



A robust multi-objective humanitarian relief chain network design for earthquake response, with evacuation assumption under uncertainties

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Abstract

In this paper, we have proposed a multi-objective mathematical model for the humanitarian supply chain design problem that minimizes: (1) total number of the injured not transferred to hospitals and total number of the homeless not evacuated from the affected area, and (2) total unmet relief commodity needs. In this model, such parameters as the demand and travel time have been considered as uncertain and two discrete robust counterpart models (with “ellipsoidal” and “box and polyhedral” uncertainty sets) have been developed to model uncertainties. Results found from Tehran Case Study have revealed that the one with the “box and polyhedral” uncertainty set performs better than the “ellipsoidal” set.

Keywords Disaster response · Humanitarian supply chain network design · Multi-objective · Robust optimization

1 Introduction

Annually, people all over the world suffer from enormous life/financial losses due to such natural/unnatural (man-made) disasters as earthquakes, floods, tsunamis, terrorist attacks, and so on [1]. In recent years, much attention has been paid to disasters [2], but considering the progress in the science and technology, man has not yet been able to foresee and prevent them [3]. Materials commonly and mostly used in disasters include pharmaceuticals, canned food, water bottles, and tents [4]. In a disaster, rapid resource distribution is quite necessary for the minimization of the damage and fatalities. Since time and resources are limited, the relief logistics will decide on the allocation

of time, budget, and other resources [5]. In a disaster, relief commodities are quite vital to reduce fatalities. Generally, very serious challenges of the crisis management include relief and logistic, reducing financial costs, and decreasing fatalities [6]. The main objective of the relief logistics is to set up shelters to help the victims and the disaster-affected people in the shortest possible time [7]. After a disaster, shelters and hospitals are appropriate places for taking care of the injured and the homeless to stay protected from different viruses, cold weather, and so on. The purposeful transferring of the affected people to these places can considerably reduce the fatalities. Since disasters are uncertain and unpredictable, flexibility in logistic activities is of the utmost importance. In a disaster, considering such issues as the uncertain demands, facilities capacities (to be used in the distribution process), transfer capacity, and the available resources is quite important for the decision makers [8]. The obvious problem in effective pre-disaster planning is the uncertainty about the disaster itself because the information about it is scarce [9]. It can be stated that before a disaster, there are no certainties about its occurrence, severity, and losses, and this is why the post-disaster relief logistics planning is usually associated with disorder and perturbation. Under such circumstances, the robust optimization (RO) has proven to be a very powerful tool to model uncertainties.

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Although researches about the relief supply chain are not numerous, they can still be classified into two groups one of which has focused on evacuation planning and the other on the relief logistics; hence, their main differences lie in “man” and “commodity.” In one, the main goal is to evacuate people from the affected area, but in the other, it is logistic/commodity delivery.

1.1 Evacuation planning

In these problems, a large number of people reside in accident-prone areas and the aim is to evacuate and send them to safe places. For this purpose, people should usually cover a distance to reach the nearest evacuation station where vehicles are ready to move them. Some examples of such researches are as follows:

An et al. [10] have presented a model to locate evacuation transportation facilities under disruption conditions in order to plan the evacuation of a large number of people residing at accident-prone places. The objective of the research is to minimize the costs of the facility setup, transfer, and transportation through a preplanned evacuating facility disruption risk model. Guan [11] has proposed a model for locating emergency facilities to rescue people in accident-prone areas; the main objective is to minimize the average costs of facility setup and travel time. The travel time between distribution centers and demand points is uncertain, and the use of the stochastic programming approach has been made. Kulshrtha et al. [12] have presented a model for locating evacuation facilities and allocating buses under uncertain demand conditions. The objective of this paper is to minimize the total shipping time. Its demand is uncertain, and the uncertainty modeling has been done using the RO [13] approach. Gama et al. [14] have proposed a multi-period model of locating–allocating shelters for the evacuation of the people experiencing flood and storm. The primary concern of this research is to minimize the total transportation time, and it uses a heuristic approach to solve the proposed model.

1.2 Relief commodity delivery

The humanitarian supply chain consists of several main layers wherein the accident-prone areas that need emergency services are specified. The goals are to locate centers for the distribution of relief commodities, store pre-disaster supplies, and manage their distribution in post-disaster situations. Examples of such researches are as follows:

