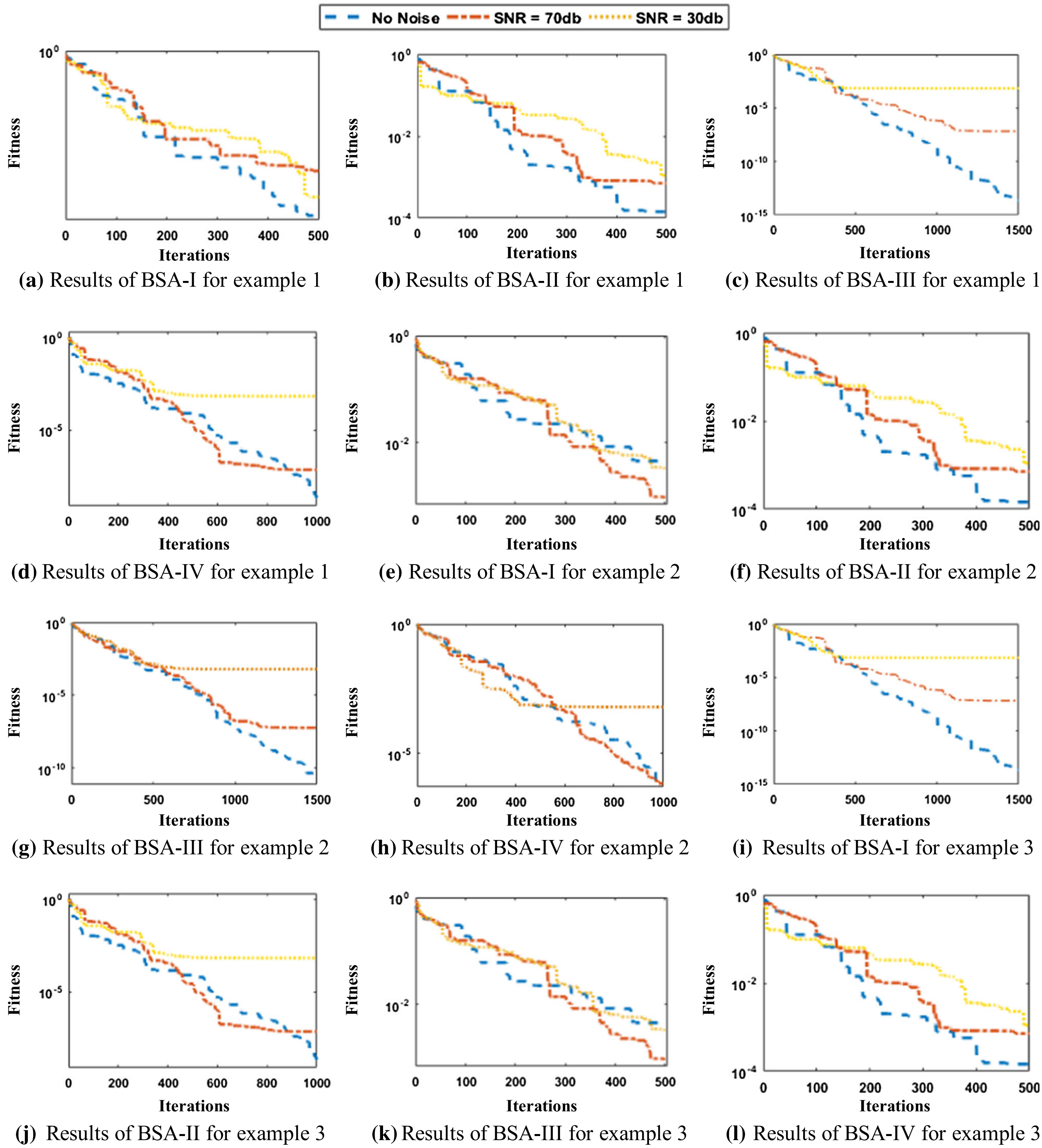
$$e_1 = \frac{1}{N} \sum_{n=1}^{N} (s_n - \hat{s}_n)^2, \quad (14)$$

while e_2 is an error term associated with parameter vector,

$$e_2 = \frac{1}{N} \sum_{n=1}^{N} \left(\vartheta_n - \hat{\vartheta}_n\right)^2. \quad (15)$$

Table 2 Description of six BSA variants

BSA variant	Example 1		Example 2		Example 3	
	PS	GENS	PS	GENS	PS	GENS
I	90	500	60	1000	20	500
II	90	1000	60	1500	20	700
III	90	1500	60	2000	20	1000
IV	20	1000	40	1000	10	500
V	40	1000	70	1000	30	500
VI	60	1000	80	1000	40	500



(a) Results of BSA-I for example 1 **(b)** Results of BSA-II for example 1 **(c)** Results of BSA-III for example 1

(d) Results of BSA-IV for example 1 **(e)** Results of BSA-I for example 2 **(f)** Results of BSA-II for example 2

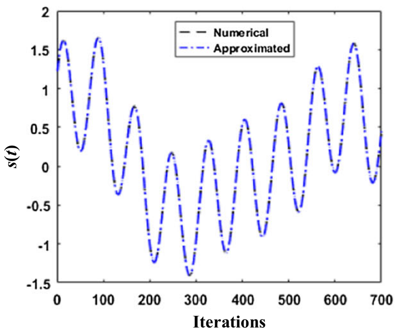
(g) Results of BSA-III for example 2 **(h)** Results of BSA-IV for example 2 **(i)** Results of BSA-I for example 3

(j) Results of BSA-II for example 3 **(k)** Results of BSA-III for example 3 **(l)** Results of BSA-IV for example 3

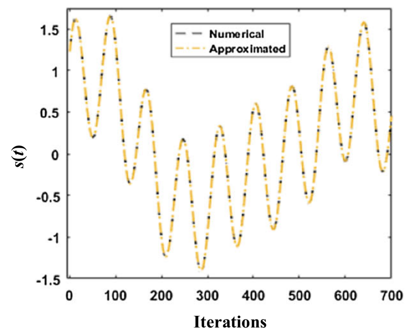
Fig. 2 Convergence graphs of BSA for parameter estimation of power signal modeling problem with noise variation scenarios

For example, the fitness function formulation for power signal model represented in Eq. (3) with decision variables as given in Eq. (5) in case of e_1 is given as:

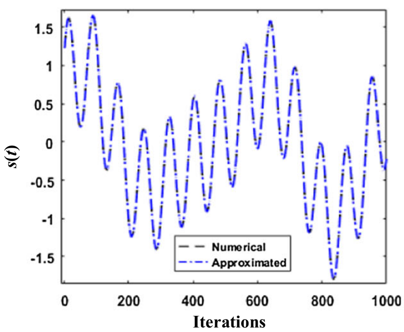
$$e_1 = \frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l} \sum\limits_{k=1}^{K}(c_k \cos k\omega t_n + d_k sin\, k\omega t_n) + v_n \\ -\sum\limits_{k=1}^{K}\left(\hat{c}_k \cos k\omega t_n + \hat{d}_k sin\, k\omega t_n\right)\end{array}\right)^2. \tag{16}$$



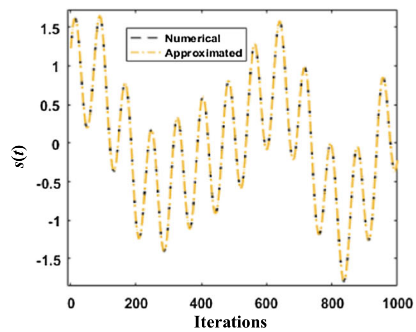
(a) Comparison of results BSA-III, Exp: 1, SNR 0db



