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Novel multi-objective, multi-item and four-dimensional transportation problem with vehicle speed in *LR*-type intuitionistic fuzzy environment

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Abstract

In this paper, we present some novel multi-objective, multi-item and four-dimensional transportation problems in *LR*-type intuitionistic fuzzy environment. Here, for the first time, the speed of different vehicles and rate of disturbance of speed due to the road condition of different routes for the time minimization objective are introduced. Furthermore, three models are presented under three different conditions. The reduced deterministic models are obtained on implementation of a defuzzification approach by using the accuracy function. Moreover, a new method for converting multi-objective problem into single-objective one is proposed and also we use convex combination method. The models are illustrated by some numerical examples and optimal results are presented.

Keywords Four-dimensional transportation problem · Intuitionistic fuzzy number · *LR*-type intuitionistic fuzzy number · Accuracy function

1 Introduction

It is well known that the traditional transportation problem [1] is an optimization problem, which offers huge potential in decreasing costs involving two kinds of constraints taken into consideration, i.e., availability constraint and demand constraint. Afterward, many researchers have studied this problem from various viewpoints and acquired some important results such as [2–5]. However, in real-life situation, we have to deal with another constraint besides source constraint and destination constraint, which is conveyance constraint. To obtain minimum total cost, an appropriate mode of transportation is to be resolved at each source. Subsequently, as an extension of the traditional TP,

the TP with the conveyance constraint is known as solid transportation problem (STP), which was introduced by Haley [6]. Thereafter, the STP has been taken much into consideration and numerous models have been investigated. In the real situation, the solid transportation problem (STP) plays a very important role in global competition for minimizing solid transportation cost, time, distance, road condition and providing service. For example, Bhatia [7] offered an algorithm to solve a STP with indefinite quadratic objective function. Again Pandian et al. [8] presented a new method applying the principle of zero-point method to find an optimal solution of the STP. Pandian et al. [9] also offered a new method named as bound technique to find cost sensitivity ranges of STP (cf. Pramanik et al. [10]).

In real-life problems, there may be various routes/paths to move from a source to a destination. Some of the roads may be in good condition, whereas others may be in bad condition. Distance may differ in a wide range. Again if the transported item in a transportation system is breakable, it may depend on the type of the vehicle, condition of the road and the distance of the path. So it is more realistic to consider the choice of routes along with the vehicles in a STP. Thus, if along with different vehicles, different routes are also considered in a TP, then the STP is changed to a

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four-dimensional TP (4DTP) (cf. Bera et al. [11]). As a result, the distance of different routes from different sources to different destinations is necessary to be considered in 4DTPs, which is not considered in traditional TPs or STPs. In addition, if one has to deal with a time minimization problem, then it is obvious to consider the speed of different vehicles as it is not exact for all types of vehicles. So far, none has considered this fact. If time is to be minimized, then short route is to be considered. Again it may happen that the short routes are in bad condition or may be in traffic jump problem most of the time. Accordingly, the speed of the vehicle may be disturbed due to this. Hence, rate of disturbance of speed due to different routes plays an important role in case of time minimization problem in TPs. This phenomenon is also not considered by any researchers till now. Moreover, in a 4DTP, if different types of item are considered, then the TP becomes a multi-item 4DTP (MI4D-TP). For example, normally a constraint on a resource, say, capacity for expenditure, is forced in the model.

Many researchers studied the TP in deterministic environment, and the related parameters are considered as constant numbers. But, in real life, due to the complexity of the social and economic environment, the relevant parameters in the STPs are often treated as uncertain variables to manage the practical situations. In reality, there exists uncertainty everywhere in practical life problems. As a result, it is not suitable to describe the relevant parameters as constant numbers. Nowadays, the researchers are in a rising interest to model and solve the TPs in which all of the parameters are considered in imprecise environment such as random environment and fuzzy environment. The fuzzy programming approach was introduced by Zimmermann [12] for solving linear programming problem with several objective functions. Numerous works on TP have been executed in imprecise environment.

In real-life optimization problems, one may suppose that an object belongs to a set to a certain degree; however, it is possible that he is not sure regarding this. In other words, there may be hesitation or uncertainty about the membership degree. The main point is that the parameters' demands across the problem are uncertain. However, they are known to fall within a prescribed uncertainty set with some attributed degree. In fuzzy set theory, there is no means to incorporate this hesitation in the membership degree. To incorporate the hesitation in the membership degree, intuitionistic fuzzy sets (IFSs) proposed by Atanassov in 1986 are an extension of type 1 fuzzy set theory. For example, Jimanez et al. [13] developed uncertain solid transportation problems where uncertainty appears in the problem as interval solid transportation problem and fuzzy solid transportation problem. A multi-objective STP in imprecise environments was presented and solved by Pramanik et al. [14]. Again Pramanik et al. [15] considered a fixed-charge multi-objective STP in random fuzzy environment and got the solution. Pramanik et al.[16] also developed a multi-objective solid transportation problem in fuzzy, bi-fuzzy environment via genetic algorithm. Samanta et al. [17], Jana et al. [18] have made some contributions in this field.

Nowadays, fuzzy set theory has been broadly developed and much effort has been given to generalize the ordinary fuzzy set theory concept. As a result, different generalizations and modifications have appeared. For example, an extension of the ordinary fuzzy set theory concept, namely the intuitionistic fuzzy set theory concept, was introduced by Atanassov [19]. Such a concept generalizes the idea of ordinary fuzzy sets by separately considering not only the degree of membership of the elements to a given set, but also the degree of nonmembership along with the degree of hesitation. Thus, the intuitionistic fuzzy set (IFS) concept offers a richer representation to handle the present uncertainty as compared to the ordinary fuzzy sets concept. Angelov [20] introduced an application of the IFSs to optimization problems based on the technique of maximizing the degree of membership and minimizing the degree of nonmembership. Several researchers like Yager [21], Guha et al. [22], Beliakov [23], Hajiagha et al. [24], Chakraborty et al. [25] worked on IFSs. Using chance operator, Chakraborty et al. [26] offered a new approach for solving multi-objective multi-choice, multi-item Atanassova's intuitionistic fuzzy TP. Again Chakraborty et al. [27] also developed arithmetic operations on generalized intuitionistic fuzzy number and applied them to TP. Chakraborty et al. [28] developed the expected value of intuitionistic fuzzy number (IFN) using credibility measures and used this to solve a multi-objective, multi-item, intuitionistic fuzzy solid transportation problems for damageable items. Jana [29] developed novel arithmetic operations on type 2 intuitionistic fuzzy environment and used them to solve a transportation problem. So far we know, there is no method in the literature for solving STP in *LR*-type intuitionistic fuzzy environments, though Singh et al. [30] developed and optimized the unrestricted *LR*-type intuitionistic fuzzy mathematical programming problems. Again Kaur et al. [31] solved fully fuzzy linear programming problems in which the relevant parameters are represented by unrestricted $L-R$ flat fuzzy numbers. Ghanbari et al. [32] offered a method for characterization and also presented an approach to solve the fuzzy linear systems with $L-R$ fuzzy variables.

The main contributions of this paper are summarized as follows:

- For the first time, some multi-objective, multi-item 4DTPs in *LR*-type intuitionistic fuzzy environment are formulated.
- Also for the first time, the speed of different vehicles and rate of disturbance of speed due to different routes for the time minimization objective function are considered.
- We present three models; the first model is about transportation of breakable items which depends on different conveyances along different routes and the type of the item, the second one is about transportation of damageable items which depends on only its type, and the third one considers nonbreakable/damageable items.
- The reduced deterministic models are obtained on implementation of a defuzzification approach by using the accuracy function..
- We propose a new method for converting multi-objective optimization problem into single-objective one and also use convex combination method
- The models are illustrated by some numerical examples, and optimal results are presented in tabular forms.

The rest of the paper is organized as follows.

In Sect. 2, the proposed method for converting multi-objective optimization problem into single-objective problem is described. In Sect. 3, notations for the proposed models are listed. In Sect. 4, we formulate the models considered in this paper. Equivalent crisp models are obtained in Sect. 5. The numerical illustration is provided in Sect. 6, and a set of optimal solutions are shown here. In Sect. 7, the practical implication has been given. The concluding remarks are given in Sect. 8. The paper ends with "Appendix," which reviews some basic definitions and theorems related to IFNs and *LR*-type IFNs and the convex combination method.

2 Novel method for conversion of multi-objective into single objective

Preliminary ideas on *LR*-type TIFN, arithmetic operations on *LR*-type TIFN and convex combination method (CCM) are given in the "Appendix" section.

In this section, a new approach for multi-objective optimization problem is offered. The proposed approach has been motivated by Zimmermann [12] concepts and the CCM.

Let us consider the following multi-objective decision-making model in Eq. (1) as:

$$\begin{cases} \max & f_i(x); \ i = 1, 2, \ldots, m' \\ \min & f_j(x), \ j = m' + 1, m' + 2, \ldots, m \\ \text{s.t} & \begin{cases} g_r(x) \le c_r; & r = 1, 2, \ldots, n' \\ g_r(x) \ge c_r; & r = n' + 1, n' + 2, \ldots, n \\ x \ge 0 \end{cases} \end{cases} \quad (1)$$

The objective functions $f_i(x); i = 1, 2, \ldots, m'$, of model in Eq. (1) are to be maximized. So for each objective, we introduce the membership function as

$$\mu_i(f_i(x)) = \begin{cases} 1 & \text{for} \quad f_i(x) > U^i \\ \dfrac{f_i(x) - L^i}{U^i - L^i} & \text{for} \quad L^i \le f_i(x) \le U^i \\ 0 & \text{for} \quad f_i(x) < L^i \end{cases} \quad (2)$$

where L^i and U^i are the lower and upper bounds for each objective. Again some of the objective functions $f_j(x); j = m' + 1, m' + 2, \ldots, m$ of model (1) are to be minimized, so for each objective we introduce the membership function as

$$\mu_j(f_j) = \begin{cases} 0 & \text{for} \ f_j(x) \ge U^j \\ \dfrac{U^j - f_j(x)}{U^j - L^j} & \text{for} \ L^j \le f_j(x) \le U^j \\ 1 & \text{for} \ f_j \le L^j \end{cases} \quad (3)$$

Currently, our major objective is to increase the level of satisfaction of the decision maker, i.e., the membership functions for both the maximizing objective functions and minimizing objective functions. So problem (1) can be modeled as:

$$\begin{cases} \max & [\mu_i(f_i(x)), \quad i = 1, 2, \ldots, m] \\ \text{s.t} & \begin{cases} g_r(x) \le c_r; & r = 1, 2, \ldots, n' \\ g_r(x) \ge c_r; & r = n' + 1, n' + 2, \ldots, n \\ x \ge 0 \end{cases} \end{cases} \quad (4)$$

This model can be reduced to the single-objective problem as follows:

$$\begin{cases} \max & \sum_{i=1}^{m} \alpha_i \mu_i(f_i(x)), \quad \text{where} \ \sum_{i=1}^{m} \alpha_i = 1, 0 < \alpha_i < 1; i = 1, 2, \ldots, m \\ \text{s.t} & \begin{cases} g_r(x) \le c_r; & r = 1, 2, \ldots, n' \\ g_r(x) \ge c_r; & r = n' + 1, n' + 2, \ldots, n \\ x \ge 0 \end{cases} \end{cases}$$

$$(5)$$

where α_i is the weight function of membership function of the *i*th objective.

