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S.I. : APPLYING ARTIFICIAL INTELLIGENCE TO THE INTERNET OF THINGS

Evidence of power-law behavior in cognitive IoT applications

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Abstract

The motivations induced due to the presence of scale-free characteristics of neural systems governed by the well-known power-law distribution of neuronal activities have led to its convergence with the Internet of things (IoT) framework. The IoT is one such framework, where the self-organization of the connected devices is a momentous aspect. The devices involved in these networks inherently relate to the collection of several consolidated devices like the sensory devices, consumer appliances, wearables, and other associated applications, which facilitate a ubiquitous connectivity among the devices. This is one of the most significant prerequisites of IoT systems as several interconnected devices need to be included in the convolution for the uninterrupted execution of the services. Thus, in order to understand the scalability and the heterogeneity of these interconnected devices, the exponent of power-law plays a significant role. In this paper, an analytical framework to illustrate the ubiquitous power-law behavior of the IoT devices is derived. An emphasis regarding the mathematical insights for the characterization of the dynamic behavior of these devices is conceptualized. The observations made in this direction are illustrated through simulation results. Further, the traits of the wireless sensor networks, in context with the contemporary scale-free architecture, are discussed.

Keywords Internet of things · Wireless sensor networks · Power-law · Scalability · Interconnectivity

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1 Introduction

The Internet of things (IoT) is experiencing momentous growth in recent years, to administer the requirements of several applications worldwide by providing a platform for the interconnection of physical world devices. The convergences of the IoT and its associated services have evidenced the expansion in several sectors like health care, academia, and business industries to name a few [1–4]. However, in order to achieve an entirely functional model for the IoT ecosystem, several crucial performance benchmarks are to be considered such as heterogeneity of the devices and interconnection network, topological constraints, efficient management of resources, interoperability among the devices, and optimizing the power consumption [5–9]. These factors are extensively dependent on the density of the network and the diverse collection of devices involved in the network.

Several real-world systems have evidenced the presence of complex topological structures arising from the heterogeneity observed in their underlying traits. These systems typically encompass the devices, which possess diverse computational and technological capabilities. In relation to mathematical notions, these complex structures or networks were conceptualized in terms of graph-theoretic models with explicitly higher degrees of connectivity and randomness in their topological structures. The inherent dynamics of these large-scale complex networks still lacked in the requirements of its design principles and anatomy. The studies made in [10–13] provide more admissible principles for the conceptualization of these networks.

At present, several networks (like complex networks) are portraying an extensive degree of heterogeneity in nodes. Some popular examples where these patterns can be distinctively observed include the World Wide Web (WWW), the Internet, and social networks, and aircraft networks. There have been several studies on complex networks, which reveal that in the course of progression in the topological support of network, certain types of emanating networks, referred to as the scale-free networks, have emerged. These networks encompass several peculiarities in their coherent traits and parameters by displaying intrinsic heterogeneity in their behavior. This heterogeneous behavior is observed due to the preferential attachment, which enables the nodes to establish several connections with the set of nodes initially present in the network [10, 14–16]. The degree of the nodes evidently exhibit heterogeneity in their degree distribution, such that the network's degree follows the power-law distribution. The nodes involved in these networks have relatively short distance of interconnectivity, which complies with the small-world property. These systems are very robust toward node failures, due to the densely interconnected network of nodes.

In this paper, we report a scale-free WSN framework for large-scale IoT systems. A brief summary of some of the evolving dynamic network technologies is discussed. We then provide an insight to the basic terminologies relevant to our study like the random graph theory concepts, the small-world concept, and the scale-free networks. The convergence of these technologies with IoT is discussed, and we conceptualize the scale-free traits of IoT. The proposed IoT-based framework for handling dynamic networks is provided. These networks are observed to provide more befitting solutions as compared to the conventional theories for achieving robustness in highly complex communication networks. We simulate the proposed framework to show the scale-free behavior of wireless sensor networks. The power-law distribution, which characterizes the simulated data, is provided. The estimation procedure for the parameters involved in the proposed model, which characterize the simulated data, is reported. The proposed system is believed to work well for networks with high levels of disorder. The presence of the power-law exponent and the scale-free characteristics is evident of the reliability, scalability, and fault tolerance of the system.

The remainder of this paper is as follows: Sect. 2 deals with some of the studies made in the direction of random graph theory, and the scale-free and small-world scenarios along with the applicabilities of the power-law distribution. In Sect. 3, we provide some basic concepts relevant to our problem of interest. The proposed framework is introduced in Sect. 4. The corresponding IoT-based framework for depicting the role of the power-law distribution in characterizing the scale-free behavior of IoT systems is provided in Sect. 5. In Sect. 6, we provide the simulation results obtained for generating the scale-free WSNs along with some insights toward estimating the parameters for the least square curve fitted with the simulated data. Finally, Sect. 7 provides the conclusive remarks and future scopes on the evolving IoT frameworks and the role of the power-law exponent in handling the high degrees of disorder observed in the IoT systems.

2 Background

Most of the physical world objects, irrespective of their capabilities, can be efficiently converged into a single functional unit by utilizing the networking abilities of IoT. In 1960, Erdös and Rênyi [17] proposed the random graph model for large-scale networks, which was designed by combining the conventional approaches of the graph-theoretic concepts and the notion of statistical physics. In [18], Watts and Strogatz studied the importance of the small-world networks for some dynamical systems. They extended the regular graph model by introducing some irregularities to its structure, to better study the versatility of dynamical systems. Their model showed properties analogous to the random graphs, in terms of path lengths.