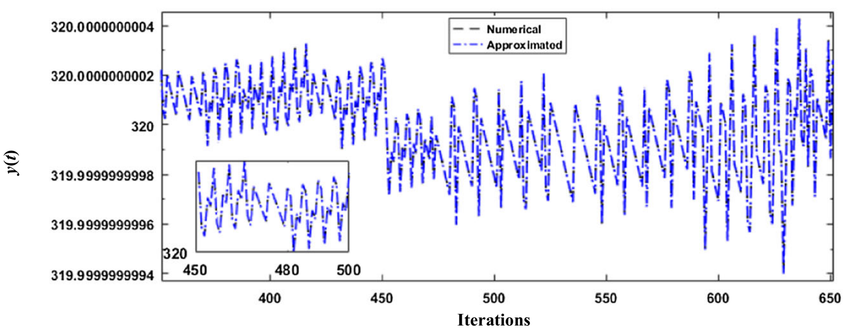
(b) Comparison of results BSA-III, Exp: 1, SNR 30db



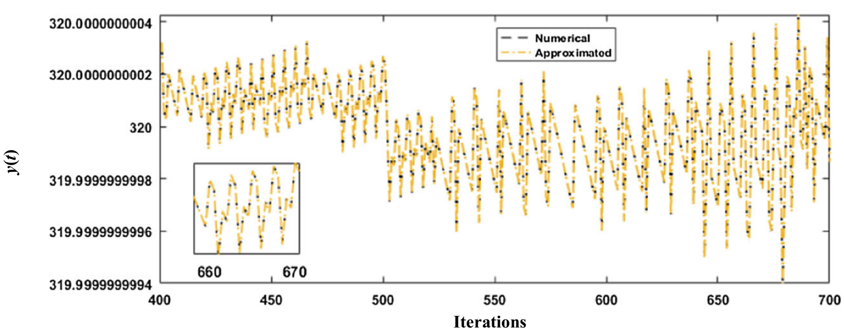
(c) Comparison of results BSA-III, Exp: 2, SNR 0db



(d) Comparison of results BSA-III, Exp: 2, SNR 30db



(e) Comparison of results BSA-III, Exp: 3, SNR 0db



(f) Comparison of results BSA-III, Exp: 3, SNR 30db

Fig. 3 Comparison plots of BSA for parameter estimation of power signal modeling problem with noise variation scenarios

Table 3 Comparison of true parameters with the proposed results of parameter estimation of power signal models model Example 1 for all noise variances

Method	σ^2	Parameter vector $\hat{\vartheta}$					
		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
BSA-I	0	1.803	1.903	0.301	0.750	0.680	0.566
	70	1.799	1.899	0.293	0.748	0.679	0.561
	30	1.803	1.900	0.303	0.751	0.677	0.560
BSA-II	0	1.800	1.900	0.300	0.750	0.680	0.560
	70	1.800	1.900	0.300	0.750	0.680	0.560
	30	1.800	1.901	0.302	0.747	0.679	0.559
BSA-III	0	1.800	1.900	0.300	0.750	0.680	0.560
	70	1.800	1.900	0.300	0.750	0.680	0.560
	30	1.798	1.899	0.300	0.747	0.680	0.559
BSA-IV	0	1.800	1.900	0.300	0.750	0.680	0.560
	70	1.800	1.900	0.300	0.750	0.680	0.560
	30	1.797	1.902	0.303	0.747	0.679	0.560
BSA-V	0	1.800	1.900	0.300	0.750	0.680	0.560
	30	1.800	1.900	0.300	0.750	0.680	0.560
	10	1.798	1.902	0.303	0.747	0.679	0.560
BSA-VI	0	1.800	1.900	0.300	0.750	0.680	0.560
	30	1.800	1.900	0.300	0.750	0.680	0.560
	10	1.797	1.902	0.304	0.747	0.679	0.560
True ϑ		1.800	1.900	0.300	0.750	0.680	0.560

Accordingly, e_2 is error function associated with the parameter vector in case of Eq. (5) is given as:

$$e_2 = \frac{1}{N}\sum_{n=1}^{N}\left(\vartheta_{cd_n} - \hat{\vartheta}_{cd_n}\right)^2, \tag{17}$$

using Eq. (5), we have

$$e_2 = \frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}(c_1, c_2, \ldots, c_k, d_1, d_2, \ldots, d_k)_n \\ -(\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_k, \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_k)_n\end{array}\right)^2. \tag{18}$$

Now, the fitness function e as given in Eq. (13) is written as:

$$e = \left[\begin{array}{l}\frac{1}{N}\sum_{k=1}^{K}\left(\begin{array}{l}\sum_{k=1}^{K}(c_k\cos k\omega t_n + d_k sin\, k\omega t_n) \\ +v_n - \sum_{k=1}^{K}(\hat{c}_k\cos k\omega t_n + \hat{d}_k sin\, k\omega t_n)\end{array}\right)^2 \\ +\frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}(c_1, c_2, \ldots, c_k, d_1, d_2, \ldots, d_k)_n \\ -(\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_k, \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_k)_n\end{array}\right)^2\end{array}\right]. \tag{19}$$

Similarly, the fitness function formulation for power signal model represented in Eq. (4) with decision variables as given in Eq. (11) is given as:

$$e = \left[\begin{array}{l}\frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}\sum_{i=1}^{k} r_i\sin(\omega_i t_n + \varphi_i) + v_n \\ -\sum_{i=1}^{k} r_i\sin(\hat{\omega}_i t_n + \hat{\varphi}_i)\end{array}\right)^2 \\ +\frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}(\omega_1, \omega_2, \ldots, \omega_k, \varphi_1, \varphi_2, \ldots, \varphi_k)_n \\ -(\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_k, \hat{\varphi}_1, \hat{\varphi}_2, \ldots, \hat{\varphi}_k)_n\end{array}\right)^2\end{array}\right]. \tag{20}$$

Accordingly, the fitness function formulation for power signal model represented in Eq. (4) with decision variables as given in Eq. (11) is given as:

$$e = \left[\begin{array}{l}\frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}\sum_{i=1}^{k} r_i\sin(\omega_i t_n + \varphi_i) + v_n \\ -\sum_{i=1}^{k} \hat{r}_i\sin(\hat{\omega}_i t_n + \hat{\varphi}_i)\end{array}\right)^2 \\ +\frac{1}{N}\sum_{n=1}^{N}\left(\begin{array}{l}(r_1, r_2, \ldots, r_k, \omega_1, \omega_2, \ldots, \omega_k, \varphi_1, \varphi_2, \ldots, \varphi_k)_n \\ -(\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_k, \hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_k, \hat{\varphi}_1, \hat{\varphi}_2, \ldots, \hat{\varphi}_k)_n\end{array}\right)^2\end{array}\right]. \tag{21}$$

On a similar pattern, the rest of the fitness functions are constructed for different power signal models.

Now, objective is optimization of the error functions given in Eqs. (19)–(21) in such a way that as e approaches zero, the adaptive parameter vectors of decision variable $\hat{\boldsymbol{\vartheta}}_{cd}$, $\hat{\boldsymbol{\vartheta}}_{\omega,\varphi}$ and $\hat{\boldsymbol{\vartheta}}_{r,\omega,\varphi}$ matches the desired variables $\boldsymbol{\vartheta}_{cd}$, $\boldsymbol{\vartheta}_{\omega,\varphi}$ and $\boldsymbol{\vartheta}_{r,\omega,\varphi}$ of the power signal models, respectively.

3.2 Learning method: variants of BSA

The second phase of designed methodology, we introduced BSA for optimization of decision variable of power signal model.