Corresponding x and $f_i(x)$ of Eq. (5) are the solutions of the problem in Eq. (1).

3 Notations

To formulate the multi-objective, multi-item and four-dimensional transportation model, the following notations are used:

(i) m = number of sources.

(ii) n = number of destinations.

(iii) K = number of conveyances, i.e., different modes of the transportation problem.

(iv) Q = number of routes.

(v) T = number of items.

(vi) Z_k^I = the objective functions, where $k = 1, 2$.

(vii) p_{it}^I = the purchasing price of tth item at the ith origin.

(viii) s_{jt}^I = the selling price of tth item at the jth destination.

(ix) c_{ijkqt}^I = the unit transportation cost of tth item from ith source to jth destination by kth conveyance via qth route per unit distance.

(x) x_{ijkqt} = the decision variable which is the amount of the tth item to be transported from ith source to jth destination by kth conveyance via qth route.

(xi) β_{ijkqt} = the rate of breakability per unit distance of the tth item from ith source to jth destination by kth conveyance via qth route.

(xii) d_{ijq} = distance from ith source to jth destination via qth route.

(xiii) v_k = speed of the kth conveyance .

(xiv) δ_{ijq} = Rate of disturbance of speed due to qth route from ith source to jth destination.

(xv) γ_{ijkt}^I = loading and unloading time of the tth item with respect to the transportation activity from ith source to jth destination by kth conveyance.

(xvi) a_{it}^I = the amount of the tth item available at the ith origin.

(xvii) b_{jt}^I = the demand of the tth item at the jth destination.

(xviii) e_k^I = capacity of a single vehicle of kth conveyance.

4 Model formulation

We assume that there are m origins (or sources) O_i $(i = 1, 2, \ldots, m)$, n destinations (or demands) $D_j (j = 1, 2, \ldots, n)$, K conveyances $E_k (k = 1, 2, \ldots, K)$, i.e., different modes of transport may be trucks, cargo flights, goods trains, ships, etc., Q routes $G_q (q = 1, 2, \ldots, Q)$ and

T items $P_t (t = 1, 2, \ldots, T)$. Moreover, in this model, the objectives are to maximize the profit incurred by the transportation activities and also to minimize the total transportation time. At the present situation, it is not so simple for anyone to estimate the exact amount of related parameters. Practically, in application of TP, decision makers may face different uncertainties such as availability of raw materials in the sources, demands in the destinations and unit transportation cost due to various unmanageable factors. These unmanageable factors in a TP may be as follows: (1) There exists uncertainty regarding the product availability at a source due to the time factors, (2) there exists some sort of vagueness about the total demand of a newly launched product to the market, (3) there may not be a decision maker who exactly knows the unit transportation cost of the first-time transportation operation.

Considering this fact, we take the problem under *LR*-type Intuitionistic fuzzy environment, which aims to make service strategies on the tactical planning level. So, we formulate the problem assuming that parameters are all *LR*-type trapezoidal intuitionistic fuzzy variables.

Furthermore, the transported item is likely to be broken or damaged during the time of transportation. There are several causes for this damageability.

- This damageability may depend on different conveyances along different routes and the type of the items. There are some items mainly made of glass such as ceramics and China clay. These types of items depend on the above conditions.
- Moreover, there are some items where the damageability depends on only its type.
- Again there are a number of items which are not damageable.

So, keeping all these in mind, here we present three models. The first model is about transportation of breakable items which depend on different conveyances along different routes and its type. So here we use β_{ijkqt} as the rate of breakability per unit distance of the tth item from ith source to jth destination by kth conveyance via qth route. The second one is about damageable items which depend on only their type. Here, we use β_t^I as the rate of breakability per unit distance of the tth item. And the third one considers nonbreakable/damageable items.

4.1 Model 1: model with breakable items which depend on conveyances, routes and items

The First objective of the problem is to maximize the total profit, which is as follows:

$$\max \quad Z_1^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \sum_{t=1}^{T}$$
$$\left[(s_{jt}^I (1 - \beta_{ijkqt} d_{ijq}) - p_{it}^I - c_{ijkqt}^I d_{ijq}) \; x_{ijkqt} \right]$$

The other objective is to minimize the total transportation time, which is as follows:

$$\min \quad Z_2^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \frac{1}{v_k(1 - \delta_{ijq})} d_{ijq} y_{ijkq}$$
$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \sum_{t=1}^{T} \gamma_{ijkt}^I x_{ijkqt}$$

Here, if the items are transported from ith source to jth destination by kth conveyance via qth route, then only the transporting time will be taken. So, we introduce a binary relation as

$$y_{ijkq} = \begin{cases} 1, & \text{if } \sum_{t=1}^{T} x_{ijkqt} > 0; \\ 0, & \text{if } \sum_{t=1}^{T} x_{ijkqt} = 0 \end{cases}$$

Now, the constraints are the supply constraints, demand constraints and capacity constraints. As the quantity of a item from a source cannot exceed the supply capacity, so we have

$$\sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ijkqt} \le a_{it}^I, \quad i = 1, 2, 3, \ldots, m; t = 1, 2, \ldots, T$$

Again the quantity of a item transported to a destination should not be less than its demand, that is

$$\sum_{i=1}^{m} \sum_{k=1}^{K} \sum_{q=1}^{Q} (1 - \beta_{ijkqt} d_{ijq}) x_{ijkqt} \ge b_{jt}^I,$$
$$j = 1, 2, 3, \ldots, n; t = 1, 2, \ldots, T$$

The third constraint requires the total amount of items transported from different sources to different destinations by conveyance k to be not greater than its transportation capacity. So we have

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{q=1}^{Q} \sum_{t=1}^{T} x_{ijkqt} \ge e_k^I, \quad k = 1, 2, \ldots, K$$

It is usual to have the nonnegativity of decision variable x_{ijkqt}, that is

$$x_{ijkqt} \ge 0, \quad i = 1, 2, 3; j = 1, 2, 3, \ldots, n;$$
$$k = 1, 2, \ldots, K; q = 1, 2, \ldots, Q; t = 1, 2, \ldots, T$$

So, the multi-objective, multi-item and four-dimensional transportation problem can be written as:

$$\max \quad Z_1^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \sum_{t=1}^{T}$$
$$\left[(s_{jt}^I (1 - \beta_{ijkqt} d_{ijq}) - p_{it}^I - c_{ijkqt}^I d_{ijq}) \; x_{ijkqt} \right]$$
$$(6)$$

$$\min \quad Z_2^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \frac{1}{v_k(1 - \delta_{ijq})} d_{ijq} y_{ijkq}$$
$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} \sum_{t=1}^{T} \gamma_{ijkt}^I x_{ijkqt}$$
$$(7)$$

where

$$y_{ijkq} = \begin{cases} 1, & \text{if } \sum_{t=1}^{T} x_{ijkqt} > 0; \\ 0, & \text{if } \sum_{t=1}^{T} x_{ijkqt} = 0 \end{cases} \qquad (8)$$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{q=1}^{Q} x_{ijkqt} \le a_{it}^I, \quad i = 1, 2, 3, \ldots, m; t = 1, 2, \ldots, T$$
$$(9)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} \sum_{q=1}^{Q} (1 - \beta_{ijkqt} d_{ijq}) x_{ijkqt} \ge b_{jt}^I,$$
$$j = 1, 2, 3, \ldots, n; t = 1, 2, \ldots, T$$
$$(10)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{q=1}^{Q} \sum_{t=1}^{T} x_{ijkqt} \ge e_k^I, \quad k = 1, 2, \ldots, K \qquad (11)$$

$$x_{ijkqt} \ge 0, \quad i = 1, 2, 3 \ldots, m; j = 1, 2, 3, \ldots, n; k = 1, 2, \ldots, K;$$
$$q = 1, 2, \ldots, Q; t = 1, 2, \ldots, T$$
$$(12)$$

4.2 Model 2: model with breakable items which depend on only their type

Now, we formulate a problem where we select some items where the damageability depends on only its type. So here we use β_t as the rate of breakability per unit distance of the tth item instead of β_{ijkqt} which is taken in the previous model. So the first objective and the second constraint are changed. The rest are same as the previous model. As a

result, the multi-objective, multi-item and four-dimensional transportation problem can be written as:

$$\begin{cases} \max \quad Z_1^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T} \\ \left[\left(s_{jt}^I(1-\beta_t d_{ijq}) - p_{it}^I - c_{ijkqt}^I d_{ijq} \right) x_{ijkqt} \right] \\ \text{with Eqs. (7) and (8)} \end{cases} \quad (13)$$

subject to

$$\begin{cases} \text{Equations (9), (11), (12) and} \\ \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q}(1-\beta_t d_{ijq})x_{ijkqt} \geq b_{jt}^I, \quad j=1,2,3,\ldots,n; t=1,2,\ldots,T \end{cases}$$
$$(14)$$

4.3 Model 3: model with no breakable items

Here, we present a model where we select some items where no damageability occurs. So, we do not consider any rate of breakability in this case. The multi-objective, multi-item and four-dimensional transportation problem becomes:

$$\begin{cases} \max \quad Z_1^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T} \left[\left(s_{jt}^I - p_{it}^I - c_{ijkqt}^I d_{ijq} \right) x_{ijkqt} \right] \\ \text{with Eqs. (7) and (8)} \end{cases}$$
$$(15)$$

subject to

$$\begin{cases} \text{Equations (9), (11), (12) and} \\ \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q}x_{ijkqt} \geq b_{jt}^I, \quad j=1,2,3,\ldots,n; \quad t=1,2,\ldots,T \end{cases}$$
$$(16)$$

5 Equivalent crisp problem

The above TP with imprecise market supplies, demands, capacities, costs and loading and unloading time can be stated as equivalent deterministic problem by using the accuracy function, which is given as follows.

