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ORIGINAL ARTICLE

Multi-objective multi-item fixed-charge solid transportation problem under twofold uncertainty

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Abstract
In this paper, we investigate a multi-objective multi-item fixed-charge solid transportation problem (MOMIFCSTP) with fuzzy-rough variables as coefficients of the objective functions and of the constraints. The main focus of the paper is to analyze MOMIFCSTP under a fuzzy-rough environment for a transporting system. In practical situations, the parameters of a MOMIFCSTP are imprecise in nature, due to several uncontrollable factors. For these reasons, we introduce the fuzzy-rough variables in MOMIFCSTP to tackle vague data which are different from fuzziness and roughness. Fuzzy-rough expected-value operator is employed to convert fuzzy-rough MOMIFCSTP into deterministic MOMIFCSTP. Thereafter, we develop a methodology to solve the deterministic MOMIFCSTP by technique for order preference by similarity to ideal solution (TOPSIS). Three distinct approaches, namely extended TOPSIS, weighted goal programming (WGP) and fuzzy programming, are used to derive Pareto-optimal solution from the suggested model. A comparison is drawn among the optimal solutions which are derived from different approaches. It is observed from the extracted results that TOPSIS provides a better optimal solution than WGP and fuzzy programming. TOPSIS also overcomes some difficulties which arise in WGP. Finally, a real-world (industrial) problem is incorporated to show the applicability and feasibility of the proposed problem.

Keywords Fixed-charge solid transportation problem · Multi-objective programming · Fuzzy-rough variable · Twofold uncertainty · TOPSIS · Fuzzy programming · Weighted goal programming

1 Introduction

In the classical transportation problem (TP), it is generally computed the minimum cost to transport a certain type of commodity from a set of source points to a set of destination points. Generally, TP deals with two types of constraints, namely supply constraint and demand constraint. Solid transportation problem (STP) is an extended version of the classical TP. STP mainly deals with three types of constraint, namely supply constraint, demand constraint and conveyance constraint. The third constraint (i.e., conveyance constraint) is involved due to different transportation modes to ship the product from sources to destinations. In STP, three constraints are taken into account as three-dimensional aspects; based on this fact it is called *three-dimensional STP*. It was first introduced by Haley [1].

A special structure of the classical TP is fixed-charge transportation problem (FCTP). In FCTP, a fixed cost is associated with each route if the route is opened and a transportation cost per unit commodity is to be shipped. The fixed-charge solid transportation problem (FCSTP) is a modified version of STP. In FCSTP, fixed cost is incurred in the origin for each route from source to destination by a specified transportation mode.

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One of the important roles of the economy of FCSTP is the determination of the efficient distributions of the product from the factories to the destinations by different transportation modes. In such an industrial problem, generally two or more commodities are produced at plants of a company and those products are transported to different destinations through various transportation modes. Due to that reason, the three-dimensional STP is extended to a multi-item fixed-charge solid transportation problem (MIFCSTP).

In an industrial problem, two or multi-criteria are more relevant rather than one criterion. To accommodate these criteria, several objective functions are treated simultaneously in the STP. For example, the objective functions to be minimized may be the total (shipping and fixed) cost, packing cost of goods, the transporting time, the deterioration rate of goods during transportation, under-used capacity of goods, etc. To tackle the problem for many real-world situations, we consider multi-objective multi-item FCSTP (MOMIFCSTP) in our proposed work.

In practical situations, the parameters in a STP or an FCSTP are uncertain in nature due to various aspects such as insufficient input information, fluctuation of financial market, weather condition, bad statistical analysis and other uncontrollable factors. To tackle uncertainty in real-life transporting systems, researchers have studied STP and FCSTP in several uncertain environments, such as stochastic, interval, fuzzy, rough, etc. But some of the cases arise in an industrial transporting systems where an uncertain environment is not adequate to tackle the situation. Based on this fact, the parameters of the formulated model are treated as fuzzy-rough variables.

Literature survey reveals that a large number of papers has been published on STP and FCSTP. But until now, no one did consider the fuzzy-rough variable approach with multi-item concept in MOSTP and MOFCSTP. So, in our proposed model, we incorporate the MOMIFCSTP that may include all possible criteria of an industrial problem.

The main contributions of our proposed work are summarized as follows:

- Fuzzy-rough MOMIFCSTP model is developed.
- Fuzzy-rough MOMIFCSTP is converted into a deterministic form by using the expected-valued operator.
- Deterministic MOMIFCSTP is solved using three different approaches.
- TOPSIS approach is extended to analyze MOMIFCSTP.
- To resolve MOMIFCSTP by TOPSIS, we decompose the proposed MOMIFCSTP into two subproblems. The optimal solutions of these subproblems provide the optimal solution of MOMIFCSTP.

To the best of our knowledge, the proposed work is the first contribution to solve MOMIFCSTP under a fuzzy-rough environment.

The remainder of the paper is organized as follows. A related literature review is discussed in Sect. 2. Motivation of this study is presented in Sect. 3. In Sect. 4, basic knowledge of rough set, fuzzy number and fuzzy-rough variables is depicted. Notations and assumptions are described in Sect. 5. In Sect. 6, MOMIFCSTP and the deterministic model of MOMIFCSTP are provided. In Sect. 7, the solution procedure for multi-objective FCSTP is stated. In Sect. 8, an application example on MOMIFCSTP is included, and results and discussion are reflected in Sect. 9. Finally, concluding remarks with future research directions are given in Sect. 10.

2 Related literature review

FCTP was first initiated by Hirsch and Dantzig [2]. Thereafter, various developments and modifications on FCTP have been made. In recent years, numerous research papers have been published on multi-objective FCTP and TP by several researchers. For instance, Midya and Roy [3] presented a single-sink fixed-charge multi-objective multi-index stochastic transportation problem and solved the model by a fuzzy programming technique. Upmanyu and Saxena [4] solved a multi-objective fixed-charge problem with imprecise fractional objective functions. Maity et al. [5] studied a multi-objective transportation problem under stochastic environments. Roy et al. [6] investigated a multi-objective two-stage gray transportation problem using a utility function with goals. Li and Lai [7] solved a multi-objective TP by fuzzy programming. Midya and Roy [8] analyzed interval programming in different environments and its application to fixed-charge transportation problem. Roy et al. [9] introduced conic scalarization approach to solve a multi-objective transportation problem with interval goal under multi-choice environment.

Furthermore, Effati et al. [10] discussed a new fuzzy neural network model for solving fuzzy linear programming problems with applications. Roy et al. [11] presented a new approach for solving intuitionistic fuzzy multi-objective transportation problem. Roy and Maity [12] studied a minimizing cost and time through a single objective function in multi-choice interval-valued transportation problem. Maity et al. [13] introduced a new approach for solving dual hesitant fuzzy transportation problem with restrictions. Recently, Moghaddam et al. [14] analyzed the fixed-charge transportation problem in a fuzzy environment using a metaheuristics technique. Roy et al. [15] studied a multi-objective fixed-charge transportation problem under rough and random rough environments.

Uncertainty in TP as well as in STP is a common phenomenon. To tackle uncertainty in STP, many researchers have studied STP and FCSTP in several uncertain environments. A few of them are described as follows.

Yang and Feng [16] solved a bi-criteria STP with fixed charge under stochastic environment. Kundu et al. [17] studied a multi-objective multi-item STP in a fuzzy environment. Roy and Mahapatra [18] proposed and solved a solid transportation problem with multi-choice cost and stochastic supply and demand. Zhang et al. [19] proposed an algorithm to solve a fixed-charge solid transportation problem in an uncertain environment. Jimenez and Verdegay [20] solved STP under two types of uncertain environments, which are interval-number valued and fuzzy-number valued, respectively, and they solved a fuzzy STP by a parametric approach [21]. Zavardehi et al. [22] presented a fuzzy fixed-charge solid transportation problem and solved it by metaheuristics. Tao and Xu [23] proposed and solved a class of rough multiple-objective programming and its application to STP. Later on, Gupta et al. [24] studied a fully fuzzy fixed-charge multi-item solid transportation problem. Recently, Roy and Midya [25] solved a multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment.

Furthermore, Xu and Zhao [26] presented a class of fuzzy-rough expected-value multi-objective decision-making model and its application to inventory problems. Xu and Yao [27] investigated a class of expected-value multi-objective programming problem with random rough coefficients. Ebrahimnejad [28] proposed a new method for solving a fuzzy transportation problem with LR flat fuzzy numbers. Atteya [29] introduced a rough multiple-objective programming problem and solved it.

Generally, TOPSIS is used to convert a given multi-objective optimization problem into a bi-objective problem. Abo-Sinna et al. [30] applied TOPSIS method to solve multi-objective large-scale nonlinear programming problems with a block-angular structure. Later on, Li [31] developed a TOPSIS-based nonlinear programming methodology for multi-attribute decision-making problems. Damghani et al. [32] solved multi-period project selection problems with fuzzy goal programming, based on TOPSIS. A summary of some recent literature connected with TP is given in Table 1.

3 Motivation for this study

Though many investigations have been done on STP and FCSTP under an uncertain environment, yet there are some real-world situations which occur in an industrial problem where a single uncertain environment is not enough to tackle the situation. Under these circumstances, we consider a twofold uncertain environment in our proposed model. An example is provided to describe the situation subsequently.

A seasonal disease such as cold–cough is widespread in the summer and the rainy season and less in other seasons, i.e., demand of medicine for the disease cold–cough is seasonal. When we predict the demand of the medicine in a period, we may use a fuzzy variable to estimate it. For example, we address a middle value γ, a left spread γ_L and a right spread γ_R of the fuzzy variable. Further, the middle value γ is usually not a crisp number, due to an integrated logistic network within the period. Definitely, we consider the demand of such medicine to cover the whole season. Therefore, it is appropriate to use a rough variable to describe the middle value γ. In fact, we assume a twofold uncertain environment [36]. In this case, fuzziness and roughness appear simultaneously. Furthermore, if we collect the statistical data of previous years on a certain parameter from a large data set, generally all the points are not equally possible. Fuzzy-rough set is approximated for such a type of linguistic information. On the other hand, in real life a STP occurs where the decision maker (DM) has no deterministic information about data, and then the parameters in the STP are estimated values. For this phenomenon, the feasible region of the STP will be changeable. Using rough set approximations, the feasible region of the STP makes the decision-making process more flexible. Thus, a fuzzy-rough multi-objective environment in STP has a realistic background, which is the main motivation of our investigated problem. So, we propose a fuzzy-rough variable in our formulated MOMIFCSTP model to describe the demand of medicine of seasonal diseases and other parameters associated with the proposed model.

4 Preliminaries

In this section, we describe the basic definitions of rough set and fuzzy set. Besides, we present several useful definitions and theorems in connection with a fuzzy-rough variable.

Definition 4.1 [37] Let Λ be a nonempty set, $\mathcal{A}$ be a σ-algebra of subsets of Λ, Δ be an element in $\mathcal{A}$, and π be a nonnegative, real-valued and additive set function. Then, $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a ***rough space***.

Trust theory is the foundation for rough programming as the possibility theory for fuzzy programming. Liu [37] combined the trust measure both the probability measure and the possibility measure to describe twofold uncertain

Table 1 Summary of related literature for TP

References	Nature of problem	Additional function	Environments	No. of items	No. of objectives
Jimenez and Verdegay [20]	STP	–	Interval and fuzzy	Single	Single
Yang and Feng [16]	STP	–	Stochastic	Single	Bi
Tao and Xu [23]	STP	–	Rough	Single	Multi
Zavardehi et al. [22]	STP	Fixed-charge	Fuzzy	Single	Single
Kundu et al. [17]	STP	–	Fuzzy	Multi	Multi
Roy and Mahapatra [18]	STP	–	Stochastic	Single	Single
Gupta et al. [24]	STP	–	Fuzzy	Multi	Single
Aggarwal and Gupta [33]	STP	–	Intuitionistic fuzzy	Single	Single
Zhang et al. [19]	STP	Fixed-charge	Uncertain	Single	Single
Rani and Gulati [34]	STP	–	Fuzzy	Multi	Multi
Midya and Roy [8]	TP	Fixed-charge	Interval and rough	Single	Single
Roy et al. [6]	TP	–	Gray number	Single	Multi
Roy et al. [15]	TP	Fixed-charge	Random rough	Single	Multi
Maity et al. [13]	TP	–	Hesitant fuzzy	Single	Single
Moghaddam et al. [14]	TP	Fixed-charge	Fuzzy	Single	Single
Singh et al. [35]	STP	–	Stochastic	Single	Multi
Roy and Midya [25]	STP	Fixed-charge	Intuitionistic fuzzy	Single	Multi
Proposed model	STP	Fixed-charge	Fuzzy-rough	Multi	Multi

events, such as random rough variable and fuzzy-rough variable.

4.1 Rough sets and their approximations

We consider a set of objects U which is called the "universe," and an indiscernibility relation, $R \subseteq U \times U$, is also treated, representing our lack of knowledge about the elements of U. For the sake of simplicity, we consider that R is an equivalence relation. Let X be a subset of U. We want to characterize the set X with respect to R. The fundamental concepts of rough set theory are presented as follows:

- The lower approximation of a set X with respect to R is the set of all objects, which can be certainly classified as X with respect to R (are certainly X with respect to R).
- The upper approximation of a set X with respect to R is the set of all objects that can be possibly classified as X with respect to R (are possibly X in view of R).
- The boundary region of a set X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R.

Definition 4.2 [38] The set X is called ***crisp*** (exact with respect to R), if the boundary region of X is empty. The set X is called ***rough*** (inexact with respect to R), if the boundary region of X is nonempty.