BSA is an initial population-based stochastic algorithm introduced by Civicioglu [39] in 2012 for the solution of constraint and unconstraint optimization problem. In BSA, trial population is generated using three basic recombination operators including selection, mutation and crossover. BSA employs random mutation process and non-uniform crossover strategy which is relatively complex than traditional ones. BSA belongs to the class of global optimization technique designed to solve high dimensional multimodal optimization problems having simple structure, single control parameter and additional benefit of possessing a memory. Few recent potential applications of BSA include beach realignment [40], solving constrained engineering problem [41], parameter estimation [42], economic load dispatch problem [43], wireless communication [44, 45], photovoltaic models [46] and nonlinear system identification [47]. Graphical flow chart describing the procedural steps of BSA is shown in Fig. 1, while necessary further detailed of these steps is provide in the pseudocode presented in Table 1.

Table 4 Comparison of true parameters with the proposed results of parameter estimation of power signal models model Examples 2 and 3 for all noise variances

Method	σ^2	Example 2 Parameter vector $\hat{\vartheta}$									Example 3 Parameter vector $\hat{\vartheta}$			
		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
BSA-I	0	0.697	0.799	0.301	0.060	0.400	0.100	0.855	0.696	0.663	300.0045	40.0000	20.0000	10.0008
	70	0.704	0.801	0.302	0.060	0.400	0.099	0.858	0.698	0.668	300.0546	39.9972	20.0756	9.5891
	30	0.708	0.797	0.299	0.060	0.400	0.100	0.853	0.701	0.667	300.0466	40.0981	20.0003	10.5677
BSA-II	0	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	70	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	30	0.699	0.798	0.299	0.060	0.400	0.100	0.851	0.701	0.660	299.9998	40.0000	19.9998	10.0000
BSA-III	0	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0056	40.0045	20.0006	10.0010
	70	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	9.9998
	30	0.699	0.798	0.300	0.060	0.400	0.100	0.851	0.700	0.660	300.0001	39.9992	20.0003	10.0050
BSA-IV	0	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	70	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	30	0.698	0.799	0.300	0.060	0.400	0.100	0.850	0.701	0.661	300.0000	40.0000	19.9996	10.0000
BSA-V	0	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	30	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0000	40.0000	20.0000	10.0000
	10	0.699	0.798	0.300	0.060	0.400	0.100	0.849	0.700	0.659	300.0056	39.9989	19.9996	10.0000
BSA-VI	0	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0006	40.0000	20.0002	10.0000
	30	0.700	0.800	0.300	0.060	0.400	0.100	0.850	0.700	0.660	300.0034	40.0024	19.9929	9.9964
	10	0.699	0.798	0.300	0.060	0.400	0.100	0.851	0.700	0.660	299.9998	39.9924	19.9998	9.9968
True ϑ		0.7	0.8	0.3	0.06	0.4	0.1	0.85	0.7	0.66	300	40	20	10

The performance analysis for parameter estimation of signal modeling problem has also been carried out based on fitness function, normalizing error calculation, root mean squared error (RMSE) and Thiel's inequality coefficient (TIC). The mathematical definitions of these performance indices can be seen in [47] for interested readers.

4 Numerical experimentation

Simulations are performed for three different examples of power signals parameter estimation problems through the evolutionary computing heuristics of BSA under varying noise scenarios. The variants of BSA are designed by means of different population size (PS) and generations (GENS) as tabulated in Table 2.

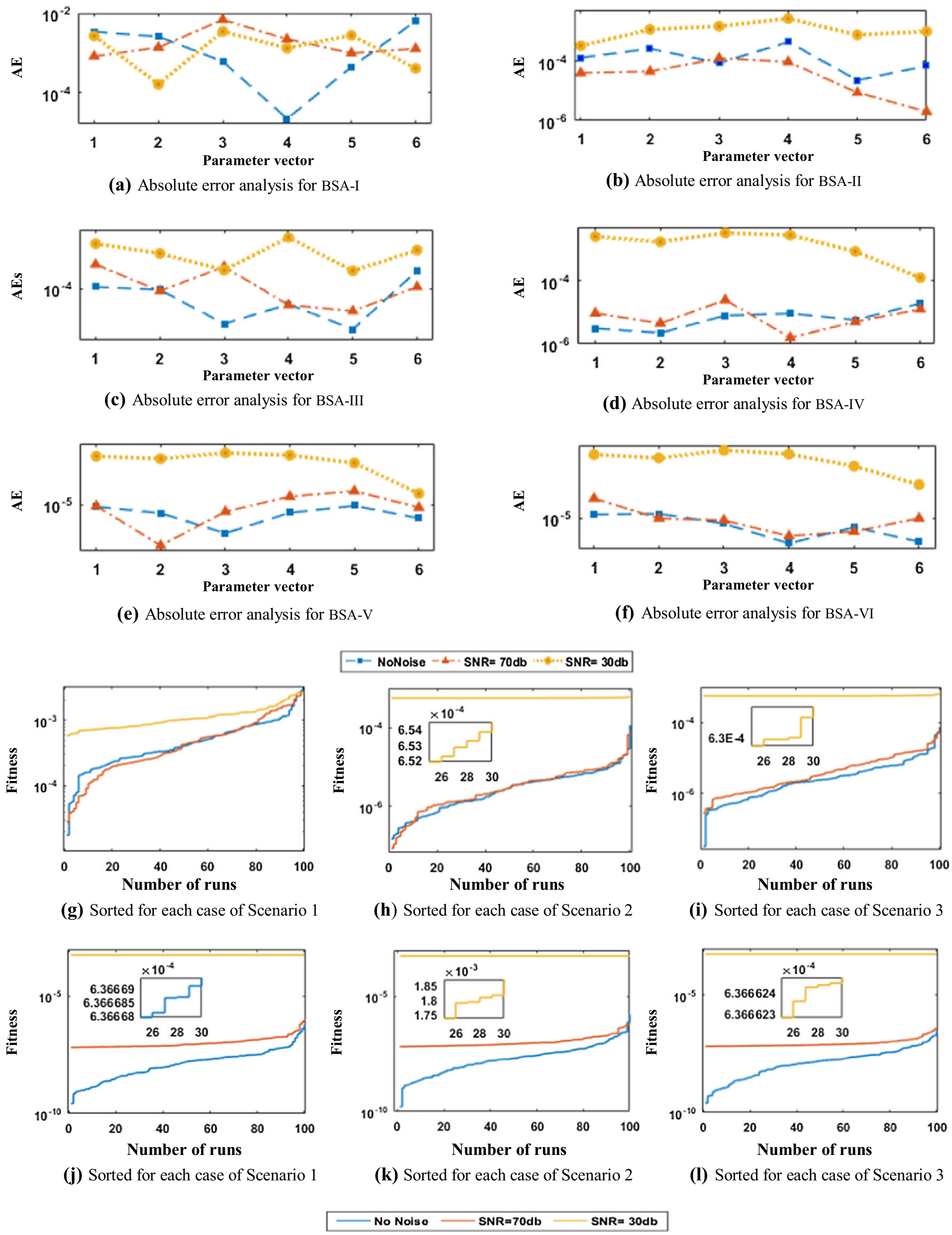
Example 1 In this case study, the power signal estimation problem with known amplitude while, unknown frequency and phase parameters is taken. The mathematical expressions for Example 1 are written as [8]:

$$\begin{aligned} s(t) &= r_1 \sin(\omega_1 t + \varphi_1) + r_2 \sin(\omega_2 t + \varphi_2) + r_3 \sin(\omega_3 t + \varphi_3) \\ \vartheta &= [\omega_1, \omega_2, \omega_3, \varphi_1, \varphi_2, \varphi_3]^{\mathrm{T}} \\ \vartheta &= [0.07, 0.1, 0.2, 0.95, 0.8, 0.76]^{\mathrm{T}}. \end{aligned} \tag{22}$$

Example 2 The power signal modeling problem with unknown amplitude, frequency and phase in the parameter vector is taken. The mathematical expressions for Example 2 are given as [8]:

$$\begin{aligned} s(t) &= r_1 \sin(\omega_1 t + \varphi_1) + r_2 \sin(\omega_2 t + \varphi_2) + r_3 \sin(\omega_3 t + \varphi_3) \\ \vartheta &= [r_1, r_2, r_3, \omega_1, \omega_2, \omega_3, \varphi_1, \varphi_2, \varphi_3]^{\mathrm{T}} \\ \vartheta &= [0.07, 0.1, 0.2, 0.95, 0.8, 0.76]^{\mathrm{T}}. \end{aligned} \tag{23}$$