Several studies have been conducted in the direction of analyzing the dynamic behavior of neuronal networks in context to small-world scenarios. Perotti et al. [19] introduced a neurogenesis algorithm for characterization of neural network activities. The framework defined a power-law distribution among the connectivity of the neural networks, which portray scale-free and small-world characteristics for the addition of new neuron links to the existing ones. Faqeeh et al. [20] addressed the critical factors associated with systems replicating the neuronal dynamics like optimal transmission of information and network stability. They studied the presence of power-law behavior for neuronal avalanches in context to structural heterogeneity. Barábasi et al. [21] studied the complex network behavior for scientific collaborations. Further, the role of the links in defining the scale-free property of the network was explored.

Barábasi and Albert [15] studied the scale-free properties of large-scale networks. They put forward two generic rules: (i) The new vertices associated with a network followed preferential attachment; and (ii) the network topology displayed continuous expansion with the inclusion of new vertices. These features gave rise to the concept of the scale-free distributions in large networks. This model illustrates the self-organizing properties of the vertices involved in a network, which resembles the fundamental properties of real WSNs. The applications of several probabilistic models can be observed in the studies [22–25]. These systems worked well under the influence of growing uncertainties in the system. Barrat et al. [26] studied a network model, which considered the edge weights of each connected node. These weight parameters enable the addition of several essential functionalities associated with a network. Some of them include the variations in the connection strength, factors affecting the connection, and intensity of the connection. This type of network is most suitable for evaluating the performance of the WSNs involved in the IoT.

3 Basic concepts

In this section, some of the primary concepts associated with the evolution of the scale-free networks are discussed. A brief account on the evolution and applicabilities of the random graphs is provided, which resulted in the invention of various evolving technologies. We start with the elementary concepts of the random graphs and then quickly upsurge to the unfolding traits of the scale-free networks.

3.1 Random networks

The evolution of the random graphs has been an elemental revelation to many developing technologies. Over the years, several patterns in nature as well as complex systems have evolved around this theory. In [17], Erdös and Rényi were the first to bring into focus the concept of random graphs. Random graphs have evolved as a consequence of the irregularities observed in the topological arrangements of the regular graphs [14, 17, 27, 28]. The emergence of random graphs has induced a rich set of concepts associated with the degree distribution of the graphs and has favored the growth of several associated perceptions. The complexity of arrangements observed specifically in the random networks and the uncertainty of their organization have made their study prominent while dealing with complex networks. The random graph theory has several applications in modeling complex networks [29].

3.2 Small-world networks

The small-world systems are mostly observed to be highly clustered, which is quite similar to the behavior portrayed by regular lattices [15]. This phenomenon can be observed in several research applications like the spread of contagious diseases, social networks, and the Internet [10, 30–32]. A peculiar example of the small-world networks can be observed in the electricity grid distribution lines, as the distribution lines need to be highly resilient toward the occurrence of any failures in the transmission grid. The small-world networks show an increasingly growing amount of disorder in the network topology, which makes it suitable for modeling the behavior of dynamical systems [33].

3.3 Scale-free networks

In 1999, Barábasi and Albert [15, 34] initially studied the evolution of scale-free networks, which were based on the behavior of real-world networks. A large number of real-world networks display the scale-free property. The scale-free networks are initially constituted of a fixed number of nodes in the network, which shows continuous growth with the addition of randomly incoming nodes. These newly added nodes require to be connected to the existing network, without interfering with the existing topology of the nodes.

Figure 1 shows the topology of the scale-free network, where the newly added nodes are connected to the central node. The large scale-free networks may be constituted of

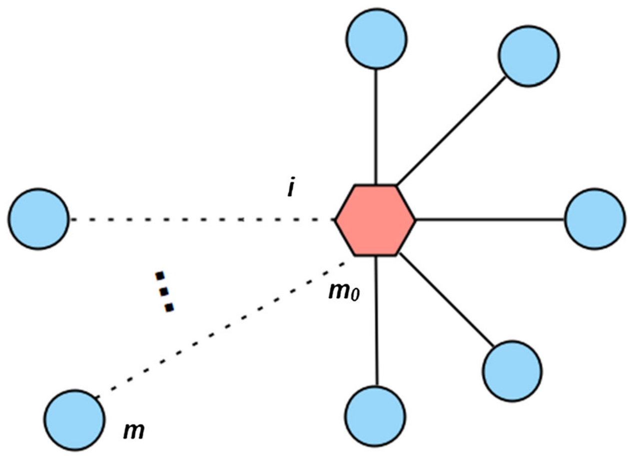

Fig. 1 A topological representation of the scale-free network with the newly added nodes

several central nodes. These central nodes usually have a higher number of connections as compared to the other nodes present in the network.

There are two important features of the scale-free network:

- *Topological growth* As the new incoming nodes are continuously added to the network, the network scales up. A system constituting of an initial number of nodes (say m_0) is considered. If we consider a random set of incoming nodes (say R_m), such that the new node m (for $m \in R_m$) with e_m edges, such that $e_m \leq m_0$.
- *Preferential attachment of nodes* While selecting from the existing set of nodes with which the newly added nodes can be connected, a probability-based attachment principle is used [14, 15], which enables the new node to connect to the existing set of nodes (say N) depending upon the degree (say κ_i) of the node N nodes.

4 Proposed work

In this section, we introduce the proposed scale-free WSN architecture for IoT systems. We also discuss the compliance of the proposed model with some of the conventional network theories.