The equivalence class of R is determined by anyone of its elements x and is denoted by $R(x)$. The indiscernibility relation in a certain sense describes our lack of knowledge about the universe. Equivalence classes of indiscernibility relation, called *granules*, are generated by R, and they represent an elementary portion of knowledge.

Definition 4.3 [39] The ***lower approximation*** of X with respect to R is denoted by $\underline{R}(X)$ and is defined as follows:

$$\underline{R}(X) := \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}.$$

The ***upper approximation*** of X with respect R is denoted by $\overline{R}(X)$ and is described as stated below:

$$\overline{R}(X) := \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}.$$

The ***boundary region*** of X with respect R is denoted by $BN_R(X)$ and is depicted as follows:

$$BN_R(X) := \overline{R}(X) - \underline{R}(X).$$

The diagrammatic representation of rough set is shown in Fig. 1.

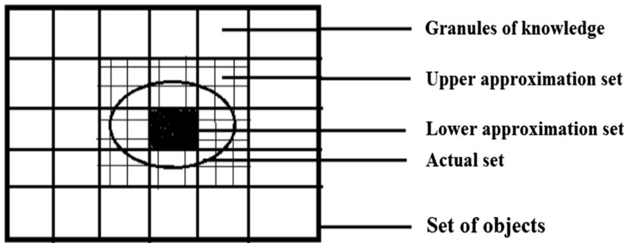

Fig. 1 Diagrammatic representation of rough set

4.2 Arithmetics of rough intervals

The arithmetic operations on rough intervals (*RIs*) are similar to arithmetic operations of crisp intervals. The arithmetic of *RIs* [40] is expressed as follows:

Let $RI = ([\underline{RI}_1, \overline{RI}_1], [\underline{RI}_2, \overline{RI}_2])$ and $RI' = ([\underline{RI}'_1, \overline{RI}'_1], [\underline{RI}'_2, \overline{RI}'_2])$ be two rough intervals, $o \in \{+, -, \times, /\}$ be a binary operation on the set of crisp intervals. Then, the rough interval arithmetic operations are defined by $RIoRI' := ([\underline{RIoRI'}], [\overline{RIoRI'}])$ $(o \in \{+, -, \times\})$, where $RIoRI'$ is also a rough interval.

Addition $RI + RI' := ([\underline{RI} + \underline{RI'}], [\overline{RI} + \overline{RI'}])$.

Other arithmetic operations such as **subtraction** and **multiplication** on rough intervals are defined similarly as addition.

Division $RI/RI' := \left([\underline{RI}_1/\overline{RI}'_1, \overline{RI}_1/\underline{RI}'_1], [\underline{RI}_2/\overline{RI}'_2, \overline{RI}_2/\underline{RI}'_2]\right)$ if $0 \notin [\underline{RI}'_2, \overline{RI}'_2]$.

4.3 Fuzzy numbers

Definition 4.4 [41] A ***fuzzy set*** $\tilde{A}$ defined on the set of real numbers, $\mathbb{R}$, is said to be a fuzzy number, if its membership function $\mu_{\tilde{A}} : \mathbb{R} \to [0, 1]$ has the following characteristics:

1. $\mu_{\tilde{A}}$ is convex, i.e., $\mu_{\tilde{A}}\{(1-\lambda)x_1 + \lambda x_2\} = \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in \mathbb{R}, 0 \le \lambda \le 1$.
2. $\mu_{\tilde{A}}$ is normal, i.e., there is an $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.
3. $\mu_{\tilde{A}}$ is piecewise continuous.

4.4 Trapezoidal fuzzy numbers

Definition 4.5 [41] ***Trapezoidal fuzzy number*** is a fuzzy number represented by a quadruple $\tilde{A} = (a_1, a_2, a_3, a_4)$. This quadruple is interpreted as a membership function and satisfies the following conditions:

1. The line segment over the interval from a_1 to a_2 is an increasing function.
2. The line segment over the interval from a_2 to a_3 is a constant function.
3. The line segment over the interval from a_3 to a_4 is a decreasing function, and
4. $a_1 < a_2 \le a_3 < a_4$.

The membership function of $\tilde{A}$ is denoted as $\mu_{\tilde{A}}(x)$ (as shown in Fig. 2) and defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for} \quad x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & \text{for} \quad a_1 \le x \le a_2, \\ 1, & \text{for} \quad a_2 \le x \le a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & \text{for} \quad a_3 \le x \le a_4, \\ 0, & \text{for} \quad x > a_4. \end{cases}$$

4.5 Fuzzy-rough variables

Definition 4.6 [37] A ***fuzzy-rough variable*** is a function ξ from a rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ to a collection of fuzzy variables such that for any Borel set B of $\mathbb{R}$, the function

$$\xi^*(B)(\lambda) = \text{Pos}\{\xi(\lambda) \in B\}$$

is a measurable mapping of λ, where the abbreviation Pos represents possibility [37].

Example 4.1 Let $\xi = (\rho - 2, \rho - 1, \rho + 1, \rho + 2)$ with $\rho = ([a, b], [c, d]), 0 \le c \le a < b \le d$, where the quadruple denotes a trapezoidal fuzzy number and ρ is a rough variable; then, ξ is a fuzzy-rough variable.

For convenience, a fuzzy-rough variable $\xi = (\rho - 2, \rho - 1, \rho + 1, \rho + 2)$, with $\rho = ([a, b], [c, d])$, is symbolically denoted by $\rho \vdash ([a, b], [c, d])$.

Definition 4.7 [37] Let ξ be a fuzzy-rough variable, defined on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$. Then, its ***expected value*** is defined as follows:

$$E[\xi] := \int_0^{\infty} \text{Tr}\{\lambda \in \Lambda : E[\xi(\lambda)] \ge r\} dr - \int_{-\infty}^{0} \text{Tr}\{\lambda \in \Lambda : E[\xi(\lambda)] \le r\} dr,$$

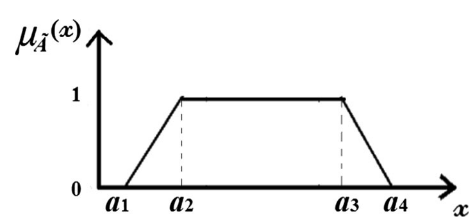

Fig. 2 Graph of a trapezoidal fuzzy number

provided that at least one of the integrals exist, where E is the expected-value operator and the abbreviation Tr represents the trust measure [37].

Theorem 4.1 [37] *Let ξ and ζ be the fuzzy-rough variables with finite expected values. Then, for any real numbers a and b, we have*

$$E[a\xi + b\zeta] = aE[\xi] + bE[\zeta].$$

Proposition 4.1 [42] *Let ξ be a trapezoidal fuzzy-rough variable $\xi = (\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4)$, where $\bar{r}_1, \bar{r}_2, \bar{r}_3$ and $\bar{r}_4$ are rough variables (i.e., rough intervals) defined on a rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$, and we have*

$$\begin{aligned} \bar{r}_1 &= ([m_2, m_3], [m_1, m_4]),\ m_1 \le m_2 < m_3 \le m_4, \\ \bar{r}_2 &= ([n_2, n_3], [n_1, n_4]),\ n_1 \le n_2 < n_3 \le n_4, \\ \bar{r}_3 &= ([s_2, s_3], [s_1, s_4]),\ s_1 \le s_2 < s_3 \le s_4, \\ \bar{r}_4 &= ([t_2, t_3], [t_1, t_4]),\ t_1 \le t_2 < t_3 \le t_4. \end{aligned}$$

Then, the expected value of ξ is given by:

$$E[\xi] = \frac{1}{16}\sum_{i=1}^{4}(m_i + n_i + s_i + t_i).$$

Theorem 4.2 [42] *If $\tilde{\bar{c}}_{ijk}$ are a trapezoidal fuzzy-rough variable, defined as: $\tilde{\bar{c}}_{ijk}(\kappa) = (\bar{c}_{ijk1}, \bar{c}_{ijk2}, \bar{c}_{ijk3}, \bar{c}_{ijk4})$ with $\bar{c}_{ijks} \vdash ([\bar{c}_{ijks2}, \bar{c}_{ijks3}], [\bar{c}_{ijks1}, \bar{c}_{ijks4}])$, for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, p$; $s = 1, 2, 3, 4$, where $c_{ijks1} \le c_{ijks2} < c_{ijks3} \le c_{ijks4}$,* then

$$E[\tilde{\bar{c}}_{ijk}] = \frac{1}{16}\sum_{s=1}^{4}\sum_{v=1}^{4} c_{ijksv}\ \forall\, i, j, k.$$

5 Notations and assumptions

The following notations and assumptions are considered to design the paper.

Notations

m	number of sources,
n	number of destinations,
p	number of conveyances (i.e., different transportation modes),
h	number of items,
x^l_{ijk}	unit amount of the product to be transported from the ith source to the jth destination by the kth conveyance for the lth item,
$\eta(x^l_{ijk})$	binary variable takes the value "1" if the source i is used, and "0" otherwise,
$\tilde{\bar{c}}^l_{ijk}$	fuzzy-rough transportation (variable) cost for unit quantity of the product from the ith source to the jth destination by the kth conveyance for the lth item,
$\tilde{\bar{f}}^l_{ijk}$	fuzzy-rough fixed cost associated with the ith source to the jth destination by the kth conveyance for the lth item,
$\tilde{\bar{t}}^l_{ijk}$	fuzzy-rough time of transportation of the product from the ith source to the jth destination by the kth conveyance for the lth item which is independent of the unit amount of the product transported,
$\tilde{\bar{d}}^l_{ijk}$	fuzzy-rough packing cost for unit pack of the product from the ith source to the jth destination by the kth conveyance for the lth item,
$\tilde{\bar{a}}^l_i$	fuzzy-rough availability of the product at the ith source for the lth item,
$\tilde{\bar{b}}^l_j$	fuzzy-rough demand of the product at the jth destination for the lth item,
$\tilde{\bar{e}}_k$	total fuzzy-rough capacity of the product which can be carried by the kth conveyance,
$\tilde{\bar{Z}}_K$	objective functions in fuzzy-rough nature $(K = 1, 2, 3)$,
Z_K	objective functions in crisp nature $(K = 1, 2, 3)$, where $Z_K = E[\tilde{\bar{Z}}_K]$, and E denotes the expected-value operator,
d^+_K, d^-_K	positive and negative deviations corresponding to the Kth goal of the objective function, respectively.

Assumptions

1. $\tilde{\bar{a}}^l_i > 0$, $\tilde{\bar{b}}^l_j > 0\ \forall\, i, j, l$.
2. No items deteriorate during transportation.
3. Transporting time from the ith origin to jth destination by using the kth conveyance is the same whatever the items are.
4. Each regarded trapezoidal fuzzy-rough variable is positive in all of its components.

6 Mathematical model

In our proposed MOMIFCSTP, we consider three objective functions in which the first objective function represents the total transportation cost (the variable cost and the fixed cost), the second objective function considers the transporting time and the last objective function refers to the packing cost; all of the three goals are to be minimized. Especially, we mention here about the nature of the second objective function in our proposed model. Actually, the second objective function is taken to maximize the customers' satisfaction level; in fact, to measure it, we treat the total transportation time. So, with respect to maximizing the customers' satisfaction level, the value of the objective function should be minimized. There are

m factories (supply points), n customers (demand points) and p conveyances (different transportation modes such as trucks, air freight, goods trains, and ships). Each of the m factories can transport to any of the n customers by the p conveyances for l items at a transporting cost of c_{ijk}^{l} per unit commodity and a fixed cost of f_{ijk}^{l}. The problem is to determine the amount x_{ijk}^{l}, for any $i, j, k, l,$ of the product conveyed from the ith source to the jth destination by the kth transportation mode for lth items, in such a way that the overall value of the three objective functions are to be minimized. The proposed MOMIFCSTP can be formulated as follows:

Model 1

$$\text{minimize } \tilde{\tilde{Z}}_1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} \left[\tilde{\tilde{c}}_{ijk}^{l} x_{ijk}^{l} + \tilde{\tilde{f}}_{ijk}^{l}\eta(x_{ijk}^{l})\right], \tag{6.1}$$

$$\text{minimize } \tilde{\tilde{Z}}_2(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} \left[\tilde{\tilde{t}}_{ijk}^{l}\eta(x_{ijk}^{l})\right], \tag{6.2}$$

$$\text{minimize } \tilde{\tilde{Z}}_3(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} \tilde{\tilde{d}}_{ijk}^{l} x_{ijk}^{l} \tag{6.3}$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk}^{l} \le \tilde{\tilde{a}}_i^{l} \quad (i = 1,2,\ldots,m;\ l = 1,2,\ldots,h), \tag{6.4}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} x_{ijk}^{l} \ge \tilde{\tilde{b}}_j^{l} \quad (j = 1,2,\ldots,n;\ l = 1,2,\ldots,h), \tag{6.5}$$

$$\sum_{l=1}^{h}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{l} \le \tilde{\tilde{e}}_k \ (k = 1,2,\ldots,p), \tag{6.6}$$

$$\begin{cases} x_{ijk}^{l} \ge 0, \\ \eta(x_{ijk}^{l}) = 0 \text{ if } x_{ijk}^{l} = 0, \\ \eta(x_{ijk}^{l}) = 1 \text{ if } x_{ijk}^{l} > 0 \\ (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ k = 1,2,\ldots,p;\ l = 1,2,\ldots,h). \end{cases} \tag{6.7}$$

It is obvious that Model 1 has a feasible solution if

$$\sum_{i=1}^{m}\tilde{\tilde{a}}_i^{l} \ge \sum_{j=1}^{n}\tilde{\tilde{b}}_j^{l}\ \forall l, \text{ and } \sum_{k=1}^{p}\tilde{\tilde{e}}_k \ge \sum_{l=1}^{h}\sum_{j=1}^{n}\tilde{\tilde{b}}_j^{l}.$$

