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ORIGINAL ARTICLE

A robust correlation coefficient for probabilistic dual hesitant fuzzy sets and its applications

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Abstract

As a generalization of the hesitant fuzzy sets (HFSs) and dual HFSs (DHFSs), probabilistic dual hesitant fuzzy sets (PDHFSs) are a strong and valuable tool to represent the imprecise information by embedding both the features of HFSs and probabilistic information instantaneously. Meanwhile, a correlation coefficient is a prominent measure to measure the relationship between two sets. Motivated by these primary characteristics, it is interesting to present some information measures to the PDHFSs and hence decision-making approach based on the correlation coefficient. In this paper, we develop a method to solve the multi-criteria decision-making (MCDM) problem under PDHFS environment. For it, firstly, we define the informational energy and the covariance between the two PDHFSs and study their properties. Secondly, we develop correlation coefficients and the weighted correlation coefficients for PDHFSs. In the formulation, DHFSs are able to represent the information in terms of their respective degrees, while the assigned probabilities give more details about the level of agreeness or disagreeness. Also, some properties of the proposed measures are also studied. Thirdly, a novel algorithm is developed based on the proposed operators to solve MCDM problems. A practical example is provided to verify the developed approach and to demonstrate its practicality and feasibility. Also, a comparative analysis with several existing studies reveals the proposed method is better during solving the decision-making problems.

Keywords Probabilistic dual hesitant fuzzy sets · Correlation coefficient · Personnel selection · Multi-criteria decision-making

1 Introduction

Multiple criteria decision-making (MCDM) problems are a huge part of human society and applied widely to practical fields like economics, management, and engineering. In the MCDM process, the task is to find the finest alternatives among the feasible ones. With the development of science and technology, the uncertainty also plays a dominant factor during the decision-making (DM) analysis. Further, the role of the decision-makers during the process is so challenging in order to collect precise data. Most of the information collected from the various resources is either uncertain or imprecise in nature and hence leads to inaccurate results. Thus, the task of the DM process is to find the finest objects among the available by utilizing this imprecise or uncertain information. To this, time-to-time, a variety of concepts have been applied to reach the correct decisions by utilizing the features such as crisp, deterministic, and precise in nature. However, to handle the uncertainty in the data, a concept of fuzzy sets (FSs) [39] and their extensions such as intuitionistic FSs (IFSs) [3], interval-valued IFSs (IVIFSs) [2], and linguistic IVIFSs [14] describe the components with membership degrees (MDs) and non-membership degrees (NMDs) with the end goal that their sum is not more than one. Later on, Torra [27] presented the concept of the hesitant fuzzy sets (HFSs) which capture the multiple discrete values of the MDs. Also, Xia and Xu [34] established the concepts of HFSs mathematically and defined their basic element, hesitant fuzzy elements (HFEs), contained in [0, 1]. Zhu et al. [43] presented the concept of dual HFSs (DHFSs) by assigning

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hesitant degrees of NMDs along with MDs into the analysis and also defined its basic element, dual HFEs (DHFEs).

Under the above-stated theories, researchers are paying more attention to solve decision-making problems (DMPs) by using different kinds of aggregation operators or information measures. Among these various ideas, one is to locate the finest alternative utilizing correlation coefficients (CCs) which assume an overwhelming job to quantify the level of dependency between the two sets. In statistical analysis, the CCs measure the linear relationship, whereas in FS theory, they determine the degree of dependency between the two variables. In that direction, Gerstenkorn and Manko [16] developed CCs for IFSs, whereas Bustince and Burillo [4] gave the CCs for IVIFSs. Park et al. [24] corroborated the CC of IVIFSs and applied them to solve the DMPs. Mitchell [23] derived simple, intuitively satisfying CC between two IFSs and elicited the relevance of obtained values by evaluating through different test stages. Enhancing the existing concepts to channelize the appropriate decision outcomes in the DM problems, Garg [8] threw light on CCs between Pythagorean fuzzy sets (PFSs). Wei et al. [33] highlighted the application of CC to IVIFSs with incomplete weight information. The CCs for the intuitionistic multiplicative sets are being presented by Garg [9]. Also, Garg and Kumar [13] presented CCs of IFSs based on the connection number of the set pair analysis.

In these above speculations, it is accepted that the ratings provided by the specialists are either a "single number" or "an interval." However, if an expert wants to give their values in terms of discrete sets, then the concept of the HFSs [27] and DHFSs [43] was introduced by the researchers. Under these environments, Chen et al. [5] and Xu and Xia [35] presented the CCs for HFSs. Farhadinia [7] and Ye [38] presented the CCs and applied them to solve the multi-attribute DMPs. Tyagi [28] presented the CCs for DHFSs. Chen et al. [6] presented an algorithm for solving DMPs under the dual hesitant fuzzy information. Meng and Chen [22] presented the Shapley weighted CCs to solve the DM problem having HFS data with incomplete weights. Arora and Garg [1] presented the CCs of dual hesitant fuzzy soft sets. Wang et al. [31] presented the correlation measures of DHFSs. Liao and Xu [20] defined the entropy and CC measures for HFSs. Recently, Sun et al. [26] presented a TOPSIS approach based on CC for HFSs. Yang et al. [37] discussed the weighted CC and applied this for solving DM problems of supplier selection and medical diagnosis. Guan et al. [17] pointed out the errors of existing CCs [5, 21, 35] and developed an improved CC measure based on the mean, variance, and length of each HFS.

Although HFSs and DHFSs are successfully applied in various DMPs, they do not address the issue regarding the occurrence of the probabilities of the elements in the HFSs or DHFSs. Here, we take the following examples as an illustration: Consider a person who gives their preferences toward the comfortless of an object in terms of HFE as {0.3, 0.5, 0.6}. During its ratings, he suggested that the comfort level corresponding to 0.5 is the most desirable as compared to others, while the comfort level associated with 0.3 is more desirable than 0.6. Thus, under such circumstance, the HFE {0.3, 0.5, 0.6} is not suitable to describe the information. Similarly, consider a rating of a person toward the evaluation of the quality of a product in terms of DHFE ({0.3, 0.4, 0.5}, {0.2, 0.3, 0.4}). During the ratings, the decision-maker believes that their comfortable toward the object rating 0.3, 0.4 is double than 0.5 in membership degrees, while triple toward the 0.4 in the non-membership degrees with respect to the others. Thus, again such DHFE ({0.3, 0.4, 0.5}, {0.2, 0.3, 0.4}) is not suitable to describe the information. To resolve such problems, the concepts of the probabilistic HFS (PHFS) and probabilistic DHFS (PDHFS) were investigated by Xu and Zhou [36] and Hao et al. [18], respectively, and defined their basic elements as probabilistic HFEs (PHFEs) and probabilistic DHFEs (PDHFEs). The PDHFE provides a more accurate description than DHFE, HFE, and PHFE and can be easily used to describe the information in the above-stated examples. Under these environments, Zhou and Xu [41], Zhu and Xu [42] defined the concept of the probabilistic hesitant fuzzy preference relation and studied its expected consistency. Wang and Li [32] presented the CCs for the PHFEs and an algorithm based on them for solving the DMPs. Kobina et al. [19] defined the concept of the probabilistic linguistic power aggregation operators for solving the DMPs. Zhou and Xu [40] defined the group consistency under uncertain probabilistic hesitant fuzzy preference environment. Ren et al. [25] presented the TODIM (an acronym in Portuguese of interactive and multi-criteria decision-making) method for PDHFSs. Garg and Kaur [12] presented aggregation operators-based algorithms with some information measures to solve the DMPs under the PDHFS environment.

Consider the fact that PDHFSs are a more strong and valuable tool to represent the information in a more profitable way. As the PDHFEs, represented as $(h|q, g|q)$, consist of two parts, that is, the membership parts (h, g) represent MD and NMD of the elements and the corresponding probabilities (p, q). Thus, this information can be treated as a probability distribution function with h, g as the random variables and p, q as the probabilities.

Therefore, keeping these points in mind, in this paper, we compute the expected mean related to the multiplication of two PDHFEs by computing the probabilities using the northwest corner rule. Apart from this, the present work also stated an algorithm to compute the probabilities of the aggregated numbers. For example, if two different experts evaluate the given object x under the DHFSs environment, one expert assigns $(\{0.2, 0.3\}, \{0.2, 0.4\})$ as PDFHE, while the other assign $(\{0.2, 0.4\}, \{0.4, 0.5\})$. In such a case, due to the loss of information, the DHFE ({0.2, 0.3, 0.4}, {0.2, 0.4, 0.5}) is not suitable to describe the information, as both the experts have different opinions regarding the object x. To address this case, the data are consolidated into PDHFS by breaking down the probabilities of a choice given by both the experts.

Therefore, motivated by the structure of the PDHFS, importance of CC, and the above-mentioned limitations, this paper focuses on the CC between the pairs of PDHFEs. As per our knowledge, the correlation measures cannot be utilized to handle the PDHFEs. Thus, we need to propose such measures to compute the relative strength between the pairs of PDHFEs. For it, we define the informational energies and the covariance between the pairs of PDHFEs. The northwest corner method has been utilized to compute the joint probabilities of the sets. The major advantage of the present method is that there is no need to match the length of the considered PDHFSs by repeating the values, as in the case of DHFEs. Then, based on these, we define four CCs, based on the inherent characteristics, and obtain some properties. Further, a novel MCDM method based on the proposed CCs is presented and illustrated it with some practical problems. The performance of the proposed measure is compared with several existing theories.

The rest of the text is organized as follows. In Sect. 2, we briefly present the concepts of HFSs, DHFSs, and PDHFS. In Sect. 3, we established the CCs to measure the strength between the pairs of PDHFSs and investigated their properties. Section 4 constructs an approach to deal with the MCDM problems. Section 5 illustrates the approach with some numerical examples and compared their results with some existing approaches. Finally, concluding remarks are given in Sect. 6.

2 Preliminaries

Some basic concepts related to HFSs, DHFSs, and PDHFSs are reviewed here over the set $\mathcal{X}$.

Definition 1 [27, 34] A HFS $\mathcal{H} : \mathcal{X} \to [0, 1]$ is defined as:

$$\mathcal{H} = \left\{(x, h_{\mathcal{H}}(x)) \mid x \in \mathcal{X}\right\} \tag{1}$$

where $h_{\mathcal{H}}$ is a discrete set of different values in [0, 1], denoting the possible MDs of $x \in \mathcal{X}$.

Definition 2 [43] A DHFS $\mathcal{D}$ on $\mathcal{X}$ is defined as

$$\mathcal{D} = \left\{(x, h_{\mathcal{D}}(x), g_{\mathcal{D}}(x)) \mid x \in \mathcal{X}\right\}, \tag{2}$$

where $h_{\mathcal{D}}(x)$ and $g_{\mathcal{D}}(x)$ are two discrete sets of some values in [0,1], denoting the possible MDs and NMDs of $x \in \mathcal{X}$ to the set $\mathcal{D}$, respectively, with the condition that $0 \le \gamma_{\mathcal{D}}, \eta_{\mathcal{D}} \le 1$ and $0 \le \gamma_{\mathcal{D}}^+ + \eta_{\mathcal{D}}^+ \le 1$ where $\gamma_{\mathcal{D}} \in h_{\mathcal{D}}$, $\eta_{\mathcal{D}} \in g_{\mathcal{D}}$, $\gamma_{\mathcal{D}}^+ \in h_{\mathcal{D}}^+ = \bigcup_{\gamma_{\mathcal{D}} \in h_{\mathcal{D}}} \max\{\gamma_{\mathcal{D}}\}, \eta_{\mathcal{D}}^+ \in g_{\mathcal{D}}^+ = \bigcup_{\eta_{\mathcal{D}} \in g_{\mathcal{D}}} \max\{\eta_{\mathcal{D}}\}$. A pair $\mathcal{D} = (h_{\mathcal{D}}, g_{\mathcal{D}}) = \bigcup_{\gamma_{\mathcal{D}} \in h_{\mathcal{D}}, \eta_{\mathcal{D}} \in g_{\mathcal{D}}} (\{\gamma_{\mathcal{D}}\}, \{\eta_{\mathcal{D}}\})$ is called as DHFE.

Definition 3 [36] A PHFS $\mathcal{P}$ on $\mathcal{X}$ can be expressed as

$$\mathcal{P} = \left\{\left(x, h_x \middle| p_x\right) \mid x \in \mathcal{X}\right\}, \tag{3}$$

where $h_x \in [0, 1]$ be the possible membership values of the set $\mathcal{P}$ and $p_x \in [0, 1]$ be their associated probabilities. For convenience, $h_x(p_x)$ is called a PHFE and is indicated as

$$h(p) = \left\{\left(\gamma_i \middle| p_i\right) \mid i = 1, 2, \ldots, \#h_{\mathcal{P}}\right\}, \tag{4}$$

with $\gamma_i \in h_i$ and p_i be the probability of γ_i such that $\sum_{i=1}^{\#h_{\mathcal{P}}} p_i = 1$. Here, $\#h_{\mathcal{P}}$ is the cardinality of $\gamma_i | p_i$.