Using goal programming, Zhan and Liu [15] have developed a multi-objective model for relief logistics under uncertain conditions. The goal of the model is to minimize unmet demand with bi-objective programming: One objective function minimizes the expected service time,

and the other minimizes the unmet demand. The demand and the maximum suppliers’ capacities are uncertain and are considered as discrete scenarios. Bozorgi-Amiri et al. [16] developed a modified particle swarm optimization model for relief logistics under uncertainty wherein the demand, suppliers’ capacities, and the costs of transportation and purchasing are assumed uncertain and the objective function is to minimize the total costs. In this problem, the objective is to locate the pre-disaster relief commodity distribution centers for the distribution of the commodities in the affected areas after the disaster. Murali et al. [17] have proposed a model for locating facilities to respond to natural disasters under uncertain demand conditions; the objective is to maximize the number of people who need service and to deal with uncertainties, the authors have used the chance constraint stochastic programming. Bozorgi-Amiri et al. [18] have presented a robust multi-objective model for the relief logistics planning under uncertainties; one objective function minimizes the total costs, and the other establishes justice in the distribution of relief goods. This paper uses the RO approach with such uncertain parameters as the demand, transfer costs, and supply preparation costs. Zhang and Jiang [19] have proposed a bi-objective robust optimization model for the planning of pharmaceutical services in emergency conditions. One objective function minimizes the total costs, and the other minimizes the response time. Here, the focus is mostly on locating the emergency facilities and regions are allocated to emergency centers. Deng and Yang [20] have developed a mathematical model for the transportation planning for the emergency operations management. The model objective is to minimize the transportation and inventory costs, and its total demand is uncertain. Rezaei Malek and Tavakoli-Moghadam [21] have developed a bi-objective RO [22] model for the logistic revival plan wherein one objective function minimizes the service time and the other minimizes the costs. The shipping time (from the warehouses to the demand points), demand, and the warehouses’ remaining capacities are considered uncertain. Bozorgi-Amiri and Khorsi [23] have presented a dynamic multi-objective model for the locating and routing of the logistic facilities under uncertain conditions. One objective function, in this 3-objective model, minimizes the maximum shortages, one minimizes the total travel time, and the third minimizes the total costs. Such parameters as the commodity purchase prices, shipping costs, demand for vital supplies, and the capacities of the suppliers and distribution centers are uncertain and are considered as discrete scenarios. Garrido et al. [24] have developed a model for the emergency planning of flood and storm with the stochastic programming approach. Its objective is to minimize transportation costs with uncertain demand. Zokaie et al. [25] have developed a RO [26] model for the design

of the humanitarian supply chain consisting of three levels of suppliers, distribution centers, and affected areas; the objective of the model is to minimize total costs. Paul and Wang [27] have presented a robust location–allocation model for earthquake response under uncertain conditions. The objective function is to minimize total social costs. It has an uncertain casualty of severity level at demand node and transportation times which are considered as different scenarios. Jha et al. [28] have developed a multi-objective programming for humanitarian relief supply chain. This paper model is a humanitarian relief chain that includes a relief goods supply chain and an evacuation chain in case of a natural disaster. The objective functions are presented in the following: 1—demand satisfaction in relief chain, 2—demand satisfaction in evacuation chain, and 3—estimating/minimizing overall logistics cost. A multi-objective genetic algorithm, NSGA-II, is used to get dataset including Pareto solution. In Vahdani et al. [29], a multi-objective optimization model has been provided based on the travel time and total cost and reliability of the routes. In this model (during) the earthquake response activities, the damaged roads can be repaired. The relief supply chain comprises a set of distribution centers and affected areas.

1.3 The paper novelties

A literature review of the humanitarian supply chain in recent years reveals that the evacuation of the people (homeless and injured) from the affected area has not been dealt with properly; most papers have addressed only the delivery of the relief commodities in such areas and the number of papers dealing simultaneously with the distribution of the relief goods and evacuation of the people from these areas is not many. In an earthquake, some people need relief supplies, injuries of some are more severe and they should be immediately transferred to emergency centers, and those who are not injured should be sent to safe areas to be protected from the subsequent tremors, wildlife, and contagious diseases. Therefore, the need for a plan that can, at the same time, organize the distribution of the relief supplies, send the injured people to emergency centers, and evacuate homeless people to shelters is highly felt. In most studies regarding modeling, the use of the scenario-based programming has been made to deal with uncertainties in disasters and accidents, while in the real world, appropriate historical data are not available based on which scenarios can be defined; the distribution function of the random variable is also unknown [30]. Additionally, data uncertainty is assumed discrete in the scenario-based approach, while in reality it is continuous. Another weak point of the scenario-based approach is the challenge in defining and generating the scenarios [18]; the solution found under certain conditions

may not be feasible for other observations and scenarios [30]. Assuming different scenarios are generated, the next problem will be solving them; when the number of scenarios is more and the problem size is large, the solution time will face a serious challenge [4]. Therefore, the reason for choosing a RO [26, 31, 32] to develop a novel mathematical model is based on the real-world assumptions and conditions. To model uncertainties, scenarios are modeled based on an observed example. What distinguishes this research from other similar ones is the simultaneous consideration of the distribution of relief goods, transferring injured people to hospitals, and evacuating the homeless people into shelters; the planning is multi-period and considers the means of transportation as well. To be as close to the real world as possible and face uncertainties, the use of two types of RO approaches has been made: box and polyhedral [26] and ellipsoidal [31] uncertainty sets. This programming is bi-objective: One minimizes the number of injured and homeless people of the affected area sent to the emergency centers and shelters, and the other minimizes the shortage of the relief supplies, emergency centers, and shelters. The main contributions of this paper can be given as follows:

- Developing a robust multi-objective model in the disaster relief supply chain;
- Developing and comparing two RO models in the relief supply chain;
- Comparing robust models with maximum constraint violation probability;
- Simultaneous addressing of evacuating, transferring the injured, and delivering commodities;
- The proposed model is applied on a real-world case study of disaster relief;

The rest of the paper is organized as follows. The concerned problem is defined in Sect. 2, and the proposed mathematical model for humanitarian relief chain network design is presented in Sect. 3. The robust counterpart models based on “box and polyhedral” and “ellipsoidal” uncertainty sets are elaborated in Sect. 4, and the employed solution method is presented in Sect. 5. Finally, an illustrative example and concluding remarks and some possible future works are presented in Sects. 6 and 7, respectively.

2 Problem description

Suppose the relief supply chain (Fig. 1) consists of four levels including distribution centers, shelters, affected areas, and hospitals. In an earthquake, successive tremors are most likely; if some buildings remain partially undamaged in the first hit, they may get totally destroyed in the next tremors. Therefore, evacuation of the people from

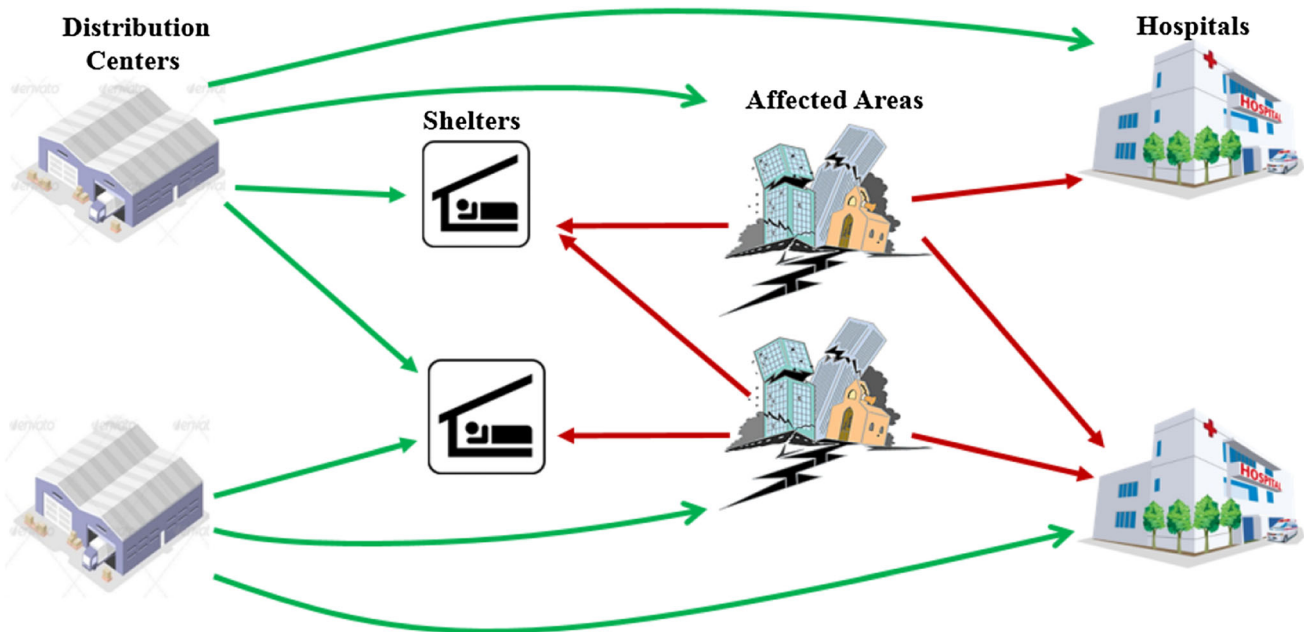


Fig. 1 Network structure of the relief supply chain

such places will become a very serious issue. Meanwhile, some people may be injured and need to be sent to emergency centers and hospitals for revival and more medication. In the proposed model, we have considered some relief–rescue revival centers and some evacuation shelters so that immediately after the disaster the injured may be sent to hospitals and the unhurt can be transferred to shelters. In a disaster, the unhurt are willing to help their relatives and other people and, for this purpose, they would require relief commodities (first aid supplies, water, food, etc.), and those who have been sent to hospitals and shelters would be in need of pharmaceuticals, drinks, and food stuff; hence, each level requires relief commodities. Here, there are some different capacity distribution centers that provide service to send relief commodities to each of these points [33].