4.1 Motivations

The ubiquitously growing applications of IoT have gained popularity in some of the most crucial areas like management of transportation services, health care, public safety, and defense systems. To facilitate these capabilities, a highly reliable network is required, which can handle the complexity of the growing network topology with the inclusion of newly added sensory devices. Thus, we propose a scale-free architecture for the IoT systems, which can dynamically handle the increasing network topology of the interconnected devices. We exploit the power-law exponent to study the scalability and reliability aspects of the proposed model. The evolved IoT network is believed to perform well against random sensor node failures and complexity of the interconnection links.

4.2 Convergence with IoT

In this section, we conceptualize the existence of the power-law behavior observed in the topological arrangement of dynamic WSNs and its relevance for characterizing the increased amount of uncertainty observed in the degree distributions of large heterogeneously spread IoT networks. These networks are exclusively tolerant toward the changes caused by the addition of new incoming sensor nodes in the network and are also more reliable in terms of the occurrence of node failures.

4.3 Conceptualizing the small-world scenario for WSNs

Wireless sensor networks (WSNs) have enormously developed in their potentiality and configurations over the years. In order to perceive the continually changing events, a densely deployed WSN is required for obtaining the factual and cumulative data of the environment. This involves the association of a huge number of coupled sensor nodes over a large area for sensing the data coming from heterogeneous sources. The WSNs usually are portrayed as self-organizing systems, which efficiently customize the network topology of the sensor nodes, for the dynamic management of the network activities. These traits distinguish the WSNs from most static, wired, and constrained systems, by making them more adaptable to faults, distance of communication, and power constraints. Due to its capabilities of being versatile, and ubiquitously clustered, the WSNs display high degree of disorder in their organization.

In recent times, the interest in the study of the small-world properties of different scenarios is expanding hugely. Several researchers have explored alternating fields of the real world and have observed intriguing outcomes, which obey the small-world property. Unlike the properties of the regular lattices, the networks that obey the small-world property have relatively higher connectivity among the nodes, show considerable variations in their degree distributions, and have reduced path lengths between the nodes. In [11, 35–37], the authors have extensively addressed the issues in genomic sequencing in terms of the small-world network. Further, the studies in [38–40] have revealed that the functionalities of the human brain network follow the small-world property in response to external stimuli.

The topology of the WSNs is said to follow the small-world property characterized by the densely clustered sensor nodes possessing diverse sensing capabilities. Previously, several issues in the WSNs prevented their extensive use in a multitude of fields. Some of the drawbacks in the static network topology of the WSNs are as follows:

- *Inaccuracy in the signal* Due to the increase in the distance between the two communicating nodes, the propagated signal usually lacks accuracy at the receiving end. These inaccuracies are induced from several conflicting situations like presence of huge obstacles in the ray of sight (ROS) of the signal, delays caused due to excessive path lengths.
- *Node failure* The sensor nodes and microcontrollers involved in a WSN have usually limited power resources. While accomplishing the exchange of information between far off nodes, these nodes usually lose their efficiency due to the constrained power supply. Hence, the network may suffer from node failures, their by minimizing the reliability of the WSN.
- *Energy wastage* In static WSNs, there is no source for regulating the activation and the deactivation of the sensor nodes. All the nodes in the network work uniformly at the same level. This sometimes results in the wastage of energy resources of the sensor nodes as in a network of sensor nodes not all the sensors need to be employed concurrently for accomplishing a certain task.
- *Network topology* The topology of the static WSNs is similar to that of the regular lattice. This topology may not be satisfying for dynamically changing requirements. Therefore, it may be unsuitable for real-world applications.

Hence, it can be said that the regular WSNs may fail to efficiently handle the increase in the number of sensor nodes and the growing interconnection network topology. Thus, bringing about the need for a more robust WSN topology, this can adaptively handle the increase in the amount of uncertainty in the network. This brings in the approach for a large network constituted of heterogeneous sensor nodes with distinct sensing capabilities. These nodes need to be organized in a highly coupled interconnection network such that they provide an accurate exchange of data. Further, this network organization is much desired as it reduces the delay in transmission caused due to lengthy communication paths, and works well against node failures, in an integrative way. These systems are ideal especially in healthcare sectors, and the defense, as they facilitate an uninterrupted flow of data throughout the system, without causing any bifurcations in the perceived data. Thus, the notion of the small-world properties of the WSN emerges, which can be appropriately advantaged as a working model for providing an integrated characterization of the dynamic WSNs. By conceptualizing the small-world properties of the WSNs, we can achieve befitting results in terms of the robustness of the network, functionality, reduced propagation delays, and improved signal quality.

4.4 Conceptualizing the scale-free networks for IoT

In this section, we extend our study toward the application of the scale-free properties of IoT. From the studies made in [26, 41], it is evident that the scale-free networks provide a better understanding of the frequently changing networks and for networks, which have varying flow of information throughout the system. The most fundamental functional modules of the IoT are constituted of the sensor nodes, which are used for the inception of multiple events. In order to infer the data from a wider area, a huge WSN is desired with immensely high-performance constraints. Large WSNs usually require to be dynamic in order to efficiently utilize the resources involved. The WSNs with dynamic properties show a high rate of clustering with the cluster head (or, the central node) being connected to a larger number of sensor nodes.

Thus, the cluster heads are rich in the number of links they establish with each sensor node in the cluster and are therefore said to have a higher degree distribution.