Definition 4 [18] A PDHFS $\mathcal{P}$ on $\mathcal{X}$ is defined as

$$\mathcal{P} = \left\{\left(x, h_x \middle| p_x, g_x \middle| q_x\right) \mid x \in \mathcal{X}\right\}. \tag{5}$$

Here, the components $h_x, g_x \in [0, 1]$ are the hesitant MDs and NMDs, while p_x, q_x be their associated probabilities with the conditions, $0 \le \gamma, \eta \le 1, 0 \le \gamma^+, \eta^+ \le 1$ such that $\gamma \in h_x, \eta \in g_x$, $\gamma^+ \in h_x^+ = \bigcup_{\gamma \in h_x} \max\{\gamma\}, \eta^+ \in g_x^+ = \bigcup_{\eta \in g_x} \max\{\eta\}, p_i \in p_x$ and $q_j \in q_x$ with $\sum_{i=1}^{\#h_{\mathcal{P}}} p_i = 1$, $\sum_{j=1}^{\#g_{\mathcal{P}}} q_j = 1$. The symbols $\#h_{\mathcal{P}}$ and $\#g_{\mathcal{P}}$ represent the cardinality of the components $h_x | p_x$ and $g_x | q_x$, respectively. A pair

$$\mathcal{A} = (h_{\mathcal{A}} \mid p_{\mathcal{A}}, g_{\mathcal{A}} \mid q_{\mathcal{A}}) = \bigcup_{\substack{\gamma_j \in h_{\mathcal{A}} \\ \eta_k \in g_{\mathcal{A}}}} \left(\{\gamma_j | p_j\}, \{\eta_k | q_k\}\right) \tag{6}$$

with $j = 1, 2, \ldots, \#h_{\mathcal{A}}; k = 1, 2, \ldots, \#g_{\mathcal{A}}$ is called as PDHFE.

Definition 5 [18] For two PDHFEs $\mathcal{A} = (h_{\mathcal{A}}|p_{\mathcal{A}}, g_{\mathcal{A}}|q_{\mathcal{A}}) = \bigcup_{\gamma_j \in h_{\mathcal{A}}, \eta_k \in g_{\mathcal{A}}} (\{\gamma_j \mid p_j\}, \{\eta_k \mid q_k\})$ and $\mathcal{B} = (h_{\mathcal{B}}|p_{\mathcal{B}}, g_{\mathcal{B}}|q_{\mathcal{B}}) = \bigcup_{\gamma'_{j'} \in h_{\mathcal{B}}, \eta'_{k'} \in g_{\mathcal{B}}} \left(\{\gamma'_{j'} \mid p'_{j'}\}, \{\eta'_{k'} \mid q'_{k'}\}\right)$, where $j = 1, 2, \ldots, \#h_{\mathcal{A}}; k = 1, 2, \ldots, \#g_{\mathcal{A}}$; $j' = 1, 2, \ldots, \#h_{\mathcal{B}}; k' = 1, 2, \ldots, \#g_{\mathcal{B}}$, the basic operations on $\mathcal{A}$ and $\mathcal{B}$ are defined as

(i) $\mathcal{A} = \mathcal{B}$ if and only if

$$\bigcup_{\gamma_j \in h_{\mathcal{A}}} \{\gamma_j\} = \bigcup_{\gamma'_{j'} \in h_{\mathcal{B}}} \left\{\gamma'_{j'}\right\}, \quad \bigcup_{\eta_k \in g_{\mathcal{A}}} \{\eta_k\} = \bigcup_{\eta'_{k'} \in g_{\mathcal{B}}} \{\eta'_{k'}\},$$

$$\bigcup_{p_j \in p_{\mathcal{A}}} \{p_j\} = \bigcup_{p'_{j'} \in p_{\mathcal{B}}} \{p'_{j'}\}, \quad \bigcup_{q_k \in q_{\mathcal{A}}} \{q_k\} = \bigcup_{q'_{k'} \in q_{\mathcal{B}}} \{q'_{k'}\}.$$

(ii) $\mathcal{A} \subseteq \mathcal{B}$, if

$$\bigcup_{\gamma_j \in h_{\mathcal{A}}} \{\gamma_j\} \leq \bigcup_{\gamma'_{j'} \in h_{\mathcal{B}}} \{\gamma'_{j'}\}, \quad \bigcup_{\eta_k \in g_{\mathcal{A}}} \{\eta_k\} \geq \bigcup_{\eta'_{k'} \in g_{\mathcal{B}}} \{\eta'_{k'}\}$$

$$\bigcup_{p_j \in p_{\mathcal{A}}} \{p_j\} = \bigcup_{p'_{j'} \in p_{\mathcal{B}}} \{p'_{j'}\}, \quad \bigcup_{q_k \in q_{\mathcal{A}}} \{q_k\} = \bigcup_{q'_{k'} \in q_{\mathcal{B}}} \{q'_{k'}\}$$

(iii) The complement of PDHFE $\mathcal{A}$ is

$$\mathcal{A}^c = \begin{cases} \bigcup\limits_{\gamma_j \in h_{\mathcal{A}}, \eta_k \in g_{\mathcal{A}}} (\{\eta_k|q_k\}, \{\gamma_j|p_j\}), & \text{if } h_{\mathcal{A}} \neq \phi \text{ and } g_{\mathcal{A}} \neq \phi \\ \bigcup\limits_{\gamma_j \in h_{\mathcal{A}}} (\{1-\gamma_j|p_j\}, \{\phi\}), & \text{if } h_{\mathcal{A}} \neq \phi \text{ and } g_{\mathcal{A}} = \phi \\ \bigcup\limits_{\eta_k \in g_{\mathcal{A}}} (\{\phi\}, \{1-\eta_k|q_k\}), & \text{if } h_{\mathcal{A}} = \phi \text{ and } g_{\mathcal{A}} \neq \phi \end{cases}$$

3 Correlation coefficient on PDHFSs

For a universal set $\mathcal{X} = \{x_i \mid i = 1, , 2\ldots, n\}$, we define the concept of the informational energy, covariance, and CC for PDHFSs. For it, we use the following notations throughout the paper.

Notations	Meaning	Notations	Meaning
n	Number of elements in $\mathcal{X}$	N'	Number of elements in $g_{\mathcal{B}}$
$h_{\mathcal{A}}$	Hesitant membership values of set $\mathcal{A}$	p	Probability for hesitant membership of set $\mathcal{A}$
$g_{\mathcal{A}}$	Hesitant non-membership values of set $\mathcal{A}$	q	Probability for hesitant non-membership of set $\mathcal{A}$
$h_{\mathcal{B}}$	Hesitant membership values of set $\mathcal{B}$	p'	Probability for hesitant membership of set $\mathcal{B}$
$g_{\mathcal{B}}$	Hesitant non-membership values of set $\mathcal{B}$	q'	Probability for hesitant non-membership of set $\mathcal{B}$
M	Number of elements in $h_{\mathcal{A}}$	ω	Weight vector
N	Number of elements in $g_{\mathcal{A}}$	m	Number of alternatives
M'	Number of elements in $h_{\mathcal{B}}$	t	Number of criteria

Definition 6 Let two PDHFEs $\mathcal{A} = (h_{\mathcal{A}}|p_{\mathcal{A}}, g_{\mathcal{A}}|q_{\mathcal{A}}) = \bigcup_{\gamma_{i,j} \in h_{\mathcal{A}}, \eta_{i,k} \in g_{\mathcal{A}}} (\{\gamma_{i,j}|p_{i,j}\}, \{\eta_{i,k}|q_{i,k}\})$ and $\mathcal{B} = (h_{\mathcal{B}}|p_{\mathcal{B}}, g_{\mathcal{B}}|q_{\mathcal{B}}) = \bigcup_{\gamma'_{i,j'} \in h_{\mathcal{B}}, \eta'_{i,k'} \in g_{\mathcal{B}}} \left(\{\gamma'_{i,j'}|p'_{i,j'}\}, \{\eta'_{i,k'}|q'_{i,k'}\}\right)$ where $j = 1, 2, \ldots, M_i$; $k = 1, 2, \ldots, N_i; j' = 1, 2, \ldots, M'_i; k = 1, 2, \ldots, N'_i$, the informational energies of them are defined as

$$\mathcal{I}(\mathcal{A}) = \sum_{i=1}^{n} \left(\sum_{j=1}^{M_i} (\gamma_{i,j})^2 p_{i,j} + \sum_{k=1}^{N_i} (\eta_{i,k})^2 q_{i,k} \right) \tag{7}$$

and

$$\mathcal{I}(\mathcal{B}) = \sum_{i=1}^{n} \left(\sum_{j'=1}^{M'_i} (\gamma'_{i,j'})^2 p'_{i,j'} + \sum_{k'=1}^{N'_i} (\eta'_{i,k'})^2 q'_{i,k'} \right). \tag{8}$$

Further, the covariance between $\mathcal{A}$ and $\mathcal{B}$ is given as:

$$\mathcal{C}(\mathcal{A}, \mathcal{B}) = \sum_{i=1}^{n} \left(\sum_{j=1}^{M_i} \sum_{j'=1}^{M'_i} \gamma_{i,j} \gamma'_{i,j'} p_{i,jj'} + \sum_{k=1}^{N_i} \sum_{k'=1}^{N'_i} \eta_{i,k} \eta_{i,k'} q_{i,kk'} \right) \tag{9}$$

where $p_{i,jj'}$ and $q_{i,kk'}$ are the joint probabilities of $\mathcal{A}$ and $\mathcal{B}$ calculated as below:

Joint probability distribution between $\mathcal{A}$ and $\mathcal{B}$

Membership values						Non-membership values					
	$\gamma'_{i,1'}$	$\gamma'_{i,2'}$	$\cdots$	γ'_{i,M'_i}			$\eta'_{i,1'}$	$\eta'_{i,2'}$	$\cdots$	η'_{i,N'_i}	
$\gamma_{i,1}$	$p_{i,11'}$	$p_{i,12'}$	$\cdots$	$p_{i,1M'_i}$	$p_{i,1}$	$\eta_{i,1}$	$q_{i,11'}$	$q_{i,12'}$	$\cdots$	$q_{i,1N'_i}$	$q_{i,1}$
$\gamma_{i,2}$	$p_{i,21'}$	$p_{i,22'}$	$\cdots$	$p_{i,2M'_i}$	$p_{i,2}$	$\eta_{i,2}$	$q_{i,21'}$	$q_{i,22'}$	$\cdots$	$q_{i,2N'_i}$	$q_{i,2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
γ_{i,M_i}	$p_{i,M_i1'}$	$p_{i,M_i2'}$	$\cdots$	$p_{i,M_iM'_i}$	p_{i,M_i}	η_{i,N_i}	$q_{i,N_i1'}$	$q_{i,N_i2'}$	$\cdots$	$q_{i,N_iN'_i}$	q_{i,N_i}
	$p'_{i,1}$	$p'_{i,2}$	$\cdots$	p'_{i,M'_i}	1		$q'_{i,1}$	$q'_{i,2}$	$\cdots$	q'_{i,N'_i}	1

From Eq. (9), it is seen that $\mathcal{C}(\mathcal{A},\mathcal{B}) = \mathcal{C}(\mathcal{B},\mathcal{A})$ and $\mathcal{C}(\mathcal{A},\mathcal{A}) = \mathcal{I}(\mathcal{A})$. However, it is also being noted that when $\mathcal{A} = \mathcal{B}$, the joint probability distribution of $\mathcal{C}(\mathcal{A},\mathcal{B})$ is equal to $\mathcal{I}(\mathcal{A})$, i.e., $p_{i,jj'} = p_{i,j}$ which is computed by using the northwest corner rule as demonstrated below:

Joint probabilities computed using northwest corner rule

Membership values						Non-membership values					
	$\gamma_{i,1}$	$\gamma_{i,2}$	$\cdots$	γ_{i,M_i}			$\eta_{i,1}$	$\eta_{i,2}$	$\cdots$	η_{i,N_i}	
$\gamma_{i,1}$	$p_{i,1}$	0	$\cdots$	0	$p_{i,1}$	$\eta_{i,1}$	$q_{i,1}$	0	$\cdots$	0	$q_{i,1}$
$\gamma_{i,2}$	0	$p_{i,2}$	$\cdots$	0	$p_{i,2}$	$\eta_{i,2}$	0	$q_{i,2}$	$\cdots$	0	$q_{i,2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
γ_{i,M_i}	0	0	$\cdots$	p_{i,M_i}	p_{i,M_i}	η_{i,N_i}	0	0	$\cdots$	q_{i,N_i}	q_{i,N_i}
	$p_{i,1}$	$p_{i,2}$	$\cdots$	p_{i,M_i}	1		$q_{i,1}$	$q_{i,2}$	$\cdots$	q_{i,N_i}	1