In an earthquake, some of these service centers may be destroyed, and since no exact information is available about the earthquake intensity, we cannot specify the remaining capacities after it hits. To make sure not to miss some potential of the relief chain in a disaster, hospitals, shelters, and distribution centers are considered outside the disaster-prone areas. Under such circumstances, the service time (commodity distribution, people transportation, etc.) may be prolonged; if this service time exceeds a specified limit, the provided service may totally lose its desirability. This is why a maximum coverage time is considered for every service so that we may not face such adverse conditions.

The main assumptions of the relief supply chain design problem are as follows:

1. The planning horizon has three periods (3×24);
2. The number of candidate points for setting up distribution centers and shelters is known;
3. Capacities of the distribution centers, hospitals, and shelters are limited;
4. Three relief commodities (drinking water, food stuff, and pharmaceuticals) are distributed;
5. The number of the affected and homeless people, relief commodity demand, and the travel time between all nodes are considered uncertain;
6. The service time is not to exceed a specified amount;
7. The distribution centers can be established at only one level (large, medium, or small);
8. The number and capacities of the transportation vehicles are limited;
9. The planning budget is limited;

Considering the above assumptions, we have developed a bi-objective relief optimization model that minimizes, as its first objective, the number of people who have not been sent to the hospitals and shelters and, as its second objective, the total shortages of the relief commodities through making such decisions as locating shelters and distribution centers, determining the capacity level of the distribution centers, determining the number of people sent to hospitals and shelters, determining the flow of the relief commodities from distribution centers to the affected area, hospitals, and shelters, and dispatching transportation vehicles.

3 Model formulation

3.1 Indices and sets

- t Planning horizon
- i Set of the affected points
- j Set of the candidate locations for setting up distribution centers
- p Set of the shelters
- k Set of the relief commodities
- l Set of the distribution centers
- v Set of the transportation vehicles
- h Set of the losses and injuries

3.2 Parameters

- fp_p Fixed cost of setting up a shelter at location p
- ff_{jl} Fixed cost of setting up a distribution center with capacity l at location j
- di_{hi} Number of injured people type h available at affected area i
- dh_i Number of people at the affected area i that are to be evacuated
- dk_{kit} Demand for the relief commodity type k at the affected area i at time t
- ds_{kst} Demand for the relief commodity type k at the hospital s at time t
- Ci_s Capacity of the hospital s for the injured people
- Cw_v Weight capacity of the vehicle type v
- Cv_v Volume capacity of the vehicle type v
- Civ_v Capacity of the vehicle type v for shipping injured people
- Ch_v Capacity of the vehicle type v for evacuating the homeless people
- Cap_l Capacity of distribution center level l
- Cps_p Capacity of shelter p
- dt_{vis} Travel time of vehicle v between the affected area i and the hospital s
- dt_{vip} Travel time of vehicle v between the affected area i and the shelter p
- dt_{vjs} Travel time of vehicle v between the distribution center j and the hospital s
- dt_{vjp} Travel time of vehicle v between the distribution center j and the shelter p
- dt_{vji} Travel time of vehicle v between the distribution center j and the affected area i
- Wt_k Weight of one unit relief commodity type k
- Vl_k Volume of one unit relief commodity type k
- nk_k Number of the relief commodity type k required per person in a shelter

- A_{jv} Number of the vehicle type v available at the distribution center j when the planning horizon begins
- B_{iv} Number of the vehicle type v available at the affected area i at the beginning of the planning horizon
- B_o Maximum accessible budget
- Tc Coverage time
- M_{big} A very big positive number

3.3 Variables

- y_p A binary variable to set up a shelter at location p
- y_{li} A binary variable to set up a distribution center with capacity l at the location j
- x_{kfst} No. of the relief commodity type k sent from the distribution center j to the hospital s at time t
- x_{kjpt} No. of the relief commodity type k sent from the distribution center j to the shelter p at time t
- x_{kjit} No. of the commodity type k sent from the distribution center j to the affected area i at time t
- Ni_{hist} No. of injured people type h sent from the affected area i to the hospital s at time t
- Nh_{ipt} No. of people sent from the affected area i to the shelter p at time t
- Nvh_{ipvt} No. of the vehicle type v sent from the affected area i to the shelter p at time t
- Nvi_{isvt} No. of the vehicle type v sent from the affected area i to the hospital s at time t
- Nvk_{jpv} No. of the vehicle type v sent from the distribution center j to the shelter p at time t
- Nvj_{jivt} No. of the vehicle type v sent from the distribution center j to the affected area i at time t
- $Nvjs_{jsvt}$ No. of the vehicle type v sent from the distribution center j to the hospital s at time t
- dv_{kpt} Demand for the relief commodity type k in the shelter p at time t
- av_{ivt} No. of the vehicle type v available at the affected area i at time t