6.1 Equivalent crisp model of Model 1

We cannot deal with the proposed MOMIFCSTP model directly due to the presence of fuzzy-rough variables. So, we employ the expected-value operator E to transform Model 1 into a fuzzy-rough expected-value model (i.e., Model 2) by using Theorems 4.1 and 4.2, which are stated in Sect. 4.5:

Model 2

$$\begin{aligned} \text{minimize } E\left[\tilde{\tilde{Z}}_1(x)\right] &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} E\left[\tilde{\tilde{c}}_{ijk}^{l} x_{ijk}^{l} + \tilde{\tilde{f}}_{ijk}^{l}\eta(x_{ijk}^{l})\right], \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} \left[E\left[\tilde{\tilde{c}}_{ijk}^{l}\right] x_{ijk}^{l} + E\left[\tilde{\tilde{f}}_{ijk}^{l}\right]\eta(x_{ijk}^{l})\right], \end{aligned} \tag{6.8}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_2(x)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} E\left[\tilde{\tilde{t}}_{ijk}^{l}\right]\eta(x_{ijk}^{l}), \tag{6.9}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_3(x)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{h} E\left[\tilde{\tilde{d}}_{ijk}^{l}\right] x_{ijk}^{l} \tag{6.10}$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk}^{l} \le E\left[\tilde{\tilde{a}}_i^{l}\right] \quad (i = 1,2,\ldots,m;\ l = 1,2,\ldots,h), \tag{6.11}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} x_{ijk}^{l} \ge E\left[\tilde{\tilde{b}}_j^{l}\right] \quad (j = 1,2,\ldots,n;\ l = 1,2,\ldots,h), \tag{6.12}$$

$$\sum_{l=1}^{h}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{l} \le E\left[\tilde{\tilde{e}}_k\right] \ (k = 1,2,\ldots,p), \tag{6.13}$$

$$\text{constraint (6.7).} \tag{6.14}$$

Herewith, Model 2 has a feasible solution if

$$\sum_{i=1}^{m} E\left[\tilde{\tilde{a}}_i^{l}\right] \ge \sum_{j=1}^{n} E\left[\tilde{\tilde{b}}_j^{l}\right] \forall l, \text{ and } \sum_{k=1}^{p} E\left[\tilde{\tilde{e}}_k\right] \ge \sum_{l=1}^{h}\sum_{j=1}^{n} E\left[\tilde{\tilde{b}}_j^{l}\right].$$

Definition 6.1 A feasible solution $x^* = (x_{ijk}^{*l} : l = 1,2,\ldots, h;\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ k = 1,2,\ldots,p)$ is said to be a ***Pareto-optimal (non-dominated) solution*** of Model 2 if there exists no other feasible solution $x = (x_{ijk}^{l} : l = 1,2,\ldots,h;\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ k = 1,2,\ldots,p)$ such that

$$E[\tilde{\tilde{Z}}_K(x)] \le E[\tilde{\tilde{Z}}_K(x^*)] \quad \text{for } K = 1,2,3, \text{ and}$$
$$E[\tilde{\tilde{Z}}_K(x)] < E[\tilde{\tilde{Z}}_K(x^*)] \quad \text{for at least one } K.$$

Definition 6.2 If x^* is a Pareto-optimal solution of Model 2, then we treat it as a fuzzy-rough expected-value Pareto-optimal solution.

6.2 Decomposition of Model 2

Generally, a large number of variables is involved in an industrial problem. We want to extract the Pareto-optimal solution from Model 2 using TOPSIS approach by LINGO iterative scheme. Here, Model 2 contains a high number of variables which generally is computationally difficult when providing an optimal solution. So with the help of decomposition principle, we resolve Model 2 into Model 3 and Model 4. Additionally, in LINGO iterative scheme, there is a limitation on the total number of variables for solving mathematical optimization models. To overcome this difficulty also, we decompose Model 2 into Model 3 and Model 4 indeed. In this subsection, we decompose the proposed MOMIFCSTP (i.e., Model 2) accordingly. For this decomposition, we assume two types of commodities (i.e., $h = 2$) to be shipped from source points to destinations. Therefore, Model 3 and Model 4 are achieved, looking as follows:

Model 3

$$\text{minimize } E\left[\tilde{\tilde{Z}}_1(x^1)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[E\left[\tilde{\tilde{c}}^1_{ijk}\right]x^1_{ijk} + E\left[\tilde{\tilde{f}}^1_{ijk}\right]\eta(x^1_{ijk})\right], \tag{6.15}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_2(x^1)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} E\left[\tilde{\tilde{t}}^1_{ijk}\right]\eta(x^1_{ijk}), \tag{6.16}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_3(x^1)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} E\left[\tilde{\tilde{d}}^1_{ijk}\right]x^1_{ijk} \tag{6.17}$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{k=1}^{p} x^1_{ijk} \le E\left[\tilde{\tilde{a}}^1_i\right] \; (i = 1,2,\ldots,m), \tag{6.18}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} x^1_{ijk} \ge E\left[\tilde{\tilde{b}}^1_j\right] \; (j = 1,2,\ldots,n), \tag{6.19}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x^1_{ijk} \le E\left[\tilde{\tilde{e}}_k\right] \; (k = 1,2,\ldots,p), \tag{6.20}$$

$$\begin{cases} x^1_{ijk} \ge 0, \\ \eta(x^1_{ijk}) = 0 \text{ if } x^1_{ijk} = 0, \\ \eta(x^1_{ijk}) = 1 \text{ if } x^1_{ijk} > 0 \\ (i = 1,2,\ldots,m;\, j = 1,2,\ldots,n;\, k = 1,2,\ldots,p); \end{cases} \tag{6.21}$$

Model 4

$$\text{minimize } E\left[\tilde{\tilde{Z}}_1(x^2)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[E\left[\tilde{\tilde{c}}^2_{ijk}\right]x^2_{ijk} + E\left[\tilde{\tilde{f}}^2_{ijk}\right]\eta(x^2_{ijk})\right], \tag{6.22}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_2(x^2)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} E\left[\tilde{\tilde{t}}^2_{ijk}\right]\eta(x^2_{ijk}), \tag{6.23}$$

$$\text{minimize } E\left[\tilde{\tilde{Z}}_3(x^2)\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} E\left[\tilde{\tilde{d}}^2_{ijk}\right]x^2_{ijk} \tag{6.24}$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{k=1}^{p} x^2_{ijk} \le E\left[\tilde{\tilde{a}}^2_i\right] \; (i = 1,2,\ldots,m), \tag{6.25}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} x^2_{ijk} \ge E\left[\tilde{\tilde{b}}^2_j\right] \; (j = 1,2,\ldots,n), \tag{6.26}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x^2_{ijk} \le E\left[\tilde{\tilde{e}}_k\right] \; (k = 1,2,\ldots,p), \tag{6.27}$$

$$\begin{cases} x^2_{ijk} \ge 0, \\ \eta(x^2_{ijk}) = 0 \text{ if } x^2_{ijk} = 0, \\ \eta(x^2_{ijk}) = 1 \text{ if } x^2_{ijk} > 0 \\ (i = 1,2,\ldots,m;\, j = 1,2,\ldots,n;\, k = 1,2,\ldots,p). \end{cases} \tag{6.28}$$

Models 3 and 4 have feasible solutions if

$$\sum_{i=1}^{m} E\left[\tilde{\tilde{a}}^l_i\right] \ge \sum_{j=1}^{n} E\left[\tilde{\tilde{b}}^l_j\right] \text{ for } l = 1,2, \text{ and}$$

$$\sum_{k=1}^{p} E\left[\tilde{\tilde{e}}_k\right] \ge \sum_{l=1}^{h}\sum_{j=1}^{n} E\left[\tilde{\tilde{b}}^l_j\right].$$

Definition 6.3 A feasible solution $x^{*1} = (x^{*1}_{ijk} : i = 1,2,\ldots,m;\, j = 1,2,\ldots,n;\, k = 1,2,\ldots,p)$ is said to be a ***Pareto-optimal solution*** of Model 3 if there exists no other feasible solution $x^1 = (x^1_{ijk} : i = 1,2,\ldots,m;\, j = 1,2,\ldots, n;\, k = 1,2,\ldots,p)$ such that

$$E[\tilde{\tilde{Z}}_K(x^1)] \le E[\tilde{\tilde{Z}}_K(x^{*1})] \text{ for } K = 1,2,3, \text{ and}$$

$$E[\tilde{\tilde{Z}}_K(x^1)] < E[\tilde{\tilde{Z}}_K(x^{*1})] \text{ for at least one } K.$$

Definition 6.4 A feasible solution $x^{*2} = (x_{ijk}^{*2} : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ is said to be a ***Pareto-optimal solution*** of Model 4 if there does not exist any other feasible solution $x^2 = (x_{ijk}^2 : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ such that

$$E[\tilde{\tilde{Z}}_K(x^2)] \le E[\tilde{\tilde{Z}}_K(x^{*2})] \quad \text{for } K = 1, 2, 3, \text{ and}$$
$$E[\tilde{\tilde{Z}}_K(x^2)] < E[\tilde{\tilde{Z}}_K(x^{*2})] \quad \text{for at least one } K.$$

7 Solution methodology

In order to solve MOMIFCSTP (i.e., Model 2), we consider three approaches, namely:

- Fuzzy programming,
- Weighted goal programming, and
- TOPSIS.

7.1 Fuzzy programming

Fuzzy programming was initiated by Zimmermann [41] to solve multi-objective linear programming problems. He found that fuzzy linear programming always provides an efficient solution. Fuzzy programming approach is adopted to obtain a Pareto-optimal solution from the suggested model. Moreover, it is easy to apply for solving MOMIFCSTP. The solution of MOMIFCSTP (i.e., Model 2) using fuzzy programming can be obtained by the following steps:

Step 1 Solve the MOMIFCSTP as a single objective MIFCSTP, using at each time only one objective function $Z_K\,(K = 1, 2, 3)$ and ignoring the other ones. The optimal solution for the Kth $(K = 1, 2, 3)$ objective function is denoted by X^{K*}.

Step 2 Determine the values of all $K\,(K = 1, 2, 3)$ numbered of objective functions at all these X^{K*} optimal solutions.

Step 3 From Step 2, we find a lower bound (L^K) and an upper bound (U^K) for each objective function. We calculate the payoff table which is given in Table 2, from which U^K and L^K are determined in the following way:

$$U^K := \max\left\{Z_K(X^{1*}),\ Z_K(X^{2*}),\ Z_K(X^{3*})\right\},$$
$$\text{and} \quad L^K := Z_K(X^{K*}),\ K = 1, 2, 3.$$

Step 4 The initial fuzzy model can then be stated, in terms of the lower bound of each objective function, as follows: Find x_{ijk}^l for all i, j, k, l, which satisfy $Z_K \le L^K\,(K = 1, 2, 3)$ with given constraints (6.11)–(6.13) and non-negativity conditions (6.14).

Table 2 Payoff table

	Z_1	Z_2	Z_3
X_1^*	$Z_1(X^{1*})$	$Z_2(X^{1*})$	$Z_3(X^{1*})$
X_2^*	$Z_1(X^{2*})$	$Z_2(X^{2*})$	$Z_3(X^{2*})$
X_3^*	$Z_1(X^{3*})$	$Z_2(X^{3*})$	$Z_3(X^{3*})$

Step 5 Formulate a membership function $\mu_K(x)$ corresponding to Kth objective function for MOMIFCSTP which is defined as:

$$\mu_K(x) = \begin{cases} 1, & \text{if} \quad Z_K \le L^K, \\ 1 - \left(\dfrac{Z_K - L^K}{U^K - L^K}\right), & \text{if} \quad L^K \le Z_K \le U^K \ (K = 1, 2, 3), \\ 0, & \text{if} \quad Z_K \ge U^K. \end{cases}$$

Step 6 Using max–min operator, the fuzzy linear programming problem can be written in the following way:

$$\begin{aligned} &\text{maximize } \lambda_1 \\ &\text{subject to } \lambda_1 \le \frac{U^K - Z_K}{U^K - L^K} \ (K = 1, 2, 3), \\ &\qquad \text{constraints } (6.11) - (6.14), \\ &\qquad 0 \le \lambda_1 \le 1. \end{aligned} \tag{7.1}$$

Here, $\lambda_1 = \min\{\mu_K(x) : K = 1, 2, 3\}$ is the satisfaction level for the objective functions. This linear programming problem can further be simplified as:

$$\begin{aligned} &\text{maximize } \lambda_1 \\ &\text{subject to } Z_K + \lambda_1(U^K - L^K) \le U^K \ (K = 1, 2, 3), \\ &\qquad \text{constraints } (6.11) - (6.14), \\ &\qquad 0 \le \lambda_1 \le 1. \end{aligned} \tag{7.2}$$

Theorem 7.1 *If* $x^* = (x_{ijk}^{*l} : l = 1, 2, \ldots, h;\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ *is an optimal solution of linear programming problem* (7.2), *then it is also a Pareto-optimal* (*non-dominated*) *solution of Model* 2, *i.e.,* there is no other feasible solution $x = (x_{ijk}^l : l = 1, 2, \ldots, h;\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ *such that*

$$E\left[\tilde{\tilde{Z}}_K(x)\right] \le E\left[\tilde{\tilde{Z}}_K(x^*)\right] \quad \text{for } K = 1, 2, 3, \text{ and} \tag{1}$$
$$E\left[\tilde{\tilde{Z}}_K(x)\right] < E\left[\tilde{\tilde{Z}}_K(x^*)\right] \quad \text{for at least one } K. \tag{2}$$

Proof We prove this theorem by contradiction. Let x^* is a Pareto-optimal (non-dominated) solution of Model 2, i.e., there exists a feasible solution x such that inequalities (1) and (2) hold.