By utilizing the above concept, we define the CC for PDHFEs $\mathcal{A}$ and $\mathcal{B}$ as follows:

Definition 7 For two PDHFSs $\mathcal{A} = (h_{\mathcal{A}}|p_{\mathcal{A}}, g_{\mathcal{A}}|q_{\mathcal{A}}) = \bigcup_{\gamma_{i,j}\in h_{\mathcal{A}},\eta_{i,k}\in g_{\mathcal{A}}}\left(\{\gamma_{i,j}|p_{i,j}\},\{\eta_{i,k}|q_{i,k}\}\right)$ and $\mathcal{B} = (h_{\mathcal{B}}|p_{\mathcal{B}}, g_{\mathcal{B}}|q_{\mathcal{B}}) = \bigcup_{\gamma'_{i,j'}\in h_{\mathcal{B}},\eta'_{i,k'}\in g_{\mathcal{B}}}\left(\{\gamma'_{i,j'}|p'_{i,j'}\},\{\eta'_{i,k'}|q'_{i,k'}\}\right)$ defined on $\mathcal{X}$, the correlation coefficient denoted by $\mathcal{K}_1(\mathcal{A},\mathcal{B})$ is defined as

$$\mathcal{K}_1(\mathcal{A},\mathcal{B}) = \frac{\mathcal{C}(\mathcal{A},\mathcal{B})}{\sqrt{\mathcal{I}(\mathcal{A})\times\mathcal{I}(\mathcal{B})}} = \frac{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M'_i}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'} + \sum_{k=1}^{N_i}\sum_{k'=1}^{N'_i}\eta_{i,k}\eta_{i,k'}q_{i,kk'}\right)}{\sqrt{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}(\gamma_{i,j})^2p_{i,j} + \sum_{k=1}^{N_i}(\eta_{i,k})^2q_{i,k}\right)}\sqrt{\sum_{i=1}^{n}\left(\sum_{j'=1}^{M'_i}(\gamma'_{i,j'})^2p'_{i,j'} + \sum_{k'=1}^{N'_i}(\eta'_{i,k'})^2q'_{i,k'}\right)}} \quad (10)$$

Theorem 1 *The correlation coefficient $\mathcal{K}_1$ satisfies the following properties for PDHFEs $\mathcal{A}$ and $\mathcal{B}$:*

(P1) $0 \le \mathcal{K}_1(\mathcal{A},\mathcal{B}) \le 1$.
(P2) $\mathcal{K}_1(\mathcal{A},\mathcal{B}) = \mathcal{K}_1(\mathcal{B},\mathcal{A})$.
(P3) If $\mathcal{A} = \mathcal{B}$, then $\mathcal{K}_1(\mathcal{A},\mathcal{B}) = 1$.

Proof For two PDHFSs $\mathcal{A} = (h_{\mathcal{A}}|p_{\mathcal{A}}, g_{\mathcal{A}}|q_{\mathcal{A}}) = \bigcup_{\gamma_{i,j}\in h_{\mathcal{A}},\eta_{i,k}\in g_{\mathcal{A}}}\left(\{\gamma_{i,j}|p_{i,j}\},\{\eta_{i,k}|q_{i,k}\}\right)$ and $\mathcal{B} = (h_{\mathcal{B}}|p_{\mathcal{B}}, g_{\mathcal{B}}|q_{\mathcal{B}}) = \bigcup_{\gamma'_{i,j'}\in h_{\mathcal{B}},\eta'_{i,k'}\in g_{\mathcal{B}}}\left(\{\gamma'_{i,j'}|p'_{i,j'}\},\{\eta'_{i,k'}|q'_{i,k'}\}\right)$ defined on $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$

(P1) The inequality $\mathcal{K}_1(\mathcal{A},\mathcal{B}) \ge 0$ holds straightforward, and therefore, $\mathcal{K}_1(\mathcal{A},\mathcal{B}) \ge 0$. Now, from Eq. (9), we have

$$
\begin{aligned}
\mathcal{C}(\mathcal{A},\mathcal{B}) &= \sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M_i'}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'} + \sum_{k=1}^{N_i}\sum_{k'=1}^{N_i'}\eta_{i,k}\eta'_{i,k'}q_{i,kk'}\right)\\
&= \sum_{i=1}^{n}\left(\begin{array}{l}\left\{\sum_{j=1}^{M_i}\Big((\gamma_{i,j})(\gamma'_{i,1})p_{i,j1}\Big) + \sum_{j=1}^{M_i}\Big((\gamma_{i,j})(\gamma'_{i,2})p_{i,j2}\Big) + \cdots + \sum_{j=1}^{M_i}\Big((\gamma_{i,j})(\gamma'_{i,M_i'})p_{i,jM_i'}\Big)\right\}\\ + \left\{\sum_{k=1}^{N_i}\Big((\eta_{i,k})(\eta'_{i,1})q_{i,k1}\Big) + \sum_{k=1}^{N_i}\Big((\eta_{i,k})(\eta'_{i,2})q_{i,k2}\Big) + \cdots + \sum_{k=1}^{N_i}\Big((\eta_{i,k})(\eta'_{i,N_i'})q_{i,kN_i'}\Big)\right\}\end{array}\right)\\
&= \sum_{i=1}^{n}\left(\begin{array}{l}\Big((\gamma_{i,1})(\gamma'_{i,1})p_{i,11}\Big) + \Big((\gamma_{i,2})(\gamma'_{i,1})p_{i,21}\Big) + \cdots + \Big((\gamma_{i,M_i})(\gamma'_{i,1})p_{i,M_i1}\Big) + \cdots\\ + \Big((\gamma_{i,1})(\gamma'_{i,M_i'})p_{i,1M_i'}\Big) + \Big((\gamma_{i,2})(\gamma'_{i,M_i'})p_{i,2M_i'}\Big) + \cdots + \Big((\gamma_{i,M_i})(\gamma'_{i,M_i'})p_{i,M_iM_i'}\Big)\end{array}\right)\\
&\quad + \sum_{i=1}^{n}\left(\begin{array}{l}\Big((\eta_{i,1})(\eta'_{i,1})q_{i,11}\Big) + \Big((\eta_{i,2})(\eta'_{i,1})q_{i,21}\Big) + \cdots + \Big((\eta_{i,N_i})(\eta'_{i,1})q_{i,N_i1}\Big) + \cdots\\ + \Big((\eta_{i,1})(\eta'_{i,N_i'})q_{i,1N_i'}\Big) + \Big((\eta_{i,2})(\eta'_{i,N_i'})q_{i,2N_i'}\Big) + \cdots + \Big((\eta_{i,N_i})(\eta'_{i,N_i'})q_{i,N_iN_i'}\Big)\end{array}\right)\\
&= \begin{pmatrix}\gamma_{1,1}\gamma'_{1,1}p_{1,11}+\\ \cdots+\gamma_{n,1}\gamma'_{n,1}p_{n,11}\end{pmatrix} + \cdots + \begin{pmatrix}\gamma_{1,M_1}\gamma'_{1,1}p_{1,M_11}+\\ \cdots+\gamma_{n,M_n}\gamma'_{n,1}p_{n,M_n1}\end{pmatrix} + \begin{pmatrix}\gamma_{1,1}\gamma'_{1,M_1'}p_{1,1M_1'}+\\ \cdots+\gamma_{n,1}\gamma'_{n,M_n'}p_{n,1M_n'}\end{pmatrix}\\
&\quad + \cdots + \begin{pmatrix}\gamma_{1,M_1}\gamma'_{1,M_1'}p_{1,M_1M_1'}+\\ \cdots+\gamma_{n,M_n}\gamma'_{n,M_n'}p_{n,M_nM_1'}\end{pmatrix} + \begin{pmatrix}\eta_{1,1}\eta'_{1,1}q_{1,11}+\\ \cdots+\eta_{n,1}\eta'_{n,1}q_{n,11}\end{pmatrix} + \cdots + \begin{pmatrix}\eta_{1,N_1}\eta'_{1,1}q_{1,N_11}+\\ \cdots+\eta_{n,N_n}\eta'_{n,1}q_{n,N_n1}\end{pmatrix}\\
&\quad + \begin{pmatrix}\eta_{1,1}\eta'_{1,N_1'}q_{1,1N_1'}+\\ \cdots+\eta_{n,1}\eta'_{n,N_n'}q_{n,1N_n'}\end{pmatrix} + \cdots + \begin{pmatrix}\eta_{1,N_1}\eta'_{1,N_1'}q_{1,N_1N_1'}+\\ \cdots+\eta_{n,N_n}\eta'_{n,N_n'}q_{n,N_nN_1'}\end{pmatrix}\\
&= \begin{pmatrix}\gamma_{1,1}\sqrt{p_{1,11}}\gamma'_{1,1}\sqrt{p_{1,11}}+\\ \cdots+\gamma_{n,1}\sqrt{p_{n,11}}\gamma'_{n,1}\sqrt{p_{n,11}}\end{pmatrix} + \cdots + \begin{pmatrix}\gamma_{1,M_1}\sqrt{p_{1,M_11}}\gamma'_{1,1}\sqrt{p_{1,M_11}}+\\ \cdots+\gamma_{n,M_n}\sqrt{p_{n,M_n1}}\gamma'_{n,1}\sqrt{p_{n,M_n1}}\end{pmatrix}\\
&\quad + \begin{pmatrix}\gamma_{1,1}\sqrt{p_{1,1M_1'}}\gamma'_{1,M_1'}\sqrt{p_{1,1M_1'}}+\\ \cdots+\gamma_{n,1}\sqrt{p_{n,1M_n'}}\gamma'_{n,M_n'}\sqrt{p_{n,1M_n'}}\end{pmatrix} + \cdots + \begin{pmatrix}\gamma_{1,M_1}\sqrt{p_{1,M_1M_1'}}\gamma'_{1,M_1'}\sqrt{p_{1,M_1M_1'}}+\\ \cdots+\gamma_{n,M_n}\sqrt{p_{n,M_nM_1'}}\gamma'_{n,M_n'}\sqrt{p_{n,M_nM_1'}}\end{pmatrix}\\
&\quad + \begin{pmatrix}\eta_{1,1}\sqrt{q_{1,11}}\eta'_{1,1}\sqrt{q_{1,11}}+\\ \cdots+\eta_{n,1}\sqrt{q_{n,11}}\eta'_{n,1}\sqrt{q_{n,11}}\end{pmatrix} + \cdots + \begin{pmatrix}\eta_{1,N_1}\sqrt{q_{1,N_11}}\eta'_{1,1}\sqrt{q_{1,N_11}}+\\ \cdots+\eta_{n,N_n}\sqrt{q_{n,N_n1}}\eta'_{n,1}\sqrt{q_{n,N_n1}}\end{pmatrix}\\
&\quad + \begin{pmatrix}\eta_{1,1}\sqrt{q_{1,1N_1'}}\eta'_{1,N_1'}\sqrt{q_{1,1N_1'}}+\\ \cdots+\eta_{n,1}\sqrt{q_{n,1N_n'}}\eta'_{n,N_n'}\sqrt{q_{n,1N_n'}}\end{pmatrix} + \cdots + \begin{pmatrix}\eta_{1,N_1}\sqrt{q_{1,N_1N_1'}}\eta'_{1,N_1'}\sqrt{q_{1,N_1N_1'}}+\\ \cdots+\eta_{n,N_n}\sqrt{q_{n,N_nN_1'}}\eta'_{n,N_n'}\sqrt{q_{n,N_nN_1'}}\end{pmatrix}
\end{aligned}
$$