Example 3 The power signal modeling problem with unknown amplitude in the parameter vector is presented mathematically as [34]:



(a) Absolute error analysis for BSA-I
(b) Absolute error analysis for BSA-II
(c) Absolute error analysis for BSA-III
(d) Absolute error analysis for BSA-IV
(e) Absolute error analysis for BSA-V
(f) Absolute error analysis for BSA-VI
(g) Sorted for each case of Scenario 1
(h) Sorted for each case of Scenario 2
(i) Sorted for each case of Scenario 3
(j) Sorted for each case of Scenario 1
(k) Sorted for each case of Scenario 2
(l) Sorted for each case of Scenario 3

Fig. 4 Comparison of the accuracy of BSA variants for parameter estimation of power signal modeling problem with noise variation scenarios in case of Example 1

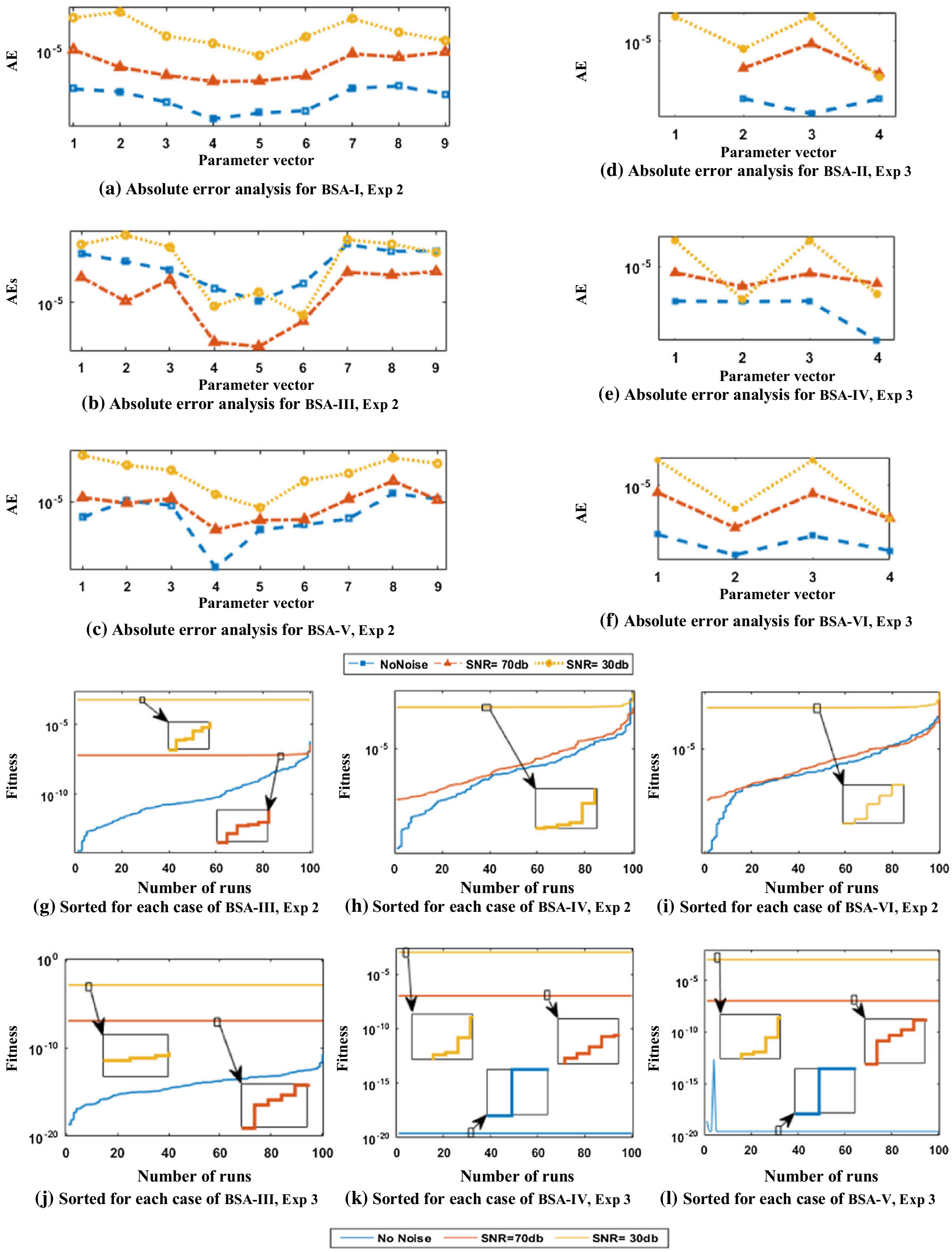


Fig. 5 Comparison of the accuracy of BSA variants for parameter estimation of power signal modeling problem with noise variation scenarios in case of Examples 2 and 3

Table 5 Performance comparison of all BSA variants through satistical and complexity operators