5.1 For Model 1

The deterministic form is as follows:

$$\max f[Z_1^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}$$
$$\left(f[s_{jt}^I](1-\beta_{ijkqt} d_{ijq}) - f[p_{it}^I] - f[c_{ijkqt}^I]d_{ijq} \right) x_{ijkqt}$$
$$(17)$$

$$\min f[Z_2^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\frac{1}{v_k(1-\delta_{ijq})}d_{ijq}y_{ijkq}$$
$$+ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}f[\gamma_{ijkt}^I]x_{ijkqt}$$
$$(18)$$

where

$$y_{ijkq} = \begin{cases} 1, & \text{if } \sum_{t=1}^{T}x_{ijkqt} > 0; \\ 0, & \text{if } \sum_{t=1}^{T}x_{ijkqt} = 0 \end{cases} \quad (19)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}x_{ijkqt} \leq f[a_{it}^I], \quad i=1,2,3,\ldots,m; t=1,2,\ldots,T$$
$$(20)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q}(1-\beta_{ijkqt}d_{ijq})x_{ijkqt} \geq f[b_{jt}^I], \quad j=1,2,3,\ldots,n;$$
$$t=1,2,\ldots,T$$
$$(21)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{Q}\sum_{t=1}^{T}x_{ijkqt} \geq f[e_k^I], \quad k=1,2,\ldots,K \quad (22)$$

$$x_{ijkqt} \geq 0, \quad i=1,2,3; j=1,2,3,\ldots,n; k=1,2,\ldots, K; q=1(Q); t=1(T).$$
$$(23)$$

Here, s_{jt}^I p_{it}^I, c_{ijkqt}^I, γ_{ijkt}^I, a_{it}^I, b_{jt}^I and e_k^I are considered as *LR*-type TIFNs; these can be denoted as: $s_{jt}^I = (s_{jt}^1, s_{jt}^2; s_{jt}^l, s_{jt}^r; s_{jt}^{l'}, s_{jt}^{r'})$, $p_{it}^I = (p_{it}^1, p_{it}^2; p_{it}^l, p_{it}^r; p_{it}^{l'}, p_{it}^{r'})$, $c_{ijkqt}^I = (c_{ijkqt}^1, c_{ijkqt}^2; c_{ijkqt}^l, c_{ijkqt}^r; c_{ijkqt}^{l'}, c_{ijkqt}^{r'})$, $\gamma_{ijkt}^I = (\gamma_{ijkt}^1, \gamma_{ijkt}^2; \gamma_{ijkt}^l, \gamma_{ijkt}^r; \gamma_{ijkt}^{l'}, \gamma_{ijkt}^{r'})$, $a_{it}^I = (a_{it}^1, a_{it}^2; a_{it}^l, a_{it}^r; a_{it}^{l'}, a_{it}^{r'})$, $b_{jt}^I = (b_{jt}^1, b_{jt}^2; b_{jt}^l, b_{jt}^r; b_{jt}^{l'}, b_{jt}^{r'})$, $e_k^I = (e_k^1, e_k^2; e_k^l, e_k^r; e_k^{l'}, e_k^{r'})$, respectively. Then, the above model in Eqs. (17)–(23) can be written in the following form by using Eq. (55):

$$\max \ f[Z_1^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}$$

$$\left[\frac{2s_{jt}^1 + 2s_{jt}^2 - \frac{s_{jt}^l}{2} + \frac{s_{jt}^r}{2} - \frac{s_{jt}^{l'}}{2} + \frac{s_{jt}^{r'}}{2}}{4} \times \left(1 - \beta_{ijkqt}d_{ijq}\right) \right.$$

$$- \frac{2p_{it}^1 + 2p_{it}^2 - \frac{p_{it}^l}{2} + \frac{p_{it}^r}{2} - \frac{p_{it}^{l'}}{2} + \frac{p_{it}^{r'}}{2}}{4}$$

$$\left. - \left(\frac{2c_{ijkqt}^1 + 2c_{ijkqt}^2 - \frac{c_{ijkqt}^l}{2} + \frac{c_{ijkqt}^r}{2} - \frac{c_{ijkqt}^{l'}}{2} + \frac{c_{ijkqt}^{r'}}{2}}{4} \right) \times \left(d_{ijq}\right) \right] x_{ijkqt} \tag{24}$$

$$\min \ f[Z_2^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q} \frac{1}{v_k(1-\delta_{ijq})} d_{ijq} y_{ijkq}$$

$$+ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}$$

$$\frac{2\gamma_{ijkt}^1 + 2\gamma_{ijkt}^2 - \frac{\gamma_{ijkt}^l}{2} + \frac{\gamma_{ijkt}^r}{2} - \frac{\gamma_{ijkt}^{l'}}{2} + \frac{\gamma_{ijkt}^{r'}}{2}}{4} x_{ijkqt} \tag{25}$$

where

$$y_{ijkq} = \begin{cases} 1, & \text{if } \sum_{t=1}^{T} x_{ijkqt} > 0; \\ 0, & \text{if } \sum_{t=1}^{T} x_{ijkqt} = 0 \end{cases} \tag{26}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q} x_{ijkqt} \leq \frac{2a_{it}^1 + 2a_{it}^2 - \frac{a_{it}^l}{2} + \frac{a_{it}^r}{2} - \frac{a_{it}^{l'}}{2} + \frac{a_{it}^{r'}}{2}}{4}, \tag{27}$$
$$i = 1(m); t = 1, 2, \ldots, T$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q} (1 - \beta_{ijkqt}d_{ijq}) x_{ijkqt}$$

$$\geq \frac{2b_{jt}^1 + 2b_{jt}^2 - \frac{b_{jt}^l}{2} + \frac{b_{jt}^r}{2} - \frac{b_{jt}^{l'}}{2} + \frac{b_{jt}^{r'}}{2}}{4}, \quad j = 1(n); t = 1(T) \tag{28}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{Q}\sum_{t=1}^{T} x_{ijkqt} \geq \frac{2e_k^1 + 2e_k^2 - \frac{e_k^l}{2} + \frac{e_k^r}{2} - \frac{e_k^{l'}}{2} + \frac{e_k^{r'}}{2}}{4},$$
$$k = 1, 2, \ldots, K \tag{29}$$

$$x_{ijkqt} \geq 0, \quad i = 1, 2, 3; j = 1, 2, 3, \ldots, n; k = 1, 2, \ldots,$$
$$K; q = 1, 2, \ldots, Q. \tag{30}$$

5.2 For Model 2

Here, all the parameters are the same as Model 1 except the rate of breakability, i.e., β_t^I. We consider the rate of breakability as $\beta_t^I = (\beta_t^1, \beta_t^2; \beta_t^l, \beta_t^r; \beta_t^{l'}, \beta_t^{r'})$, By using the accuracy function values as stated in Eq. (55), the deterministic equivalent form of Model 2 can be written as follows:

$$\max \ f[Z_1^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}$$

$$\left[\left(\frac{2s_{jt}^1 + 2s_{jt}^2 - \frac{s_{jt}^l}{2} + \frac{s_{jt}^r}{2} - \frac{s_{jt}^{l'}}{2} + \frac{s_{jt}^{r'}}{2}}{4} \right) \times (1 - \beta_t d_{ijq}) \right.$$

$$- \frac{2p_{it}^1 + 2p_{it}^2 - \frac{p_{it}^l}{2} + \frac{p_{it}^r}{2} - \frac{p_{it}^{l'}}{2} + \frac{p_{it}^{r'}}{2}}{4}$$

$$\left. - \left(\frac{2c_{ijkqt}^1 + 2c_{ijkqt}^2 - \frac{c_{ijkqt}^l}{2} + \frac{c_{ijkqt}^r}{2} - \frac{c_{ijkqt}^{l'}}{2} + \frac{c_{ijkqt}^{r'}}{2}}{4} \right) \times \left(d_{ijq}\right) \right] x_{ijkqt} \tag{31}$$

with Eqs. (25) and (26) subject to Eqs. (27), (29), (30) and

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q} (1 - \beta_t d_{ijq}) x_{ijkqt} \geq \frac{2b_{jt}^1 + 2b_{jt}^2 - \frac{b_{jt}^l}{2} + \frac{b_{jt}^r}{2} - \frac{b_{jt}^{l'}}{2} + \frac{b_{jt}^{r'}}{2}}{4},$$
$$j = 1, 2, 3, \ldots, n; t = 1, 2, \ldots, T \tag{32}$$

5.3 For Model 3

Here, the deterministic equivalent form of Model 3 is as follows:

$$\max \ f[Z_1^I] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{t=1}^{T}$$

$$\left[\frac{2s_{jt}^1 + 2s_{jt}^2 - \frac{s_{jt}^l}{2} + \frac{s_{jt}^r}{2} - \frac{s_{jt}^{l'}}{2} + \frac{s_{jt}^{r'}}{2}}{4} \right.$$

$$- \frac{2p_{it}^1 + 2p_{it}^2 - \frac{p_{it}^l}{2} + \frac{p_{it}^r}{2} - \frac{p_{it}^{l'}}{2} + \frac{p_{it}^{r'}}{2}}{4}$$

$$\left. - \left(\frac{2c_{ijkqt}^1 + 2c_{ijkqt}^2 - \frac{c_{ijkqt}^l}{2} + \frac{c_{ijkqt}^r}{2} - \frac{c_{ijkqt}^{l'}}{2} + \frac{c_{ijkqt}^{r'}}{2}}{4} \right) \times \left(d_{ijq}\right) \right] x_{ijkqt} \tag{33}$$

with Eqs. (25) and (26) subject to Eqs. (27), (29), (30) and

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{q=1}^{Q} x_{ijkqt} \geq \frac{2b_{jt}^1 + 2b_{jt}^2 - \frac{b_{jt}^l}{2} + \frac{b_{jt}^r}{2} - \frac{b_{jt}^{l'}}{2} + \frac{b_{jt}^{r'}}{2}}{4},$$
$$j = 1(n); t = 1(T).$$

6 Numerical experiment

In this segment, the following example of a multi-objective, multi-item and four-dimensional transportation problem to illustrate the efficiency and effectiveness of the proposed approach is considered. The selling prices, purchasing costs of different items, availabilities of these items in the corresponding origins, demands in the destinations, capacity of each vehicle, unit transportation costs of different items per unit distance and loading and unloading time are assumed as LR-type TIFNs, which are as follows:

6.1 Input data

In this experiment, we assume two origins or sources, two destinations, two conveyances, two routes from each source to each destination and two items, i.e., $m = 2$, $n = 2$, $K = 2$, $Q = 2$ and $T = 2$.

The availabilities of items in the origins, the demands of items in the destinations and the capacity of conveyances are given in Tables 1, 2 and 3.

Now, the purchasing price, selling price of different items and unit transportation cost of different items per unit distance between different origins and destinations by different conveyances via different routes are given in Tables 4, 5 and 6.

Again rate of breakability of different items per unit distance, distance between different origins and

Table 1 Availabilities in the sources

i	t	a_{it}^I
1	1	(287, 292; 1, 3; 3, 5)
	2	(207, 212; 2, 4; 4, 6)
2	1	(179, 181; 4, 3; 5, 6)
	2	(194, 200; 2, 3; 6, 5)

Table 2 Demands in the destinations

j	t	b_{jt}^I
1	1	(78, 85; 1, 3; 3, 5)
	2	(48, 52; 2, 2; 4, 4)
2	1	(32, 38; 1, 1; 3, 3)
	2	(68, 72; 1, 1; 2, 2)

Table 3 Capacities of the two types for conveyances

e_1^I	e_2^I
(345, 350; 1, 3; 4, 6)	(363, 366; 2, 2; 4, 8)

Table 4 Purchasing prices for different items

i	t	p_{it}^I
1	1	(6, 12; 1, 1; 3, 4)
	2	(5, 10; 2, 3; 3, 6)
2	1	(4, 8; 1, 1; 3, 3)
	2	(5, 10; 2, 1; 5, 2)

Table 5 Selling prices for different items

j	t	s_{jt}^I
1	1	(54, 60; 2, 3; 5, 4)
	2	(82, 87; 1, 3; 4, 6)
2	1	(80, 83; 2, 2; 4, 8)
	2	(70, 75; 3, 1; 5, 3)

destinations via different routes, speed of the different conveyances, rate of disturbance of speed due to different routes between different origins and destinations are given in Tables 7, 8, 9 and 10.

On addition, the loading and unloading times with respect to the transportation activity of two items from different sources to different destinations by different conveyances are given in Table 11.

6.2 Optimum results

The generalized reduced gradient method (GRG) technique is a method for solving NLP and LPP problems for handling equality as well as inequality constraints in an optimization problem. The solution techniques used here are the generalized reduced gradient (GRG) technique (using LINGO-14.0 solver).