In densely clustered networks like the one desired for modeling the IoT with a heterogeneous set of sensor nodes [15, 34, 41], a huge number of links between the sensor nodes exist. This makes the network more resilient toward link failures and can therefore be used for dynamic systems. When the number of sensor nodes in the IoT architecture scales up continually, the system becomes more vulnerable to failures. Thus, the dynamic capabilities of the scale-free networks make it suitable to model the IoT with huge number of nodes.

Figure 2 shows a scenario for the deployment of several sensor nodes in a city with each central node connected to the sink or, the gateways. This particular case is quite similar to the study in [41], with the gateways acting as the central nodes for several other central nodes (cluster heads) present in the cluster of the sensor nodes. These gateways have a higher degree of connection as observed in [26, 41].

5 The IoT framework

In this section, we present two holistic frameworks based on the conventional small-world phenomena and the scale-free networks [14, 15, 26]. The applicability of these two scenarios has been previously discussed in Sect. 4.2. It was observed

Fig. 2 Deployment scenario for various IoT devices

that large network structures like the IoT, which are characteristically constituted of a relatively heterogeneous set of sensor nodes, converge with the dynamical systems with temporally changing configurations. Thus, by accomplishing a scale-free model for the IoT, the resources can be better utilized (like power supply and memory constraints), which consequently provide a system with enhanced capabilities. It is observed that the degree distribution of the WSNs, under the influence of uncertainties in the dynamic network, follows a power-law distribution. Some other applications of the power-law distribution can be found in [42, 43].

5.1 The small-world scenario

If we consider a WSN to have node set, $v = \{v_1, v_2, \ldots, v_m\}$ where $m = |v|$ denotes the total number of nodes in the WSN. In the random or small-world WSN topology, the connection probability p at ith node having κ_i degrees follows binomial distribution with parameters m and p, i.e.,

$$\kappa_i \sim B(m, p). \tag{1}$$

So,

$$F(\kappa_i = \kappa) = C^{m-1} p^{\kappa} (1-p)^{m-1-\kappa}. \tag{2}$$

The probability of the ith node connected to κ existing nodes is p^{κ}; similarly, the probability that the ith node will not connect to the network is given as $(1-p)^{m-1-\kappa}$. Let X_{κ} be any random variable, representing the number of nodes in the WSN with degree κ. Hence, the expectation value for the number of nodes with degree κ can be obtained from Eq. (2) as follows,

$$E(X_{\kappa}) = mF(\kappa_i = \kappa) = \alpha_{\kappa}, \tag{3}$$

where

$$\alpha_{\kappa} = mC^{m-1} p^{\kappa} (1-p)^{m-1-\kappa}. \tag{4}$$

When $m \gg 0$, and p is very small, the Eq. (2) can be rewritten as,

$$p(\kappa) = \lim_{m \to \infty} \frac{(m-1)(m-2)\ldots(m-2-\kappa)(m-1-\kappa)!}{\kappa!(m-1-\kappa)!} \times \left(\frac{\alpha_{\kappa}}{m}\right)^{\kappa} \left(1 - \frac{\alpha_{\kappa}}{m}\right)^{m-1-\kappa} \tag{5}$$

$$p(\kappa) = \lim_{m \to \infty} \frac{m-1}{m-1} \cdot \frac{m-2}{m-1} \cdot \frac{m-3}{m-1} \cdots \frac{m-1-(1+\kappa)}{m-1} \times \left(\frac{\alpha_{\kappa}^{\kappa}}{\kappa!}\right) \left(1 - \frac{\alpha_{\kappa}}{m}\right)^{m-1} \left(1 - \frac{\alpha_{\kappa}}{m}\right)^{-\kappa} \tag{6}$$

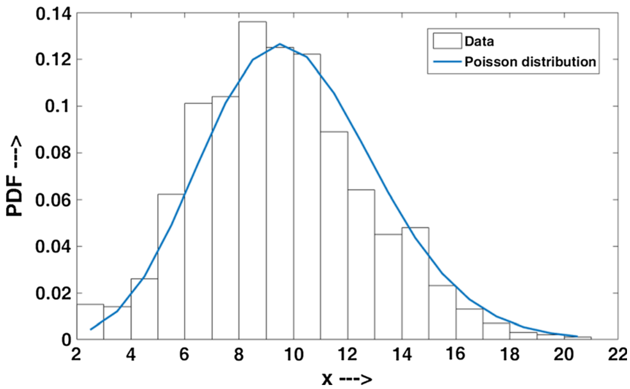

Fig. 3 Representation of the degree distribution with parameters N = 1000, and κ_{avg} = 10.0368 for a random network

Thus,

$$\lim_{m\to\infty}\left(1-\frac{\alpha_\kappa}{m}\right)^{m-1}=e^{-\alpha_\kappa} \tag{7}$$

Hence, the degree distribution of small-world WSNs is given by,

$$F(X_\kappa=\kappa)=\frac{e^{-\alpha_\kappa}\alpha_\kappa^\kappa}{\kappa!}=\frac{e^{-\langle\kappa\rangle}\langle\kappa\rangle^\kappa}{\kappa!} \tag{8}$$

where $\langle\kappa\rangle$ is the average degree of the random networks or small-world WSN topologies. Therefore, from Eq. (8), it is evident that for large number of nodes, the degree distribution of the random or small-world WSN topology follows Poisson distribution.

In Fig. 3, we present the Poisson distribution in compliance with the degree distribution of the simulated data with 1000 nodes and an average degree distribution of κ_{avg} = 10.0368. We have fitted the conventional Poisson distribution over the simulated data on the basis of the parameters estimated using the maximum likelihood estimate.