Example	Method	Noise σ^2	Accuracy operators					Complexity operators		
			e	δ	MAE	RMSE	TIC	Iterations	Time (s)	FC
1	BSA-I	0	1.78E−05	2.67E−03	2.24E−03	3.14E−03	1.33E−03	500	1.2734	6060
		70	3.84E−05	2.42E−03	2.26E−03	2.84E−03	1.21E−03	500	1.2367	6060
		30	6.09E−04	1.87E−03	1.80E−03	2.19E−03	9.33E−04	500	1.1500	6060
	BSA-II	0	3.14E−07	1.94E−04	1.74E−04	2.27E−04	9.68E−05	1000	2.2283	12,060
		70	8.70E−08	6.01E−05	5.40E−05	7.06E−05	3.00E−05	1000	2.3732	12,060
		30	5.69E−04	1.28E−03	1.29E−03	1.51E−03	6.41E−04	1000	2.1849	12,060
	BSA-III	0	3.34E−08	2.97E−03	9.69E−05	1.46E−04	6.20E−05	1500	2.1891	12,060
		70	2.89E−07	1.24E−03	1.93E−04	2.68E−04	1.14E−04	1500	2.2160	12,060
		30	5.77E−04	2.01E−03	1.27E−03	1.55E−03	6.59E−04	1500	2.2175	12,060
	BSA-IV	0	2.72E−10	2.38E−05	7.92E−06	9.73E−06	4.14E−06	1000	2.3077	12,040
		70	6.60E−08	2.75E−05	9.66E−06	1.23E−05	5.24E−06	1000	2.1440	12,040
		30	5.63E−04	2.02E−03	1.95E−03	2.25E−03	9.57E−04	1000	1.5425	12,040
	BSA-V	0	1.67E−10	5.61E−05	4.41E−06	5.29E−06	2.25E−06	1000	2.2201	12,040
		30	6.78E−08	5.97E−05	1.50E−05	1.87E−05	7.95E−06	1000	2.2503	12,040
		10	5.63E−04	1.94E−03	1.92E−03	2.22E−03	9.43E−04	1000	1.6232	12,040
	BSA-VI	0	2.56E−10	1.36E−04	7.04E−06	8.94E−06	3.81E−06	1000	4.3601	24,080
		30	6.76E−08	1.89E−04	1.50E−05	2.05E−05	8.72E−06	1000	4.5045	24,080
		10	5.63E−04	1.95E−03	1.96E−03	2.24E−03	9.54E−04	1000	3.0472	24,080
2	BSA-I	0	3.10E−05	4.40E−03	1.84E−03	2.56E−03	2.20E−03	500	7.3440	45,090
		70	1.52E−04	7.07E−03	2.91E−03	4.11E−03	3.53E−03	500	7.9400	45,090
		30	9.17E−04	6.97E−03	2.81E−03	4.05E−03	3.48E−03	500	7.2318	45,090
	BSA-II	0	2.10E−09	4.26E−05	1.49E−05	2.48E−05	2.13E−05	1000	14.8126	90,090
		70	6.68E−08	4.60E−05	1.75E−05	2.68E−05	2.30E−05	1000	15.1012	90,090
		30	6.16E−04	1.49E−03	5.89E−04	8.65E−04	7.44E−04	1000	14.5493	90,090
	BSA-III	0	8.56E−15	6.84E−08	2.93E−08	3.98E−08	3.42E−08	1500	22.1965	135,090
		70	6.14E−08	1.02E−05	3.95E−06	5.93E−06	5.10E−06	1500	23.2409	135,090
		30	6.15E−04	1.66E−03	5.52E−04	9.62E−04	8.28E−04	1500	21.9351	135,090
	BSA-IV	0	5.94E−10	1.81E−05	6.80E−06	1.05E−05	9.05E−06	1000	6.7722	40,040
		70	6.39E−08	5.97E−05	1.87E−05	3.47E−05	2.98E−05	1000	8.7852	40,040
		30	6.18E−04	1.37E−03	5.57E−04	7.98E−04	6.87E−04	1000	6.4256	40,040
	BSA-V	0	3.44E−10	1.34E−05	4.62E−06	7.78E−06	6.69E−06	1000	7.9962	40,040
		30	6.14E−08	1.64E−05	6.93E−06	9.55E−06	8.21E−06	1000	8.5442	40,040
		10	6.16E−04	1.41E−03	5.32E−04	8.19E−04	7.05E−04	1000	6.4890	40,040
	BSA-VI	0	1.26E−09	3.42E−05	1.36E−05	1.99E−05	1.71E−05	1000	13.5810	80,080
		30	6.17E−08	2.63E−05	1.14E−05	1.53E−05	1.31E−05	1000	12.8295	80,080
		10	6.15E−04	1.52E−03	5.81E−04	8.81E−04	7.58E−04	1000	12.7947	80,080
3	BSA-I	0	2.27E−19	2.91E−12	3.00E−10	4.41E−10	1.45E−12	1000	6.818	20,040
		70	1.04E−07	2.46E−08	1.96E−06	3.73E−06	1.23E−08	1000	6.780	20,040
		30	1.02E−03	1.05E−06	7.98E−05	1.59E−04	5.25E−07	1000	6.753	20,040
	BSA-II	0	2.52E−20	1.85E−13	1.46E−11	2.81E−11	9.27E−14	2000	13.539	28,040
		70	1.04E−07	1.85E−08	1.79E−06	2.81E−06	9.26E−09	2000	10.165	28,040
		30	1.05E−03	1.07E−06	1.14E−04	1.62E−04	5.33E−07	2000	10.716	28,040
	BSA-III	0	2.51E−20	2.01E−13	1.53E−11	3.06E−11	1.01E−13	3000	13.568	40,040
		70	1.04E−07	2.74E−08	2.08E−06	4.16E−06	1.37E−08	3000	13.501	40,040
		30	1.02E−03	1.02E−06	7.98E−05	1.60E−04	5.26E−07	3000	13.430	40,040

Table 5 continued

Example	Method	Noise σ^2	Accuracy operators					Complexity operators		
			e	δ	MAE	RMSE	TIC	Iterations	Time (s)	FC
	BSA-IV	0	2.59E−17	3.28E−11	3.75E−09	4.97E−09	1.64E−11	1000	1.838	5010
		70	1.04E−07	1.78E−08	1.79E−06	2.70E−06	8.90E−09	1000	2.323	5010
		30	1.50E−03	1.07E−06	1.03E−04	1.61E−04	5.30E−07	1000	1.949	5010
	BSA-V	0	5.83E−19	1.22E−12	1.27E−10	1.85E−10	6.10E−13	1000	3.972	10,020
		30	1.04E−07	1.86E−08	1.79E−06	2.82E−06	9.28E−09	1000	3.737	10,020
		10	1.35E−03	1.06E−06	1.03E−04	1.61E−04	5.33E−07	1000	5.249	10,020
	BSA-VI	0	4.49E−17	4.18E−11	4.21E−09	6.34E−09	2.09E−11	1000	5.894	15,030
		30	1.04E−07	1.84E−08	1.80E−06	2.78E−06	9.18E−09	1000	5.439	15,030
		10	1.33E−02	1.04E−06	1.14E−04	1.61E−04	5.32E−07	1000	7.035	15,030

Table 6 Convergence analysis of all examples using BSA variants

Exp	Fitness10^{xx}	BSA-I			BSA-II			BSA-III			BSA-IV			BSA-V			BSA-VI		
		0	70	30	0	70	30	0	70	30	0	70	30	0	70	30	0	70	30
1	−03	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−04	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−05	57	56	0	100	100	0	100	100	0	100	100	0	100	100	0	100	100	0
	−06	1	4	0	99	98	0	98	98	0	100	100	0	100	100	0	100	100	0
2	−03	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−04	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−05	57	56	0	100	100	0	100	100	0	100	100	0	100	100	0	100	100	0
	−06	1	4	0	99	98	0	98	98	0	100	100	0	100	100	0	100	100	0
3	−04	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−05	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	−06	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	08	100	100	30	100	100	30	100	100	30	100	100	30	100	100	30	100	100	30

$$\begin{aligned} s(t) &= c_1\cos\omega t + d_1 sin\,\omega t + c_2\cos 2\omega t + d_2 sin\,2\omega t \\ \vartheta &= [c_1, c_2, d_1, d_2]^{\mathrm{T}} \\ \vartheta &= [300,\ 40,\ 20,\ 10]^{\mathrm{T}}. \end{aligned} \tag{24}$$

In the simulations, $s(t)$ is taken as the input signal and $v(t)$ represents noise signal having zero mean and three different noise levels i.e., no noise, 30 db and 70 db. The power signal modeling problem as described in Eqs. (22)–(24) is executed with the proposed scheme and the objective function is implemented for $N = 20$ snap shots. The meta-heuristic algorithm, BSA together with all its six variants is performed for 100 independent runs. The results of each BSA variant against the values of fitness function are shown graphically in Fig. 2, for all three signal models given in Examples 1–3 and noise levels. It is observed from the results presented in Fig. 2 that all the variants of BSA are convergent for all noise levels, but convergence of the variant-III of BSA is slightly better than all others. Comparison of the actual signals is also made with the approximated signal, and the resultant graphs are presented in Fig. 3 for all three examples. The actual and estimation decision variables are listed in Table 3 for Example 1 in case of all three noise levels, while these results for Examples 2 and 3 are provided in Table 4 for each noise variation. The estimated signals overlap the actual signals consistently, as well as, relatively small difference between actual and estimation parameters which prove the accuracy of the scheme. However, with the rise in the noise level,