6.2.1 For Model 1

The deterministic optimization problems of Model 1 given by Eqs. (24)–(30) are solved for the above data. We have calculated the values of U^i and L^i, $i = 1, 2$. for each objective separately. These values are given as follows:

$$U^1 = 23512.71, \quad L^1 = 13634.39, \quad U^2 = 103.07, \quad L^2 = 43.85$$

As the first objective is to maximize and the second objective is to minimize, using Eqs. (2) and (3) the membership functions $\mu_1(Z_1(x))$ and $\mu_2(Z_2(x))$ are defined as follows:

Table 6 Unit transportation costs of different items per unit distance

i	j	k	q	c^I_{ijkq1}	c^I_{ijkq2}
1	1	1	1	(0.13, 0.15; 0.01, 0.02; 0.04, 0.03)	(0.22, 0.26; 0.02, 0.01; 0.03, 0.04)
			2	(0.12, 0.2; 0.01, 0.02; 0.05, 0.04)	(0.17, 0.23; 0.02, 0.02; 0.03, 0.03)
		2	1	(0.12, 0.14; 0.03, 0.02; 0.04, 0.05)	(0.11, 0.15; 0.01, 0.03; 0.06, 0.04)
			2	(0.17, 0.2; 0.03, 0.02; 0.05, 0.06)	(0.06, 0.14; 0.03, 0.01; 0.04, 0.06)
	2	1	1	(0.09, 0.17; 0.02, 0.03; 0.05, 0.04)	(0.08, 0.16; 0.01, 0.02; 0.06, 0.05)
			2	(0.1, 0.14; 0.03, 0.01; 0.05, 0.07)	(0.15, 0.17; 0.01, 0.02; 0.04, 0.03)
		2	1	(0.08, 0.12; 0.02, 0.02; 0.05, 0.05)	(0.2, 0.28; 0.01, 0.03; 0.07, 0.05)
			2	(0.17, 0.23; 0.02, 0.03; 0.06, 0.05)	(0.36, 0.38; 0.02, 0.01; 0.03, 0.04)
2	1	1	1	(0.12, 0.16; 0.03, 0.02; 0.06, 0.07)	(0.12, 0.15; 0.02, 0.01; 0.05, 0.06)
			2	(0.12, 0.2; 0.01, 0.02; 0.04, 0.03)	(0.12, 0.2; 0.03, 0.02; 0.05, 0.06)
		2	1	(0.06, 0.14; 0.03, 0.01; 0.04, 0.06)	(0.18, 0.22; 0.02, 0.03; 0.05, 0.04)
			2	(0.17, 0.19; 0.01, 0.02; 0.04, 0.03)	(0.36, 0.4; 0.02, 0.02; 0.05, 0.05)
	2	1	1	(0.08, 0.16; 0.02, 0.03; 0.06, 0.05)	(0.082, 0.142; 0.02, 0.01; 0.03, 0.04)
			2	(0.15, 0.17; 0.03, 0.02; 0.06, 0.07)	(0.1, 0.14; 0.02, 0.02; 0.03, 0.03)
		2	1	(0.09, 0.12; 0.03, 0.02; 0.04, 0.05)	(0.16, 0.24; 0.01, 0.03; 0.06, 0.04)
			2	(0.16, 0.24; 0.01, 0.02; 0.04, 0.03)	(0.06, 0.14; 0.01, 0.03; 0.07, 0.05)

Table 7 Rate of breakability of different items per unit distance

i	j	k	q	β_{ijkq1}	β_{ijkq2}
1	1	1	1	0.015	0.025
			2	0.015	0.016
		2	1	0.01	0.011
			2	0.25	0.015
	2	1	1	0.016	0.026
			2	0.025	0.016
		2	1	0.015	0.015
			2	0.012	0.013
2	1	1	1	0.015	0.025
			2	0.025	0.025
		2	1	0.02	0.02
			2	0.022	0.01
	2	1	1	0.013	0.017
			2	0.015	0.011
		2	1	0.013	0.02
			2	0.013	0.012

Table 8 Distance from the sources to the destinations via different routes

i	j	d_{ij1}	d_{ij2}
1	1	30	40
	2	42	30
2	1	40	35
	2	53	40

Table 9 Speed of the two types of conveyances

$v_1 = 30$	$v_2 = 20$

Table 10 Rate of disturbance of speed due to two routes

i	j	δ^I_{ij1}	δ^I_{ij2}
1	1	0.0022	0.01
	2	0.003	0.002
2	1	0.02	0.005
	2	0.001	0.02

$\mu_1(Z_1(x))$

$$= \begin{cases} 1 & \text{for } Z_1(x) \geq 23512.71 \\ \dfrac{Z_1(x) - 13634.39}{23512.71 - 13634.39} & \text{for } 13634.39 \leq Z_1(x) \leq 23512.71 \\ 0 & \text{for } Z_1(x) \leq 13634.39 \end{cases}$$

$\mu_1(Z_2(x))$

$$= \begin{cases} 0 & \text{for } Z_2(x) \geq 103.07 \\ \dfrac{103.07 - Z_2(x)}{103.07 - 43.85} & \text{for } 43.85 \leq Z_2(x) \leq 103.07 \\ 1 & \text{for } Z_2(x) \leq 43.85 \end{cases}$$

Now to get an efficient solution, we compute the following reduced deterministic single-objective problem with the constraints in Eqs. (27)–(30).

$$\max \quad \alpha_1\mu_1(Z_1(x)) + \alpha_2\mu_2(Z_2(x)), \quad \text{where} \quad \alpha_1 + \alpha_2 = 1, 0 < \alpha_i < 1 \tag{34}$$

After solving this, we obtain the solutions for total profit (Z_1) and total transportation time (Z_2) of the proposed

Table 11 Loading and unloading times

i	j	k	γ^I_{ijk1}	γ^I_{ijk2}
1	1	1	(0.08, 0.12; 0.03, 0.01; 0.04, 0.06)	(0.17, 0.23; 0.03, 0.02; 0.05, 0.06)
		2	(0.24, 0.26; 0.03, 0.01; 0.04, 0.06)	(0.13, 0.17; 0.02, 0.03; 0.05, 0.04)
	2	1	(0.27, 0.33; 0.01, 0.02; 0.04, 0.03)	(0.21, 0.23; 0.02, 0.02; 0.05, 0.05)
		2	(0.09, 0.13; 0.01, 0.03; 0.07, 0.05)	(0.18, 0.24; 0.02, 0.03; 0.06, 0.05)
2	1	1	(0.31, 0.35; 0.02, 0.03; 0.06, 0.05)	(0.47, 0.53; 0.02, 0.01; 0.03, 0.04)
		2	(0.19, 0.21; 0.03, 0.02; 0.06, 0.07)	(0.78, 0.82; 0.02, 0.02; 0.03, 0.03)
	2	1	(0.87, 0.93; 0.03, 0.02; 0.04, 0.05)	(0.09, 0.11; 0.02, 0.01; 0.03, 0.04)
		2	(0.09, 0.13; 0.01, 0.02; 0.05, 0.04)	(0.2, 0.26; 0.01, 0.03; 0.07, 0.05)

Table 12 Optimum results with different values of α_1, α_2 via proposed method for Model 1

α_1	α_2	μ_1	μ_2	Z_1	Z_2	x_{ijkqt}
0.95	0.05	0.995	0.153	23,461.33	94.03	$x_{11111} = 135.00$, $x_{11211} = 1.93$, $x_{11212} = 210.00$, $x_{12221} = 153.07$, $x_{21111} = 16.00$, $x_{22122} = 197.00$
0.90	0.10	0.992	0.158	23,435.01	93.73	$x_{11111} = 144.00$, $x_{11212} = 210.00$, $x_{12221} = 146.00$, $x_{21111} = 7.00$, $x_{22122} = 197.00$, $x_{22221} = 9.00$
0.85	0.15	0.988	0.208	23,398.82	90.76	$x_{11111} = 151$, $x_{11212} = 210.00$, $x_{12221} = 139.00$, $x_{22122} = 197.00$, $x_{22221} = 16.00$
0.5	0.5	0.987	0.211	23,387.31	90.57	$x_{11111} = 149.09$, $x_{11212} = 210.00$, $x_{12221} = 140.91$, $x_{22122} = 197.00$, $x_{22221} = 14.09$
0.4	0.6	0.951	0.272	23,029.96	86.98	$x_{11111} = 149.09$, $x_{11212} = 210.00$, $x_{12221} = 140.91$, $x_{22122} = 197.00$
0.35	0.65	0.126	0.736	14,877.96	59.47	$x_{11111} = 149.09$, $x_{11212} = 74.63$, $x_{12221} = 140.91$, $x_{22122} = 125.00$
0.3	0.7	0	0.798	13,634.39	55.81	$x_{11111} = 149.09$, $x_{11212} = 74.63$, $x_{12221} = 107.62$, $x_{22122} = 125.00$

Table 13 Optimum results via convex combination method for Model 1

w_1	w_2	Z_1	Z_2	x_{ijkqt}
0.9	0.1	23, 512.71	103.07	$x_{11111} = 135.00$, $x_{11211} = 11.07$, $x_{11212} = 210.00$, $x_{12221} = 143.93$, $x_{22121} = 16.00$, $x_{22122} = 197.00$
0.1	0.9	23, 461.33	94.03	$x_{11111} = 135.00$, $x_{11211} = 1.93$, $x_{11212} = 210.00$ $x_{12221} = 153.07$, $x_{21111} = 16.00$, $x_{22122} = 197.00$

Table 14 Optimum results with different values of α_1, α_2 via proposed method for Model 2

α_1	α_2	μ_1	μ_2	Z_1	Z_2	x_{ijkqt}
0.95	0.05	0.994	0.251	36,196.62	146.76	$x_{11111} = 96.47$, $x_{11212} = 210.00$, $x_{12111} = 54.53$, $x_{12211} = 139.00$, $x_{21112} = 101.76$, $x_{22112} = 95.24$, $x_{22211} = 16.00$
0.80	0.20	0.9449195	0.5644063	35,047.92	101.7604	$x_{11111} = 96.47$, $x_{11112} = 156.29$, $x_{11212} = 53.71$, $x_{12211} = 193.53$, $x_{22112} = 95.24$, $x_{22211} = 117.76$
0.60	0.40	0.92	0.61	34,496.27	95.65	$x_{11111} = 96.47$, $x_{11112} = 54.53$, $x_{11212} = 155.47$, $x_{12211} = 193.53$, $x_{22112} = 197.00$, $x_{22211} = 16.00$
0.4	0.6	0.89	0.63	33,785.75	92.38	$x_{11111} = 151.00$, $x_{11212} = 210.00$, $x_{12211} = 139.00$, $x_{22112} = 197.00$, $x_{22211} = 16.00$
0.35	0.65	0.82	0.67	32,025.30	86.34	$x_{11111} = 151.00$, $x_{11212} = 58.82$, $x_{12211} = 139.00$, $x_{22112} = 197.00$, $x_{22211} = 167.18$
0.30	0.70	0.739	0.710	30,259.91	80.88	$x_{11111} = 96.47$, $x_{11212} = 58.82$, $x_{12211} = 193.53$, $x_{22112} = 197.00$, $x_{22211} = 112.65$
0.1	0.9	0.000000056	1	13,026.18	39.25	$x_{11111} = 96.47$, $x_{11212} = 58.82$, $x_{12211} = 44.30$, $x_{22112} = 95.24$