Using Cauchy–Schwarz inequality, $(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 \leq (x_1^2 + x_2^2 + \cdots + x_n^2) \cdot (y_1^2 + y_2^2 + \cdots + y_n^2)$, where $(x_1 + x_2 + \cdots + x_n)$ and $(y_1 + y_2 + \cdots + y_n) \in \mathcal{R}^n$, we get

$$
\begin{aligned}
&\left(\mathcal{C}(\mathcal{A},\mathcal{B})\right)^2\\
&\leq \left(\begin{array}{l}
\begin{pmatrix}(\gamma_{1,1})^2 p_{1,11}+\\ \cdots+(\gamma_{n,1})^2 p_{n,11}\end{pmatrix}+\cdots+\begin{pmatrix}(\gamma_{1,M_1})^2 p_{1,M_11}+\\ \cdots+(\gamma_{n,M_n})^2 p_{n,M_n1}\end{pmatrix}+\begin{pmatrix}(\gamma_{1,1})^2 p_{1,1M'_1}+\\ \cdots+(\gamma_{n,1})^2 p_{n,1M'_n}\end{pmatrix}+\cdots\\
+\begin{pmatrix}(\gamma_{1,M_1})^2 p_{1,M_1M'_1}+\\ \cdots+(\gamma_{n,M_n})^2 p_{n,M_nM'_n}\end{pmatrix}+\begin{pmatrix}(\eta_{1,1})^2 q_{1,11}+\\ \cdots+(\eta_{n,1})^2 q_{n,11}\end{pmatrix}+\cdots+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_11}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_n1}\end{pmatrix}\\
+\begin{pmatrix}(\eta_{1,1})^2 q_{1,1N'_1}+\\ \cdots+(\eta_{n,1})^2 q_{n,1N'_n}\end{pmatrix}+\cdots+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_1N'_1}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_nN'_n}\end{pmatrix}
\end{array}\right)\\
&\times\left(\begin{array}{l}
\begin{pmatrix}(\eta_{1,1})^2 q_{1,11}+\\ \cdots+(\eta_{n,1})^2 q_{n,11}\end{pmatrix}+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_11}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_n1}\end{pmatrix}+\begin{pmatrix}(\eta_{1,1})^2 q_{1,1N'_1}+\\ \cdots+(\eta_{n,1})^2 q_{n,1N'_n}\end{pmatrix}+\cdots\\
+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_1N'_1}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_nN'_n}\end{pmatrix}+\begin{pmatrix}(\eta_{1,1})^2 q_{1,11}+\\ \cdots+(\eta_{n,1})^2 q_{n,11}\end{pmatrix}+\cdots+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_11}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_n1}\end{pmatrix}\\
+\begin{pmatrix}(\eta_{1,1})^2 q_{1,1N'_1}+\\ \cdots+(\eta_{n,1})^2 q_{n,1N'_n}\end{pmatrix}+\cdots+\begin{pmatrix}(\eta_{1,N_1})^2 q_{1,N_1N'_1}+\\ \cdots+(\eta_{n,N_n})^2 q_{n,N_nN'_n}\end{pmatrix}
\end{array}\right).\\
&=\left(\begin{array}{l}
\sum_{i=1}^{n}\left(\begin{array}{l}\sum_{j'=1}^{M'_i}(\gamma_{i,1})^2 p_{i,1j'}+\sum_{j'=1}^{M'_i}(\gamma_{i,2})^2 p_{i,2j'}+\cdots+\sum_{j'=1}^{M'_i}(\gamma_{i,M_i})^2 p_{i,M_ij'}\\ +\sum_{k'=1}^{N'_i}(\eta_{i,1})^2 q_{i,1k'}+\sum_{k'=1}^{N'_i}(\eta_{i,2})^2 q_{i,2k'}+\cdots+\sum_{k'=1}^{N'_i}(\eta_{i,N_i})^2 q_{i,N_ik'}\end{array}\right)\\
\times\sum_{i=1}^{n}\left(\begin{array}{l}\sum_{j'=1}^{M'_i}(\gamma'_{i,1})^2 p_{i,1j'}+\sum_{j'=1}^{M'_i}(\gamma'_{i,2})^2 p_{i,2j'}+\cdots+\sum_{j'=1}^{M'_i}(\gamma'_{i,M_i})^2 p_{i,M_ij'}\\ +\sum_{k'=1}^{N'_i}(\eta'_{i,1})^2 q_{i,1k'}+\sum_{k'=1}^{N'_i}(\eta'_{i,2})^2 q_{i,2k'}+\cdots+\sum_{k'=1}^{N'_i}(\eta'_{i,N_i})^2 q_{i,N_ik'}\end{array}\right)
\end{array}\right)\\
&=\left(\begin{array}{l}
\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}(\gamma_{i,j})^2(p_{i,jj})+\sum_{k=1}^{N_i}(\eta_{i,k})^2(q_{i,kk})\right)\\
\times\sum_{i=1}^{n}\left(\sum_{j'=1}^{M'_i}(\gamma'_{i,j'})^2(p_{i,j'j'})+\sum_{k'=1}^{N'_i}(\eta_{i,k'})^2(q_{i,k'k'})\right)
\end{array}\right)\\
&=\mathcal{I}(\mathcal{A})\times\mathcal{I}(\mathcal{B})
\end{aligned}
$$

Therefore, $\left(\mathcal{C}(\mathcal{A},\mathcal{B})\right)^2 \leq \mathcal{I}(\mathcal{A}) \times \mathcal{I}(\mathcal{B})$. Thus, from Eq. (10), it follows that $\mathcal{K}_1(\mathcal{A},\mathcal{B}) \leq 1$. Hence, $0 \leq \mathcal{K}_1(\mathcal{A},\mathcal{B}) \leq 1$.

(P2) Proof is obvious, so we omit it here.

(P3) Since $\mathcal{A} = \mathcal{B}$, i.e., for all $j = 1, 2, \ldots, M_i; k = 1, 2, \ldots, N_i; j' = 1, 2, \ldots, M'_i; k' = 1, 2, \ldots, N'_i$, we have $\gamma_{i,j} = \gamma'_{i,j'}$, $p_{i,j} = p'_{i,j'}$, $\eta_{i,k} = \eta'_{i,k'}$, $q_{i,k} = q'_{i,k'}$; therefore,

$$\mathcal{I}(\mathcal{A}) = \sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\left(\gamma_{i,j}\right)^2 p_{i,j} + \sum_{k=1}^{N_i}\left(\eta_{i,k}\right)^2 q_{i,k}\right) = \mathcal{I}(\mathcal{B})\,.$$

Since $\mathcal{C}(\mathcal{A},\mathcal{A}) = \mathcal{I}(\mathcal{A})$, we obtain $\mathcal{K}_1(\mathcal{A},\mathcal{B}) = 1$. □.

In order to incorporate the pessimistic feature of the decision -maker toward the process, we define a new correlation coefficient by taking the maximum among the energies of the set. This is defined as below.

Definition 8 For two PDHFSs $\mathcal{A}$ and $\mathcal{B}$, the correlation coefficient $\mathcal{K}_2$ is defined as:

$$\begin{aligned}\mathcal{K}_2(\mathcal{A},\mathcal{B}) &= \frac{\mathcal{C}(\mathcal{A},\mathcal{B})}{\max\{\mathcal{I}(\mathcal{A}),\mathcal{I}(\mathcal{B})\}}\\ &= \frac{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M_i'}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'} + \sum_{k=1}^{N_i}\sum_{k'=1}^{N_i'}\eta_{i,k}\eta_{i,k'}q_{i,kk'}\right)}{\max\left\{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\left(\gamma_{i,j}\right)^2 p_{i,j} + \sum_{k=1}^{N_i}\left(\eta_{i,k}\right)^2 q_{i,k}\right), \sum_{i=1}^{n}\left(\sum_{j'=1}^{M_i'}\left(\gamma'_{i,j'}\right)^2 p'_{i,j'} + \sum_{k'=1}^{N_i'}\left(\eta'_{i,k'}\right)^2 q'_{i,k'}\right)\right\}}\end{aligned} \quad (11)$$

Theorem 2 *The correlation coefficient $\mathcal{K}_2$ has the following properties:*

(P1) $0 \le \mathcal{K}_2(\mathcal{A},\mathcal{B}) \le 1$.
(P2) $\mathcal{K}_2(\mathcal{A},\mathcal{B}) = \mathcal{K}_2(\mathcal{B},\mathcal{A})$.
(P3) $\mathcal{K}_2(\mathcal{A},\mathcal{B}) = 1$, if $\mathcal{A} = \mathcal{B}$.

Proof By Cauchy–Schwarz inequality:

$$\begin{aligned}\sum_{j=1}^{n} x_j y_j &\le \sqrt{\left(\sum_{j=1}^{n} x_j^2\right)\cdot\left(\sum_{j=1}^{n} y_j^2\right)}\\ &\le \sqrt{\left(\max\left\{\sum_{j=1}^{n} x_j^2, \sum_{j=1}^{n} y_j^2\right\}\right)^2} = \max\left\{\sum_{j=1}^{n} x_j^2, \sum_{j=1}^{n} y_j^2\right\}\end{aligned}$$

with equality if and only if the two vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ are linearly dependent.

Thus, by Eq. (11), we get $0 \le \mathcal{K}_2(\mathcal{A},\mathcal{B}) \le 1$.

Also,

$$\begin{aligned}\mathcal{K}_2(\mathcal{A},\mathcal{B}) &= \frac{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M_i'}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'} + \sum_{k=1}^{N_i}\sum_{k'=1}^{N_i'}\eta_{i,k}\eta_{i,k'}q_{i,kk'}\right)}{\max\left\{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\left(\gamma_{i,j}\right)^2 p_{i,j} + \sum_{k=1}^{N_i}\left(\eta_{i,k}\right)^2 q_{i,k}\right), \sum_{i=1}^{n}\left(\sum_{j'=1}^{M_i'}\left(\gamma'_{i,j'}\right)^2 p'_{i,j'} + \sum_{k'=1}^{N_i'}\left(\eta'_{i,k'}\right)^2 q'_{i,k'}\right)\right\}}\\ &= \frac{\sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M_i'}\gamma'_{i,j'}\gamma_{i,j}p_{i,jj'} + \sum_{k=1}^{N_i}\sum_{k'=1}^{N_i'}\eta_{i,k'}\eta_{i,k}q_{i,kk'}\right)}{\max\left\{\sum_{i=1}^{n}\left(\sum_{j'=1}^{M_i'}\left(\gamma'_{i,j'}\right)^2 p'_{i,j'} + \sum_{k'=1}^{N_i'}\left(\eta'_{i,k'}\right)^2 q'_{i,k'}\right), \sum_{i=1}^{n}\left(\sum_{j=1}^{M_i}\left(\gamma_{i,j}\right)^2 p_{i,j} + \sum_{k=1}^{N_i}\left(\eta_{i,k}\right)^2 q_{i,k}\right)\right\}}\\ &= \mathcal{K}_2(\mathcal{B},\mathcal{A})\end{aligned}$$

□.

To illustrate the working of it, we give a numerical example as follows.

Example 1 Let $\mathcal{A} = \Big\{\big(x_1, (\{0.6|0.2, 0.2|0.8\}, \{0.3|0.6, 0.4|0.4\})\big), \big(x_2, (\{0.2|0.3, 0.4|0.7\}, \{0.5|1\})\big)\Big\}$ and $\mathcal{B} = \Big\{\big(x_1, (\{0.3|0.5, 0.4|0.5\}, \{0.4|1\})\big), \big(x_2, (\{0.4|1\}, \{0.3|\ 0.6, 0.5|0.4\})\big)\Big\}$ be two PDHFSs defined over $\mathcal{X} = \{x_1, x_2\}$. Then, by Eq. (7), we get

$$\begin{aligned}\mathcal{I}(\mathcal{A}) &= \sum_{i=1}^{2}\left(\sum_{j=1}^{M_i}\left(\gamma_{i,j}\right)^2 p_{i,j} + \sum_{k=1}^{M_i'}\left(\eta_{i,k}\right)^2 q_{i,k}\right)\\ &= (0.6)^2 \times 0.2 + (0.2)^2 \times 0.8\\ &\quad + (0.3)^2 \times 0.6 + (0.4)^2 \times 0.4\\ &\quad + (0.2)^2 \times 0.3 + (0.4)^2 \times 0.7 + (0.5)^2 \times 1\\ &= 0.5960\end{aligned}.$$

Similarly, we get

$$\begin{aligned}\mathcal{I}(\mathcal{B}) &= (0.3)^2 \times 0.5 + (0.4)^2 \times 0.5 + (0.4)^2 \times 1 + (0.4)^2 \times 1\\ &\quad + (0.3)^2 \times 0.6 + (0.5)^2 \times 0.4\\ &= 0.5990\end{aligned}.$$

Furthermore, by using northwest corner rule, we compute joint probabilities corresponding to x_1 and x_2 as

Joint probability calculation for membership values						
x_1	0.3	0.4	p_{1,M_i} $(i=1,2)$	x_2	0.4	p_{2,M_i} $(i=1,2)$
0.6	0.2	0	0.2	0.3	0.6	0.6
0.2	0.3	0.5	0.8	0.4	0.4	0.4
p'_{1,M'_i} $(i=1,2)$	0.5	0.5		$p'_{2,M'_i}(i=1)$	1	

Based on it,

$$\sum_{i=1}^{n}\sum_{j=1}^{M_i}\sum_{j'=1}^{M'_i}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'} = (0.6\times0.3\times0.2)+(0.2\times0.3\times0.3) + (0.2\times0.4\times0.5)+(0.3\times0.4\times0.6) + (0.4\times0.4\times0.4) = 0.23$$