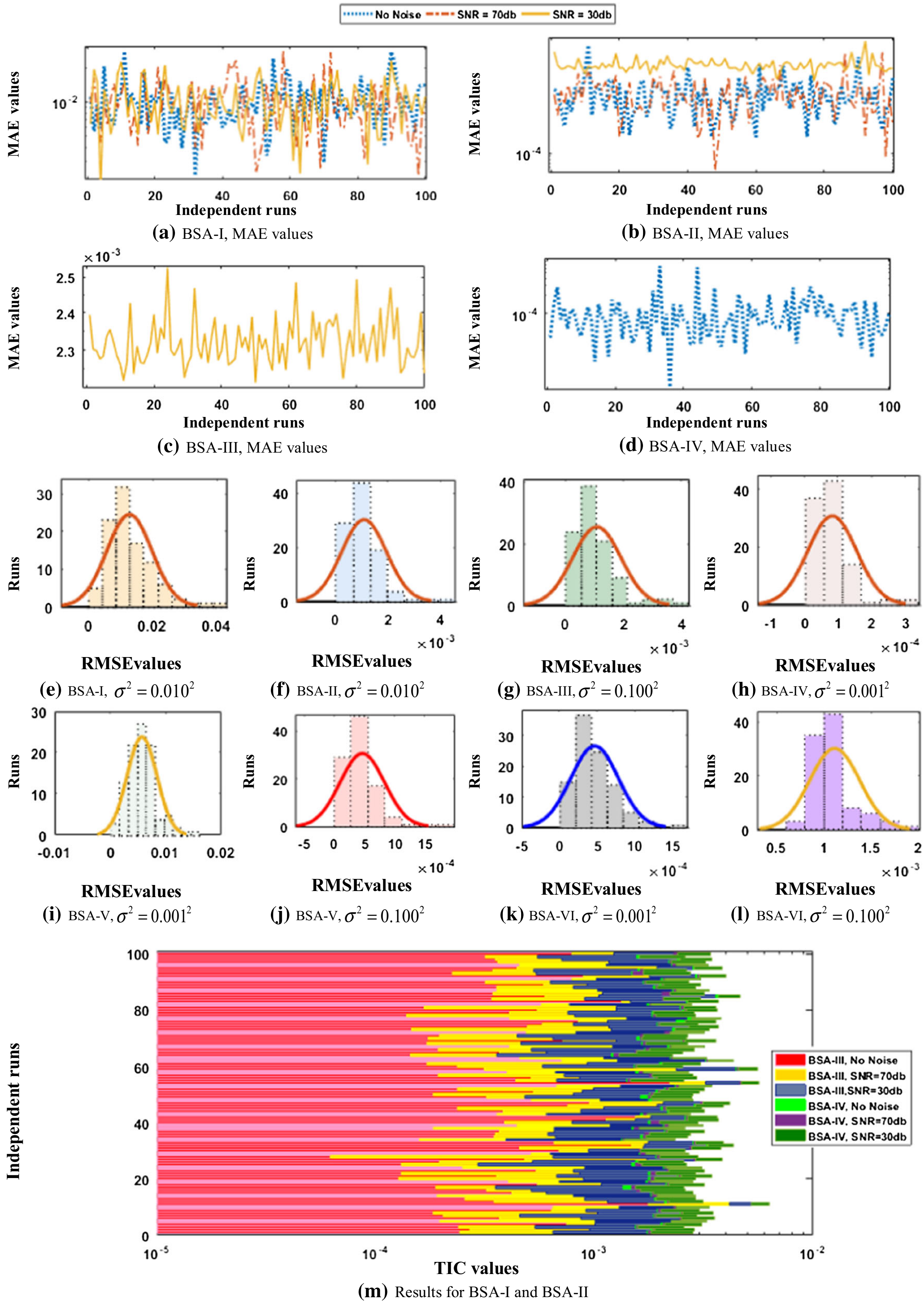


Fig. 6 Comparative study of BSA variants on the basis of different performance indices for parameter estimation of power signal modeling problem

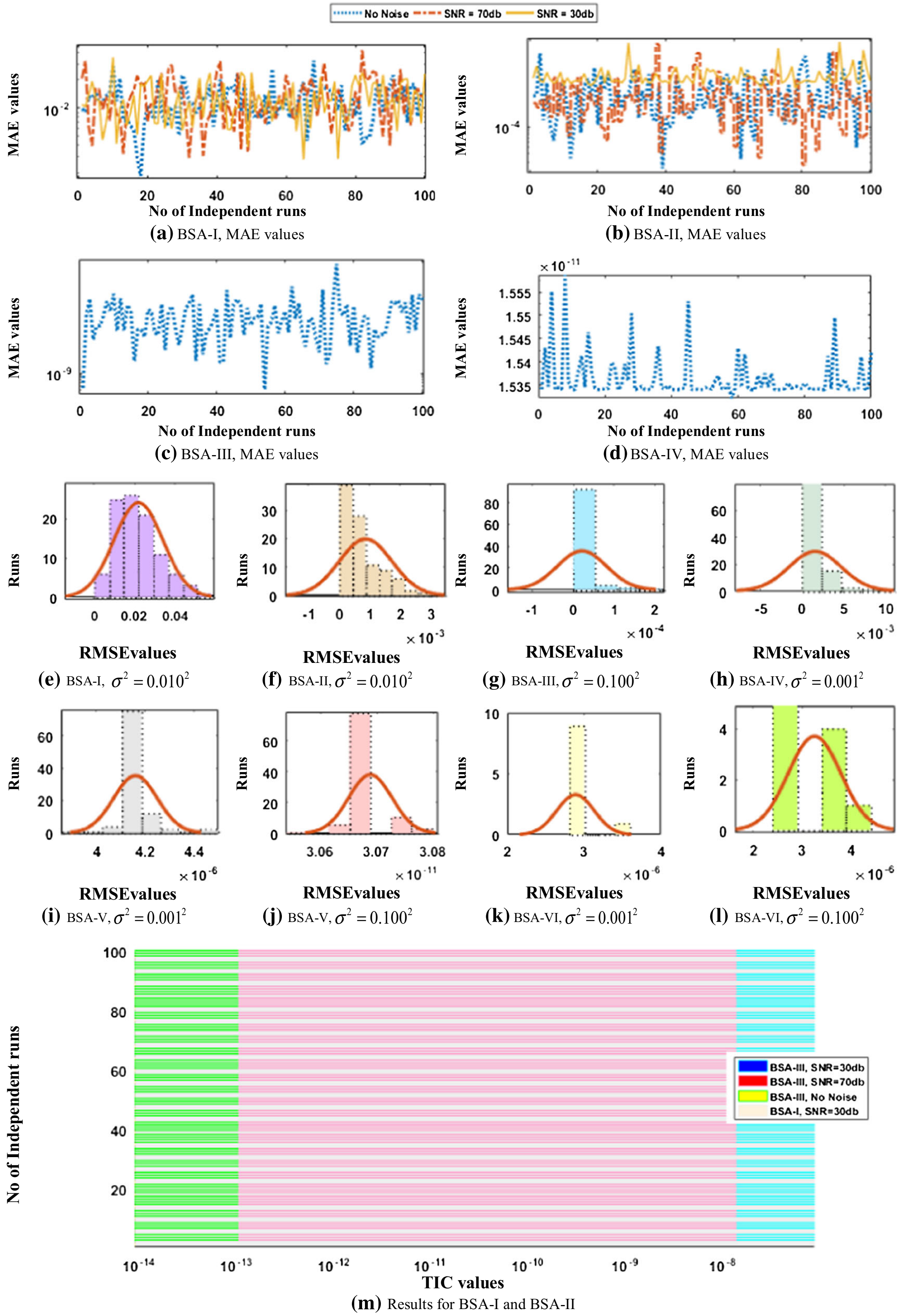


Fig. 7 Comparative study of BSA variants on the basis of different performance indices for parameter estimation of power signal modeling problem

Table 7 ANOVA test results for Examples 1, 2 and 3 of power signal model with 70 dB SNR

Scheme	Method Null hypothesis All means are equal Alternative hypothesis At least one mean is different Significance level α = 0.05 Equal variances were assumed for the analysis.
Example 1	**Analysis of Variance** Source DF F-Value P-Value Factor 2 1.28 0.283 Error 87 Total 89 **Model Summary** R-sq ..2.86% **Means** Factor N Mean StDev 95% CI C1 30 0.000865 0.000444 (0.000677, 0.001053) C2 30 0.000756 0.000475 (0.000569, 0.000944) C3 30 0.000970 0.000617 (0.000782, 0.001158) Pooled StDev = 0.000517625
Example 2	**Analysis of Variance** Source DF F-Value P-Value Factor 2 0.2 0.811 Error 90 Total 92 **Model Summary** R-sq 0.47% **Means** Factor N Mean StDev 95% CI C1 31 0.000772 0.000002 (0.000772, 0.000773) C2 31 0.000772 0.000001 (0.000772, 0.000772) C3 31 0.000772 0.000001 (0.000772, 0.000772) Pooled StDev = 1.157001E-06
Example 3	**Analysis of Variance** Source D F-Value P-Value Factor 18 0.01 0.928 Error 58 Total 59 **Model Summary** R-sq 0.01% **Means** Factor N Mean StDev 95% CI C1 30 0.000001 0.000000 (0.000001, 0.000001) C2 30 0.000001 0.000000 (0.000001, 0.000001) Pooled StDev = 4.403870E-13

there is a decrease in the accuracy level is observed for all six variants of BSA.