Table 15 Optimum results via convex combination method for Model 2

w_1	w_2	Z_1	Z_2	x_{ijkqt}
0.90	0.10	36,331.59	182.76	$x_{11111} = 62.39$, $x_{11211} = 19.02$, $x_{11212} = 121.39$, $x_{12112} = 88.61$, $x_{12211} = 208.59$, $x_{21112} = 197.00$, $x_{21211} = 16.00$
0.50	0.50	36,320.66	172.17	$x_{11111} = 81.41$, $x_{11212} = 140.41$, $x_{12112} = 69.59$, $x_{12211} = 208.59$, $x_{21112} = 176.56$, $x_{21211} = 16.00$, $x_{22112} = 20.44$
0.10	0.90	36,196.62	146.76	$x_{11111} = 96.47$, $x_{11212} = 210.00$, $x_{12111} = 54.53$, $x_{12211} = 139.00$, $x_{21112} = 101.76$, $x_{22112} = 95.24$, $x_{22211} = 16.00$

Table 16 Optimum results with different values of α_1, α_2 via proposed method for Model 3

α_1	α_2	μ_1	μ_2	Z_1	Z_2	x_{ijkqt}
0.95	0.05	0.995	0.500	48,063.95	156.83	$x_{11111} = 82.00$, $x_{11212} = 210.00$, $x_{12121} = 69.00$, $x_{21112} = 127.00$, $x_{22122} = 70.00$, $x_{22211} = 155.00$
0.90	0.10	0.981	0.641	47,588.18	126.57	$x_{11111} = 82.00$, $x_{11112} = 25.00$, $x_{11212} = 185.00$, $x_{12121} = 171.00$, $x_{22122} = 70.00$, $x_{22211} = 180.00$
0.50	0.50	0.925	0.817	45,640.12	88.97	$x_{11111} = 82.00$, $x_{11112} = 69.00$, $x_{12211} = 185.00$, $x_{22122} = 197.00$, $x_{22211} = 180.00$
0.35	0.65	0.910	0.826	45,143.75	87.07	$x_{11111} = 101.00$, $x_{11112} = 50.00$, $x_{12211} = 185.00$, $x_{22122} = 197.00$, $x_{22211} = 180.00$
0.30	0.70	0.879	0.833	44,058.75	85.58	$x_{11111} = 151.00$, $x_{11212} = 50.00$, $x_{12211} = 135.00$, $x_{22122} = 197.00$, $x_{22211} = 180.00$

Table 17 Optimum results via convex combination method for Model 3

w_1	w_2	Z_1	Z_2	x_{ijkqt}
0.95	0.50	48,250.60	198.42	$x_{11212} = 210.00$, $x_{12121} = 126.00$, $x_{21112} = 127.00$, $x_{21211} = 82.00$, '$x_{22111} = 25.00$, $x_{22122} = 70.00$, $x_{22211} = 73.00$
0.60	0.40	48,241.48	181.65	$x_{11212} = 210.00$, $x_{12121} = 151.00$, $x_{21112} = 127.00$, $x_{21211} = 82.00$, $x_{22122} = 70.00$, $x_{22211} = 73.00$
0.10	0.90	48,063.95	156.83	$x_{11111} = 82.00$, $x_{11212} = 210.00$, $x_{12121} = 69.00$, $x_{21112} = 127.00$, $x_{22122} = 70.00$, $x_{22211} = 155.00$

Model 1 Eqs. (6)–(12) for the above data. The solutions are listed in Table 12. We also obtain the solutions for Model 1 after conversion of multi-objective into single objective by using CCM, which is given in Table 13.

6.2.2 For Model 2

To get the optimum results for this model, we use the above data except the rate of breakability, i.e., β_{ijkqt}. The corresponding values are $\beta_1 = 0.005$, $\beta_2 = 0.01$. With respect to the deterministic optimization problems of Model 2 given by Eqs. (25)–(27) and (29)–(32) using these data, the calculated values of U^i and L^i, $i = 1, 2$. are given as follows:

$$U^1 = 36331.59, \quad L^1 = 13026.18, \quad U^2 = 182.76, \quad L^2 = 39.25$$

Consequently, we formulate the membership functions $\mu_1(Z_1)$ and $\mu_2(Z_2)$. Then, after getting the reduced

deterministic single-objective problem, we obtain the solutions of Model 2 (Eqs. (13) and (14)), which are listed in Table 14. Again Table 15 contains the solutions of Model 2 after conversion of multi-objective into single objective by using CCM.

6.2.3 For Model 3

At last, we get the optimum results for Model 3 by using the same input data as given in Sect. 6.1 except the rate of breakability, because in this model we consider non-breakable/damageable items. In this case, the calculated values of U^i and L^i, $i = 1, 2$. are given as follows:

$$U^1 = 48250.95, \quad L^1 = 13634.39, \quad U^2 = 263.50, \quad L^2 = 49.99$$

Here also, we formulate the membership functions $\mu_1(Z_1)$ and $\mu_2(Z_2)$. Then, after getting the reduced deterministic single-objective problem, we obtain the solutions of Model

3 given by Eqs. (15) and (16), which are listed in Table 16. Again Table 17 contains the solutions of Model 3 after conversion of multi-objective into single objective by using CCM.

6.3 Discussion

Tables 12, 14 and 16 give the optimum results for Model 1, Model 2 and Model 3, respectively, by our proposed method of conversion of multi-objective optimization problem into single-objective optimization problem. Here, the optimum results are presented for different values of the weight functions of the membership functions of two objectives. While we increase the weight function of the membership function of the first objective, i.e., α_1, and decrease the weight function of the membership function of the second objective, i.e., α_2, we observe that the profit increases and the time also increases which is an expected phenomenon as we have to maximize profit and minimize time. Also we observe that the membership functions of the objective functions change as we desire. Again Tables 13, 15 and 17 give the optimum results for Model 1, Model 2 and Model 3, respectively, by CCM.

In Model 1, considering the highest value of α_1 and the lowest value of α_2 among all the taken values, the optimum profit is \$23,461.33 and optimum time is 94.03 min. Again considering the lowest value of α_1 and the highest value of α_2 among all the taken values, the optimum profit is \$13,634.39 and optimum time is 55.81 min. However, in the second model for the highest value of α_1 and for the lowest value of α_2 among all the considering values, the optimum profit is \$36,196.62 and optimum time is 146.76 min. Again for the lowest value of α_1 and the highest value of α_2 among all the taken values, the optimum profit is \$13,026.18 and optimum time is 39.25 min. In Model 3 for the highest value of α_1 and the lowest value of α_2 among all the taken values, the optimum profit is \$48,063.95 and optimum time is 156.83 min. Again for the lowest value of α_1 and the highest value of α_2 among all the considering values, the optimum profit is \$44,058.75 and optimum time is 85.58 min. From these results, we have observed that for Model 1, we get maximum profit. Again considering Model 3, we get minimum profit, whereas a moderate profit is obtained for Model 2, which is expected. This is because in Model 1, damage of the items occurs and it depends on different conveyances along different routes and the type of the item, in Model 2 damage of the items occurs but it depends on only its type and in Model 3, no damage occurs.

7 Practical implication

The proposed model is very much helpful in the real-world business sectors. A retailer, Mr. Parimal Das, has outlets of different types of glass containers at three different places Haldia, Kharagpur and Panskura in West Bengal, India. He collects the products from different manufacturing companies situated in Kolkata, Howrah and Durgapur in West Bengal, India. The products can be transported from different manufacturing companies to the retailer through different paths such as NH-4, NH-6 (NH: National Highway) and other village paths using different conveyances like lorry, tempo, truck, etc. Since there are different paths between one manufacturing company and one retailer, the distances will differ. Moreover, as glass container is a breakable item, depending on the path and the type of the conveyance, rate of breakability will be different. On the other hand, as there are different types of conveyances between one manufacturing company and one retailer, the speed of each conveyance is different, which will affect the time minimization objective. Furthermore, depending on the path, the speed of each conveyance will vary due to the road condition. As a result, rate of disturbance of speed occurs. As customers' mood is unpredictable, the availabilities at different manufacturing companies vary, different prices, times vary; these parameters can be represented as uncertain or imprecise quantities by LR-type TIFNs. Hence, the present investigation will be helpful to execute the above real-life phenomena.

8 Conclusions

Here, we concentrate on dealing with novel multi-objective, multi-item 4DTP with the relevant parameters in $L-R$-type intuitionistic fuzzy environment. To the best of our knowledge, this is not done so far. It is more realistic to consider the choice of routes along with the vehicles in a TP, as we have done here. As a result, the distance of different routes from different sources to different destinations is taken into consideration. For the first time, the speed of different vehicles and rate of disturbance of speed due to different routes for the time minimization objective are introduced which are quite realistic. The reduced deterministic models are obtained on implementation of a defuzzification approach by using the accuracy function. We propose a new method for converting multi-objective problem into single objective one. Here, we present three models: The first model is about transportation of breakable items which depends on different conveyances along different routes and the type of the item, the second one is about transportation of damageable items which depends

on only its type, and the third one considers nonbreakable/damageable items. The models are illustrated by some numerical examples, and optimal results are presented in tabular forms for all the models. In the future, this approach can be applied for different realistic problems such as supply chain network design, transportation problems including space constraints, price discounts on the basis of amount of transported units. As a direction for future research, more investigation can be carried out to develop a multi-choice, multi-objective 4DTP and also to include the aspects related to the sustainability in the 4DTP as well.

Compliance with ethical standards

Appendix

Preliminaries

In this section, we recall some basic definitions and results which will be used in the next sections of this paper.

Definition 1 An intuitionistic fuzzy set (IFS) is a generalization of the ordinary fuzzy sets, which is characterized by a membership function and a nonmembership function. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a collection of some objects, then an intuitionistic fuzzy set A^I in X is defined as the form of an ordered triplet $A^I = \{\langle x_i, \mu_{A^I}(x_i), v_{A^I}(x_i)\rangle / x_i \in X\}$, where $\mu_{A^I}(x_i) : X \to [0, 1]$ is called the membership function or grade of membership of x_i in A^I and $v_{A^I}(x_i) : X[0, 1]$ is called the nonmembership function or grade of nonmembership of x_i in A^I satisfying the condition $0 \leq \mu_{A^I}(x_i) + v_{A^I}(x_i) \leq 1$. $\pi_{A^I}(x) = 1 - \mu_{A^I}(x) - v_{A^I}(x)$ represents the degree of hesitation or the degree of indeterminacy of x_i being in A^I in X and $0 \leq \pi_{A^I} \leq 1$.