Similarly, for non-membership values of $\mathcal{A}$ and $\mathcal{B}$, we have

Joint probability calculation for non-membership values						
x_1	0.4	q_{1,N_i} $(i=1,2)$	x_2	0.3	0.5	q_{2,N_i} $(i=1)$
0.3	0.6	0.6	0.5	0.6	0.4	1
0.4	0.4	0.4				

Joint probability calculation for non-membership values				
q'_{1,N'_i} $(i=1)$	1	$q'_{2,N'_i}(i=1,2)$	0.6	0.4

Thus,

$$\sum_{i=1}^{n}\sum_{k=1}^{N_i}\sum_{k'=1}^{N'_i}\eta_{i,k}\eta_{i,k'}q_{i,kk'} = (0.3\times0.4\times0.6)+(0.4\times0.4\times0.4) + (0.5\times0.3\times0.6)+(0.5\times0.5\times0.4) = 0.3260$$

Hence, by Eq. (9), we obtain $\mathcal{C}(\mathcal{A},\mathcal{B}) = 0.2300 + 0.3260 = 0.5560$. Therefore, Eq. (10) and Eq. (11) become $\mathcal{K}_1(\mathcal{A},\mathcal{B}) = \frac{0.5560}{\sqrt{0.5990}\sqrt{0.5960}} = 0.9305$ and $\mathcal{K}_2(\mathcal{A},\mathcal{B}) = \frac{0.5560}{\max\{0.5990, 0.5960\}} = 0.9282.$

In all the above-stated correlation formulae, equal priority is given to all the elements of universal set. This may not be relevant to the real-life scenario, as we often come across such entities which are given more weightage as compared to the other ones. To tackle such cases, we assign the weight $\omega_i > 0$ with $\sum_{i=1}^{n}\omega_i = 1$ to each of the element of $\mathcal{X}$ and define weighted CCs between two PDHFSs as follows:

$$\mathcal{K}_3(\mathcal{A},\mathcal{B}) = \frac{C_\omega(\mathcal{A},\mathcal{B})}{\sqrt{\mathcal{I}_\omega(\mathcal{A})}\sqrt{\mathcal{I}_\omega(\mathcal{B})}} = \frac{\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M'_i}\left(\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'}\right)+\sum_{k=1}^{N_i}\sum_{k'=1}^{N'_i}\left(\eta_{i,k}\eta'_{i,k'}q_{i,kk'}\right)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{M_i}\left(\left(\gamma_{i,j}\right)^2p_{i,j}\right)+\sum_{k=1}^{N_i}\left(\left(\eta_{i,k}\right)^2q_{i,k}\right)\right)}\sqrt{\sum_{i=1}^{n}\omega_i\left(\sum_{j'=1}^{M'_i}\left(\left(\gamma'_{i,j'}\right)^2p'_{i,j'}\right)+\sum_{k'=1}^{N'_i}\left(\left(\eta'_{i,k'}\right)^2q'_{i,k'}\right)\right)}} \tag{12}$$

and

$$\mathcal{K}_4(\mathcal{A},\mathcal{B}) = \frac{C_\omega(\mathcal{A},\mathcal{B})}{\max\{\mathcal{I}_\omega(\mathcal{A}),\mathcal{I}_\omega(\mathcal{B})\}} = \frac{\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{M_i}\sum_{j'=1}^{M'_i}\gamma_{i,j}\gamma'_{i,j'}p_{i,jj'}+\sum_{k=1}^{N_i}\sum_{k'=1}^{N'_i}\eta_{i,k}\eta'_{i,k'}q_{i,kk'}\right)}{\max\left\{\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{M_i}\left(\left(\gamma_{i,j}\right)^2p_{i,j}\right)+\sum_{k=1}^{N_i}\left(\left(\eta_{i,k}\right)^2q_{i,k}\right)\right),\sum_{i=1}^{n}\omega_i\left(\sum_{j'=1}^{M'_i}\left(\left(\gamma'_{i,j'}\right)^2p'_{i,j'}\right)+\sum_{k'=1}^{N'_i}\left(\left(\eta'_{i,k'}\right)^2q'_{i,k'}\right)\right)\right\}} \tag{13}$$

Also, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then Eqs. (12) and (13) reduce to Eqs. (10) and (11), respectively.

Theorem 3 *The coefficient defined in Eq. (12) has the following properties:*

(P1) $0 \leq \mathcal{K}_3(\mathcal{A}, \mathcal{B}) \leq 1$.
(P2) $\mathcal{K}_3(\mathcal{A}, \mathcal{B}) = \mathcal{K}_3(\mathcal{B}, \mathcal{A})$.
(P3) $\mathcal{K}_3(\mathcal{A}, \mathcal{B}) = 1$, *if* $\mathcal{A} = \mathcal{B}$.

Proof The properties (P2) and (P3) are straightforward, so we omit their proofs. Also, the inequality $\mathcal{K}_3(\mathcal{A}, \mathcal{B}) \geq 0$ is evident since $\mathcal{C}_w(\mathcal{A}, \mathcal{B}) \geq 0$, we shall show that $\mathcal{K}_3(\mathcal{A}, \mathcal{B}) \leq 1$. For it, by definition of $\mathcal{C}_w(\mathcal{A}, \mathcal{B})$ and by Cauchy–Schwarz inequality, we can easily deduce that

$$\begin{aligned}
\mathcal{C}_w(\mathcal{A}, \mathcal{B}) &= \sum_{i=1}^{n} \omega_i \left(\sum_{j=1}^{M_i} \sum_{j'=1}^{M'_i} \left(\gamma_{i,j} \gamma'_{i,j'} p_{i,jj'} \right) \right. \\
&\quad \left. + \sum_{k=1}^{N_i} \sum_{k'=1}^{N'_i} \left(\eta_{i,k} \eta'_{i,k'} q_{i,kk'} \right) \right) \\
&\leq \sqrt{ \sum_{i=1}^{n} \omega_i \left(\sum_{j=1}^{M_i} \left((\gamma_{i,j})^2 p_{i,j} \right) + \sum_{k=1}^{N_i} \left((\eta_{i,k})^2 q_{i,k} \right) \right) } \\
&\quad \times \sqrt{ \sum_{i=1}^{n} \omega_i \left(\sum_{j'=1}^{M'_i} \left((\gamma'_{i,j'})^2 p'_{i,j'} \right) + \sum_{k'=1}^{N'_i} \left((\eta'_{i,k'})^2 q'_{i,k'} \right) \right) } \\
&\leq \sqrt{\mathcal{I}_w(\mathcal{A}) \times \mathcal{I}_w(\mathcal{B})}
\end{aligned}$$

Therefore, $\mathcal{K}_3(\mathcal{A}, \mathcal{B}) \leq 1$. Hence, (P1) holds. □

Theorem 4 *The correlation coefficient* $\mathcal{K}_4$ *also satisfies the same properties as of Theorem 3.*

Proof Similar to above. □

From Definitions 7 and 8, it is observed that the CCs formulated in Eq. (10) use the geometric mean of the informational energies of PDHFSs, whereas Eq. (11) considers the maximum energy possessing PDHFS. Thus, for the decision-maker who is adopting an optimistic behavior, the CCs are given in Eq. (10) which works well for without weighted criterion information and Eq. (12) works appropriately under the weighted criteria information. However, if the expert possesses pessimistic behavior, then Eqs. (11) and (13) work efficiently for the non-weighted and weighted criterion information, respectively.

Furthermore, in the DM process, an expert may provide their information either in terms of DHFSs or in terms of PDHFSs. So in order to integrate their values into the PDHFSs, we assign the probabilities to each element and then aggregate their values according to the procedure described in Algorithm 1.

Algorithm 1 Aggregating probabilities for more than one Probabilistic fuzzy sets

Input: $\mathscr{A}^{(1)}, \mathscr{A}^{(2)}, \ldots, \mathscr{A}^{(d)}$ where $\mathscr{A}^{(d)} = \left(h^{(d)} \middle| p^{(d)}\right)$ where $d = 1, 2, \ldots, D$ such that D is the total number of elements to be fused together.

Output: $\mathscr{A}^{(out)} = \left(h^{(out)} \middle| p^{(out)}\right)$

1: Let $u = \frac{1}{D}$, be the normalized unit.
2: List all the probabilistic membership values in a set and represent it as $M = \{m_l | s_l\}$, where $m_l | s_l = h^{(d)} | p^{(d)}$, $\forall d = 1, 2, \ldots, D$, and $l = 1, 2, \ldots, \#L$, such that $\#L$ is the total number of probabilistic membership values of all the considered elements.
3: Set $i = 1$
4: Set $m_e = m_i$
5: $f^{(l)}_{(mem)} = \begin{cases} 1, & \text{if} \quad m_e = m_l \\ 0, & \text{if} \quad m_e \neq m_l \end{cases}$
6: Set $l = l + 1$ and repeat Step 5, until $l = \#L$
7: Set $h^{(out)} = \bigcup_i m_e$
8: $p^{(out)} = \left(\sum_l \left(f^{(l)}_{(mem)} \cdot s_l \right) \cdot u \right)$
9: Set $i = i + 1$ and goto Step 4, until $i = \#L$

To illustrate the working of this algorithm, we provide an example as below.

Example 2 Let $\mathcal{A}^{(1)} = (\{0.1|0.1, 0.2|0.5, 0.3|0.4\}, \{0.5|1\})$; $\mathcal{A}^{(2)} = (\{0.2|0.4, 0.3|0.6\}, \{0.5|0.2, 0.6|0.8\})$; and $\mathcal{A}^{(3)} = (\{0.1|0.4, 0.2|0.4, 0.6 |0.2\}, \{0.1|1\})$ be three PDHFEs to be fused together. Thus, for the membership part of PDHFEs, we have

$$\left(h^{(1)}, p^{(1)}\right) = (\{0.1|0.1, 0.2|0.5, 0.3|0.4\}),$$
$$\left(h^{(2)}, p^{(2)}\right) = (\{0.2|0.4, 0.3|0.6\}),$$
$$\left(h^{(3)}, p^{(3)}\right) = (\{0.1|0.4, 0.2|0.4, 0.6|0.2\})$$

Now, construct $M = \{0.1|0.1, 0.2|0.5, 0.3|0.4, 0.2|0.4, 0.3|0.6, 0.1|0.4, 0.2|0.4, 0.6|0.2\}$ according to Algorithm 1 and hence $\#L = 8$ and $D = 3$. Thus, by implementing the steps of Algorithm 1 for it, we get the output values corresponding to membership degrees which are $\{0.1|0.1667, 0.2|0.4333, 0.3| \ 0.3333, 0.6|0.066\}$. Similarly, by applying Algorithm 1 for non-membership degrees, we get $\{0.5|0.4, 0.6|0.2666, 0.1|0.3333\}$. Hence, the aggregated PDHFE becomes

$$\mathcal{A}^{(out)} = \left(\left\{ \begin{matrix} 0.1|0.1667, 0.2|0.4333, \\ 0.3|0.3333, 0.6|0.066 \end{matrix} \right\}, \left\{ \begin{matrix} 0.5|0.4, 0.6|0.2666, \\ 0.1|0.3333 \end{matrix} \right\} \right).$$

4 Decision-making approach based on PDHF information

This section presents an approach for solving the DMPs under the PDHFS environment.

For this, let $\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_m$ be the m alternatives which are evaluated under the different criteria $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_t$ by a set of "d" decision-makers. Each decision-maker evaluates $\mathcal{V}_r$ under $\mathcal{G}_t$ and provides their preferences in terms of PDHFEs $\alpha_{rv}^{(d)} = \left(h_{rv}^{(d)}|p_{rv}^{(d)}, g_{rv}^{(d)}|q_{rv}^{(d)}\right)$ where $r = 1, 2, \ldots, m$; $v = 1, 2, \ldots, t$. Then, the rating of each alternative $\mathcal{V}_r$ under $\mathcal{G}_v$ is expressed as

$$\mathcal{V}_r = \left\{ (\mathcal{G}_1, \alpha_{r1}), (\mathcal{G}_2, \alpha_{r2}), \ldots, (\mathcal{G}_t, \alpha_{rt}) \right\}. \tag{14}$$

Let $\omega_v > 0$ be the normalized weight vector of criteria $\mathcal{G}_v$. Then, the following steps are executed to compute the best alternative based on the proposed measure.