In order to access the minute difference, the values of absoluter error (AE), i.e., difference between actual and estimated parameter, is calculated for each case. The values of AE are plotted in Fig. 4 for Example 1 in case of all six variants of BSA, while the results for Examples 2 and 3 are shown in Fig. 5. It is seen that the range of AE values lie around 10^{-02} to 10^{-04} for BSA-I while, for BSA-II, range is around 10^{-05} to 10^{-06} and similar trend for rest of the variants is observed.

The performance indices magnitudes based on fitness e, normalized error δ, MAE, RMSE and TIC for best independent run of the scheme are listed in Table 5 for each noise variation in case of all six variants of BSA. Near-to-optimal magnitudes of all these metrics achieved for all three examples of power signal estimation problem which evidently demonstrate the accuracy of the proposed scheme. Additionally, the computational complexity measures in terms of mean execution time, iteration consumed and function counts (FC) during the whole optimization procedure are also tabulated in Table 5 for all three example for each scenario. The complexity analysis shows that BSA-III consumed more iterations and time than other variants but is comparatively accurate from rest of the methodologies.

The results of each variant of BSA against the fitness values are graphically presented in Fig. 4g–l for several independent runs, i.e., 100 trials, all three noise levels in case of Example 1, while these illustrations for Examples 2 and 3 are shown in Fig. 5g–l. All these graphs are given in sorted and zoomed plots for better assessment of the results. It is evident from the plots that all the variants of the BSA converge but accuracy degrades as noise increases.

Convergence analysis is performed to evaluate reliability of the scheme for attaining the various accuracy levels on the basis of fitness gauges, i.e., fitness $\varepsilon \leq 10^{-03}$, 10^{-04}, 10^{-05} and 10^{-06}. The results of percentage independent runs fulfilling these criterions are listed in Table 6 for each scenario of all three example of power signal estimation problem. It is seen that nearly 100% of the independent runs meet the primary level of the basic fitness measure and also even a few trials attained relatively tough criteria. The results are comparatively more accurate and convergent for BSA-IV and BSA-V for rest of the scheme.

The statistical performances in terms of MAE, RMSE and TIC metrics are also evaluated and results of these indices are shown in Figs. 6 and 7 for each scenario of Examples 1 and 2, respectively. In Fig. 6a–d, MAE magnitudes are plotted for 100 independent trials of all the variants of BSA algorithm on semi log scale. The results show that MAE are near 10^{-3} to 10^{-2}, 10^{-4} to 10^{-2}, 10^{-3} and 10^{-3} to 10^{-4} for BSA I, II, III and IV, respectively. Histogram plots are also shown in Fig. 6e–l for all six variants of BSAs. The smaller magnitudes of RMSE verify the accuracy of the designed methodology. In Fig. 6m, TIC magnitudes as stacked bar graph are shown for BSA-III and BSA-IV each noise variance-based scenario of Example 1. These results demonstrate that magnitudes of TIC lie around 10^{-5} to 10^{-2}. Similarly plots for Examples 2 and 3 are given in Fig. 7a–d, e–l and m in case of MAE, RMSE and TIC values, respectively. The similar trend of results is seen as in case of Example 1.

Further evaluation about the performance of the BSA variants for periodic signals estimation is made using ANOVA test. The results are computed for all three examples and presented in Table 7 for SNR 70 dB. With the consideration of assumption of uniform variances, the null hypothesis of homogeneous variances at the significance level $\alpha = 0.05$ is accepted, as the respective probability values i.e., p values, attained for absolute errors for Examples 1, 2 and 3 are 0.283, 0.811 and 0.828, respectively. The result of ANOVA established that the expected values show uniformity, and there is no strong evidence against the null hypothesis. Therefore, it is quite evident that all the means are equivalent.

5 Conclusions

Evolutionary computational paradigm through variants of BSA are exploited for effective, viable and reliable solution of parameter estimation of power signal models with various noise level-based scenarios. Comparative study of the designed algorithms for both noise less and noisy environment, i.e., no noise, SNR = 70 db and 30 db, depict the accuracy of all six variants of BSA technique. The study shows that the accuracy level decreases as noise increases for all variants of BSA; despite, all the results are quite precise. Analysis based on performance measures, i.e., normilizing error, AEs, MAD, RMSE and TIC metrics, demonstrated the efficacy of the proposed scheme. While, the complexity analysis in terms of time consumed, iterations executed and functions count show that the results of BSA-III and BSA-IV are relatively higher than the other variants of the BSA optimization technique; however, their relatively better accuracy for the rest of scheme overshadows this aspect. Power signal modeling with increase degrees of freedom slight deteriorates the performance of the proposed BSA in terms of accuracy and complexity.

One may apply the proposed methodology for solving real-world complex power and energy-related optimization problems [48–52]. Recently introduced meta-heuristic techniques such as firefly, particle swarm optimization and cuckoo search algorithm along with their fractional variants can be exploited to improve the accuracy and convergence in power signal estimation problems.

Compliance with ethical standards

Conflict of interest All the authors of the manuscript declare that there is no potential conflict of interests.

References

1. Van Cutsem T, Vournas C (2007) *Voltage stability of electric power systems*. Springer, Berlin
2. Dugan RC, McGranaghan MF, Beaty HW, Santoso S (1996) Electrical power systems quality, vol 2. Mcgraw-Hill, New York
3. Phan AT, Wira P, Hermann G (2018) A dedicated state space for power system modeling and frequency and unbalance estimation. Evol Syst 9(1):57–69