Definition 2 The α-cut of an IFS A^I is denoted as A_α^I and is given by:

$$A_\alpha^I = \{x \in X : \mu_{A^I}(x) \geq \alpha\}, \quad \forall \alpha \in [0, 1]$$

Definition 3 The β-cut of an IFS A^I is denoted as A_β^I and is given by:

$$A_\beta^I = \{x \in X : v_{A^I}(x) \leq \beta\}, \quad \forall \beta \in [0, 1]$$

Definition 4 The (α, β)-cut of an IFS A^I is denoted as $A_{\alpha,\beta}^I$ and is given by:

$$A_{\alpha,\beta}^I = \{x \in X : \mu_{A^I}(x) \geq \alpha, v_{A^I}(x) \leq \beta; \alpha + \beta \leq 1\},$$
$$\forall \alpha, \beta \in [0, 1]$$

Definition 5 An IFS $A^I = \{\langle x, \mu_{A^I}(x), v_{A^I}(x)\rangle / x \in \mathfrak{R}\}$ is called an intuitionistic fuzzy number (IFN) if the following hold

(i) There exists $m \in \mathfrak{R}$ such that $\mu_{A^I}(m) = 1$ and $v_{A^I}(m) = 0$ (m is called the mean value of A^I)

(ii) The membership function μ_{A^I} and nonmembership function v_{A^I} are piecewise continuous functions from R to the closed interval $[0, 1]$ and $0 \leq \mu_{A^I}(x) + v_{A^I}(x) \leq 1$, $\forall x \in \mathfrak{R}$. μ_{A^I}, v_{A^I} are of the following forms:

$$\mu_{A^I}(x) = \begin{cases} f_1(x), & \text{for } m - l < x < m \\ 1, & \text{for } x = m \\ f_2(x), & \text{for } m < x < m + r \\ 0, & \text{otherwise} \end{cases}$$

and

$$v_{A^I}(x) = \begin{cases} g_1(x), & \text{for } m - l' < x < m; 0 \leq f_1(x) + g_1(x) \leq 1 \\ 0, & \text{for } x = m \\ g_2(x), & \text{for } m < x < m + r'; 0 \leq f_2(x) + g_2(x) \leq 1 \\ 1, & \text{otherwise} \end{cases}$$

Here, f_1 and f_2 are piecewise continuous, strictly increasing and strictly decreasing functions in $(m - l, m)$ and $(m, m + r)$, respectively. Again g_1 and g_2 are piecewise continuous, strictly decreasing and strictly increasing functions in $(m - l', m)$ and $(m, m + r')$, respectively. l and r are the left and right spreads of membership function μ_{A^I}, respectively. Again l' and r' are the left and right spreads of nonmembership function v_{A^I}, respectively. The IFN A^I is represented by $(m; l, r; l', r')$.

Definition 6 An IFN A^I is called trapezoidal intuitionistic fuzzy number (TIFN) if its membership function μ_{A^I} and nonmembership function v_{A^I} are as follows:

$$\mu_{A^I}(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1}, & \text{for } r_1 \leq x \leq r_2 \\ 1, & \text{for } r_2 \leq x \leq r_3 \\ \dfrac{r_4 - x}{r_4 - r_3}, & \text{for } r_3 \leq x \leq r_4 \\ 0, & \text{otherwise} \end{cases}$$

and

$$v_{A^I}(x) = \begin{cases} \dfrac{r_2 - x}{r_2 - r_1'}, & \text{for } r_1' \leq x \leq r_2 \\ 0, & \text{for } r_2 \leq x \leq r_3 \\ \dfrac{x - r_3}{r_4' - r_3}, & \text{for } r_3 \leq x \leq r_4' \\ 1, & \text{otherwise} \end{cases}$$

where $r_1' \leq r_1 \leq r_2 \leq r_3 \leq r_4 \leq r_4'$. The TIFN A^I in $\Re$ is represented as $(r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ with its membership function μ_{A^I} and nonmembership function v_{A^I}.

Definition 7 A function $f : [0, \infty) \rightarrow [0, 1]$ is called a shape function if the following conditions hold:

(a) $f(0) = 1$
(b) f is continuous function on $[0, \infty)$
(c) f is decreasing on $[0, \infty)$ and
(d) $\lim_{x \to \infty} f(x) = 0$.

Definition 8 An IFN A^I is called *LR*-type IFN, so that for membership function $\mu_{A^I}(x)$ and nonmembership function $v_{A^I}(x)$, $0 \leq \mu_{A^I}(x) + v_{A^I}(x) \leq 1$ holds and is defined as follows:

$$\mu_{A^I}(x) = \begin{cases} L\left(\dfrac{m - x}{l}\right), & \text{for } x \leq m \\ R\left(\dfrac{x - m}{r}\right), & \text{for } x \geq m \end{cases}$$

and

$$v_{A^I}(x) = \begin{cases} 1 - L\left(\dfrac{m - x}{l'}\right), & \text{for } x \leq m \\ 1 - R\left(\dfrac{x - m}{r'}\right), & \text{for } x \geq m \end{cases}$$

Definition 9 The *LR*-type representation of a TIFN $A^I = (r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ is given by $A^I = (r_2, r_3; r_2 - r_1, r_4 - r_3; r_2 - r_1', r_4' - r_3)$, and its membership function μ_{A^I} and nonmembership function v_{A^I} are defined by

$$\mu_{A^I}(x) = \begin{cases} L\left(\dfrac{r_2 - x}{r_2 - r_1}\right), & \text{for } x \leq r_2 \\ 1, & \text{for } r_2 \leq x \leq r_3 \\ R\left(\dfrac{x - r_3}{r_4 - r_3}\right), & \text{for } x \geq r_3 \end{cases}$$

and

$$v_{A^I}(x) = \begin{cases} 1 - L\left(\dfrac{r_2 - x}{r_2 - r_1'}\right), & \text{for } x \leq r_2 \\ 0, & \text{for } r_2 \leq x \leq r_3 \\ 1 - R\left(\dfrac{x - r_3}{r_4' - r_3}\right), & \text{for } x \geq r_3 \end{cases}$$

where $L(x) = R(x) = \max\{0, 1 - x\}$; $r_2 - r_1$ and $r_4 - r_3$ are the left spread and right spread of the membership function μ_{A^I}, respectively, whereas $r_2 - r_1'$ and $r_4' - r_3$ are the left spread and right spread of the nonmembership function v_{A^I}, respectively.

Theorem 1 *The α-cut and β-cut of a TIFN $A^I = (r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ are given by $A_\alpha^I = [r_1 + (r_2 - r_1)\alpha, r_4 - (r_4 - r_3)\alpha]$ and $A_\beta^I = [r_2 - (r_2 - r_1')\beta, r_3 + (r_4' - r_3)\beta]$, $\forall \alpha, \beta \in (0, 1]]$*

Proof For $\alpha \in (0, 1]$,

$$\mu_{A^I}(x) \geq \alpha \Rightarrow \frac{x - r_1}{r_2 - r_1} \geq \alpha, \frac{r_4 - x}{r_4 - r_3} \geq \alpha$$
$$\Rightarrow x \geq r_1 + (r_2 - r_1)\alpha, x \leq r_4 - (r_4 - r_3)\alpha$$
$$\Rightarrow r_1 + (r_2 - r_1)\alpha \leq x \leq r_4 - (r_4 - r_3)\alpha$$
$$\Rightarrow A_\alpha^I = [r_1 + (r_2 - r_1)\alpha, r_4 - (r_4 - r_3)\alpha]$$

Now for $\beta \in (0, 1]$,

$$v_{A^I}(x) \leq \beta \Rightarrow \frac{r_2 - x}{r_2 - r_1'} \leq \beta, \frac{x - r_3}{r_4' - r_3} \leq \beta$$
$$\Rightarrow x \geq r_2 - (r_2 - r_1')\beta, x \leq r_3 + (r_4' - r_3)\beta$$
$$\Rightarrow r_2 - (r_2 - r_1')\beta \leq x \leq r_3 + (r_4' - r_3)\beta$$
$$\Rightarrow A_\beta^I = [r_2 - (r_2 - r_1')\beta, r_3 + (r_4' - r_3)\beta]$$

Hence, it is proved. □

Theorem 2 *The (α, β)-cut of a TIFN $A^I = (r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ is given by $A_{\alpha, \beta}^I = [r_1 + (r_2 - r_1)\alpha, r_4 - (r_4 - r_3)\alpha] \cap [r_2 - (r_2 - r_1')\beta, r_3 + (r_4' - r_3)\beta]$, $\forall \alpha, \beta \in (0, 1]]$*

Proof For $\alpha \in (0, 1]$, the α-cut of a TIFN $A^I = (r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ is given by $A_\alpha^I = [r_1 + (r_2 - r_1)\alpha, r_4 - (r_4 - r_3)\alpha]$.

For $\beta \in (0, 1]$, the β-cut of the TIFN $A^I = (r_1, r_2, r_3, r_4; r_1', r_2, r_3, r_4')$ is given by $A_\beta^I = [r_2 - (r_2 - r_1')\beta, r_3 + (r_4' - r_3)\beta]$, $\forall \beta \in (0, 1]]$.

So by Definition 4, $A_{\alpha, \beta}^I = [r_1 + (r_2 - r_1)\alpha, r_4 - (r_4 - r_3)\alpha] \cap [r_2 - (r_2 - r_1')\beta, r_3 + (r_4' - r_3)\beta]$, $\forall \alpha \in (0, 1], \alpha + \beta \leq 1$.

Hence, it is proved. □

Theorem 3 *Let $A^I = (m, n; l, r; l', r')$ be a LR-type TIFN, where l and r are the left spread and right spread of the membership function μ_{A^I}, l' and r' are the left spread and*

right spread of the nonmembership function v_{A^I}. Then, its α-cut and β-cut are given by $A_\alpha^I = [m - lL^{-1}(\alpha), n + rR^{-1}(\alpha)]$ and $A_\beta^I = [m - l'L^{-1}(1-\beta), n + rR^{-1}(1-\beta)]$, $\forall \alpha, \beta \in (0, 1]$.

Proof For $\alpha \in (0, 1]$,

$$\mu_{A^I}(x) \geq \alpha \Rightarrow L\left(\frac{m-x}{l}\right) \geq \alpha, R\left(\frac{x-n}{r}\right) \geq \alpha$$
$$\Rightarrow x \geq m - lL^{-1}(\alpha), x \leq n + rR^{-1}(\alpha)$$
$$\Rightarrow m - lL^{-1}(\alpha) \leq x \leq n + rR^{-1}(\alpha)$$
$$\Rightarrow A_\alpha^I = [m - lL^{-1}(\alpha), n + rR^{-1}(\alpha)]$$

Now for $\beta \in (0, 1]$,

$$v_{A^I}(x) \leq \beta \Rightarrow 1 - L\left(\frac{m-x}{l'}\right) \leq \beta, 1 - R\left(\frac{x-n}{r'}\right) \leq \beta$$
$$\Rightarrow x \geq m - l'L^{-1}(1-\beta), x \leq n + r'R^{-1}(1-\beta)$$
$$\Rightarrow m - l'L^{-1}(1-\beta) \leq x \leq n + r'R^{-1}(1-\beta)$$
$$\Rightarrow A_\beta^I = [m - l'L^{-1}(1-\beta), n + r'R^{-1}(1-\beta)]$$

Hence, it is proved. □

Theorem 4 Let $A^I = (m, n; l, r; l', r')$ be a LR-type TIFN. Then, its (α, β)-cut is given by $A_{\alpha,\beta}^I = [m - lL^{-1}(\alpha), n + rR^{-1}(\alpha)] \cap [m - l'L^{-1}(1-\beta), n + r'R^{-1}(1-\beta)]$, $\forall \alpha, \beta \in (0, 1]$ and $\alpha + \beta \leq 1$.