Step 1: Arrange the information of each decision-maker toward $\mathcal{V}_r$ in terms of decision matrices $\mathcal{R}^{(d)}$ as:

$$\mathcal{R}^{(d)} = \begin{matrix} & \mathcal{G}_1 & \mathcal{G}_2 & \cdots & \mathcal{G}_t \\ \mathcal{V}_1 & \left(h_{11}^{(d)}|p_{11}^{(d)}, g_{11}^{(d)}|q_{11}^{(d)}\right) & \left(h_{12}^{(d)}|p_{12}^{(d)}, g_{12}^{(d)}|q_{12}^{(d)}\right) & \cdots & \left(h_{1t}^{(d)}|p_{1t}^{(d)}, g_{1t}^{(d)}|q_{1t}^{(d)}\right) \\ \mathcal{V}_2 & \left(h_{21}^{(d)}|p_{21}^{(d)}, g_{21}^{(d)}|q_{21}^{(d)}\right) & \left(h_{22}^{(d)}|p_{22}^{(d)}, g_{22}^{(d)}|q_{22}^{(d)}\right) & \cdots & \left(h_{2t}^{(d)}|p_{2t}^{(d)}, g_{2t}^{(d)}|q_{2t}^{(d)}\right) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{V}_m & \left(h_{m1}^{(d)}|p_{m1}^{(d)}, g_{m1}^{(d)}|q_{m1}^{(d)}\right) & \left(h_{m2}^{(d)}|p_{m2}^{(d)}, g_{m2}^{(d)}|q_{m2}^{(d)}\right) & \cdots & \left(h_{mt}^{(d)}|p_{mt}^{(d)}, g_{mt}^{(d)}|q_{mt}^{(d)}\right) \end{matrix}$$

Step 2: If $d = 1$, then $\left(h_{rv}^{(d)}|p_{rv}^{(d)}, g_{rv}^{(d)}|q_{rv}^{(d)}\right) = (h_{rv}|p_{rv}, g_{rv}|q_{rv})$. On the other hand, if there are more than one decision-maker, i.e., when $d \geq 2$, then applying Algorithm 1 to obtain the aggregated decision matrix $\mathcal{R} = (\alpha_{rv})$ from $\mathcal{R}^{(d)}$ as

$$\mathcal{R} = \begin{matrix} & \mathcal{G}_1 & \mathcal{G}_2 & \cdots & \mathcal{G}_t \\ \mathcal{V}_1 & (h_{11}|p_{11}, g_{11}|q_{11}) & (h_{12}|p_{12}, g_{12}|q_{12}) & \cdots & (h_{1t}|p_{1t}, g_{1t}|q_{1t}) \\ \mathcal{V}_2 & (h_{21}|p_{21}, g_{21}|q_{21}) & (h_{22}|p_{22}, g_{22}|q_{22}) & \cdots & (h_{2t}|p_{2t}, g_{2t}|q_{2t}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{V}_m & (h_{m1}|p_{m1}, g_{m1}|q_{m1}) & (h_{m2}|p_{m2}, g_{m2}|q_{m2}) & \cdots & (h_{mt}|p_{mt}, g_{mt}|q_{mt}) \end{matrix}$$

where $\alpha_{rv} = (h_{rv}|p_{rv}, g_{rv}|q_{rv}) = \bigcup_{\gamma_{rv,j} \in h_{rv}, \eta_{rv,k} \in g_{rv}} (\{\gamma_{rv,j}|p_{rv,j}\}, \{\eta_{rv,k}|q_{rv,k}\})$, where $r = 1, 2, \ldots, m$; $v = 1, 2, \ldots, t$; $j = 1, 2, \ldots, M_{rv}$ and $k = 1, 2, \ldots, N_{rv}$.

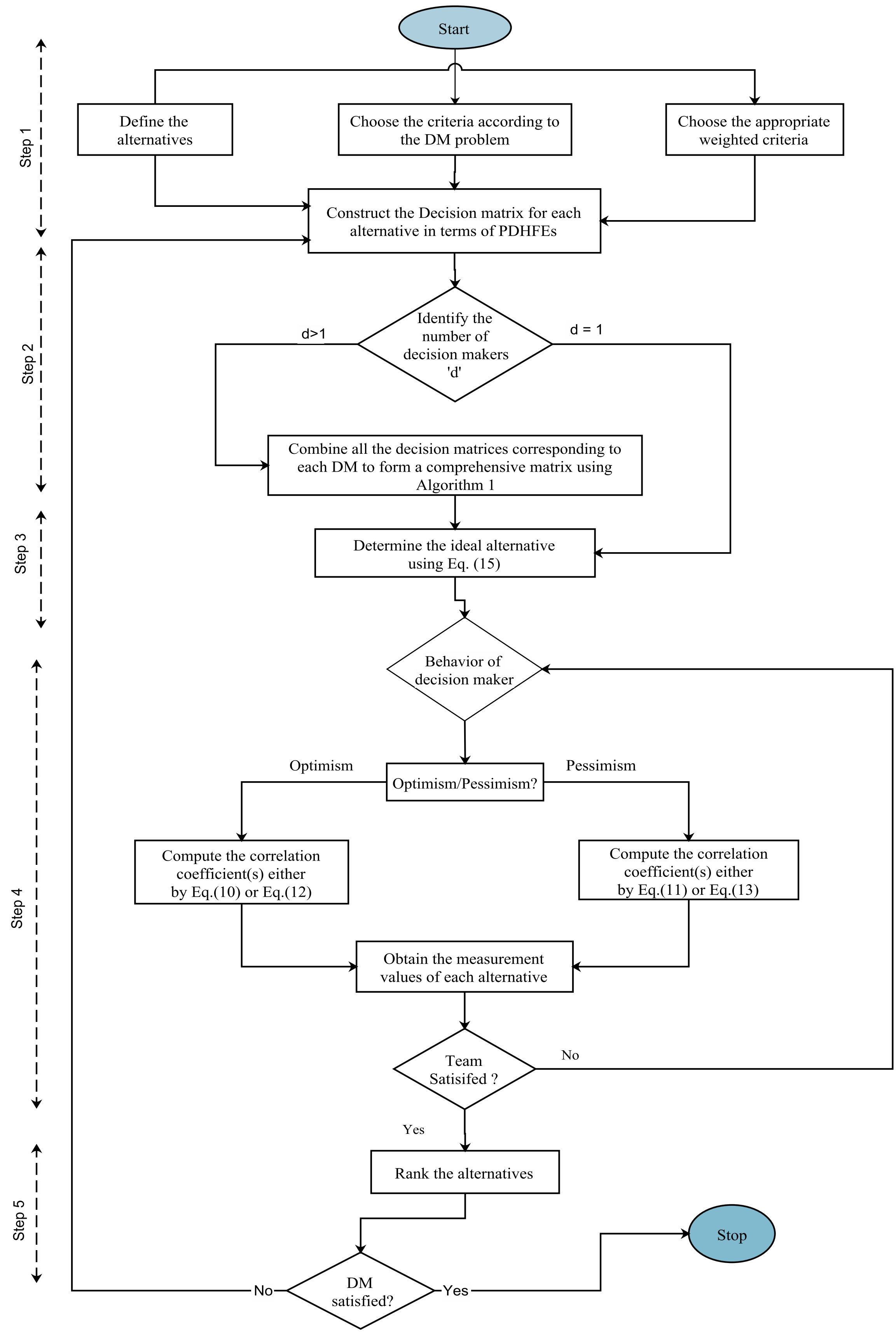

Fig. 1 Flowchart of the proposed approach

Step 3: Construct the ideal alternative $\mathcal{V}^*$ under the criteria $\mathcal{G}_v$ as:

$$\mathcal{V}^* = \bigcup_{\gamma_{rv,j} \in h_{rv}, \eta_{rv,k} \in g_{rv}} \left(\max\{(\gamma_{rv,j} \cdot p_{rv,j})\}, \min\{(\eta_{rv,k} \cdot q_{rv,k})\} \right) \tag{15}$$

where $j = 1, 2, \ldots, \#h_{rv}$ and $k = 1, 2, \ldots, \#g_{rv}$.

Step 4: Compute the measurement values between $\mathcal{V}_r$ and $\mathcal{V}^*$ by utilizing either $\mathcal{K}_1$ or $\mathcal{K}_2$ or $\mathcal{K}_3$ or $\mathcal{K}_4$ as given in Eqs. (10), (11), (12), and (13), respectively.

Step 5: Ordering the alternatives with the maximum value of "$\arg\max \mathcal{K}$."

A pictorial representation of the proposed approach is given as a flowchart and illustrated in Fig. 1.

5 Case study

For justifying the practical applicability of the approach proposed above, a case study based on personnel selection is considered in which a decision-making panel has to select a prospective candidate who suits best for the job. Personnel selection is a very prominent area of DM problems. In the practical DM processes, there arise many cases in which the best candidate has to be selected among a pool of contenders. Due to the complexity in rating criteria, there arise a lot of uncertain data which are needed to be addressed carefully to reach the desired accurate results.

Recruiting a prospective candidate for the survey projects is a prominent task carried out by multi-national companies. Such kind of projects is basically survey oriented which can be broadly classified into two types: external survey projects and internal survey projects. In the external survey projects, the company analyzes the position of the external environment in accordance with which the company has to adapt itself to survive in the business market, whereas the internal survey projects, thoroughly, focus on the internal environment of the company. In this, internal analysis of the company is conducted in figuring out several issues faced by the company, such as employee turnover, job satisfaction level, company's revenue returns. Preferably, for the unbiased internal survey, often a company hires an individual from outside the company so that an honest evaluation of the company's internal working can be made.

Suppose a software company desired to hire a project manager to pay his services in fulfillment of an internal survey project. In order to select the prospective candidate for the job, three experts were decided to give their assessment values. From a pool of applicants, four prospective candidates were shortlisted for the personal interviews. The panel has decided to evaluate the candidates $\mathcal{V}_i; (i = 1, 2, 3, 4)$ based on four criteria, namely $\mathcal{G}_1$: "Educational qualification"; $\mathcal{G}_2$: "Technical knowledge"; $\mathcal{G}_3$: "Communication skills"; $\mathcal{G}_4$: "Work experience." All these criteria are accessed under the weighted criteria $\omega = (0.30, 0.40, 0.20, 0.10)^T$. The aim of the company is to recruit the best candidate for the post of project manager so that the project can be assigned to him and an internal survey can be conducted smoothly in the company. For it, the assessment ratings of applicants were provided by a panel of three experts in the form of PDHFEs which are given in Tables 1, 2, and 3.

Since the number of decision-makers is more than one, by using Algorithm 1, the group PDHFEs are obtained and summarized in Table 4.

The set of information given in $\mathcal{V}^*$ is considered as reference standards. For it, by utilizing Eq. (15), we compute the rating values of this set and are summarized in Table 5, which is used to compute the correlation indices for the alternatives.

By taking these preferences, the indices values corresponding to $\mathcal{K}_1$ and $\mathcal{K}_2$ are computed from set $\mathcal{V}^*$ to $\mathcal{V}_r$, $(r = 1, 2, 3, 4)$ and get:

$$\mathcal{K}_1(\mathcal{V}_1, \mathcal{V}^*) = 0.8920, \quad \mathcal{K}_1(\mathcal{V}_2, \mathcal{V}^*) = 0.9245,$$
$$\mathcal{K}_1(\mathcal{V}_3, \mathcal{V}^*) = 0.9196, \quad \mathcal{K}_1(\mathcal{V}_4, \mathcal{V}^*) = 0.9057$$

and

$$\mathcal{K}_2(\mathcal{V}_1, \mathcal{V}^*) = 0.9119, \quad \mathcal{K}_2(\mathcal{V}_2, \mathcal{V}^*) = 0.9170,$$
$$\mathcal{K}_2(\mathcal{V}_3, \mathcal{V}^*) = 0.9329, \quad \mathcal{K}_2(\mathcal{V}_4, \mathcal{V}^*) = 0.8939$$

Thus, the ordering is $\mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4 \succ \mathcal{V}_1$ when $\mathcal{K}_1$ correlation coefficient index has been used, while $\mathcal{V}_3 \succ \mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_4$ when $\mathcal{K}_2$ correlation coefficient index has been utilized, where "$\succ$" refer "preferred to." As the ranking order is different by both the coefficients, so based on the inherent properties of these proposed coefficients, the decision-maker may choose their goals according to their desire.