4. Acha E, Madrigal M (2001) Power systems harmonics. Wiley, New York
5. Xu L, Ding F (2018) Iterative parameter estimation for signal models based on measured data. Circuits Syst Signal Process 37:1–24
6. Xu L, Xiong W, Alsaedi A, Hayat T (2018) Hierarchical parameter estimation for the frequency response based on the dynamical window data. Int J Control Autom Syst 16(4):1756–1764
7. Li H, Zhou M, Wu R, Zhang Z, Zheng J (2018) Parameter estimation of air maneuvering target for multi-antenna system via reconstructing time samples and signal. Multidimens Syst Signal Process 29(2):621–641
8. Cao Y, Liu Z (2010) Signal frequency and parameter estimation for power systems using the hierarchical identification principle. Math Comput Model 52(5–6):854–861
9. Qian H, Zhao R, Chen T (2007) Interharmonics analysis based on interpolating windowed FFT algorithm. IEEE Trans Power Deliv 22(2):1064–1069
10. Zhang Q, Liu H, Chen H, Li Q, Zhang Z (2008) A precise and adaptive algorithm for interharmonics measurement based on iterative DFT. IEEE Trans Power Deliv 23(4):1728–1735
11. Zubair S, Chaudhary NI, Khan ZA, Wang W (2018) Momentum fractional LMS for power signal parameter estimation. Signal Process 142:441–449
12. Chaudhary NI et al (2017) A new computing approach for power signal modeling using fractional adaptive algorithms. ISA Trans 68:189–202
13. Chaudhary NI et al (2013) Identification of input nonlinear control autoregressive systems using fractional signal processing approach. Sci World J 2013:467276. https://doi.org/10.1155/2013/467276
14. Wan L, Ding F (2019) Decomposition-and gradient-based iterative identification algorithms for multivariable systems using the multi-innovation theory. Circuits Syst Signal Process 38:1–21
15. Huang C, Liu L, Yuen C (2018) Asymptotically optimal estimation algorithm for the sparse signal with arbitrary distributions. IEEE Trans Veh Technol 67(10):10070–10075
16. Xia H, Ji Y, Xu L, Hayat T (2019) Maximum likelihood-based recursive least-squares algorithm for multivariable systems with colored noises using the decomposition technique. Circuits Syst Signal Process 38:1–19
17. Shuai Z, Zhang J, Tang L, Teng Z, Wen H (2018) Frequency shifting and filtering algorithm for power system harmonic estimation. IEEE Trans Ind Inform 15:1554–1565
18. Liu S, Ding F, Xu L, Hayat T (2019) Hierarchical principle-based iterative parameter estimation algorithm for dual-frequency signals. Circuits Syst Signal Process 38:1–18
19. Hu B, Gharavi H (2018) A fast recursive algorithm for spectrum tracking in power grid systems. IEEE Trans Smart Grid 10:2882–2891
20. Hu Y, Zhou Q, Yu H, Zhou Z, Ding F (2018) Two-stage generalized projection identification algorithms for stochastic systems. Circuits Syst Signal Process 38:1–17
21. El Maani R, Radi B, El Hami A (2019) Multiobjective backtracking search algorithm: application to FSI. Struct Multidiscip Optim 59:1–21
22. Mallick S, Kar R, Mandal D, Ghoshal SP (2016) CMOS analogue amplifier circuits optimisation using hybrid backtracking search algorithm with differential evolution. J Exp Theor Artif Intell 28(4):719–749
23. Nazri NN, Malik NN, Idoumghar L, Latiff NM, Ali S (2018) Backtracking search optimization for collaborative beamforming in wireless sensor networks. Telkomnika 16(4):1801–1808
24. Askarzadeh A, dos Santos Coelho L (2014) A backtracking search algorithm combined with Burger's chaotic map for parameter estimation of PEMFC electrochemical model. Int J Hydrog Energy 39(21):11165–11174
25. Yang H, Yu J, Qiu Y, Li Q, Chen W (2018) A coordinated optimization method considering time-delay effect of islanded photovoltaic microgrid based on modified backtracking search algorithm. J Renew Sustain Energy 10(2):023503
26. Zhou J, Zhang C, Peng T, Xu Y (2018) Parameter identification of pump turbine governing system using an improved backtracking search algorithm. Energies 11(7):1668
27. Yan S, Zhou J, Zheng Y, Li C (2018) An improved hybrid backtracking search algorithm based T–S fuzzy model and its implementation to hydroelectric generating units. Neurocomputing 275:2066–2079
28. Modiri-Delshad M, Kaboli SHA, Taslimi-Renani E, Rahim NA (2016) Backtracking search algorithm for solving economic dispatch problems with valve-point effects and multiple fuel options. Energy 116:637–649
29. Hannan MA, Lipu MSH, Hussain A, Saad MH, Ayob A (2018) Neural network approach for estimating state of charge of lithium-ion battery using backtracking search algorithm. IEEE Access 6:10069–10079
30. Khan WU et al (2018) Backtracking search integrated with sequential quadratic programming for nonlinear active noise control systems. Appl Soft Comput 73:666–683
31. Agarwal SK, Shah S, Kumar R (2015) Classification of mental tasks from EEG data using backtracking search optimization based neural classifier. Neurocomputing 166:397–403
32. bin Mohd Zain MZ, Kanesan J, Kendall G, Chuah JH (2018) Optimization of fed-batch fermentation processes using the backtracking search algorithm. Expert Syst Appl 91:286–297
33. Lin J (2019) Backtracking search based hyper-heuristic for the flexible job-shop scheduling problem with fuzzy processing time. Eng Appl Artif Intell 77:186–196
34. Bian J, Xing J, Liu J, Li Z, Li H (2016) An adaptive and computationally efficient algorithm for parameters estimation of superimposed exponential signals with observations missing randomly. Digit Signal Process 48:148–162
35. Li X, Ding F (2013) Signal modeling using the gradient search. Appl Math Lett 26(8):807–813
36. Mehmood A, Chaudhary NI, Zameer A et al (2020) Novel computing paradigms for parameter estimation in power signal models. Neural Comput Appl 32:6253–6282
37. Zhou L, Li X, Xu H, Zhu P (2016) Multi-innovation stochastic gradient method for harmonic modelling of power signals. IET Signal Process 10(7):737–742
38. Pan J, Yang X, Cai H, Mu B (2016) Image noise smoothing using a modified Kalman filter. Neurocomputing 173:1625–1629
39. Civicioglu P (2013) Backtracking search optimization algorithm for numerical optimization problems. Appl Math Comput 219(15):8121–8144
40. Chatzipavlis A, Tsekouras GE, Trygonis V, Velegrakis AF, Tsimikas J, Rigos A, Hasiotis T, Salmas C (2019) Modeling beach realignment using a neuro-fuzzy network optimized by a novel backtracking search algorithm. Neural Comput Appl 31(6):1747–1763
41. Wang H, Hu Z, Sun Y, Su Q, Xia X (2019) A novel modified BSA inspired by species evolution rule and simulated annealing principle for constrained engineering optimization problems. Neural Comput Appl 31(8):4157–4184
42. Mehmood A et al (2019) Backtracking search optimization heuristics for nonlinear Hammerstein controlled auto regressive auto regressive systems. ISA Trans 91:99–113
43. Bhattacharjee K (2018) Economic dispatch problems using backtracking search optimization. Int J Energy Optim Eng (IJEOE) 7(2):39–60

44. Zaman F et al (2019) Backtracking search optimization paradigm for pattern correction of faulty antenna array in wireless mobile communications. Wirel Commun Mob Comput 2019:9046409. https://doi.org/10.1155/2019/9046409
45. Raja MAZ, Akhtar R, Chaudhary NI, Khan WU, Zhiyu Z, Jamil A, Zaman F (2020) Design of backtracking search optimization paradigm for joint amplitude-angle measurement of sources lying in fraunhofer zone. Measurement 149:106977
46. Yu K, Liang JJ, Qu BY, Cheng Z, Wang H (2018) Multiple learning backtracking search algorithm for estimating parameters of photovoltaic models. Appl Energy 226:408–422
47. Mehmood A, Zameer A, Chaudhary NI, Raja MAZ (2019) Backtracking search heuristics for identification of electrical muscle stimulation models using Hammerstein structure. Appl Soft Comput 24:105705
48. Kayri I, Gencoglu MT (2019) Predicting power production from a photovoltaic panel through artificial neural networks using atmospheric indicators. Neural Comput Appl 31(8):3573–3586
49. El-Sattar SA, Kamel S, El Sehiemy RA et al (2019) Single- and multi-objective optimal power flow frameworks using Jaya optimization technique. Neural Comput Appl 31:8787–8806. https://doi.org/10.1007/s00521-019-04194-w
50. Mohamed MA, Diab AAZ, Rezk H et al (2019) A novel adaptive model predictive controller for load frequency control of power systems integrated with DFIG wind turbines. Neural Comput Appl. https://doi.org/10.1007/s00521-019-04205-w
51. Mishra SP, Dash PK (2019) Short-term prediction of wind power using a hybrid pseudo-inverse Legendre neural network and adaptive firefly algorithm. Neural Comput Appl 31(7):2243–2268
52. Wang T, He X, Deng T (2019) Neural networks for power management optimal strategy in hybrid microgrid. Neural Comput Appl 31(7):2635–2647