5.2 The scale-free scenario

The scaling up of the WSN in the support of the IoT infrastructure falls into the category of random networks. The dynamical changes of the ad hoc sensor nodes to the existing WSNs satisfy the scale-free network properties, and the degree distribution of sensor nodes follows a power-law behavior. If we initially consider a network of m_0 sensor nodes, such that m represents the cumulative set of sensor nodes after each time step t, and κ_i represents the degree corresponding to the ith sensor nodes. Finally, this procedure leads to the generation of N set of sensor nodes depicting the scale-free scenario. In Algorithm 1, a procedure for generating a random collection of sensor nodes is provided, which is in line with [14]. The algorithm generates a random set of sensor nodes based on the probability of the degree distribution of the sensor nodes given as,

$$\prod(\kappa_i)=\frac{\kappa_i}{\sum_j\kappa_j} \tag{9}$$

where $\sum_j\kappa_j$ denotes the cumulative degree of nodes present in the network up to stage i.

Algorithm 1: An algorithm for random node generation in scale-free WSNs

Input : m_0, κ_i, m, N

Output : $\prod(\kappa_i)$

$m_0 \leftarrow$ set of sensor nodes initially present in the network such that $m_0 \in N$.

$\kappa_i \leftarrow$ degree distribution of all the sensor nodes in the network.

$m \leftarrow$ set of cumulative number of sensor nodes in the network.

$t \leftarrow$ length of each time step.

$\kappa_j \leftarrow$ degree distribution of newly added sensor nodes.

$N \leftarrow$ set of all generated sensor nodes.

for $m \leq N$ **do**

  $m = m_0 + t$

  Compute:

  $\prod(\kappa_i)=\frac{\kappa_i}{\sum_j \kappa_j}$

end

Following Barabasi and Albert [14], the degree distribution can be modeled as,

$$\frac{\partial \kappa_i}{\partial t} = m \prod(\kappa_i) = \frac{n\kappa_i}{\sum_{j=1}^{n} \kappa_j} \tag{10}$$

After t time steps, the WSN creates $m = t + m_0$ nodes and mt edges. Here, m_0 is the number of initial nodes in the WSN. At each time step, a new node is added to the existing WSN and is connected to nodes present in the network, where $m \leq m_0$. The denominator sum is calculated as,

$$\sum_j \kappa_j = 2mt - m \tag{11}$$

Using Eqs. (10) and (11), we get,

$$\frac{\partial \kappa_i}{\partial t} = \frac{\kappa}{2t} \tag{12}$$

The above equation can be solved under initial condition at the ith node, $\kappa_i\ (t_i) = m$ is,

$$\kappa_i(t) = m\sqrt{\frac{t}{t_i}} \tag{13}$$

The probability that a node having $\kappa_i\ (t_i)$ degree less than κ degree.

$$F(\kappa_i(t) < k) = F\left(t_i > \left(\frac{m}{\kappa}\right)^2 t\right) \tag{14}$$

The probability density function (PDF) for the time step t_i is given by,

$$p(t_i) = \frac{1}{n_o + t} \tag{15}$$

$$F\left(t_i > \left(\frac{m}{\kappa}\right)^2 t\right) = 1 - F\left(t_i \leq t\left(\frac{m}{\kappa}\right)^2\right) \tag{16}$$

$$= 1 - t\left(\frac{m}{\kappa}\right)^2 \frac{1}{n_0 + t} \tag{17}$$

The probability density function can be obtained as,

$$f(\kappa) = \frac{\partial F(k_i(t) < k)}{\partial k} = \frac{2m^2 t}{m_0 + t} \frac{1}{\kappa^3} \tag{18}$$

For very large value of t, i.e., $t \gg 0$,

$$f(\kappa) \sim 2m^2 \kappa^{-3} \tag{19}$$

From Eq. (19), we obtain the degree distribution for a dynamic network of wireless sensor nodes. This behavior is typically portrayed by systems, which are resilient to high degrees of disorder. Thus, by providing a power-law degree distribution for the WSNs, we intend to add more reliability to the network. This can provide a futuristic framework for building the IoT applications.

6 Results and discussions

In this section, we provide the simulation results obtained using MATLAB for illustrating the power-law behavior portrayed by the WSNs [15, 44]. We first generate a scale-free WSN for some specific node values; we then exploit the power-law properties inherently observed in the degree distribution of the sensor nodes.

6.1 Generation of scale-free WSN

We have considered three specific cases for which we generate the scale-free sensor networks. In the first case, we have considered a network, which is constituted of $N = 100$ sensor nodes, with the average degree distribution of the nodes as $\langle \kappa \rangle = 4$, and the interconnection probability of the sensor nodes is $p = 0.5$, which gives rise to 400 links (or edges, E). In [14], it is said that with the increase in the probability of the interconnection link p, the amount of uncertainty of the network increases up to 1. Figure 4 shows the simulated sensor network with the above parameters. In the second case, we consider a WSN with 50 sensor nodes, and the total number of links obtained is 200, with the average degree distribution as 4 and the interconnection probability as 0.6. This network architecture can be observed in Fig. 5. In the third case, we consider a WSN with 20 nodes, and the degree distribution of the nodes is considered as 4 with an interconnection probability of 0.4 giving rise to 120 links, which can be observed in Fig. 6. These networks favor the dynamic behavior of the IoT, by augmenting the performance of the system in terms of reliability, ubiquity, and robustness. Table 1