Proof For $\alpha \in (0, 1]$, The α-cut of a LR-type TIFN $A^I = (m, n; l, r; l', r')$ is given by $A_\alpha^I = [m - lL^{-1}(\alpha), n + rR^{-1}(\alpha)]$ For $\beta \in (0, 1]$, The β-cut of the LR-type TIFN $A^I = (m, n; l, r; l', r')$ is given by $A_\beta^I = [m - l'L^{-1}(1-\beta), n + rR^{-1}(1-\beta)]$, $\forall \beta \in (0, 1]]$.

So by definition, $A_{\alpha,\beta}^I = [m - lL^{-1}(\alpha), n + rR^{-1}(\alpha)] \cap [m - l'L^{-1}(1-\beta), n + r'R^{-1}(1-\beta)]$, $\forall \alpha, \beta \in (0, 1], \alpha + \beta \leq 1$. Hence, it is proved. □

Arithmetic Operations on LR -type TIFN

Proposition 1 (Addition) If $A_1^I = (m_1, n_1; l_1, r_1; l'_1, r'_1)$ and $A_2^I = (m_2, n_2; l_2, r_2; l'_2, r'_2)$ are two LR-type TIFNs, then $A_1^I + A_2^I = (m_1 + m_2, n_1 + n_2; l_1 + l_2, r_1 + r_2; l'_1 + l'_2, r'_1 + r'_2)$.

Proof For $\alpha \in (0, 1]$ and $\beta \in (0, 1]$, the α-cut and β-cut of the LR-type TIFNs $A_1^I = (m_1, n_1; l_1, r_1; l'_1, r'_1)$ and $A_2^I = (m_2, n_2; l_2, r_2; l'_2, r'_2)$ are given by $A_{1\alpha}^I = [m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)]$, $A_{2\alpha}^I = [m_2 - l_2 L^{-1}(\alpha), n_2 + r_2 R^{-1}(\alpha)]$; $A_{1\beta}^I = [m_1 - l'_1 L^{-1}(1-\beta), n_1 + r'_1 R^{-1}(1-\beta)]$, $A_{2\alpha}^I = [m_2 - l'_2 L^{-1}(1-\beta), n_2 + r'_2 R^{-1}(1-\beta)]$, respectively Now,

$$(A_1^I + A_2^I)_\alpha = A_{1\alpha}^I + A_{2\alpha}^I$$
$$= [m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)]$$
$$+ [m_2 - l_2 L^{-1}(\alpha), n_2 + r_2 R^{-1}(\alpha)]$$
$$= [(m_1 + m_2) - (l_1 + l_2)L^{-1}(\alpha), (n_1 + n_2)$$
$$+ (r_1 + r_2)R^{-1}(\alpha)] \tag{35}$$

Since L and R are decreasing functions on $[0, \infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\alpha = \alpha_0 \in (0, 1]$ such that $L^{-1}(\alpha_0) = R^{-1}(\alpha_0) = 1$.

Therefore,

$$(A_1^I + A_2^I)_{\alpha_0} = [(m_1 + m_2) - (l_1 + l_2), (n_1 + n_2) + (r_1 + r_2)] \tag{36}$$

Now, by putting $\alpha = 1$ in Eq. (35), we get the model point of $A_1^I + A_2^I$, which is given by

$$(A_1^I + A_2^I)_1 = [m_1 + m_2, n_1 + n_2] \tag{37}$$

Again

$$(A_1^I + A_2^I)_\beta = A_{1\beta}^I + A_{2\beta}^I$$
$$= [m_1 - l'_1 L^{-1}(1-\beta), n_1 + r'_1 R^{-1}(1-\beta)]$$
$$+ [m_2 - l'_2 L^{-1}(1-\beta), n_2 + r'_2 R^{-1}(1-\beta)] \tag{38}$$
$$= [(m_1 + m_2) - (l'_1 + l'_2)L^{-1}(1-\beta), (n_1 + n_2)$$
$$+ (r'_1 + r'_2)R^{-1}(1-\beta)]$$

Since L and R are decreasing functions on $[0, \infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\beta = \beta_0 \in (0, 1]$ such that $L^{-1}(1-\beta_0) = R^{-1}(1-\beta_0) = 1$.

Therefore,

$$(A_1^I + A_2^I)_{\beta_0} = [(m_1 + m_2) - (l'_1 + l'_2), (n_1 + n_2) + (r'_1 + r'_2)] \tag{39}$$

Since A_1^I, A_2^I are two LR-type TIFNs, $0 \leq l_1 \leq l'_1$, $0 \leq l_2 \leq l'_2$, $0 \leq r_1 \leq r'_1$ and $0 \leq r_2 \leq r'_2$, so $0 \leq l_1 + l_2 \leq l'_1 + l'_2$ and $0 \leq r_1 + r_2 \leq r'_1 + r'_2$

So, from Eqs. (36), (37) and (39), we have $A_1^I + A_2^I = (m_1 + m_2, n_1 + n_2; l_1 + l_2, r_1 + r_2; l'_1 + l'_2, r'_1 + r'_2)$.

Hence, it is proved. □

Proposition 2 (Subtraction) If $A_1^I = (m_1, n_1; l_1, r_1; l'_1, r'_1)$ and $A_2^I = (m_2, n_2; l_2, r_2; l'_2, r'_2)$ are two LR-type TIFNs, then $A_1^I - A_2^I = (m_1 - n_2, n_1 - m_2; l_1 + r_2, r_1 + l_2; l'_1 + r'_2, r'_1 + l'_2)$.

Proof The α-cut and β-cut of the LR-type TIFNs $A_1^I = (m_1, n_1; l_1, r_1; l'_1, r'_1)$ and $A_2^I = (m_2, n_2; l_2, r_2; l'_2, r'_2)$ are given by

$$A_{1\alpha}^I = [m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)],$$
$$A_{2\alpha}^I = [m_2 - l_2 L^{-1}(\alpha), n_2 + r_2 R^{-1}(\alpha)];$$
$$A_{1\beta}^I = [m_1 - l_1' L^{-1}(1-\beta), n_1 + r_1' R^{-1}(1-\beta)],$$
$$A_{2\alpha}^I = [m_2 - l_2' L^{-1}(1-\beta), n_2 + r_2' R^{-1}(1-\beta)],$$

respectively. Now,

$$
\begin{aligned}
(A_1^I - A_2^I)_\alpha &= A_{1\alpha}^I - A_{2\alpha}^I \\
&= [m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)] \\
&\quad - [m_2 - l_2 L^{-1}(\alpha), n_2 + r_2 R^{-1}(\alpha)] \\
&= [m_1 - n_2 - (l_1 L^{-1}(\alpha) + r_2 R^{-1}(\alpha)), n_1 \\
&\quad - m_2 + (r_1 R^{-1}(\alpha) + l_2 L^{-1}(\alpha))]
\end{aligned}
\tag{40}
$$

Since L and R are decreasing functions on $[0,\infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\alpha = \alpha_0 \in (0,1]$ such that $L^{-1}(\alpha_0) = R^{-1}(\alpha_0) = 1$.

Therefore,

$$(A_1^I - A_2^I)_{\alpha_0} = [(m_1 - n_2) - (l_1 + r_2), (n_1 - m_2) + (r_1 + l_2)] \tag{41}$$

Now, by putting $\alpha = 1$ in Eq. (40), we get the model point of $A_1^I - A_2^I$, which is given by

$$(A_1^I - A_2^I)_1 = [m_1 - n_2, n_1 - m_2] \tag{42}$$

Again

$$
\begin{aligned}
(A_1^I - A_2^I)_\beta &= A_{1\beta}^I - A_{2\beta}^I \\
&= [m_1 - l_1' L^{-1}(1-\beta), n_1 + r_1' R^{-1}(1-\beta)] \\
&\quad - [m_2 - l_2' L^{-1}(1-\beta), n_2 + r_2' R^{-1}(1-\beta)] \\
&= [m_1 - n_2 - (l_1' L^{-1}(1-\beta) + r_2' R^{-1}(1-\beta)), n_1 \\
&\quad - m_2 + (r_1' R^{-1}(1-\beta) + l_2' L^{-1}(1-\beta))]
\end{aligned}
\tag{43}
$$

Since L and R are decreasing functions on $[0,\infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\beta = \beta_0 \in (0,1]$ such that $L^{-1}(1-\beta_0) = R^{-1}(1-\beta_0) = 1$.

Therefore,

$$(A_1^I - A_2^I)_{\beta_0} = [(m_1 - n_2) - (l_1' + r_2'), (n_1 - m_2) + (r_1' + l_2')] \tag{44}$$

Since A_1^I, A_2^I are two LR-type TIFNs, $0 \le l_1 \le l_1'$, $0 \le l_2 \le l_2'$, $0 \le r_1 \le r_1'$ and $0 \le r_2 \le r_2'$, so $0 \le l_1 + r_2 \le l_1' + r_2'$ and $0 \le r_1 + l_2 \le r_1' + l_2'$.

Therefore, from Eqs. (41), (42) and (44), we have

$$A_1^I - A_2^I = (m_1 - n_2, n_1 - m_2; l_1 + r_2, r_1 + l_2; l_1' + r_2', r_1' + l_2').$$

Hence, it is proved.

Proposition 3 (Scalar multiplication) *If* $A_1^I = (m_1, n_1; l_1, r_1; l_1', r_1')$ *is a LR-type TIFN and* λ *is any real number, then* $\lambda A_1^I = (\lambda m_1, \lambda n_1; \lambda l_1, \lambda r_1; \lambda l_1', \lambda r_1')$, *when* $\lambda \ge 0$.

$\lambda A_1^I = (\lambda n_1, \lambda m_1; -\lambda r_1, -\lambda l_1; -\lambda r_1', -\lambda l_1')$, *when* $\lambda < 0$.

Proof The α-cut and β-cut of the LR-type TIFN $A_1^I = (m_1, n_1; l_1, r_1; l_1', r_1')$ are given by $A_{1\alpha}^I = [m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)]$, $A_{1\beta}^I = [m_1 - l_1' L^{-1}(1-\beta), n_1 + r_1' R^{-1}(1-\beta)]$, respectively

Case I $\lambda \ge 0$

$$
\begin{aligned}
(\lambda A_1^I)_\alpha &= \lambda A_{1\alpha}^I \\
&= \lambda[m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)] \\
&= [\lambda m_1 - \lambda l_1 L^{-1}(\alpha), \lambda n_1 + \lambda r_1 R^{-1}(\alpha)]
\end{aligned}
\tag{45}
$$

Since L and R are decreasing functions on $[0,\infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\alpha = \alpha_0 \in (0,1]$ such that $L^{-1}(\alpha_0) = R^{-1}(\alpha_0) = 1$.

Therefore,

$$(\lambda A_1^I) = [\lambda m_1 - \lambda l_1, \lambda n_1 + \lambda r_1] \tag{46}$$

Now, by putting $\alpha = 1$ in Eq. (45), we get the model point of λA_1^I, which is given by

$$(\lambda A_1^I)_1 = [\lambda m_1, \lambda n_1] \tag{47}$$

Again

$$
\begin{aligned}
(\lambda A_1^I)_\beta &= \lambda A_{1\beta}^I \\
&= [\lambda m_1 - \lambda l_1' L^{-1}(1-\beta), \lambda n_1 + \lambda r_1' R^{-1}(1-\beta)]
\end{aligned}
\tag{48}
$$

Since L and R are decreasing functions on $[0,\infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\beta = \beta_0 \in (0,1]$ such that $L^{-1}(1-\beta_0) = R^{-1}(1-\beta_0) = 1$.