Since $\mu_K(x)$ decreases strictly with respect to the corresponding objective value $E[\tilde{\tilde{Z}}_K(x)]$ in [0, 1], we have $\mu_K(x) \geq \mu_K(x^*) \, \forall K$, and $\mu_K(x) > \mu_K(x^*)$ for at least one $K = 1, 2, 3$.

Hence, it follows from the strict monotonicity of the weight root-power mean operator [7] and using product–min operator [41], we can write $\mu_K(x) > \mu_K(x^*)$. This inequality provides a contradiction to the fact that x^* is an optimal solution of problem (7.2). This completes the proof of the theorem. □

7.2 Weighted goal programming

Goal programming (GP) was first introduced by Charnes and Cooper [43] to solve multi-objective linear programming problems. The overall concepts of GP are to minimize the deviations between the achievement of the goal and their aspiration levels. After that, the concept of relative importance of weights of the objective functions is introduced in GP, which is known as weighted goal programming [44]. WGP is used to find a Pareto-optimal (non-dominated) solution from the formulated model according to priorities of the objective functions.

The mathematical model of WGP to solve the MOMIFCSTP (i.e., Model 2) is presented as follows:

Model 5

$$\begin{aligned}
&\text{minimize } \sum_{K=1}^{3} \frac{W_K}{N_K}(d_K^+ + d_K^-),\\
&\text{subject to } E[\tilde{\tilde{Z}}_K(x)] - d_K^+ + d_K^- = g_K \; (K = 1, 2, 3),\\
&\qquad d_K^+ \times d_K^- = 0,\\
&\qquad d_K^+ \geq 0, d_K^- \geq 0 \; (K = 1, 2, 3),\\
&\qquad \text{constraints } (6.11)–(6.14).
\end{aligned}$$

Model 5 can be solved along the following steps:

Step 1 Assign the goal $g_K \; (K = 1, 2, 3)$ associated with each objective function. We consider the target values $g_1 = 820$, $g_2 = 860$ and $g_3 = 400$, corresponding to the objective functions, respectively.

Step 2 Select the weight $W_K \; (K = 1, 2, 3)$ corresponding to the objective function according to the degree of importance of the objective function in Model 5. Here, we choose the weights $W_1 = 0.5$, $W_2 = 0.4$ and $W_3 = 0.1$, specifying the objective functions.

Step 3 Substitute the value of normalization constant $N_K = g_K/100 \; (K = 1, 2, 3)$, measured by a percentage scale [45].

Step 4 Afterward, substituting the values of g_K, W_K and $N_K \; (K = 1, 2, 3)$, we obtain the optimal solution of Model 5 by using LINGO iterative scheme.

Theorem 7.2 *Let $W_K > 0$ and $N_K > 0$ for $K = 1, 2, 3$. If $x^* = (x_{ijk}^{*l} : l = 1, 2, \ldots, h; \, i = 1, 2, \ldots, m; \, j = 1, 2, \ldots, n; \, k = 1, 2, \ldots, p)$ is an optimal solution of Model 5, then it is also a Pareto-optimal (non-dominated) solution of Model 2, i.e., there is no other feasible solution $x = (x_{ijk}^{l} : l = 1, 2, \ldots, h; \, i = 1, 2, \ldots, m; \, j = 1, 2, \ldots, n; \, k = 1, 2, \ldots, p)$ such that*

$$E[\tilde{\tilde{Z}}_K(x)] \leq E[\tilde{\tilde{Z}}_K(x^*)] \text{ for } K = 1, 2, 3, \text{ and}$$
$$E[\tilde{\tilde{Z}}_K(x)] < E[\tilde{\tilde{Z}}_K(x^*)] \text{ for at least one } K.$$

Proof Let x^* subsequently be any non-dominated solution of Model 2, i.e., there exists a feasible solution x such that the above inequalities hold.

As d_K^+ and d_K^- are positive and negative deviations corresponding to the Kth goal of the objective function, we can write:

$$(d_K^+ + d_K^-)_x \leq (d_K^+ + d_K^-)_{x^*} \text{ for } K = 1, 2, 3, \text{ and}$$
$$(d_K^+ + d_K^-)_x < (d_K^+ + d_K^-)_{x^*} \text{ for at least one } K.$$

Furthermore, as $W_K > 0$ and $N_K > 0$ for $K = 1, 2, 3$, so, $\frac{W_K}{N_K} > 0$, $K = 1, 2, 3$, from the above inequalities we have

$$\left[\frac{W_K}{N_K}(d_K^+ + d_K^-)\right]_x \leq \left[\frac{W_K}{N_K}(d_K^+ + d_K^-)\right]_{x^*} \text{ for } K = 1, 2, 3, \text{ and} \tag{i}$$
$$\left[\frac{W_K}{N_K}(d_K^+ + d_K^-)\right]_x < \left[\frac{W_K}{N_K}(d_K^+ + d_K^-)\right]_{x^*} \text{ for at least one } K. \tag{ii}$$

Taking sum on inequalities (i) and (ii), we can write:

$$\left[\min_x \sum_{K=1}^{3} \frac{W_K}{N_K}(d_K^+ + d_K^-)\right] < \left[\min_{x^*} \sum_{K=1}^{3} \frac{W_K}{N_K}(d_K^+ + d_K^-)\right],$$

which contradicts the fact that x^* is an optimal solution to Model 5. Thus, x^* is a non-dominated solution of Model 2. This concludes the proof of the theorem. □

7.3 TOPSIS

Hwang and Yoon [46] initiated TOPSIS for obtaining Pareto-optimal solutions to multi-attribute decision-making problems. It is based on the idea that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). Classical TOPSIS has been competently used for solving many selection and ranking problems. It basically focuses on three kinds of decisions: (1) find the rank of all alternatives, (2) the alternatives are ranked from the best to the worst and (3) select the best alternative. The extended TOPSIS approach gives the Pareto-optimal solution for Model 2, and it generates a better Pareto-optimal solution. Extended TOPSIS approach can accommodate a large number of variables involved in a

MOMIFCSTP by decomposing it into subproblems. Moreover, it ensures that the Pareto-optimal solutions obtained from different subproblems are also the Pareto-optimal solution of the original MOMIFCSTP.

Definition 7.1 If a multi-objective optimization problem is reduced to a bi-objective problem by TOPSIS approach, then the reduced problem is called a ***TOPSIS-based bi-objective optimization problem***.

We extend the concept of TOPSIS approach in our proposed study to obtain a Pareto-optimal solution of MOMIFCSTP (i.e., Model 2), which is described by the following steps:

Step 1 Decompose our MOMIFCSTP into two subproblems, namely Model 3 and Model 4, which are introduced in Sect. 6.2.
Step 2 Determine the individual minimum and maximum values of all the objective functions for the subproblems subject to the constraints (6.18)–(6.21) and (6.25)–(6.28), respectively.
Step 3 Identify PIS (Z^+) and NIS (Z^-) for Model 3, which are given as follows:

$$Z^+ = (Z_1^+, Z_2^+, Z_3^+),$$
$$Z^- = (Z_1^-, Z_2^-, Z_3^-),$$
$$\text{where} \begin{cases} Z_K^+ = \min\{Z_K : K = 1,2,3;\ \text{subject to}\ (6.18){-}(6.21)\}, \\ Z_K^- = \max\{Z_K : K = 1,2,3;\ \text{subject to}\ (6.25){-}(6.28)\}. \end{cases}$$

We note that the size of range of the objective functions in MOMIFCSTP with $Z_K^+ < Z_K^-$ can be estimated by $Z_K^- - Z_K^+$ $(K = 1,2,3)$. The concepts of PIS and NIS are depicted in Fig. 3.
Step 4 Using the PIS and NIS, we calculate the distance function from PIS $\left(\text{i.e.,}\, d_q^{PIS}(x^1)\right)$ and the distance function from NIS $\left(\text{i.e.,}\, d_q^{NIS}(x^1)\right)$ as follows:

$$d_q^{PIS}(x^1) = \left[\sum_{K=1}^{3} \left[W_K \frac{Z_K(x^1) - Z_K^+}{Z_K^- - Z_K^+}\right]^q\right]^{\frac{1}{q}}, \tag{7.3}$$

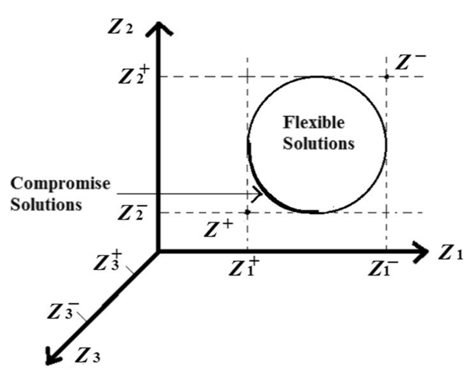

Fig. 3 Graphical representation of PIS and NIS

$$d_q^{NIS}(x^1) = \left[\sum_{K=1}^{3} \left[W_K \frac{Z_K^- - Z_K(x^1)}{Z_K^- - Z_K^+}\right]^q\right]^{\frac{1}{q}},$$
$$\sum_{K=1}^{3} W_K = 1,\ W_K \geq 0\ \forall K. \tag{7.4}$$

The parameters W_K $(K = 1,2,3)$ in Eqs. (7.3) and (7.4) denote the weights of the objective functions. Here, we consider the priorities by weights, and they are $W_1 = 0.5$, $W_2 = 0.4$ and $W_3 = 0.1$, for three objective functions, respectively, of Model 3. The indices $q = 1,2,\ldots,\infty$ are employed to control the compromise solution in TOPSIS. In general, $q = 1$, $q = 2$, and $q = \infty$ are widely used to deal with multi-objective optimization problems. Different values of q refer to different distances, e.g., $q = 1$ refers to the Manhattan distance (the farthest distance in the geometrical sense), $q = 2$ refers to the Euclidean distance (the least distance in the geometrical sense), and $q = \infty$ refers to the Tchebycheff distance (the shortest distance in the numerical sense). Other distances are used less because they have no concrete meaning in practice. Furthermore, $q = 1$ implies an equal weight for all the deviations, $q = 2$ implies that these deviations are weighted proportionality with the largest deviation having the largest weight, and $q = \infty$ means that the largest deviation completely dominates the distance determination.
Step 5 Set $q = 2$ into Eqs. (7.3) and (7.4). To obtain a Pareto-optimal solution, we transform Model 3 into the following bi-objective problem:

$$\text{minimize}\ d_2^{PIS}(x^1), \tag{7.5}$$

$$\text{maximize}\ d_2^{NIS}(x^1) \tag{7.6}$$

$$\text{subject to constraints}\ (6.18){-}(6.21). \tag{7.7}$$

Step 6 Calculate the individual values to minimize $d_2^{PIS}(x^1)$ and maximize $d_2^{NIS}(x^1)$ subject to the constraints (6.18)–(6.21). Now, we construct the payoff table which is shown in Table 3. For convenience, we introduce notation as: $(d_2^{PIS})^* := d_2^{PIS}(x^{1\,PIS})$, $(d_2^{NIS})^* := d_2^{NIS}(x^{1\,NIS})$, $(d_2^{PIS})' := d_2^{PIS}(x^{1\,NIS})$, $(d_2^{NIS})' := d_2^{NIS}(x^{1\,PIS})$.
Step 7 On the basis of the preference concept, we formulate the membership functions $\mu_1(x^1)$ and $\mu_2(x^1)$ of two objective functions from Step 5. They are defined as follows:

$$\mu_1(x^1) = \begin{cases} 1, & \text{if} \quad d_2^{PIS}(x^1) \leq (d_2^{PIS})^*, \\ 1 - \left(\dfrac{d_2^{PIS}(x^1) - (d_2^{PIS})^*}{(d_2^{PIS})' - (d_2^{PIS})^*}\right), & \text{if} \quad (d_2^{PIS})^* \leq d_2^{PIS}(x^1) \leq (d_2^{PIS})', \\ 0, & \text{if} \quad (d_2^{PIS})' \leq d_2^{PIS}(x^1), \end{cases}$$

and

$$\mu_2(x^1) = \begin{cases} 1, & \text{if} \quad (d_2^{NIS})^* \le d_2^{NIS}(x^1), \\ 1 - \left(\frac{(d_2^{NIS})^* - d_2^{NIS}(x^1)}{(d_2^{NIS})^* - (d_2^{NIS})'} \right), & \text{if} \quad (d_2^{NIS})' \le d_2^{NIS}(x^1) \le (d_2^{NIS})^*, \\ 0, & \text{if} \quad d_2^{NIS}(x^1) \le (d_2^{NIS})'. \end{cases}$$

Step 8 Using the max–min operator, introduced by Bellman and Zadeh [47] and further extended by Zimmermann, we solve the bi-objective problem from Step 5. Finally, we obtain a Pareto-optimal solution of Model 3 by solving the simplified and equivalent form of Eqs. (7.5)–(7.7), which gives us the following form of our model:

$$\begin{aligned} &\text{maximize } \lambda_2 \\ &\text{subject to } \mu_1(x^1) \ge \lambda_2,\ \mu_2(x^1) \ge \lambda_2 \\ &\qquad\qquad \text{constraints } (6.18)-(6.21), \\ &\qquad\qquad 0 \le \lambda_2 \le 1. \end{aligned} \tag{7.8}$$

where $\lambda_2 = \min\{\mu_r(x^1) : r = 1, 2\}$ is the satisfactory level for both the criteria of the minimum distance from the PIS and of the maximum distance from the NIS of Model 3.

Step 9 Repeat Step 3 in similar way for Model 4.