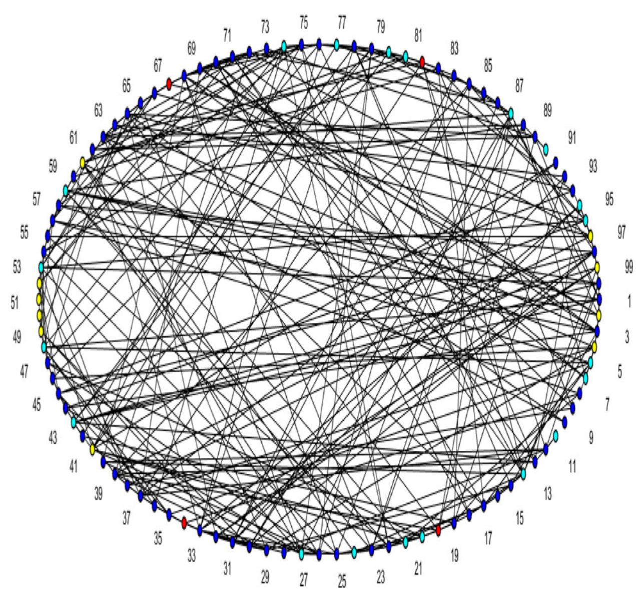

Fig. 4 Generation of the scale-free WSN with 400 edges for the inputs $N = 100$, $p = 0.5$, $\langle \kappa \rangle = 4$

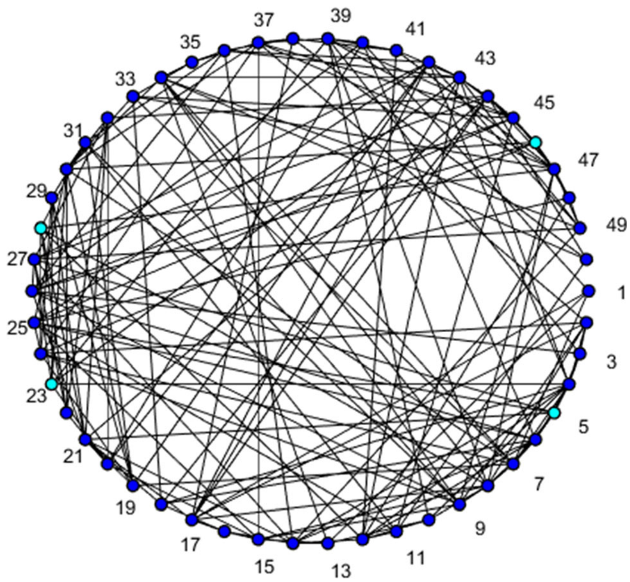

Fig. 5 Generation of the scale-free WSN with 200 edges for the inputs $N = 50$, $p = 0.6$, $\langle \kappa \rangle = 4$

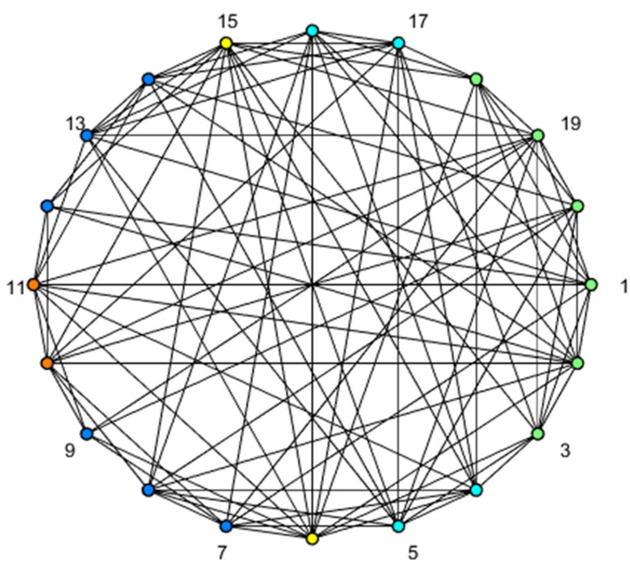

Fig. 6 Generation of the scale-free WSN with 120 edges for the inputs $N = 20$, $p = 0.6$, $\langle \kappa \rangle = 6$

Table 1 Parameters for generation of random nodes for the scale-free IoT framework

Nodes (N)	Probability (p)	Node degree (κ)	Edges (E)
100	0.4	4	400
50	0.6	4	200
20	0.4	6	120

provides a comprehensive account of the specific simulation parameters used for generating the three scale-free WSNs.

6.2 Simulation results

In this section, we provide the simulation results obtained in compliance with the methods used in [44]. We build the simulation environment by initially considering $m_0 = m = 5$ and $m_0 = m = 7$ heterogeneous sensor nodes giving rise to two separate WSNs. Here, the heterogeneity of the sensor nodes is independent of the network topology and is only concerned with the coherent properties of the sensor nodes and the type of data perceived. The two simulated networks considered in this study are constituted of a total of 5000 and 7000 sensor nodes, obtained by incrementally making addition of new sensor nodes to the network. At every step function, a new node is added to the existing m sensor nodes in the WSN. The equation for the line, which entails the simulated data, is obtained in Eq. (20). The line coefficients a and b obtained in Eq. (20) can be estimated using Eqs. (21) and (22), and the corresponding value for each estimated coefficient for the two simulated WSNs is provided in Tables 2 and 3. Further, the confidence intervals of the line coefficients a and b are obtained using Eqs. (25) and (26) corresponding to each set of simulated scale-free WSNs. A detailed account of all the simulation parameters and the fitting parameters are listed in Tables 2 and 3.