3.4 Objective functions and constraints

The proposed mathematical model for humanitarian relief chain network design (HRCND) is as follows:

HRCND:

$$\text{Min } z_1 = \sum_h \sum_i \left(di_{hi} - \sum_s \sum_t Ni_{hist} \right) + \sum_s \left(dh_i - \sum_p \sum_t Nh_{ipt} \right) \tag{1}$$

$$\begin{aligned} \text{Min } z_2 = & \sum_k \sum_i \sum_t \left(dk_{kit} - \sum_j x_{kjit} \right) \\ & + \sum_k \sum_s \sum_t \left(ds_{kst} - \sum_j x_{kjst} \right) \\ & + \sum_k \sum_p \sum_t \left(dv_{kpt} - \sum_j x_{kjpt} \right) \end{aligned}$$

subject to

$$\sum_p f_p y_p + \sum_j \sum_l f_{jl} y_{jl} \leq B_o \tag{3}$$

$$\sum_l y_{lj} \leq 1 \quad \forall j \in J \tag{4}$$

$$\sum_i \sum_t N h_{ipt} \leq C p s_p \cdot y_p \quad \forall p \in P \tag{5}$$

$$\sum_i \sum_t \sum_h N i_{hist} \leq C i_s \quad \forall s \in S \tag{6}$$

$$\sum_i N h_{ipt} n k_k = d v_{kpt} \quad \forall k \in K, p \in P, t \in T \tag{7}$$

$$\begin{aligned} & \sum_k \sum_p \sum_t x_{kjpt} + \sum_k \sum_s \sum_t x_{kjst} \\ & + \sum_k \sum_i \sum_t x_{kjit} \leq \sum_l C a p_l \cdot y_{jl} \end{aligned} \tag{8}$$

$\forall j \in J$

$$\sum_p \sum_t N h_{ipt} \leq d h_i \quad \forall i \in I \tag{9}$$

$$\sum_s \sum_t N i_{hist} \leq d i_{hi} \quad \forall h \in H, i \in I \tag{10}$$

$$\sum_j x_{kjit} \leq d k_{kit} \quad \forall k \in K, i \in I, t \in T \tag{11}$$

$$\sum_j x_{kjst} \leq d s_{kst} \quad \forall k \in K, s \in S, t \in T \tag{12}$$

$$\sum_j x_{kjpt} \leq d v_{kpt} \quad \forall k \in K, p \in P, t \in T \tag{13}$$

$$\sum_k w t_k x_{kjpt} \leq \sum_v C w_v \cdot N v k_{jpvt} \quad \forall j \in J, p \in P, t \in T \tag{14}$$

$$\sum_k v l_k x_{kjpt} \leq \sum_v C v_v \cdot N v k_{jpvt} \quad \forall j \in J, p \in P, t \in T \tag{15}$$

$$\sum_k w t_k x_{kjst} \leq \sum_v C w_v \cdot N v j s_{jsvt} \quad \forall j \in J, s \in S, t \in T \tag{16}$$

$$\sum_k v l_k \cdot x_{kjst} \leq \sum_v C v_v \cdot N v j s_{jsvt} \quad \forall j \in J, s \in S, t \in T \tag{17}$$

$$\sum_k w t_k x_{kjit} \leq \sum_v C w_v \cdot N v j i_{jivt} \quad \forall j \in J, i \in I, t \in T \tag{18}$$

$$\sum_k v l_k x_{kjit} \leq \sum_v C v_v \cdot N v j i_{jivt} \quad \forall j \in J, i \in I, t \in T \tag{19}$$

$$\sum_h N i_{hist} \leq \sum_v C i v_v \cdot N v i_{isvt} \quad \forall i \in I, s \in S, t \in T \tag{20}$$

$$N h_{ipt} \leq \sum_v C h v_v \cdot N v h_{ipvt} \quad \forall i \in I, p \in P, t \in T \tag{21}$$

$$N v k_{jpvt} \leq M_{big, y_p} \quad \forall j \in J, p \in P, v \in V, t \in T \tag{22}$$

$$N v k_{jpvt} \leq \sum_l M_{big, y_{jl}} \quad \forall j \in J, p \in P, v \in V, t \in T \tag{23}$$

$$N v j i_{jivt} \leq \sum_l M_{big, y_{jl}} \quad \forall j \in J, i \in I, v \in V, t \in T \tag{24}$$

$$N v j s_{jsvt} \leq \sum_l M_{big, y_{jl}} \quad \forall j \in J, p \in P, v \in V, t \in T \tag{25}$$

$$d t_{vis} N v i_{isvt} \leq t c \quad \forall i \in I, s \in S, v \in V, t \in T \tag{26}$$