Step 10 Using the PIS and NIS from Model 4, we calculate the distance function from PIS $\left(\text{i.e., } d_q^{PIS}(x^2)\right)$ and the distance function from NIS $\left(\text{i.e., } d_q^{NIS}(x^2)\right)$ as follows:

$$d_q^{PIS}(x^2) = \left[\sum_{K=1}^{3} \left[W_K \frac{Z_K(x^2) - Z_K^+}{Z_K^- - Z_K^+} \right]^q \right]^{\frac{1}{q}}, \tag{7.9}$$

$$\begin{aligned} &d_q^{NIS}(x^2) = \left[\sum_{K=1}^{3} \left[W_K \frac{Z_K^- - Z_K(x^2)}{Z_K^- - Z_K^+} \right]^q \right]^{\frac{1}{q}}, \\ &\sum_{K=1}^{3} W_K = 1,\ W_K \ge 0\ \forall K. \end{aligned} \tag{7.10}$$

The parameters W_K $(K = 1, 2, 3)$ in Eqs. (7.9) and (7.10) stand for the weights of the objective functions. We consider the priorities as weights, and they are $W_1 = 0.5$, $W_2 = 0.4$ and $W_3 = 0.1$, for three objective functions, respectively, of Model 4.

Table 3 Payoff table of ideal solutions

	$d_2^{PIS}(x^1)$	$d_2^{NIS}(x^1)$
x^{1PIS}	$d_2^{PIS}(x^{1PIS})$	$d_2^{NIS}(x^{1PIS})$
x^{1NIS}	$d_2^{PIS}(x^{1NIS})$	$d_2^{NIS}(x^{1NIS})$

Step 11 Set $q = 2$ into Eqs. (7.9) and (7.10). To obtain a Pareto-optimal solution, we transform Model 4 into the following bi-objective problem:

$$\text{minimize } d_2^{PIS}(x^2), \tag{7.11}$$

$$\text{maximize } d_2^{NIS}(x^2) \tag{7.12}$$

$$\text{subject to constraints } (6.25)-(6.28). \tag{7.13}$$

Step 12 Repeat Steps 6 and 7 in a similar way for Model 4.

Step 13 Using a max–min operator, we solve the bi-objective problem from Step 11. Finally, we obtain a Pareto-optimal solution of Model 4 by solving the simplified and equivalent form of Eqs. (7.11)–(7.13), which is as follows:

$$\begin{aligned} &\text{maximize } \lambda_3 \\ &\text{subject to } \mu_3(x^2) \ge \lambda_3,\ \mu_4(x^2) \ge \lambda_3 \\ &\qquad\qquad \text{constraints } (6.25)-(6.28), \\ &\qquad\qquad 0 \le \lambda_3 \le 1. \end{aligned} \tag{7.14}$$

where $\lambda_3 = \min\{\mu_r(x^2) : r = 3, 4\}$ is the satisfactory level for both the criteria of the minimum distance from the PIS and the maximum distance from the NIS of Model 4.

Step 14 From Steps 8 and 13, we finally obtain the Pareto-optimal solution of our proposed MOMIFCSTP (i.e., Model 2).

Theorem 7.3 *If the Pareto-optimal solutions of Model* 3 *and Model* 4 *exist, then the solutions are also the Pareto-optimal solution of Model* 2.

Proof Suppose the Pareto-optimal solutions of Model 3 and Model 4 exist, and they are $x^{*1} = (x_{ijk}^{*1} : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ and $x^{*2} = (x^{*2} : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$, respectively. If there are no other feasible solutions $x^1 = (x_{ijk}^1 : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ for Model 3 and $x^2 = (x_{ijk}^2 : i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, p)$ for Model 4, then from Definitions 5.3 and 5.4, we have for all i, j and k:

$$E[\tilde{\tilde{Z}}_K(x^1)] \le E[\tilde{\tilde{Z}}_K(x^{*1})] \quad \text{for } K = 1, 2, 3, \tag{7.15}$$

$$E[\tilde{\tilde{Z}}_K(x^1)] < E[\tilde{\tilde{Z}}_K(x^{*1})] \quad \text{for at least one } K, \tag{7.16}$$

and

$$E[\tilde{\tilde{Z}}_K(x^2)] \le E[\tilde{\tilde{Z}}_K(x^{*2})] \quad \text{for } K = 1, 2, 3, \tag{7.17}$$

$$E[\tilde{\tilde{Z}}_K(x^2)] < E[\tilde{\tilde{Z}}_K(x^{*2})] \quad \text{for at least one } K. \tag{7.18}$$

Because of inequalities (7.15) and (7.17), (7.16) and (7.18), we can write:

$$E[\tilde{\tilde{Z}}_K(x)] \leq E[\tilde{\tilde{Z}}_K(x^*)] \quad \text{for } K = 1, 2, 3, \tag{7.19}$$

$$E[\tilde{\tilde{Z}}_K(x)] < E[\tilde{\tilde{Z}}_K(x^*)] \quad \text{for at least one } K. \tag{7.20}$$

The inequalities (7.19) and (7.20) follow from Definition 6.1. Thus, the proof of the theorem is completed. □

8 Application example

In this section, first we discuss about the usefulness of fuzzy-rough parameters in our model. In a real-life MOMIFCSTP, the cost parameters (c_{ijk}) are not precise always, but there exist left and right spreads of their values. So, we can consider any cost parameter as a trapezoidal fuzzy number ($\tilde{c}_{ijk}$). Further, the middle value of $\tilde{c}_{ijk}$ is not always a crisp number, because the DM in a company would to take the opinions from more than one expert due to the uncertainty of parameters. We include a discussion to better understand this fact as: Let us assume a reputed medicine company, e.g., in India, has three experts, and their opinions about the middle value of $\tilde{c}_{ijk}$ are: [4, 8] per unit (first expert's opinion); [3, 6] per unit (second expert's opinion) and [4, 7] per unit (third expert's opinion). The DM would like to minimize the transportation cost, so that he or she not only finds the optimal compromise solution, but also uses the opinions of all experts. One way to combine the opinions of the three experts is representing the middle value of $\tilde{c}_{ijk}$ by rough intervals, which is ([4, 6], [3, 8]). Thus, we can represent the cost parameter as a trapezoidal fuzzy-rough variable ($\tilde{\tilde{c}}_{ijk}$). Due to a similar reason, we treat all other parameters in our MOMIFCSTP as fuzzy-rough variables. Furthermore, we illustrate a situation for the applicability of our MOMIFCSTP. The medicine company produces different types of medicine (i.e., multi-item is considered), and to transport the medicine from factories to distribution centers through roads, the company would pay a certain amount of toll charge to National Highways Authority of India for different types of conveyances (i.e., fixed charge is considered). A detailed description of the application example is given below.

The medicine company produces different types of medicine for the seasonal disease cold–cough such as antibiotic and cough syrup. The company has three factories ($m = 3$) and three distribution centers ($n = 3$), situated at different places of India. The company transports two types of medicine ($h = 2$) from factories to distribution centers through two types of conveyances ($p = 2$) by road (small and medium size goods carriers). The DM desires that the total transporting cost (variable cost per unit and fixed cost), total transporting time of medicine (from factories to distribution centers) and packing costs of medicine are to be minimized. Furthermore, the DM decides to find a compromise solution to the problem in which the values of the objective functions are to be minimized. The relative importance of the three objective functions is considered as the weight factors which are specified by the DM. The transportation cost of unit quantity (5000 mg) and fixed charge in dollars, packing cost in dollar per medium pack (specified by the DM) and time in hours are taken into account. Data are collected from different sources which are represented in Tables 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17.

Using Proposition 4.1 which is stated in Sect. 4.5 for the expected-value operator on the trapezoidal fuzzy-rough variable, Tables 4, 5, 6 and 7 reduce to Table 18, Tables 8, 9, 10 and 11 become Table 19, Tables 12 and 13 transform to Table 20, and Tables 14, 15, 16 and 17 reduce to Table 21, respectively.

9 Results and discussion

Here, we discuss the Pareto-optimal solutions of the equivalent crisp Model 2 and of the decomposed Model 3 and Model 4.

Fuzzy programming Utilizing the crisp data of each fuzzy-rough variable from Tables 18, 19, 20, 21 and 22 in Model 2, applying the solution procedure described in Sect. 7.1 and using LINGO, we obtain the following Pareto-optimal solution by fuzzy programming which is shown in Table 23.

Weighted goal programming Using the crisp data of each fuzzy-rough variable from Tables 18, 19, 20, 21 and 22 in Model 5, considering the solution procedure presented in Sect. 7.2 and using LINGO, we derive the following Pareto-optimal solution by WGP which is listed in Table 23.

TOPSIS Considering the crisp data of each fuzzy-rough variable from Tables 18, 19, 20, 21 and 22 in Model 3 and Model 4, utilizing the solution procedure delineated in Sect. 7.3 and using LINGO, we receive the subsequent Pareto-optimal solution by TOPSIS which is displayed in Table 23.

From Table 23, we conclude that the Pareto-optimal solution for MOMIFCSTP, extracted from TOPSIS and WGP, is more preferable than fuzzy programming. As we illustrated, for the real-life problem in connection with a reputed medicine company, transporting cost and transporting time are covered by a higher portion when

Table 4 Transportation cost of first type of medicine $(\tilde{\tilde{c}}^1_{ijk})$ for conveyance $k = 1$

	D_1	D_2	D_3	Supply
S_1	$(\rho^1_{11}-2,\ \rho^1_{11}-1,\ \rho^1_{11}+1,\ \rho^1_{11}+2),\ \rho^1_{11}\vdash([4,6],[3,8])$	$(\rho^1_{12}-2,\ \rho^1_{12}-1,\ \rho^1_{12}+1,\ \rho^1_{12}+2),\ \rho^1_{12}\vdash([5,9],[4,10])$	$(\rho^1_{13}-2,\ \rho^1_{13}-1,\ \rho^1_{13}+1,\ \rho^1_{13}+2),\ \rho^1_{13}\vdash([6,8],[4,9])$	$(\rho_{a^1_1}-2,\ \rho_{a^1_1}-1,\ \rho_{a^1_1}+1,\ \rho_{a^1_1}+2),\ \rho_{a^1_1}\vdash([18,22],[15,27])$
S_2	$(\rho^1_{21}-2,\ \rho^1_{21}-1,\ \rho^1_{21}+1,\ \rho^1_{21}+2),\ \rho^1_{21}\vdash([5,7],[3,9])$	$(\rho^1_{22}-2,\ \rho^1_{22}-1,\ \rho^1_{22}+1,\ \rho^1_{22}+2),\ \rho^1_{22}\vdash([6,10],[4,12])$	$(\rho^1_{23}-2,\ \rho^1_{23}-1,\ \rho^1_{23}+1,\ \rho^1_{23}+2),\ \rho^1_{23}\vdash([4,8],[3,10])$	$(\rho_{a^1_2}-2,\ \rho_{a^1_2}-1,\ \rho_{a^1_2}+1,\ \rho_{a^1_2}+2),\ \rho_{a^1_2}\vdash([25,30],[20,35])$
S_3	$(\rho^1_{31}-2,\ \rho^1_{31}-1,\ \rho^1_{31}+1,\ \rho^1_{31}+2),\ \rho^1_{31}\vdash([6,12],[5,14])$	$(\rho^1_{32}-2,\ \rho^1_{32}-1,\ \rho^1_{32}+1,\ \rho^1_{32}+2),\ \rho^1_{32}\vdash([7,9],[5,11])$	$(\rho^1_{33}-2,\ \rho^1_{33}-1,\ \rho^1_{33}+1,\ \rho^1_{33}+2),\ \rho^1_{33}\vdash([6,10],[5,12])$	$(\rho_{a^1_3}-2,\ \rho_{a^1_3}-1,\ \rho_{a^1_3}+1,\ \rho_{a^1_3}+2),\ \rho_{a^1_3}\vdash([24,28],[22,30])$
Demand	$(\rho_{b^1_1}-2,\ \rho_{b^1_1}-1,\ \rho_{b^1_1}+1,\ \rho_{b^1_1}+2),\ \rho_{b^1_1}\vdash([16,20],[14,22])$	$(\rho_{b^1_2}-2,\ \rho_{b^1_2}-1,\ \rho_{b^1_2}+1,\ \rho_{b^1_2}+2),\ \rho_{b^1_2}\vdash([20,22],[18,26])$	$(\rho_{b^1_3}-2,\ \rho_{b^1_3}-1,\ \rho_{b^1_3}+1,\ \rho_{b^1_3}+2),\ \rho_{b^1_3}\vdash([16,18],[14,21])$	

Table 5 Transportation cost of first type of medicine $(\tilde{\tilde{c}}^1_{ijk})$ for conveyance $k = 2$