On the other hand, if $\omega = (0.30, 0.40, 0.20, 0.10)^T$ is taken, then by the expressions of $\mathcal{K}_3$ and $\mathcal{K}_4$, we get

$$\mathcal{K}_3(\mathcal{V}_1, \mathcal{V}^*) = 0.7485, \quad \mathcal{K}_3(\mathcal{V}_2, \mathcal{V}^*) = 0.8572,$$
$$\mathcal{K}_3(\mathcal{V}_3, \mathcal{V}^*) = 0.7533, \quad \mathcal{K}_3(\mathcal{V}_4, \mathcal{V}^*) = 0.7734$$

and

$$\mathcal{K}_4(\mathcal{V}_1, \mathcal{V}^*) = 0.7700, \quad \mathcal{K}_4(\mathcal{V}_2, \mathcal{V}^*) = 0.8530,$$
$$\mathcal{K}_4(\mathcal{V}_3, \mathcal{V}^*) = 0.8061, \quad \mathcal{K}_4(\mathcal{V}_4, \mathcal{V}^*) = 0.7843 .$$

Therefore, from the computed results we obtain the ranking $\mathcal{V}_2 \succ \mathcal{V}_4 \succ \mathcal{V}_3 \succ \mathcal{V}_1$ by utilizing correlation coefficient $\mathcal{K}_3$ and $\mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4 \succ \mathcal{V}_1$ by using the correlation

Table 1 Probabilistic dual hesitant decision matrix provided by the first decision-maker

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{V}_1$	$\left(\left\{\begin{matrix}0.7\mid 0.7,\\ 0.5\mid 0.3\end{matrix}\right\},\left\{\begin{matrix}0.3\mid 0.5,\\ 0.2\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.3\mid\frac{1}{3},\\ 0.2\mid\frac{1}{3},\\ 0.1\mid\frac{1}{3}\end{matrix}\right\},\left\{\begin{matrix}0.45\mid 0.5,\\ 0.40\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.4\mid\frac{1}{3},\\ 0.3\mid\frac{1}{3},\\ 0.1\mid\frac{1}{3}\end{matrix}\right\},\{0.2\mid 1\}\right)$	$\left(\{0.1\mid 1\},\left\{\begin{matrix}0.4\mid 0.5,\\ 0.3\mid 0.5\end{matrix}\right\}\right)$
$\mathcal{V}_2$	$\left(\left\{\begin{matrix}0.4\mid 0.5,\\ 0.35\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.2\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.5\mid 0.5,\\ 0.4\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.3\mid 0.5,\\ 0.2\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.4\mid 0.5,\\ 0.2\mid 0.5\end{matrix}\right\},\{0.1\mid 1\}\right)$	$\left(\left\{\begin{matrix}0.55\mid\frac{1}{3},\\ 0.5\mid\frac{1}{3},\\ 0.40\mid\frac{1}{3}\end{matrix}\right\},\left\{\begin{matrix}0.2\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\}\right)$
$\mathcal{V}_3$	$\left(\left\{\begin{matrix}0.5\mid 0.5,\\ 0.4\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.2\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.4\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.40\mid 0.5,\\ 0.25\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.40\mid 0.5,\\ 0.30\mid 0.5\end{matrix}\right\},\{0.10\mid 1\}\right)$	$\left(\left\{\begin{matrix}0.2\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.4\mid 0.5,\\ 0.3\mid 0.5\end{matrix}\right\}\right)$
$\mathcal{V}_4$	$\left(\left\{\begin{matrix}0.6\mid\frac{1}{3},\\ 0.5\mid\frac{1}{3},\\ 0.4\mid\frac{1}{3}\end{matrix}\right\},\left\{\begin{matrix}0.3\mid 0.5,\\ 0.1\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.30\mid 0.25,\\ 0.1\mid 0.75\end{matrix}\right\},\left\{\begin{matrix}0.50\mid 0.5,\\ 0.40\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.50\mid 0.5,\\ 0.40\mid 0.5\end{matrix}\right\},\left\{\begin{matrix}0.25\mid 0.5,\\ 0.20\mid 0.5\end{matrix}\right\}\right)$	$\left(\left\{\begin{matrix}0.25\mid 0.5,\\ 0.15\mid 0.5\end{matrix}\right\},\{0.30\mid 1\}\right)$

coefficient $\mathcal{K}_4$. Hence, by the maximum recognition principle, we conclude that best alternative is $\mathcal{V}_2$, while the relation between the alternatives $\mathcal{V}_3$ and $\mathcal{V}_4$ is selected according the risk aversive and the risk preferable states will give variable results in accordance with the decision-maker's attitude.

5.1 Comparative studies

To justify the superiority of our approach, this section consists of the comparative analysis with the existing approaches. It is noticeable that the probabilistic dual hesitant fuzzy sets can be compared to the existing studies [18, 30, 32, 38] under the different environments. To analyze our approach by analyzing it parallel to these approaches, a comparative analysis is listed below:

(i) The PDHFEs can be reduced to DHFEs by making probabilities of the membership and the non-membership portions equal within themselves. That is, for a PDHFE $\mathcal{A} = \bigcup(\{\gamma_i|p_i\},\{\eta_k|q_k\})$ where $i = 1, 2, \ldots, M_i$ and $k = 1, 2, \ldots, N_i$, if $p_1 = p_2 = \ldots = p_{M_i} = p$ and $q_1 = q_2 = \ldots = q_{N_i} = q$, then it reduces to a DHFE. Based on this reduction, in accordance with the approach proposed by Wang et al. [30], the correlation coefficient "$\mathcal{K}_3$" for the four alternatives is obtained as:

$$\mathcal{K}_3(\mathcal{V}_1,\mathcal{V}^*) = 0.9352, \quad \mathcal{K}_3(\mathcal{V}_2,\mathcal{V}^*) = 0.9033,$$
$$\mathcal{K}_3(\mathcal{V}_3,\mathcal{V}^*) = 0.9442, \quad \mathcal{K}_3(\mathcal{V}_4,\mathcal{V}^*) = 0.8892 .$$

Thus, alternatives are ranked as $\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4$. It is noticeable that alternatives' ranking varies with a huge difference from that of our proposed approach's outcome. This is due to the fact that in the existing theory probabilities corresponding to agreeness and disagreeness are not considered. The proposed approach is advantageous over the existing one [30] because, in the numerical evaluation of the existing theory, the length of two DHFEs is matched by repeating a particular entry in both membership and non-membership parts. This is done by first ordering the DHFEs into either descending order or ascending order and then according to the expert's optimistic or pessimistic viewpoint, the smaller value or the larger value is repeated until the length of two DHFEs under consideration becomes equal. This leads to redundancy of same data entries and increases the computational effort as well as different results from the proposed one. But, this repetition of the smaller or larger value to make the length equal is not required in our approach. It reduces the calculation overheads as each element is having its associated probability which cannot be repeated over and over again, and hence, it makes our approach inclined more toward the real-life scenarios.

(ii) Secondly, by converting the PDHFEs to DHFEs and by comparing the outcomes to that of approach given by Ye [38], it is noticed that the correlation coefficient '$\mathcal{K}_3$' is obtained as:

$$\mathcal{K}_3(\mathcal{V}_1,\mathcal{V}^*) = 0.8831, \quad \mathcal{K}_3(\mathcal{V}_2,\mathcal{V}^*) = 0.8720,$$
$$\mathcal{K}_3(\mathcal{V}_3,\mathcal{V}^*) = 0.9121, \quad \mathcal{K}_3(\mathcal{V}_4,\mathcal{V}^*) = 0.8757$$

Table 2 Probabilistic dual hesitant decision matrix provided by the second decision-maker

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{V}_1$	$(\{0.40\|1\},\{0.15\|1\})$	$(\{0.40\|0.5, 0.20\|0.5\},\{0.10\|1\})$	$(\{0.20\|0.5, 0.10\|0.5\},\{0.30\|1\})$	$(\{0.30\|0.5, 0.10\|0.5\},\{0.50\|1\})$
$\mathcal{V}_2$	$(\{0.30\|0.5, 0.10\|0.5\},\{0.60\|1\})$	$(\{0.60\|0.5, 0.20\|0.5\},\{0.15\|0.5, 0.10\|0.5\})$	$(\{0.25\|0.5, 0.15\|0.5\},\{0.10\|1\})$	$(\{0.75\|\frac{1}{3}, 0.65\|\frac{1}{3}, 0.60\|\frac{1}{3}\},\{0.25\|0.5, 0.10\|0.5\})$
$\mathcal{V}_3$	$(\{0.40\|0.5, 0.30\|0.5\},\{0.10\|1\})$	$(\{0.5\|1\},\{0.3\|0.5, 0.2\|0.5\})$	$(\{0.20\|0.5, 0.15\|0.5\},\{0.40\|0.5, 0.30\|0.5\})$	$(\{0.35\|0.50, 0.30\|0.50\},\{0.20\|1\})$
$\mathcal{V}_4$	$(\{0.30\|0.5, 0.10\|0.5\},\{0.40\|1\})$	$(\{0.30\|0.5, 0.10\|0.5\},\{0.40\|1\})$	$(\{0.20\|\frac{1}{3}, 0.15\|\frac{1}{3}, 0.10\|\frac{1}{3}\},\{0.05\|1\})$	$(\{0.30\|1\},\{0.20\|0.5, 0.10\|0.5\})$

Table 3 Probabilistic dual hesitant decision matrix provided by the third decision-maker

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{V}_1$	$(\{0.40\|1\},\{0.2\|0.5, 0.1\|0.5\})$	$(\{0.20\|0.5, 0.10\|0.5\},\{0.30\|1\})$	$(\{0.25\|0.5, 0.20\|0.5\},\{0.10\|1\})$	$(\{0.5\|\frac{1}{3}, 0.4\|\frac{1}{3}, 0.3\|\frac{1}{3}\},\{0.2\|0.5, 0.1\|0.5\})$
$\mathcal{V}_2$	$(\{0.30\|0.10, 0.20\|0.90\},\{0.10\|1\})$	$(\{0.5\|0.5, 0.4\|0.5\},\{0.3\|1\})$	$(\{0.3\|1\},\{0.5\|0.5, 0.4\|0.5\})$	$(\{0.4\|\frac{1}{3}, 0.3\|\frac{1}{3}, 0.1\|\frac{1}{3}\},\{0.15\|1\})$
$\mathcal{V}_3$	$(\{0.4\|1\},\{0.20\|0.5, 0.10\|0.5\})$	$(\{0.3\|\frac{1}{3}, 0.2\|\frac{1}{3}, 0.1\|\frac{1}{3}\},\{0.40\|1\})$	$(\{0.20\|0.5, 0.10\|0.5\},\{0.40\|1\})$	$(\{0.3\|0.5, 0.2\|0.5\},\{0.15\|1\})$
$\mathcal{V}_4$	$\{0.45\|0.5, 0.30\|0.5\},\{0.25\|0.5, 0.20\|0.5\})$	$(\{0.30\|0.5, 0.20\|0.5\},\{0.10\|1\})$	$(\{0.70\|\frac{1}{3}, 0.60\|\frac{1}{3}, 0.50\|\frac{1}{3}\},\{0.30\|1\})$	$(\{0.35\|0.5, 0.20\|0.5\},\{0.10\|1\})$

Thus, the alternatives are ranked as $\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_4 \succ \mathcal{V}_2$. It can be seen that the best alternative does not remain the same as that of our proposed approach. This difference arises due to the difference in the ideal determination technique. In the existing approach [38], the ideal alternative is taken as $(\{1\},\{0\})$, but in our approach, the ideal is determined in accordance with the informational energy. The alternatives attribute showing the largest informational energy are taken as the membership ideal value, whereas the smallest information energy possessing alternative attribute is taken as non-membership ideal value as given in Eq. (15). By choosing an ideal alternative in such a way, there is no need to repeat the values to match the length as well as the ideal alternative is chosen in such a way that it synchronizes with the associated real-life DMPs.