$$d t_{vip} N v h_{ipvt} \leq t c \quad \forall i \in I, s \in S, v \in V, t \in T \tag{27}$$

$$d t_{vjs} N v j s_{jsvt} \leq t c \quad \forall j \in J, s \in S, v \in V, t \in T \tag{28}$$

$$d t_{vjp} N v k_{jpvt} \leq t c \quad \forall j \in J, p \in P, v \in V, t \in T \tag{29}$$

$$d t_{vji} N v j i_{jivt} \leq t c \quad \forall j \in J, i \in I, v \in V, t \in T \tag{30}$$

$$\begin{aligned} & \sum_p \sum_t N v k_{jpvt} + \sum_i \sum_t N v j i_{jivt} + \sum_s \sum_t N v j s_{jsvt} \leq A_{jv} \\ & \quad \forall j \in J, v \in V \end{aligned} \tag{31}$$

$$\begin{aligned} a v_{ivt} = & a v_{ivt-1} + \sum_j N v j i_{jivt} - \sum_p N v h_{ipvt} - \sum_s N v i_{isvt} \quad \forall i \\ & \in I, v \in V, t \in T \end{aligned} \tag{32}$$

$$a v_{iv0} = B_{iv} \quad \forall i \in I, v \in V \tag{33}$$

$$\begin{aligned} & y_p, y_{lj} \in \{0, 1\}, x_{kjpt}, x_{kjst}, x_{kjit}, d v_{kpt}, N i_{hist}, N h_{ipt}, \\ & N v h_{ipvt}, N v i_{isvt}, N v k_{jpvt}, N v j i_{jivt}, N v j s_{jsvt}, a v_{ivt} \geq 0, \text{ and int} \end{aligned} \tag{34}$$

In the above model, the first objective function (1) minimizes the total number of people who have not been sent to hospitals and shelters, while the second objective function (2) minimizes the total shortages of the relief commodities in the affected area, hospitals, and shelters. Constraint (3) ensures that the setup fixed costs may not exceed the available budget. Constraint (4) specifies the setup capacity of each distribution center. Constraint (5) keeps the number of the people sent to a shelter below its capacity. Constraint (6) limits a hospital capacity for the injured people. Constraint (7) limits the shelter demand for relief commodities. Constraint (8) ensures that the total relief commodities sent from a distribution center to other nodes should be less than their capacities. Constraints (9 and 10) ensure that the total number of people sent to hospitals and shelters should not exceed the population of the affected area. Constraints (11–13) ensure that the total number of relief commodities sent from a distribution center to other nodes should not exceed

their demands. Constraints (14–19) deal with the vehicles’ weights and volumes limitations. Constraints (20 and 21) show a vehicle’s human capacity. Constraint (22) ensures that sending people to a shelter is possible only if it already exists. Constraints (23–25) ensure that the relief commodity flow from a distribution center to other nodes is possible only if they already exist. Constraints (26–30) ensure that the time of each service should not exceed the maximum coverage time Constraint (31) ensures that the number of vehicles dispatched from distribution centers to affected areas, hospitals, and shelters cannot be more than the total number of the available vehicles. Constraints (32 and 33) balance the vehicles in the affected area. And, finally, constraint (34) specifies the sign of each variable.

4 The proposed robust model

The concept of the RO was first introduced by Soyster [32], who assumed that all uncertainty data assume values in a closed interval and, with this assumption, he determined these values so that the worst possible state may be imposed on the model; if the model is feasible with this logic, then it is so in realization. Although Soyster’s logic solved the problem of the parameters’ stochasticity to some extent, it was far from reality because the probability that all the parameters may be in their worst possible state simultaneously is almost nil. To control the conservatism degree appropriately and create more conformity between the RO and reality, many researchers developed their models on this basis. To start their works, Ben-Tal and Nemirovski [31] assumed an ellipsoidal set formation for the uncertainty data, but Bertsimas and Sim’s [26] assumption was a counterpart “box and polyhedral” set. These sets show that all the parameters cannot be in their worst possible states simultaneously.

These two approaches have been proposed against Soyster’s traditional model and may yield different results. The goal is to compare them in disaster conditions and see which one, under similar situations, yields better results so that the decision maker can perform better under such circumstances.