Therefore,

$$(\lambda A_1^I)_{\beta_0} = [\lambda m_1 - \lambda l_1', \lambda n_1 + \lambda r_1'] \tag{49}$$

Since A_1^I are LR-type TIFN and $\lambda > 0$, $0 \le l_1 \le l_1'$ and $0 \le r_1 \le r_1'$. So $0 \le \lambda l_1 \le \lambda l_1'$ and $0 \le \lambda r_1 \le \lambda r_1'$

So, from Eqs. (46), (47) and (49), we have

$$\lambda A_1^I = (\lambda m_1, \lambda n_1; \lambda l_1, \lambda r_1; \lambda l_1', \lambda r_1')$$

Hence, it is proved.

Case II $\lambda < 0$

$$
\begin{aligned}
(\lambda A_1^I)_\alpha &= \lambda A_{1\alpha}^I \\
&= \lambda[m_1 - l_1 L^{-1}(\alpha), n_1 + r_1 R^{-1}(\alpha)] \\
&= [\lambda n_1 + \lambda r_1 R^{-1}(\alpha), \lambda m_1 - \lambda l_1 L^{-1}(\alpha)]
\end{aligned}
\tag{50}
$$

Since L and R are decreasing functions on $[0,\infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\alpha = \alpha_0 \in (0,1]$ such that $L^{-1}(\alpha_0) = R^{-1}(\alpha_0) = 1$.

Therefore,

$$(\lambda A_1{}^I) = [\lambda n_1 + \lambda r_1, \lambda m_1 - \lambda l_1] \qquad (51)$$

Now, by putting $\alpha = 1$ in Eq. 50, we get the model point of $\lambda A_1{}^I$, which is given by

$$(\lambda A_1{}^I)_1 = [\lambda n_1, \lambda m_1] \qquad (52)$$

Again

$$\begin{aligned}
(\lambda A_1{}^I)_\beta &= \lambda A_{1\beta}^I \\
&= [\lambda n_1 + \lambda r_1' R^{-1}(1-\beta), \lambda m_1 - \lambda l_1' L^{-1}(1-\beta)]
\end{aligned} \qquad (53)$$

Since L and R are decreasing functions on $[0, \infty]$ with $L(0) = R(0) = 1$, $\exists$ some $\beta = \beta_0 \in (0, 1]$ such that $L^{-1}(1 - \beta_0) = R^{-1}(1 - \beta_0) = 1$.

Therefore

$$(\lambda A_1{}^I)_{\beta_0} = [\lambda n_1 + \lambda r_1', \lambda m_1 - \lambda l_1'] \qquad (54)$$

Since $A_1{}^I$ are LR-type TIFN and $\lambda < 0$, $0 \le l_1 \le l_1'$ and $0 \le r_1 \le r_1'$. So $0 \le -\lambda l_1 \le -\lambda l_1'$ and $0 \le -\lambda r_1 \le -\lambda r_1'$

So, from Eqs. (51), (52) and (54), we have

$$\lambda A_1{}^I = (\lambda n_1, \lambda m_1; -\lambda r_1, -\lambda l_1; -\lambda r_1', -\lambda l_1').$$

Hence, it is proved. □

Definition 10 [33] Let the (α, β)-cut of a LR-type TIFN be given by

$$A_{\alpha,\beta}^I = [A_1(\alpha), A_2(\alpha)] \cap [A_1'(\beta), A_2'(\beta)]; \alpha + \beta \le 1, \forall \alpha, \beta \in [0, 1];$$

where $A_1(\alpha) = m - lL^{-1}(\alpha)$, $A_2(\alpha) = n + rR^{-1}(\alpha)$, $A_1'(\beta) = m - l'L^{-1}(1-\beta)$ and $A_1'(\beta) = n + r'R^{-1}(1-\beta)$. Then, by mean of (α, β)-cut method, the representation of membership function is

$$R_\mu(A^I) = \frac{1}{2} \int_0^1 \left[A_1(\alpha) + A_2(\alpha) \right] d\alpha.$$

Again by mean of (α, β)-cut method, the representation of nonmembership function is

$$R_v(A^I) = \frac{1}{2} \int_0^1 \left[A_1'(\beta) + A_2'(\beta) \right] d\beta.$$

The accuracy function of A^I is denoted by $f(A^I)$ and defined by

$$f(A^I) = \frac{R_\mu(A^I) + R_v(A^I)}{2},$$

to defuzzify the given numbers as deterministic one.

Theorem 5 *Let $A^I = (m, n; l, r; l', r')$ be a LR-type TIFN. Then, its accuracy function is given by $f(A^I) = \frac{2m+2n-\frac{l}{2}+\frac{r}{2}-\frac{l'}{2}+\frac{r'}{2}}{4}$.*

Proof By definition, the accuracy function of A^I is defined by

$$\begin{aligned}
f(A^I) &= \frac{R_\mu(A^I) + R_v(A^I)}{2} \\
&= \frac{\frac{1}{2}\int_0^1 \left[A_1(\alpha) + A_2(\alpha) \right] d\alpha + \frac{1}{2}\int_0^1 \left[A_1'(\beta) + A_2'(\beta) \right] d\beta}{2} \\
&= \frac{1}{4}\left[\int_0^1 \left\{ m - lL^{-1}(\alpha) + n + rR^{-1}(\alpha) \right\} d\alpha \right. \\
&\quad +
\end{aligned}$$

Now, by Definition 9 for a TIFN, $L(x) = R(x) = max\{0, 1-x\}, \forall x \ge 0$;

Since $\alpha \in (0, 1]$, $L(1 - \alpha) = \alpha \Rightarrow L^{-1}(\alpha) = \alpha$ and $R(1 - \alpha) = \alpha \Rightarrow R^{-1}(\alpha) = \alpha$ Similarly, $L(\beta) = 1 - \beta \Rightarrow L^{-1}(1 - \beta) = \beta$ and $R(\beta) = 1 - \beta \Rightarrow R^{-1}(1 - \beta) = \beta$

$$\begin{aligned}
f(A^I) &= \frac{1}{4}\left[\int_0^1 \left\{ m - l(1-\alpha) + n + r(1-\alpha) \right\} d\alpha \right. \\
&\quad \left. + \int_0^1 \left\{ m - l'\beta + n + r'\beta \right\} d\beta \right] \\
\Rightarrow f(A^I) &= \frac{2m + 2n - \frac{l}{2} + \frac{r}{2} - \frac{l'}{2} + \frac{r'}{2}}{4}
\end{aligned} \qquad (55)$$

Hence, it is proved. □

Theorem 6 *Let $A_1{}^I = (m_1, n_1; l_1, r_1; l_1', r_1')$, $A_2{}^I = (m_2, n_2; l_2, r_2; l_2', r_2')$ be any two LR-type TIFNs and λ_1 and λ_2 be any two real numbers, then $f(\lambda_1 A_1{}^I + \lambda_2 A_2{}^I) = \lambda_1 f(A_1{}^I) + \lambda_2 f(A_2{}^I)$*

Proof Let $\lambda \ge 0$, now by using propositions 2 and 3, $\lambda A_1{}^I + \lambda A_2{}^I = (\lambda_1 m_1 + \lambda_2 m_2, \lambda_1 n_1 + \lambda_2 n_2; \lambda_1 l_1 + \lambda_2 l_2, \lambda_1 r_1 + \lambda_2 r_2; \lambda_1 l_1' + \lambda_2 l_2', \lambda_1 r_1' + \lambda_2 r_2')$ Thus, by using Eq. (55),

$$f(\lambda A_1{}^I + \lambda A_2{}^I)$$
$$= \frac{2(\lambda_1 m_1 + \lambda_2 m_2) + 2(\lambda_1 n_1 + \lambda_2 n_2) - \frac{\lambda_1 l_1 + \lambda_2 l_2}{2} + \frac{\lambda_1 r_1 + \lambda_2 r_2}{2} - \frac{\lambda_1 l_1' + \lambda_2 l_2'}{2} + \frac{\lambda_1 r_1' + \lambda_2 r_2'}{2}}{4} \qquad (56)$$

Again $\quad f(A_1{}^I) = \frac{2m_1 + 2n_1 - \frac{l_1}{2} + \frac{r_1}{2} - \frac{l_1'}{2} + \frac{r_1'}{2}}{4}$ $\quad$ and $\quad f(A_2{}^I) =$
$\frac{2m_2 + 2n_2 - \frac{l_2}{2} + \frac{r_2}{2} - \frac{l_2'}{2} + \frac{r_2'}{2}}{4}$ So,

$$\lambda_1 f(A_1{}^I) + \lambda_2 f(A_2{}^I)$$
$$= \lambda_1 \left(\frac{2m_1 + 2n_1 - \frac{l_1}{2} + \frac{r_1}{2} - \frac{l_1'}{2} + \frac{r_1'}{2}}{4} \right) + \lambda_2 \left(\frac{2m_2 + 2n_2 - \frac{l_2}{2} + \frac{r_2}{2} - \frac{l_2'}{2} + \frac{r_2'}{2}}{4} \right)$$
$$\Rightarrow \lambda_1 f(A_1{}^I) + \lambda_2 f(A_2{}^I)$$
$$= \frac{2(\lambda_1 m_1 + \lambda_2 m_2) + 2(\lambda_1 n_1 + \lambda_2 n_2) - \frac{\lambda_1 l_1 + \lambda_2 l_2}{2} + \frac{\lambda_1 r_1 + \lambda_2 r_2}{2} - \frac{\lambda_1 l_1' + \lambda_2 l_2'}{2} + \frac{\lambda_1 r_1' + \lambda_2 r_2'}{2}}{4}$$

Thus, from Eqs. (56) and (57), for $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ $f(\lambda_1 A_1{}^I + \lambda_2 A_2{}^I) = \lambda_1 f(A_1{}^I) + \lambda_2 f(A_2{}^I)$. In the same way, it can be proved for any value of λ_1 and λ_2.

Hence, it is proved. $\qquad\square$

Conversion technique for multi-objective into single objective

In this segment, we will present a discussion about conversion techniques for multi-objective into single objective. Now we discuss about convex combination method

Convex combination method (CCM)

We consider the multi-objective optimization problem together with some constraints as follows:

$$\begin{cases} \max & [f_i(x), \ i = 1, 2, \ldots, M] \\ \text{s.t} & g_j \geq 0; \ j = 1, 2, \ldots, N \\ & x \in X \end{cases} \tag{57}$$

Subsequently, by the convex combination method (Tanino et al. [34]), we shift the above problem into the following form as:

$$\begin{cases} \max & \sum_{i=1}^{M} w_i f_i(x), \ \text{where} \ \sum_{i=1}^{M} w_i = 1, 0 < w_i < 1 \\ \text{s.t} & g_j \geq 0; j = 1, 2, \ldots, N \\ & x \in X \end{cases} \tag{58}$$

where w_i is the weight function of ith objective. Then, the corresponding x and $f_i(x)$ are the solutions of the problem in Eq. (57).

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