	D_1	D_2	D_3	Supply
S_1	$(\tau^1_{11}-2,\ \tau^1_{11}-1,\ \tau^1_{11}+1,\ \tau^1_{11}+2),\ \tau^1_{11}\vdash([5,7],[4,8])$	$(\tau^1_{12}-2,\ \tau^1_{12}-1,\ \tau^1_{12}+1,\ \tau^1_{12}+2),\ \tau^1_{12}\vdash([7,9],[5,11])$	$(\tau^1_{13}-2,\ \tau^1_{13}-1,\ \tau^1_{13}+1,\ \tau^1_{13}+2),\ \tau^1_{13}\vdash([8,10],[6,12])$	$(\rho_{a^1_1}-2,\ \rho_{a^1_1}-1,\ \rho_{a^1_1}+1,\ \rho_{a^1_1}+2),\ \rho_{a^1_1}\vdash([18,22],[15,27])$
S_2	$(\tau^1_{21}-2,\ \tau^1_{21}-1,\ \tau^1_{21}+1,\ \tau^1_{21}+2),\ \tau^1_{21}\vdash([7,9],[5,10])$	$(\tau^1_{22}-2,\ \tau^1_{22}-1,\ \tau^1_{22}+1,\ \tau^1_{22}+2),\ \tau^1_{22}\vdash([4,8],[3,9])$	$(\tau^1_{23}-2,\ \tau^1_{23}-1,\ \tau^1_{23}+1,\ \rho^1_{23}+2),\ \tau^1_{23}\vdash([6,7],[5,18])$	$(\rho_{a^1_2}-2,\ \rho_{a^1_2}-1,\ \rho_{a^1_2}+1,\ \rho_{a^1_2}+2),\ \tau_{a^1_2}\vdash([25,30],[20,35])$
S_3	$(\tau^1_{31}-2,\ \tau^1_{31}-1,\ \tau^1_{31}+1,\ \tau^1_{31}+2),\ \tau^1_{31}\vdash([7,10],[5,12])$	$(\tau^1_{32}-2,\ \tau^1_{32}-1,\ \tau^1_{32}+1,\ \tau^1_{32}+2),\ \tau^1_{32}\vdash([7,13],[6,14])$	$(\tau^1_{33}-2,\ \tau^1_{33}-1,\ \tau^1_{33}+1,\ \tau^1_{33}+2),\ \tau^1_{33}\vdash([5,10],[3,11])$	$(\rho_{a^1_3}-2,\ \rho_{a^1_3}-1,\ \rho_{a^1_3}+1,\ \rho_{a^1_3}+2),\ \rho_{a^1_3}\vdash([24,28],[22,30])$
Demand	$(\rho_{b^1_1}-2,\ \rho_{b^1_1}-1,\ \rho_{b^1_1}+1,\ \rho_{b^1_1}+2),\ \rho_{b^1_1}\vdash([16,20],[14,22])$	$(\rho_{b^1_2}-2,\ \rho_{b^1_2}-1,\ \rho_{b^1_2}+1,\ \rho_{b^1_2}+2),\ \rho_{b^1_2}\vdash([20,22],[18,26])$	$(\rho_{b^1_3}-2,\ \rho_{b^1_3}-1,\ \rho_{b^1_3}+1,\ \rho_{b^1_3}+2),\ \rho_{b^1_3}\vdash([16,18],[14,21])$	

compared to packing cost in order to transport the medicine. As the optimal values of the two objective functions (Z_1 and Z_3) extracted from TOPSIS are better than WGP, the optimal value of the second objective function (Z_2) is equal for our MOMIFCSTP; herewith, we can observe that TOPSIS is better than WGP. Moreover, in WGP the setting of proper goals to the objective functions is difficult, and if we have not chosen properly the target values for the objective functions, for that case, WGP provides a worst optimal solution. But in TOPSIS approach, there are no such cases which effect directly the optimal solution. In this regard, we may say that TOPSIS is better than WGP for our problem. From Table 23, we conclude that the better optimal solutions to the objective functions are: Total transporting cost is 1010.01 \$, total transporting time is 80 h, and packing cost is 414.78 \$ for shipping different types of medicine from the source points to the destinations through different transportation modes. A representation with a bar diagram of the optimal solutions of MOMIFCSTP extracted from different approaches is provided in Fig. 4.

Table 6 Transportation cost of second type of medicine ($\tilde{\tilde{c}}^2_{ijk}$) for conveyance $k = 1$

	D_1	D_2	D_3	Supply
S_1	$(\rho^2_{11} - 2,\ \tau^2_{11} - 1,$ $\rho^2_{11} + 1,\ \rho^2_{11} + 2),$ $\rho^2_{11} \vdash ([7, 9], [5, 10])$	$(\rho^2_{12} - 2,\ \rho^2_{12} - 1,$ $\rho^2_{12} + 1,\ \rho^2_{12} + 2),$ $\rho^2_{12} \vdash ([4, 8], [3, 11])$	$(\rho^2_{13} - 2,\ \rho^2_{13} - 1,$ $\rho^2_{13} + 1,\ \rho^2_{13} + 2),$ $\rho^2_{13} \vdash ([7, 8], [6, 9])$	$(\rho_{a^2_1} - 3,\ \rho_{a^2_1} - 2,$ $\rho_{a^2_1} + 2,\ \rho_{a^2_1} + 3),$ $\rho_{a^2_1} \vdash ([34,37], [32,39])$
S_2	$(\rho^2_{21} - 2,\ \rho^2_{21} - 1,$ $\rho^2_{21} + 1,\ \rho^2_{21} + 2),$ $\rho^2_{21} \vdash ([8, 10], [6, 12])$	$(\rho^2_{22} - 2,\ \rho^2_{22} - 1,$ $\rho^2_{22} + 1,\ \rho^2_{22} + 2),$ $\rho^2_{22} \vdash ([9, 12], [7, 14])$	$(\rho^2_{23} - 2,\ \rho^2_{23} - 1,$ $\rho^2_{23} + 1,\ \rho^2_{23} + 2),$ $\rho^2_{23} \vdash ([10, 12], [7, 13])$	$(\rho_{a^2_2} - 3,\ \rho_{a^2_2} - 2,$ $\rho_{a^2_2} + 2,\ \rho_{a^2_2} + 3),$ $\rho_{a^2_2} \vdash ([28,30], [25,33])$
S_3	$(\rho^2_{31} - 2,\ \rho^2_{31} - 1,$ $\rho^2_{31} + 1,\ \rho^2_{31} + 2),$ $\rho^2_{31} \vdash ([7, 10], [6, 12])$	$(\rho^2_{32} - 2,\ \rho^2_{32} - 1,$ $\rho^2_{32} + 1,\ \rho^2_{32} + 2),$ $\rho^2_{32} \vdash ([5, 7], [4, 9])$	$(\rho^2_{33} - 2,\ \rho^2_{33} - 1,$ $\rho^2_{33} + 1,\ \rho^2_{33} + 2),$ $\rho^2_{33} \vdash ([8, 9], [7, 10])$	$(\rho_{a^2_3} - 3,\ \rho_{a^2_3} - 2,$ $\rho_{a^2_3} + 2,\ \rho_{a^2_3} + 3),$ $\rho_{a^2_3} \vdash ([26, 30], [24, 32])$
Demand	$(\rho_{b^2_1} - 3,\ \rho_{b^2_1} - 2,$ $\rho_{b^2_1} + 2,\ \rho_{b^2_1} + 3),$ $\rho_{b^2_1} \vdash ([22, 25], [20, 28])$	$(\rho_{b^2_2} - 3,\ \rho_{b^2_2} - 2,$ $\rho_{b^2_2} + 2,\ \rho_{b^2_2} + 3),$ $\rho_{b^2_2} \vdash ([18, 20], [16, 24])$	$(\rho_{b^2_2} - 3,\ \rho_{b^2_2} - 2,$ $\rho_{b^2_2} + 2,\ \rho_{b^2_2} + 3),$ $\rho_{b^2_3} \vdash ([17, 21], [15, 25])$	

Table 7 Transportation cost of second type of medicine ($\tilde{\tilde{c}}^2_{ijk}$) for conveyance $k = 2$

	D_1	D_2	D_3	Supply
S_1	$(\tau^2_{11} - 2,\ \tau^2_{11} - 1,$ $\tau^2_{11} + 1,\ \tau^2_{11} + 2),$ $\tau^2_{11} \vdash ([9, 11], [8, 12])$	$(\tau^2_{12} - 2,\ \tau^2_{12} - 1,$ $\tau^2_{12} + 1,\ \tau^2_{12} + 2),$ $\tau^2_{12} \vdash ([7, 10], [6, 11])$	$(\tau^2_{13} - 2,\ \tau^2_{13} - 1,$ $\tau^2_{13} + 1,\ \tau^2_{13} + 2),$ $\tau^2_{13} \vdash ([5, 9], [4, 10])$	$(\rho_{a^2_1} - 3,\ \rho_{a^2_1} - 2,$ $\rho_{a^2_1} + 2,\ \rho_{a^2_1} + 3),$ $\rho_{a^2_1} \vdash ([34,37], [32,39])$
S_2	$(\tau^2_{21} - 2,\ \tau^2_{21} - 1,$ $\tau^2_{21} + 1,\ \tau^2_{21} + 2),$ $\tau^2_{21} \vdash ([8, 11], [7, 13])$	$(\tau^2_{22} - 2,\ \tau^2_{22} - 1,$ $\tau^2_{22} + 1,\ \tau^2_{22} + 2),$ $\tau^2_{22} \vdash ([6, 12], [5, 14])$	$(\tau^2_{23} - 2,\ \tau^2_{23} - 1,$ $\tau^2_{23} + 1,\ \tau^2_{23} + 2),$ $\tau^2_{23} \vdash ([5, 11], [4, 12])$	$(\rho_{a^2_2} - 3,\ \rho_{a^2_2} - 2,$ $\rho_{a^2_2} + 2,\ \rho_{a^2_2} + 3),$ $\tau_{a^2_2} \vdash ([28,30], [25,33])$
S_3	$(\tau^2_{31} - 2,\ \tau^2_{31} - 1,$ $\tau^2_{31} + 1,\ \tau^2_{31} + 2),$ $\tau^2_{31} \vdash ([7, 9], [6, 10])$	$(\tau^2_{32} - 2,\ \tau^2_{32} - 1,$ $\tau^2_{32} + 1,\ \tau^2_{32} + 2),$ $\tau^2_{32} \vdash ([8, 9], [7, 11])$	$(\tau^2_{33} - 2,\ \tau^2_{33} - 1,$ $\tau^2_{33} + 1,\ \tau^2_{33} + 2),$ $\tau^2_{33} \vdash ([7, 10], [5, 13])$	$(\rho_{a^2_3} - 3,\ \rho_{a^2_3} - 2,$ $\rho_{a^2_3} + 2,\ \rho_{a^2_3} + 3),$ $\rho_{a^2_3} \vdash ([26, 30], [24, 32])$
Demand	$(\rho_{b^2_1} - 3,\ \rho_{b^2_1} - 2,$ $\rho_{b^2_1} + 2,\ \rho_{b^2_1} + 3),$ $\rho_{b^2_1} \vdash ([22, 25], [20, 28])$	$(\rho_{b^2_2} - 3,\ \rho_{b^2_2} - 2,$ $\rho_{b^2_2} + 2,\ \rho_{b^2_2} + 3),$ $\rho_{b^2_2} \vdash ([18, 20], [16, 24])$	$(\rho_{b^2_2} - 3,\ \rho_{b^2_2} - 2,$ $\rho_{b^2_2} + 2,\ \rho_{b^2_2} + 3),$ $\rho_{b^2_3} \vdash ([17, 21], [15, 25])$	

Table 8 Fixed charge of first type of medicine ($\tilde{\tilde{f}}^1_{ijk}$) for conveyance $k = 1$

	D_1	D_2	D_3
S_1	$(\varrho^1_{11} - 3,\ \varrho^1_{11} - 2,\ \varrho^1_{11} + 2,\ \varrho^1_{11} + 3),$ $\varrho^1_{11} \vdash ([10, 12], [9, 18])$	$(\varrho^1_{12} - 3,\ \varrho^1_{12} - 2,\ \varrho^1_{12} + 2,\ \varrho^1_{12} + 3),$ $\varrho^1_{12} \vdash ([16, 22], [14, 24])$	$(\varrho^1_{13} - 3,\ \varrho^1_{13} - 2,\ \varrho^1_{13} + 2,\ \varrho^1_{13} + 3),$ $\varrho^1_{13} \vdash ([17, 21], [16, 23])$
S_2	$(\varrho^1_{21} - 3,\ \varrho_{21} - 2,\ \varrho^1_{21} + 2,\ \varrho^1_{21} + 3),$ $\varrho^1_{21} \vdash ([10, 15], [8, 16])$	$(\varrho^1_{22} - 3,\ \varrho^1_{22} - 2,\ \varrho^1_{22} + 2,\ \varrho^1_{22} + 3),$ $\varrho^1_{22} \vdash ([14, 20], [13, 26])$	$(\varrho^1_{23} - 3,\ \varrho^1_{23} - 2,\ \varrho^1_{23} + 2,\ \varrho^1_{23} + 3),$ $\varrho^1_{23} \vdash ([15, 25], [14, 26])$
S_3	$(\varrho^1_{31} - 3,\ \varrho^1_{31} - 2,\ \varrho^1_{31} + 2,\ \varrho^1_{31} + 3),$ $\varrho^1_{31} \vdash ([12, 18], [10, 20])$	$(\varrho^1_{32} - 3,\ \varrho^1_{32} - 2,\ \varrho^1_{32} + 2,\ \varrho^1_{32} + 3),$ $\varrho^1_{32} \vdash ([15, 22], [14, 28])$	$(\varrho^1_{33} - 3,\ \varrho^1_{33} - 2,\ \varrho^1_{33} + 2,\ \varrho^1_{33} + 3),$ $\varrho^1_{33} \vdash ([16, 20], [15, 21])$

Table 9 Fixed charge of first type of medicine ($\tilde{\tilde{f}}^{1}_{ijk}$) for conveyance $k = 2$