The initial connection seed matrices corresponding to Figs. 8 and 9 for generating the simulated scale-free WSN (having $N = 5000$ and $N = 7000$ nodes) are given as,

Table 2 Simulation parameters used for Fig. 8

Parameters	Values
N	5000
m_0	5
m	5
Dimension of seed matrix (M_1)	5×5
Coefficients	$a = 9.3557e+04$
	$b = -2.606$
Confidence interval of a	(8.651e+04, 1.006e+05)
Confidence interval of b	($-$ 2.649, $-$ 2.564)

Table 3 Simulation parameters used for Fig. 9

Parameters	Values
N	7000
m_0	7
m	7
Dimension of seed matrix (M_2)	7×7
Coefficients	a = 2.211e+05
	$b = -2.723$
Confidence interval of a	(2.07e+05, 2.395e+05)
Confidence interval of b	($-$ 2.762, $-$ 2.684)

$$M_1 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix}, \text{ and}$$

$$M_2 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

The line that depicts the scale-free behavior of WSNs is given by

$$y = a + bx \tag{20}$$

Applying the minimum least square method, the coefficient of the fitted line can be estimated as:

$$\hat{a} = \left[\sum_{i=0}^{n} (x_i - \bar{x})(y_i - \bar{y})\right] \left[\sum_{i=0}^{n} (x_i - \bar{x})^2\right]^{-1} \tag{21}$$

and

$$\hat{b} = \bar{y} - \hat{a}\bar{x}^2 \tag{22}$$

The estimated variances of the parameters a and b from the simulated data can be computed as,

$$\sigma_a^2 = \sigma^2 \left[\sum_{i=0}^{n} (x_i - \bar{x})^2\right]^{-1} \tag{23}$$

and

$$\sigma_b^2 = \frac{\sigma^2}{n} \left[1 + n\bar{x}^2 \left[\sum_{i=0}^{n} (x_i - \bar{x})^2\right]^{-1}\right] \tag{24}$$

If we use sample variance S^2 in place of population variance σ^2 in Eqs. (23) and (24) using Student t distribution with $(n - 2)$ degrees of freedom under the significance level $100 \times (1 - \alpha)$, the confidence interval for the coefficients a and b is given by,

$$\hat{a} \pm t_{n-2;\alpha/2} \left[\sqrt{S^2 \left[\sum_{i=0}^{n} (x_i - \bar{x})^2\right]}\right]^{-1} \tag{25}$$

and

$$\hat{b} \pm t_{n-2;\alpha/2} \sqrt{\frac{S^2}{n} \left[1 + n\bar{x}^2 \left[\sum_{i=0}^{n} (x_i - \bar{x})^2\right]^{-1}\right]} \tag{26}$$

In Fig. 7, we provide an account of the theoretical distributions [obtained in Eq. (19)] observed for different values of m_0, considered in the logarithmic scale. It is observed that the degree distribution of the simulated scale-free WSNs comes under the generic family of power-law distributions. From Figs. 8 and 9, it is observed that the simulated results provide good agreement with the least square fitted curve for the parameters listed in Tables 2 and 3. Thus, the results provided in this paper mimic the various scale-free real-world networks ranging from bacterial dynamics to large-scale complex networks like the WWW. So, it can be inferred that the hypothesis of the scale-free networks can be governed by exploiting the scalability and the scale-free property of WSNs, which leads to their applicability in massive futuristic sensor-enabled environments and complex IoT networks.

7 Conclusion and future works

The rapid evolution of the Internet of things (IoT) and its functionalities have led to the concurrence of a more intricate and interconnected network of devices. These devices specifically relate to a generic set of devices like the sensory devices, smart appliances, smart hubs, and other associated objects, which permissively provide a ubiquitous connectivity among the devices. Thus, it is inevitable for the IoT devices to obtain heterogeneity. This gives rise to certain amount of disorder in the network, which the conventional network topologies may not efficiently characterize. Thus, a more dynamic alternative approach has been provided in this paper to befit the requirements of the IoT ecosystem, by adding more reliability to the system, and thereby improving the overall performance of the system. The complexities involved due to the addition of extensively large number of devices to the static WSNs have been discussed. Thus, in order to understand the scalability and the heterogeneity of these interconnected devices, the power-law exponent is an important measure. In this paper, we have provided an