4.1 Robust optimization with ellipsoidal uncertainty set

This modeling was first introduced by Ben-Tal and Nemirovski [31]. Consider the following mathematical model:

$$\begin{aligned}
 \text{RO : } \min z &= \sum_j c_j x_j \\
 \text{Subject to} & \\
 \sum_j \tilde{a}_{ij} x_j &\leq b_i
 \end{aligned} \tag{35}$$

where the uncertain vector (\tilde{a}_{ij}) assumes value in the interval $[\bar{a}_{ij} + \hat{a}_{ij}$ and $\bar{a}_{ij} - \hat{a}_{ij}]$ (\bar{a}_{ij}) is the nominal value of the uncertain parameter, and (\hat{a}_{ij}) is the perturbation vector. Considering the ellipsoidal uncertainty set, the robust counterpart of Ben-Tal and Nemirovski’s model is shown as follows:

$$\begin{aligned}
 \min z &= \sum_j c_j x_j \\
 \text{Subject to} & \\
 \sum_j a_{ij} x_j + \Omega \sqrt{\sum_{j \in J_i} \hat{a}_{ij}^2 x_j^2} &\leq b_i
 \end{aligned} \tag{36}$$

where (Ω) corresponds to the conservatism degree of the ellipsoidal uncertainty set. For instance, consider the first part of the objective function (1), constraints (9 and 26), in problem (HRCND):

$$\text{Min } z \tag{37}$$

Subject to

$$\sum_h \sum_i \left(d_{hi} - \sum_s \sum_t N_{ihst} \right) \leq z \tag{38}$$

$$\sum_p \sum_t N_{hpt} \leq d_{hi} \quad \forall i \in I \tag{39}$$

$$d_{vis} N v_{isvt} \leq t c \quad \forall i \in I, s \in S, v \in V, t \in T \tag{40}$$

Making use of the auxiliary variables k_{hi} and k_i , the above model can be rewritten as follows:

$$\text{Min } z \tag{41}$$

Subject to

$$\sum_h \sum_i d_{hi} k_{hi} \leq \sum_h \sum_i \sum_s \sum_t N_{ihst} + z \tag{42}$$

$$k_i d_{hi} \leq - \sum_p \sum_t N_{hpt} \quad \forall i \in I \tag{43}$$

$$d_{vis} N v_{isvt} \leq t c \quad \forall i \in I, s \in S, v \in V, t \in T \tag{44}$$

The robust counterpart of the above model is as follows:

$$\text{Min } z \tag{45}$$

Subject to

$$\begin{aligned}
 \sum_h \sum_i \bar{d}_{hi} k_{hi} \\
 + \Omega \sqrt{\sum_{h \in H_0} \sum_{i \in I_0} \hat{d}_{hi}^2 k_{hi}^2} \leq \sum_h \sum_i \sum_s \sum_t N_{ihst} + z
 \end{aligned} \tag{46}$$

$$\overline{dh}_i k_i + \Omega \sqrt{\widehat{dh}_i^2 k_i^2} \leq - \sum_p \sum_t N h_{ipt} \quad \forall i \in I \tag{47}$$

$$\overline{dt}_{vis} N v i_{isvt} + \Omega \sqrt{\widehat{dt}_{vip}^2 N v i_{isvt}^2} \leq tc \quad \forall i \in I, s \in S, v \in V, t \in T \tag{48}$$

Now, substituting the values of the auxiliary variables ($k_{hi} = 1$ and $k_i = -1$), we will get:

$$\text{Min } z \tag{49}$$

Subject to

$$\sum_h \sum_i \overline{di}_{hi} + \Omega \sqrt{\sum_{h \in H_o} \sum_{i \in I_o} \widehat{di}_{hi}^2} \leq \sum_h \sum_i \sum_s \sum_t N i_{hist} + z \tag{50}$$

$$\sum_p \sum_t N h_{ipt} \leq \overline{dh}_i - \Omega \widehat{dh}_i \quad \forall i \in I \tag{51}$$

$$\overline{dt}_{vis} N v i_{isvt} + \Omega \widehat{dt}_{vis} N v i_{isvt} \leq tc \quad \forall i \in I, s \in S, v \in V, t \in T \tag{52}$$

We can see that the nonlinear model with the ellipsoidal uncertainty set has turned into a linear programming. Finally, the robust counterpart of the multi-objective relief problem with ellipsoidal uncertainty set (RCRES) is shown as follows:

RCRES:

$$\text{Min } z1 \tag{53}$$

$$\text{Min } z2 \tag{54}$$

Subject to Constrains (3–8) (13–25) and (31–34)

$$\begin{aligned} &\sum_h \sum_i \overline{di}_{hi} + \Omega \sqrt{\sum_{h \in H_o} \sum_{i \in I_o} \widehat{dh}_i^2} - \sum_h \sum_i \sum_s \sum_t N i_{hist} \\ &+ \sum_i \overline{di}_i + \Omega \sqrt{\sum_{i \in I_o} \widehat{dh}_i^2} - \sum_i \sum_p \sum_t N h_{ipt} \leq z1 \end{aligned} \tag{55}$$