	D_1	D_2	D_3
S_1	$(\sigma^1_{11} - 5,\ \sigma^1_{11} - 4,\ \sigma^1_{11} + 4,\ \sigma^1_{11} + 5)$, $\sigma^1_{11} \vdash ([15, 20], [14, 24])$	$(\sigma^1_{12} - 5,\ \sigma^1_{12} - 4,\ \sigma^1_{12} + 4,\ \sigma^1_{12} + 5)$, $\sigma^1_{12} \vdash ([20, 26], [19, 27])$	$(\sigma^1_{13} - 5,\ \sigma^1_{13} - 4,\ \sigma^1_{13} + 4,\ \sigma^1_{13} + 5)$, $\sigma^1_{13} \vdash ([21, 25], [20, 28])$
S_2	$(\sigma^1_{21} - 5,\ \sigma_{21} - 4,\ \sigma^1_{21} + 4,\ \sigma^1_{21} + 5)$, $\sigma^1_{21} \vdash ([18, 22], [16, 25])$	$(\sigma^1_{22} - 5,\ \sigma^1_{22} - 4,\ \sigma^1_{22} + 4,\ \sigma^1_{22} + 5)$, $\sigma^1_{22} \vdash ([21, 23], [20, 24])$	$(\sigma^1_{23} - 5,\ \sigma^1_{23} - 4,\ \sigma^1_{23} + 4,\ \sigma^1_{23} + 5)$, $\sigma^1_{23} \vdash ([22, 24], [20, 25])$
S_3	$(\sigma^1_{31} - 5,\ \sigma^1_{31} - 4,\ \sigma^1_{31} + 4,\ \sigma^1_{31} + 5)$, $\sigma^1_{31} \vdash ([16, 24], [15, 25])$	$(\sigma^1_{32} - 5,\ \sigma^1_{32} - 4,\ \sigma^1_{32} + 4,\ \sigma^1_{32} + 5)$, $\sigma^1_{32} \vdash ([19, 21], [18, 22])$	$(\sigma^1_{33} - 5,\ \sigma^1_{33} - 4,\ \sigma^1_{33} + 4,\ \sigma^1_{33} + 5)$, $\sigma^1_{33} \vdash ([23, 26], [22, 27])$

Table 10 Fixed charge of second type of medicine ($\tilde{\tilde{f}}^{2}_{ijk}$) for conveyance $k = 1$

	D_1	D_2	D_3
S_1	$(\varrho^2_{11} - 3,\ \varrho^2_{11} - 2,\ \varrho^2_{11} + 2,\ \varrho^2_{11} + 3)$, $\varrho^2_{11} \vdash ([11, 13], [10, 20])$	$(\varrho^2_{12} - 3,\ \varrho^2_{12} - 2,\ \varrho^2_{12} + 2,\ \varrho^2_{12} + 3)$, $\varrho^2_{12} \vdash ([15, 20], [13, 26])$	$(\varrho^2_{13} - 3,\ \varrho^2_{13} - 2,\ \varrho^2_{13} + 2,\ \varrho^2_{13} + 3)$, $\varrho^2_{13} \vdash ([16, 22], [14, 24])$
S_2	$(\varrho^2_{21} - 3,\ \varrho^2_{21} - 2,\ \varrho^2_{21} + 2,\ \varrho^2_{21} + 3)$, $\varrho^2_{21} \vdash ([11, 16], [9, 18])$	$(\varrho^2_{22} - 3,\ \varrho^2_{22} - 2,\ \varrho^2_{22} + 2,\ \varrho^2_{22} + 3)$, $\varrho^2_{22} \vdash ([13, 23], [12, 25])$	$(\varrho^2_{23} - 3,\ \varrho^2_{23} - 2,\ \varrho^2_{23} + 2,\ \varrho^2_{23} + 3)$, $\varrho^2_{23} \vdash ([14, 24], [12, 28])$
S_3	$(\varrho^2_{31} - 3,\ \varrho^2_{31} - 2,\ \varrho^2_{31} + 2,\ \varrho^2_{31} + 3)$, $\varrho^2_{31} \vdash ([11, 19], [10, 22])$	$(\varrho^2_{32} - 3,\ \varrho^2_{32} - 2,\ \varrho^2_{32} + 2,\ \varrho^2_{32} + 3)$, $\varrho^2_{32} \vdash ([16, 23], [13, 29])$	$(\varrho^2_{33} - 3,\ \varrho^2_{33} - 2,\ \varrho^2_{33} + 2,\ \varrho^2_{33} + 3)$, $\varrho^2_{33} \vdash ([17, 20], [16, 24])$

Table 11 Fixed charge of second type of medicine ($\tilde{\tilde{f}}^{2}_{ijk}$) for conveyance $k = 2$

	D_1	D_2	D_3
S_1	$(\sigma^2_{11} - 5,\ \sigma^2_{11} - 4,\ \sigma^2_{11} + 4,\ \sigma^2_{11} + 5)$, $\sigma^2_{11} \vdash ([16, 22], [13, 25])$	$(\sigma^2_{12} - 5,\ \sigma^2_{12} - 4,\ \sigma^2_{12} + 4,\ \sigma^2_{12} + 5)$, $\sigma^2_{12} \vdash ([19, 27], [18, 28])$	$(\sigma^2_{13} - 5,\ \sigma^2_{13} - 4,\ \sigma^2_{13} + 4,\ \sigma^2_{13} + 5)$, $\sigma^2_{13} \vdash ([20, 25], [19, 29])$
S_2	$(\sigma^2_{21} - 5,\ \sigma^2_{21} - 4,\ \sigma^2_{21} + 4,\ \sigma^2_{21} + 5)$, $\sigma^2_{21} \vdash ([17, 22], [15, 25])$	$(\sigma^2_{22} - 5,\ \sigma^2_{22} - 4,\ \sigma^2_{22} + 4,\ \sigma^2_{22} + 5)$, $\sigma^2_{22} \vdash ([22, 24], [21, 26])$	$(\sigma^2_{23} - 5,\ \sigma^2_{23} - 4,\ \sigma^2_{23} + 4,\ \sigma^2_{23} + 5)$, $\sigma^2_{23} \vdash ([21, 25], [19, 27])$
S_3	$(\sigma^2_{31} - 5,\ \sigma^2_{31} - 4,\ \sigma^2_{31} + 4,\ \sigma^2_{31} + 5)$, $\sigma^2_{31} \vdash ([17, 23], [15, 25])$	$(\sigma^2_{32} - 5,\ \sigma^2_{32} - 4,\ \sigma^2_{32} + 4,\ \sigma^2_{32} + 5)$, $\sigma^2_{32} \vdash ([18, 22], [17, 24])$	$(\sigma^2_{33} - 5,\ \sigma^2_{33} - 4,\ \sigma^2_{33} + 4,\ \sigma^2_{33} + 5)$, $\sigma^2_{33} \vdash ([22, 27], [20, 29])$

Table 12 Transporting time of both types of medicine ($\tilde{\tilde{t}}_{ijk}$) for conveyance $k = 1$

	D_1	D_2	D_3
S_1	$(\chi_{11} - 2,\ \chi_{11} - 1,\ \chi_{11} + 1,\ \chi_{11} + 2)$, $\chi_{11} \vdash ([8, 9], [6, 10])$	$(\chi_{12} - 2,\ \chi_{12} - 1,\ \chi_{12} + 1,\ \chi_{12} + 2)$, $\chi_{12} \vdash ([7, 9], [5, 10])$	$(\chi_{13} - 2,\ \chi_{13} - 1,\ \chi_{13} + 1,\ \chi_{13} + 2)$, $\chi_{13} \vdash ([6, 8], [5, 11])$
S_2	$(\chi_{21} - 2,\ \chi_{21} - 1,\ \chi_{21} + 1,\ \chi_{21} + 2)$, $\chi_{21} \vdash ([7, 8], [6, 12])$	$(\chi_{22} - 2,\ \chi_{22} - 1,\ \chi_{22} + 1,\ \chi_{22} + 2)$, $\chi_{22} \vdash ([9, 10], [8, 12])$	$(\chi_{23} - 2,\ \chi_{23} - 1,\ \chi_{23} + 1,\ \chi_{23} + 2)$, $\chi_{23} \vdash ([5, 6], [4, 8])$
S_3	$(\chi_{31} - 2,\ \chi_{31} - 1,\ \chi_{31} + 1,\ \chi_{31} + 2)$, $\chi_{31} \vdash ([6, 7], [5, 8])$	$(\chi_{32} - 2,\ \chi_{32} - 1,\ \chi_{32} + 1,\ \chi_{32} + 2)$, $\chi_{32} \vdash ([9, 10], [7, 11])$	$(\chi_{33} - 2,\ \chi_{33} - 1,\ \chi_{33} + 1,\ \chi_{33} + 2)$, $\chi_{33} \vdash ([10, 11], [9, 12])$

Table 13 Transporting time of both types of medicine $(\tilde{\tilde{t}}_{ijk})$ for conveyance $k=2$

	D_1	D_2	D_3
S_1	$(\phi_{11}-4,\ \phi_{11}-3,\ \phi_{11}+3,\ \phi_{11}+4)$, $\phi_{11}\vdash([6,9],[5,10])$	$(\phi_{12}-4,\ \phi_{12}-3,\ \phi_{12}+3,\ \phi_{12}+4)$, $\phi_{12}\vdash([8,10],[7,11])$	$(\phi_{13}-4,\ \phi_{13}-3,\ \phi_{13}+3,\ \phi_{13}+4)$, $\phi_{13}\vdash([9,11],[8,12])$
S_2	$(\phi_{21}-4,\ \phi_{21}-3,\ \phi_{21}+3,\ \phi_{21}+4)$, $\phi_{21}\vdash([10,11],[9,12])$	$(\phi_{22}-4,\ \phi_{22}-3,\ \phi_{22}+3,\ \phi_{22}+4)$, $\phi_{22}\vdash([8,12],[7,13])$	$(\phi_{23}-4,\ \phi_{23}-3,\ \phi_{23}+3,\ \phi_{23}+4)$, $\phi_{23}\vdash([6,10],[5,11])$
S_3	$(\phi_{31}-4,\ \phi_{31}-3,\ \phi_{31}+3,\ \phi_{31}+4)$, $\phi_{31}\vdash([7,8],[6,10])$	$(\phi_{32}-4,\ \phi_{32}-3,\ \phi_{32}+3,\ \phi_{32}+4)$, $\phi_{32}\vdash([11,12],[10,13])$	$(\phi_{33}-4,\ \phi_{33}-3,\ \phi_{33}+3,\ \phi_{33}+4)$, $\phi_{33}\vdash([9,12],[8,14])$

Table 14 Packing cost of first type of medicine $(\tilde{\tilde{d}}^1_{ijk})$ for conveyance $k=1$

	D_1	D_2	D_3
S_1	$(v^1_{11}-0.5,\ v^1_{11}-0.2,\ v^1_{11}+0.2,\ v^1_{11}+0.5)$,$v^1_{11}\vdash([2,3],[1,4])$	$(v^1_{12}-0.5,\ v^1_{12}-0.2,\ v^1_{12}+0.2,\ v^1_{12}+0.5)$,$v^1_{12}\vdash([3,4],[2,5])$	$(v^1_{13}-0.5,\ v^1_{13}-0.2,\ v^1_{13}+0.2,\ v^1_{13}+0.5)$, $v^1_{13}\vdash([3,4],[2,5])$
S_2	$(v^1_{21}-0.5,\ v^1_{21}-0.2,\ v^1_{21}+0.2,\ v^1_{21}+0.5)$, $v^1_{21}\vdash([2.5,3.5],[2,4])$	$(v^1_{22}-0.5,\ v^1_{22}-0.2,\ v^1_{22}+0.2,\ v^1_{22}+0.5)$, $v^1_{22}\vdash([2.5,3],[1.5,4])$	$(v^1_{23}-0.5,\ v^1_{23}-0.2,\ v^1_{23}+0.2,\ v^1_{23}+0.5)$, $v^1_{23}\vdash([2.5,3.5],[1.5,4])$
S_3	$(v^1_{31}-0.5,\ v^1_{31}-0.2,\ v^1_{31}+0.2,\ v^1_{31}+0.5)$, $v^1_{31}\vdash([3.5,4],[3,4.5])$	$(v^1_{32}-0.5,\ v^1_{32}-0.2,\ v^1_{32}+0.2,\ v^1_{32}+0.5)$, $v^1_{32}\vdash([4,5.5],[3,6])$	$(v^1_{33}-0.5,\ v^1_{33}-0.2,\ v^1_{33}+0.2,\ v^1_{33}+0.5)$, $v^1_{33}\vdash([3,4.5],[2.5,5.5])$

Table 15 Packing cost of first type of medicine $(\tilde{\tilde{d}}^1_{ijk})$ for conveyance $k=2$

	D_1	D_2	D_3
S_1	$(\psi^1_{11}-0.6,\ \psi^1_{11}-0.3,\ \psi^1_{11}+0.3,\ \psi^1_{11}+0.6)$, $\psi^1_{11}\vdash([2.5,3.5],[2,4])$	$(\psi^1_{12}-0.6,\ \psi^1_{12}-0.3,\ \psi^1_{12}+0.3,\ \psi^1_{12}+0.6)$, $\psi^1_{12}\vdash([3.5,5.5],[3,6])$	$(\psi^1_{13}-0.6,\ \psi^1_{13}-0.3,\ \psi^1_{13}+0.3,\ \psi^1_{13}+0.6)$, $\psi^1_{13}\vdash([4,5],[3.5,6])$
S_2	$(\psi^1_{21}-0.6,\ \psi^1_{21}-0.3,\ \psi^1_{21}+0.3,\ \psi^1_{21}+0.6)$, $\psi^1_{21}\vdash([2,4],[1.5,5.5])$	$(\psi^1_{22}-0.6,\ \psi^1_{22}-0.3,\ \psi^1_{22}+0.3,\ \psi^1_{22}+0.6)$, $\psi^1_{22}\vdash([2,3],[1.5,4.5])$	$(\psi^1_{23}-0.6,\ \psi^1_{23}-0.3,\ \psi^1_{23}+0.3,\ \psi^1_{23}+0.6)$, $\psi^1_{23}\vdash([4.5,5.5],[2,6])$
S_3	$(\psi^1_{31}-0.6,\ \psi^1_{31}-0.3,\ \psi^1_{31}+0.3,\ \psi^1_{31}+0.6)$, $\psi^1_{31}\vdash([2.5,4.5],[1,5])$	$(\psi^1_{32}-0.6,\ \psi^1_{32}-0.3,\ \psi^1_{32}+0.3,\ \psi^1_{32}+0.6)$, $\psi^1_{32}\vdash([3,6],[2,6.5])$	$(\psi^1_{33}-0.6,\ \psi^1_{33}-0.3,\ \psi^1_{33}+0.3,\ \psi^1_{33}+0.6)$, $\psi^1_{33}\vdash([3.5,4],[1.5,5])$