(iii) The PDHFEs can be reduced to PHFEs by not considering the non-membership part. So, by converting the PDHFEs into PHFEs and by evaluating the alternatives in accordance with the

Table 4 Group probabilistic dual hesitant decision matrix

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{V}_1$	$\left(\left\{0.70\vert 0.233, 0.50\vert 0.1, 0.40\vert 0.6667\right\}, \left\{0.30\vert 0.1667, 0.20\vert 0.3333, 0.15\vert 0.3333, 0.10\vert 0.1667\right\}\right)$	$\left(\left\{0.40\vert 0.1667, 0.30\vert 0.1111, 0.20\vert 0.4444, 0.10\vert 0.2778\right\}, \left\{0.45\vert 0.1667, 0.40\vert 0.1667, 0.30\vert 0.3333, 0.10\vert 0.3333\right\}\right)$
$\mathcal{V}_2$	$\left(\left\{0.40\vert 0.1667, 0.35\vert 0.1667, 0.30\vert 0.20, 0.20\vert 0.30, 0.10\vert 0.1666\right\}, \left\{0.60\vert 0.3333, 0.20\vert 0.1667, 0.10\vert 0.5000\right\}\right)$	$\left(\left\{0.60\vert 0.1667, 0.50\vert 0.3333, 0.40\vert 0.3333, 0.20\vert 0.1667,\right\}, \left\{0.30\vert 0.5, 0.20\vert 0.1667, 0.15\vert 0.1667, 0.10\vert 0.1666\right\}\right)$
$\mathcal{V}_3$	$\left(\left\{0.50\vert 0.1667, 0.40\vert 0.6667, 0.30\vert 0.1666\right\}, \left\{0.20\vert 0.3333, 0.10\vert 0.6667\right\}\right)$	$\left(\left\{0.50\vert 0.3333, 0.40\vert 0.1667, 0.30\vert 0.1111, 0.20\vert 0.1111, 0.10\vert 0.2778\right\}, \left\{0.40\vert 0.5000, 0.30\vert 0.1667, 0.25\vert 0.1667, 0.20\vert 0.1666\right\}\right)$
$\mathcal{V}_4$	$\left(\left\{0.60\vert 0.1111, 0.50\vert 0.1111, 0.45\vert 0.1667, 0.40\vert 0.1111, 0.30\vert 0.3333, 0.10\vert 0.1667\right\}, \left\{0.40\vert 0.3333, 0.30\vert 0.1667, 0.25\vert 0.1667, 0.20\vert 0.1667, 0.10\vert 0.1666\right\}\right)$	$\left(\left\{0.30\vert 0.4167, 0.20\vert 0.1667, 0.10\vert 0.4166\right\}, \left\{0.50\vert 0.1667, 0.40\vert 0.5000, 0.10\vert 0.3333\right\}\right)$

	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{V}_1$	$\left(\left\{0.40\vert 0.1111, 0.30\vert 0.1111, 0.25\vert 0.1667, 0.20\vert 0.3333, 0.10\vert 0.2778\right\}, \left\{0.30\vert 0.3333, 0.20\vert 0.3333, 0.10\vert 0.3334\right\}\right)$	$\left(\left\{0.50\vert 0.1111, 0.40\vert 0.1111, 0.30\vert 0.2778, 0.10\vert 0.5\right\}, \left\{0.50\vert 0.3333, 0.40\vert 0.1667, 0.30\vert 0.1667, 0.20\vert 0.1667, 0.10\vert 0.1666\right\}\right)$
$\mathcal{V}_2$	$\left(\left\{0.40\vert 0.1667, 0.30\vert 0.3333, 0.25\vert 0.1666, 0.20\vert 0.1667, 0.15\vert 0.1667\right\}, \left\{0.50\vert 0.1667, 0.40\vert 0.1667, 0.10\vert 0.6666\right\}\right)$	$\left(\left\{0.75\vert 0.1111, 0.65\vert 0.1111, 0.60\vert 0.1111, 0.55\vert 0.1111, 0.50\vert 0.1111, 0.40\vert 0.2223, 0.30\vert 0.1111, 0.10\vert 0.1111\right\}, \left\{0.25\vert 0.1667, 0.20\vert 0.1667, 0.15\vert 0.3333, 0.10\vert 0.3333\right\}\right)$
$\mathcal{V}_3$	$\left(\left\{0.40\vert 0.1667, 0.30\vert 0.1667, 0.20\vert 0.3333, 0.15\vert 0.1667, 0.10\vert 0.1666\right\}, \left\{0.40\vert 0.5, 0.30\vert 0.1667, 0.10\vert 0.3333\right\}\right)$	$\left(\left\{0.35\vert 0.1667, 0.30\vert 0.3333, 0.20\vert 0.3333, 0.10\vert 0.1667\right\}, \left\{0.40\vert 0.1667, 0.30\vert 0.1667, 0.20\vert 0.3333, 0.15\vert 0.3333\right\}\right)$
$\mathcal{V}_4$	$\left(\left\{0.70\vert 0.1111, 0.60\vert 0.1111, 0.50\vert 0.2778, 0.40\vert 0.1666, 0.20\vert 0.1111, 0.15\vert 0.1111, 0.10\vert 0.1111\right\}, \left\{0.30\vert 0.3333, 0.25\vert 0.1667, 0.20\vert 0.1667, 0.05\vert 0.3333\right\}\right)$	$\left(\left\{0.35\vert 0.1667, 0.30\vert 0.3333, 0.25\vert 0.1667, 0.20\vert 0.1667, 0.15\vert 0.1666\right\}, \left\{0.30\vert 0.3333, 0.20\vert 0.1667, 0.10\vert 0.5\right\}\right)$

Table 5 Rating values of the ideal set $\mathcal{V}^*$

$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\left(\left\{0.70\|0.2333, 0.50\|0.1, 0.40\|0.6667\right\}, \left\{0.20\|0.3333, 0.10\|0.6667\right\}\right)$	$\left(\left\{0.60\|0.1667, 0.50\|0.3333, 0.40\|0.3333, 0.20\|0.1667\right\}, \left\{0.30\|0.50, 0.20\|0.1666, 0.15\|0.1666, 0.10\|0.1666\right\}\right)$	$\left(\left\{0.70\|0.1111, 0.60\|0.1111, 0.50\|0.2766, 0.40\|0.1666, 0.20\|0.1111, 0.15\|0.1111, 0.10\|0.1111\right\}, \left\{0.30\|0.3333, 0.20\|0.3333, 0.10\|0.3334\right\}\right)$	$\left(\left\{0.75\|0.1111, 0.65\|0.1111, 0.60\|0.1111, 0.55\|0.1111, 0.50\|0.1111, 0.40\|0.2223, 0.30\|0.1111, 0.10\|0.1111\right\}, \left\{0.25\|0.1667, 0.20\|0.1667, 0.15\|0.3333, 0.10\|0.3333\right\}\right)$

approach proposed by Wang and Li [32], it is observed that in the existing approach, the correlation coefficients are calculated corresponding to each criterion separately. As per their outlined approach, the obtained values for weighted correlation coefficients $(\mathcal{K}_\omega)$ are summarized in Table 6. As per the ranking index (r_j) where $j = (1, 2, 3, 4)$ proposed by Wang and Li [32] in which all the correlation coefficient values corresponding to each alternative are added separately, the r_j's are obtained as:

$$r_1 = 3.5771, \quad r_2 = 3.7192, \quad r_3 = 3.3879, \quad r_4 = 3.4945.$$

Hence, the alternatives are ranked as $\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_4 \succ \mathcal{V}_3$. The best alternative coincides with our proposed approach. This is certainly because of considering the probabilistic membership values in PHFE. But the successive ranking order differs and this variation clearly signifies that by ignoring the degrees of disagreeness, results can show great deviations as compared to the case when the disagreeness is paid equal attention. Clearly, by considering the non-membership probabilistic hesitant values, the information can be knitted more closely to the practical situations giving the best alternative same as that of the existing theory. Thus, it is better to give equal priority to the non-membership values during the DM processes.

(iv) The approach followed by Hao et al. [18] is based on aggregating information available in the form of PDHFEs. According to it, the alternatives are ranked as $\mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_4$. So, the proposed approach's best alternative coincides with the existing theory. However, it is seen that the rest of the ranking obtained by this existing approach differs from that of the evaluation using the correlation coefficient "$\mathcal{K}_3$." This significant difference is because of the variation in processing the available PDHFEs in both the theories. In the existing one, an aggregation operator is used to compute the score values which leads to the ranking of the alternatives, but the proposed approach works on the correlation coefficient.

5.2 Further discussion

Below, we study the characteristic measures of the proposed approach with the existing approaches [18, 30, 32, 38] and the results are tabulated in Table 7.

In this, the symbol "✓" represents that the associated properties satisfy, while the symbol $\times$ represents the

Table 6 Correlation coefficients in accordance with [32]

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{K}_\omega(\mathcal{V}_1, \mathcal{V}^*)$	1	0.8348	0.9322	0.8100
$\mathcal{K}_\omega(\mathcal{V}_2, \mathcal{V}^*)$	0.7553	1	0.9639	1
$\mathcal{K}_\omega(\mathcal{V}_3, \mathcal{V}^*)$	0.6935	0.8684	0.9072	0.9189
$\mathcal{K}_\omega(\mathcal{V}_4, \mathcal{V}^*)$	0.7461	0.8052	1	0.9433

associated property fails. For example, it is clearly seen from Table 7 that the MCDM method mentioned in Wang and Li [32] considers more than one experts, takes into account probabilistic values, has no data redundancy, and also takes the non-trivial ideal alternative. But this approach does not consider the falsity values which are overcome in the proposed approach. On the other hand, the approaches by Wang et al. [30] and Ye [38] have considered the non-membership, but they fail to capture all other characteristic features possessed by our approach such as one of them is not a multi-expert decision-making approach and does not capture probabilistic information with repeated data values in which their ordering, as well as the non-membership values, plays a significant role, whereas the approach outlined by Hao et al. [18] models multi-expert information and also contains non-membership. But, our approach is superior in the aspects of data redundancy and ordering of data values before the evaluation. This discussion leads to the conclusion that the proposed method will efficiently handle and solve the problems with respect to the existing methods [18, 30, 32, 38].

5.3 Advantages of the proposed approach

From the computed results, we highlights the following advantages of the method under the PDHFS environment.

(i) Since PDHFEs can model the probabilities of each membership and non-membership values separately, so an expert can give a more flexible rating in which he is free to provide probabilistic preferences to the hesitant values of agreeness as well as disagreeness.

(ii) In the numerical calculation of the correlation coefficients using the proposed approach, there is no need of repeating the hesitant values of one set to match with the number of hesitant values of the second set in consideration. So, the computational overheads get reduced and the hesitant inputs become more inclined toward the real-life scenarios, i.e., the calculations are done with the elements and their respective probabilities by keeping the same as they have been acquired by the expert, but not by repeating the values again and again to get the desired results.

(iii) In the proposed approach, the ideal alternative is chosen logically by considering the alternative having maximum informational energy for the membership hesitant values and the alternative with minimum informational energy values for the non-membership ones, classified under different criteria. So, the ideal alternative is viable in accordance with the practical situational gravity rather than fixing it to the extreme ideal alternative values such as $(\{1(1)\}, \{0(1)\})$ in the case of PDHFEs and $(\{1\}, \{0\})$ in the case of DHFEs.

(iv) Since a PDHFE can be reduced to PHFE by not considering the non-membership values along with their associated probabilities and it can also be reduced to DHFE by considering the membership and non-membership hesitant values to be equi-probabilistic, the proposed approach is a generalized version of the existing approaches based on these environments [30, 32, 38].

Table 7 Characteristic comparison of the proposed approach with different methods

Methods	Whether consider more than one decision-maker	Whether considers the probabilities	No data redundancy	No ordering of data values	Non-trivial ideal alternative	Considers non-membership degrees
Wang et al. [30]	×	×	×	×	×	✓
Ye [38]	×	×	×	×	×	✓
Wang and Li [32]	✓	✓	✓	✓	✓	×
Hao et al. [18]	✓	✓	×	×	×	✓
Our proposed approach	✓	✓	✓	✓	✓	✓

(v) Often in the multi-expert decision-making approaches, there is a need of two weighted criteria: One is the subjective weighted criteria and the other one is the objective weighted criteria. The subjective weighted criteria are used to aggregate the variable decisions taken by different experts to reach one final conclusion. But in the proposed approach, although there are more than one decision-makers, still there is no need for any additional weighted criteria to figure out the collective decision taken by all the experts. Thus, it makes our approach more flexible, time-saving as well as having less computational overheads.

6 Conclusion

PDHFS is a special dual hesitant fuzzy set where the membership and non-membership degrees of the element of the set are associated with the probabilities and can more easily describe the vagueness and uncertainty in the real world. Also, the several existing sets such as DHFS, HFSs, and PHFS are considered as a special case of the PDHFSs. Thus, the PDHFS is a more generalized and successful concept for handling the uncertainties with both stochastic and fuzzy features. By taking the advantages of these, the present paper presents the concept of the correlation coefficients for measuring the relationships between two or more values. The advantages of the proposed measures are that it not only measures the strength between two or more PDHFEs but simultaneously it avoids the inconsistency of the decision-makers results due to the loss of the information. Further, in the study, a method of northwest corner rule is utilized to compute the joint probabilities of the set. Some salient properties of the proposed CCs are also addressed. Afterward, a decision-making approach is developed for MCDM problem with probabilistic dual hesitant fuzzy information. Finally, to justify the practical resilience, the proposed method has been exemplified by a case study based on personnel selection. The comparative analysis with some of the existing studies [18, 30, 32, 38] has been conducted to show the availability and advantages of the proposed method. From the study, it is concluded that PDHFS not only capture the decision-maker preferences but also the corresponding probabilities under uncertain environment. Thus, due to these probabilities, this model can keep more detailed information and valuable results to the decision-makers as compared to the other existing theories.

In the future, the research will focus on extending the theory under other uncertain and linguistic information [10, 11, 15, 29] and some information measures and relations can be studied for PDHFSs.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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