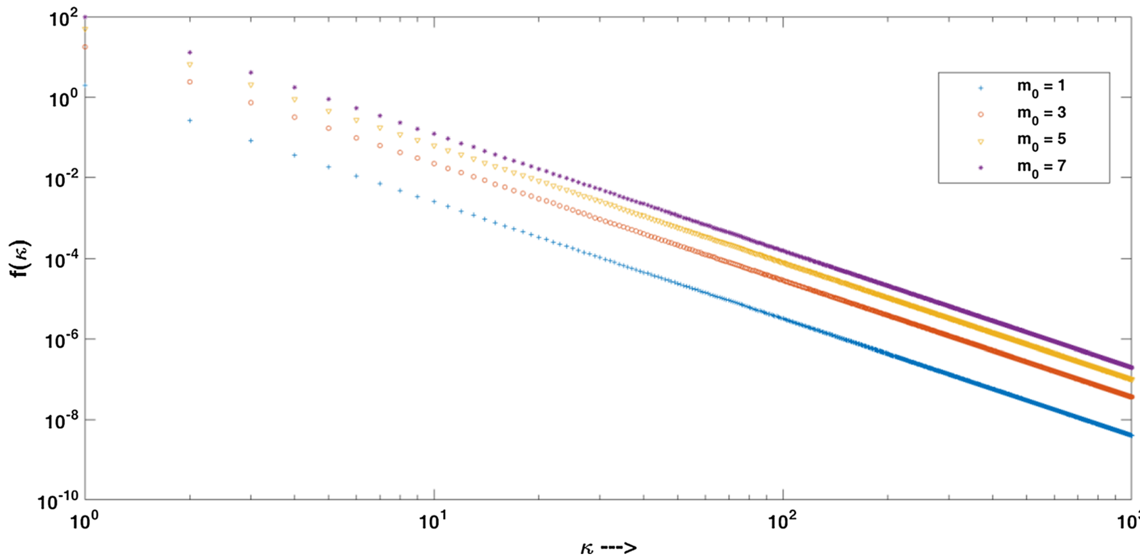

Fig. 7 Representation of the theoretical degree distribution for $m_0 = 1$, $m_0 = 3$, $m_0 = 5$, $m_0 = 7$, and b(slope) = − 3

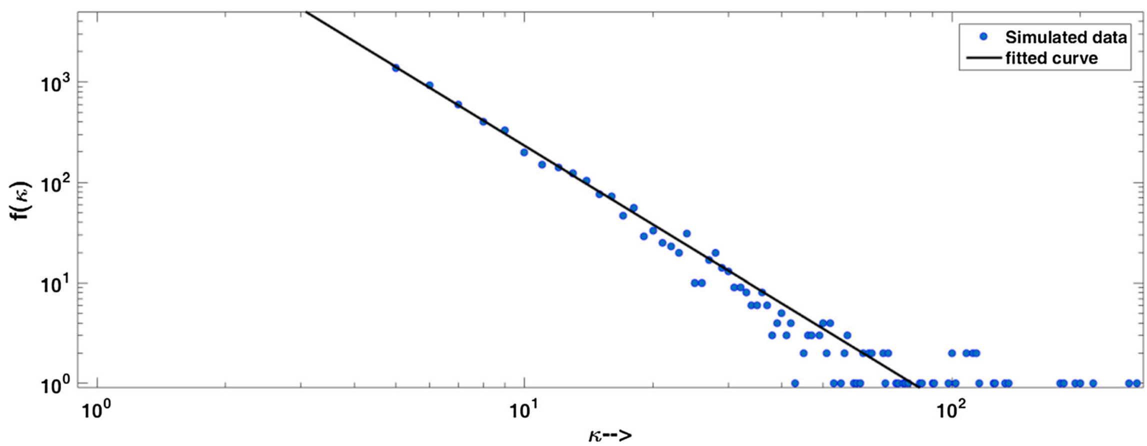

Fig. 8 Degree distribution of the empirical data with power-law distribution for $N = 5000$, $m_0 = m = 5$, $a = 9.3557e+04$, and b(slope) = − 2.606 corresponding to the seed matrix M_1

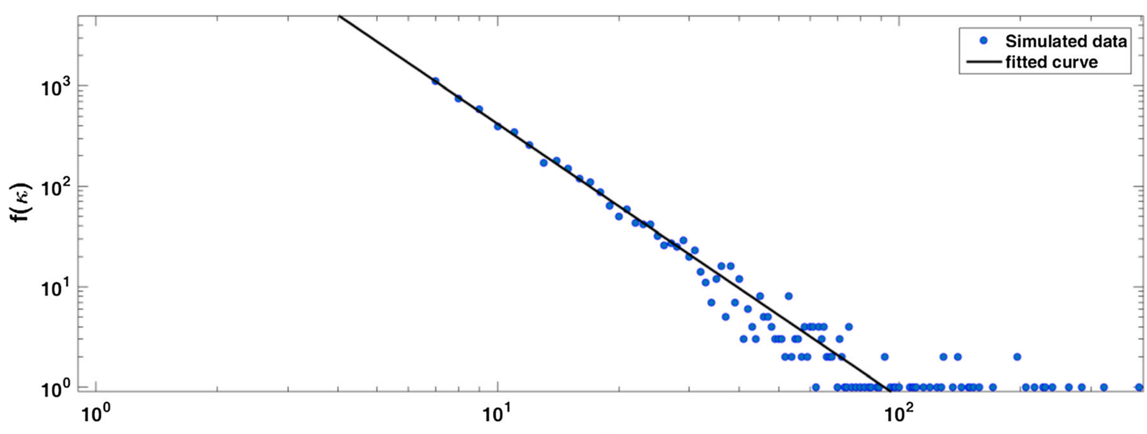

Fig. 9 Degree distribution of the empirical data with power-law distribution for $N = 7000$, $m_0 = m = 7$, $a = 2.211e+05$, and b(slope) = − 2.723 corresponding to the seed matrix M_2

analytical framework to illustrate the ubiquitous power-law behavior observed in the devices connected to the Internet. We have also provided the mathematical insights to characterize the dynamic behavior of the devices connected to the IoT by emphasizing on the small-world traits of the wireless sensor networks (WSNs). Finally, we have provided an analysis of the theoretical and simulation results obtained for the proposed model. It is observed from the simulation results that the degree distribution of the WSNs, which are a crucial element for IoT systems, follows a power-law distribution, and the evolved network is considered to be highly scale-free and reliable.

The study of scale-free networks and the small-world phenomena has led to the evolution of several constituent theories whether it may be in the characterization of the human brain cell interactions or the aircraft systems. Its applicability in new spheres of science and technology is still evolving and gaining popularity.

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