9.1 Comparison with the existing methods

In this subsection, we mainly present the advantages of the suggested model and method with respect to some existing methods and related models which are listed below:

1. Aggarwal and Gupta [33] solved a single objective STP under a intuitionistic fuzzy environment. But, our formulated model is designed under a multi-objective environment with an additional cost of fixed-charge type, which is more realistic for real-life transportation problems.
2. Zhang et al. [19] considered the parameters of FCSTP as uncertain variables to deal with indeterminacy phenomena in transporting systems. They incorporated an uncertain distribution when historical data are not valid because of unwanted events having happened. For example, when a natural disaster occurs, it may be better to consider uncertain variables as the parameters in transporting systems. However, this is a special case and would not happen in daily-life industrial transporting systems. From this stand point, we choose fuzzy-rough variables as the parameters in our

Table 16 Packing cost of second type of medicine $(\tilde{\tilde{d}}^2_{ijk})$ for conveyance $k = 1$

	D_1	D_2	D_3
S_1	$(v^2_{11}-0.5,\ v^2_{11}-0.2,\ v^2_{11}+0.2,\ v^2_{11}+0.5)$, $v^2_{11} \vdash ([3,4.5],[2.5,5.5])$	$(v^2_{12}-0.5,\ v^2_{12}-0.2,\ v^2_{12}+0.2,\ v^2_{12}+0.5)$, $v^2_{12} \vdash ([3.5,4],[2,5.5])$	$(v^2_{13}-0.5,\ v^2_{13}-0.2,\ v^2_{13}+0.2,\ v^2_{13}+0.5)$, $v^2_{13} \vdash ([2.5,4.5],[2,5.5])$
S_2	$(v^2_{21}-0.5,\ v^2_{21}-0.2,\ v^2_{21}+0.2,\ v^2_{21}+0.5)$, $v^2_{21} \vdash ([3,3.5],[2.5,4.5])$	$(v^2_{22}-0.5,\ v^2_{22}-0.2,\ v^2_{22}+0.2,\ v^2_{22}+0.5)$, $v^2_{22} \vdash ([3,4.5],[2,5])$	$(v^2_{23}-0.5,\ v^2_{23}-0.2,\ v^2_{23}+0.2,\ v^2_{23}+0.5)$, $v^2_{23} \vdash ([2.5,4],[2,4.5])$
S_3	$(v^2_{31}-0.5,\ v^2_{31}-0.2,\ v^2_{31}+0.2,\ v^2_{31}+0.5)$, $v^2_{31} \vdash ([3,4.5],[2,6.5])$	$(v^2_{32}-0.5,\ v^2_{32}-0.2,\ v^2_{32}+0.2,\ v^2_{32}+0.5)$, $v^2_{32} \vdash ([4.5,5],[3.5,6])$	$(v^2_{33}-0.5,\ v^2_{33}-0.2,\ v^2_{33}+0.2,\ v^2_{33}+0.5)$, $v^2_{33} \vdash ([4,5],[3,5.5])$

Table 17 Packing cost of second type of medicine $(\tilde{\tilde{d}}^2_{ijk})$ for conveyance $k = 2$

	D_1	D_2	D_3
S_1	$(\psi^2_{11}-0.6,\ \psi^2_{11}-0.3,\ \psi^2_{11}+0.3,\ \psi^2_{11}+0.6)$, $\psi^2_{11} \vdash ([3.5,4],[2,4.6])$	$(\psi^2_{12}-0.6,\ \psi^2_{12}-0.3,\ \psi^2_{12}+0.3,\ \psi^2_{12}+0.6)$, $\psi^2_{12} \vdash ([4,5],[3.5,6])$	$(\psi^2_{13}-0.6,\ \psi^2_{13}-0.3,\ \psi^2_{13}+0.3,\ \psi^2_{13}+0.6)$, $\psi^2_{13} \vdash ([4.5,5],[3,5.5])$
S_2	$(\psi^2_{21}-0.6,\ \psi^2_{21}-0.3,\ \psi^2_{21}+0.3,\ \psi^2_{21}+0.6)$, $\psi^2_{21} \vdash ([2.5,4.5],[2,6])$	$(\psi^2_{22}-0.6,\ \psi^2_{22}-0.3,\ \psi^2_{22}+0.3,\ \psi^2_{22}+0.6)$, $\psi^2_{22} \vdash ([3,3.6],[2,5])$	$(\psi^2_{23}-0.6,\ \psi^2_{23}-0.3,\ \psi^2_{23}+0.3,\ \psi^2_{23}+0.6)$, $\psi^2_{23} \vdash ([5,6],[3,6.5])$
S_3	$(\psi^2_{31}-0.6,\ \psi^2_{31}-0.3,\ \psi^2_{31}+0.3,\ \psi^2_{31}+0.6)$, $\psi^2_{31} \vdash ([3,5],[2.6,5.6])$	$(\psi^2_{32}-0.6,\ \psi^2_{32}-0.3,\ \psi^2_{32}+0.3,\ \psi^2_{32}+0.6)$, $\psi^2_{32} \vdash ([3.6,5.6],[2.6,6.6])$	$(\psi^2_{33}-0.6,\ \psi^2_{33}-0.3,\ \psi^2_{33}+0.3,\ \psi^2_{33}+0.6)$, $\psi^2_{33} \vdash ([4,5],[2,5.6])$

Table 18 Crisp form of transportation cost in the form $(c^1_{ij1}, c^1_{ij2}, c^2_{ij1}, c^2_{ij2})$

	D_1	D_2	D_3	Supply (a^1_i, a^2_i)
S_1	(5.25, 6, 7.75, 10)	(7, 8, 6.50, 8.5)	(6.75, 9, 7.50, 7)	(20.5, 35.5)
S_2	(6, 7.75, 9, 9.75)	(8, 6, 10.50, 9.25)	(6.50, 9, 10.50, 8)	(27.5, 29)
S_3	(9.25, 6, 8.75, 7.94)	(8, 10, 6.25, 8.75)	(8.19, 7.25, 8.50, 8.69)	(26, 28)
Demand (b^1_j, b^2_j)	(18, 23.75)	(21.5, 19.5)	(17.25, 19.5)	

Table 19 Crisp form of fixed charge in the form $(f^1_{ij1}, f^1_{ij2}, f^2_{ij1}, f^2_{ij2})$

	D_1	D_2	D_3
S_1	(12.25, 18.25, 13.56, 19)	(19, 22.92, 18.50, 23)	(19.25, 16.38, 19, 23.25)
S_2	(12.25, 20.25, 13.50, 19.75)	(18.25, 22, 18.25, 23.25)	(20, 22.75, 19.50, 23.13)
S_3	(15, 20, 15.50, 20)	(19.75, 20, 20.25, 20.25)	(18, 24.50, 19.25, 24.50)

Table 20 Crisp form of transporting time in the form (t_{ij1}, t_{ij2})

	D_1	D_2	D_3
S_1	(8.25, 7.50)	(7.75, 9)	(7.50, 10)
S_2	(8, 10.50)	(9.75, 10)	(5.75, 8)
S_3	(6.50, 7.75)	(9.25, 11.50)	(10.50, 10.75)

proposed study which efficiently deals with real-life industrial transporting systems.

3. Kundu et al. [17] solved a multi-objective multi-item solid transportation problem by fuzzy programming. In Table 23, we see that both WGP and extended TOPSIS give better results than fuzzy programming. Moreover, the suggested model is considered with an extra cost of fixed charge under twofold uncertain environment. The

Table 21 Crisp form of packing cost in the form $(d_{ij1}^1, d_{ij2}^1, d_{ij1}^2, d_{ij2}^2)$

	D_1	D_2	D_3
S_1	(2.50, 3, 3.88, 3.53)	(3.50, 4.44, 3.75, 4.69)	(3.50, 4.63, 3.63, 4.50)
S_2	(3, 3.25, 3.38, 3.75)	(2.75, 2.75, 3.63, 3.40)	(2.86, 4.50, 3.31, 5.13)
S_3	(3.75, 3.25, 4, 4.05)	(4.63, 4.38, 4.75, 4.54)	(3.88, 3.50, 4.38, 4.15)

Table 22 Capacity of kth conveyance $(\tilde{\tilde{e}}_k)$ and its expected value $(E[\tilde{\tilde{e}}_k])$

	$\tilde{\tilde{e}}_k$	$E[\tilde{\tilde{e}}_k]$
$k = 1$	$(\omega_{e_1} - 7, \omega_{e_1} - 5, \omega_{e_1} + 5, \omega_{e_1} + 7)$, $\omega_{e_1} \vdash ([70, 72], [67, 74])$	70.75
$k = 2$	$(\omega_{e_2} - 7, \omega_{e_2} - 5, \omega_{e_2} + 5, \omega_{e_2} + 7)$ $\omega_{e_2} \vdash ([60, 65], [57, 68])$	62.50

authors of [17], however, formulated a multi-objective multi-item STP under a fuzzy environment. From this viewpoint, it can be said that our model is more preferable to tackle real-life transporting systems.

4. Rani and Gulati [34] proposed a method to solve a multi-objective multi-product solid transportation problem under a fuzzy environment by fuzzy programming. But difficulties arise of their method if the number of objective functions is more than two, and a large number of variables is involved into the transporting problem. Instead, our suggested extended TOPSIS is free from such types of difficulties and it gives better results than fuzzy programming.
5. Ebrahimnejad and Verdegay [48] proposed an approach to solve intuitionistic fuzzy transportation problems. But this approach is not suitable to tackle a large number of variables in an industrial transporting system as it leads to a higher computational burden. Instead, our proposed extended TOPSIS approach can easily handle a large number of variables and it means less computational expense. In [48], the authors formulated TP under intuitionistic fuzzy environment. However, some practical situations arise where both fuzziness and roughness exist simultaneously. For

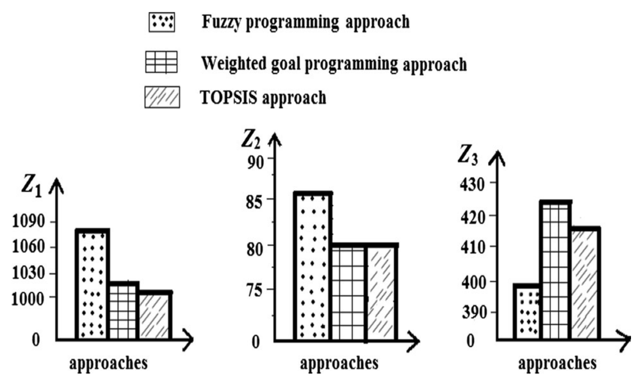

Fig. 4 Optimal values of Z_1, Z_2 and Z_3 by three approaches

these cases our suggested model can deal the situations well.

6. Singh et al. [35] solved a multi-objective solid transportation problem under a stochastic environment by fuzzy programming. From Table 23, it is concluded that both WGP and extended TOPSIS give better results than fuzzy programming. Moreover, our proposed model is formulated on multi-item ground with an extra cost of fixed-charge type, which is more preferable for an industrial transporting system.

10 Concluding remarks and future research directions

In this paper, for the first time in the course of research, we introduce the concept of fuzzy-rough variable in MOMIFCSTP by considering three objective functions which are connected with a real-life problem. We need less computational effort to generate the optimal solutions which are closest to the PIS and farthest from the NIS. In

Table 23 Pareto-optimal solutions of the proposed MOMIFCSTP

Name of the method	Optimal values of the objective functions Z_1, Z_2, Z_3	Values of $\lambda_1, \lambda_2, \lambda_3$ and d_1^+, d_2^+, d_3^+
Fuzzy programming to Model 2	1087.05, 86.25, 398.48	$\lambda_1 = 0.64$
Weighted goal programming to Model 2 (i.e., Model 5)	1014.95, 80, 422.76	$d_1^+ = 3.07, d_2^+ = 65.25$ $d_3^+ = 22.76$
TOPSIS to Model 3 and Model 4 (i.e., Model 2)	1010.01, 80, 414.78	$\lambda_2 = 0.96, \lambda_3 = 0.22$

TOPSIS approach, we address Euclidean distance measure and a max–min operator to solve the MOMIFCSTP as a bi-objective problem for getting a Pareto-optimal solution. Fuzzy programming and WGP are also employed to obtain a Pareto-optimal solution. A comparative study is drawn among the Pareto-optimal solutions from the approaches. To the best of our knowledge, we, for the first time, have applied TOPSIS approach to solve the multi-objective multi-item fixed-charge solid transportation problem, and we have seen that using of TOPSIS here produced a better result of a Pareto-optimal solution than fuzzy programming and WGP. Finally, from the applied viewpoint, we have concluded that our model is highly significant in real-life situations; this gives a new paradigm to the decision maker.

In future studies, the contents of this paper can open a new dimension to make a separate investigation for both the fixed-charge solid transportation problem and the fixed-charge transportation problem in fuzzy and rough environments. One may consider different distance measures for different values of the index q in TOPSIS approach to solve the multi-objective transportation problem under different uncertain environments. Researchers can use a portfolio of approaches for representing and handling of uncertainty, such as gray numbers, ellipsoid uncertainty and stochastic in real-life multi-objective transportation problems, and conduct the corresponding comparisons.

Compliance with ethical standards

Conflict of interest The authors have no conflict of interest for the publication of this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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