$$\begin{aligned} &\sum_k \sum_i \sum_t \overline{dk}_{kit} + \Omega \sqrt{\sum_{k \in K_o} \sum_{i \in I_o} \sum_{t \in T_o} \widehat{dk}_{kit}^2} \\ &- \sum_k \sum_j \sum_i \sum_t x_{kjit} + \sum_k \sum_s \sum_t \overline{ds}_{kst} \\ &+ \Omega \sqrt{\sum_{k \in K_o} \sum_{s \in S_o} \sum_{t \in T_o} \widehat{ds}_{kst}^2} - \sum_k \sum_j \sum_s \sum_t x_{kjst} \\ &+ \sum_k \sum_p \sum_t \left(dv_{kpt} - \sum_j x_{kjpt} \right) \leq z2 \end{aligned} \tag{56}$$

$$\sum_p \sum_t N h_{ipt} \leq \overline{dh}_i - \Omega \widehat{dh}_i \quad \forall i \in I \tag{57}$$

$$\sum_s \sum_t N i_{hist} \leq \overline{di}_{hi} - \Omega \widehat{di}_{hi} \quad \forall h \in H, i \in I \tag{58}$$

$$\sum_j x_{kjit} \leq \overline{dk}_{kit} - \Omega \widehat{dk}_{kit} \quad \forall k \in K, i \in I, t \in T \tag{59}$$

$$\sum_j x_{kjst} \leq \overline{ds}_{kst} + \Omega \widehat{ds}_{kst} \quad \forall k \in K, s \in S, t \in T \tag{60}$$

$$\overline{dt}_{vis} N v i_{isvt} + \Omega \widehat{dt}_{vis} N v i_{isvt} \leq tc \quad \forall i \in I, s \in S, v \in V, t \in T \tag{61}$$

$$\overline{dt}_{vjs} N v j_{jsvt} + \Omega \widehat{dt}_{vjs} N v j_{jsvt} \leq tc \quad \forall j \in J, s \in S, v \in V, t \in T \tag{62}$$

$$\overline{dt}_{vip} N v k_{ipvt} + \Omega \widehat{dt}_{vip} N v k_{ipvt} \leq tc \quad \forall j \in J, p \in P, v \in V, t \in T \tag{63}$$

$$\overline{dt}_{vip} N v h_{ipvt} + \Omega \widehat{dt}_{vip} N v h_{ipvt} \leq tc \quad \forall p \in P, i \in I, v \in V, t \in T \tag{64}$$

$$\overline{dt}_{vis} N v j_{ivt} + \Omega \widehat{dt}_{vij} N v j_{ivt} \leq tc \quad \forall j \in J, i \in I, v \in V, t \in T \tag{65}$$

4.2 Robust optimization with box and polyhedral uncertainty sets

This modeling was first introduced by Bertsimas and Sim [26]. In this modeling, it is assumed that some stochastic data are influenced by uncertainty and they assume value in their worst possible state.

Considering linear programming (RO), the robust counterpart of this model with box and polyhedral uncertainty sets is as follows:

$$\begin{aligned} &\text{min } z = \sum_j c_j x_j \\ &\text{Subject to} \\ &\sum_j \overline{a}_{ij} x_j + \max_{\{s_i \cup \{t_i\} | s_i \subseteq J_i, |s_i| = [\Gamma_i], t_i \in J_i \setminus s_i\}} \\ &\left\{ \sum_{j \in s_i} \hat{a}_{ij} |x_j| + (\Gamma_i - [\Gamma_i]) \hat{a}_{it_i} |x_{it_i}| \right\} \leq b_i \end{aligned} \tag{66}$$

where (J_i) is an integer, is the total number of uncertain parameters available in constraint i , and (Γ_i) is the uncertainty robustness budget ($\Gamma_i = [0, |J_i|]$). To turn the above problem into a single optimization problem, the protection function against uncertainty $\beta(x, \Gamma_i)$ is introduced as follows:

$$\beta(x, \Gamma_i) = \max_{\{s_i \cup \{t_i\} | s_i \subseteq J_i, |s_i| = d\Gamma_{it}, t_i \in J_i \setminus s_i\}} \left\{ \sum_{j \in s_i} \hat{a}_{ij} |x_j| + (\Gamma_i - d\Gamma_{it}) \hat{a}_{it_i} |x_{t_i}| \right\} \tag{67}$$

The protection function optimization problem is equivalent to the following problem:

$$\begin{aligned} & \text{Max} \sum_{j \in J_i} \hat{a}_{ij} z_{ij} |x_j| \\ & \text{Subject to} \\ & \sum_j z_{ij} \leq \Gamma_i \\ & 0 \leq z_{ij} \leq \end{aligned} \tag{68}$$

The dual of the above problem is written as follows:

$$\begin{aligned} & \text{Min} \Gamma_i p_i + \sum_{j \in J_i} q_{ij} \\ & \text{Subject to} \\ & p_i + q_{ij} \geq \hat{a}_{ij} |x_j| \quad \forall i \in I, j \in J_i \\ & p_i, q_{ij} \geq 0 \end{aligned} \tag{69}$$

q_{ij} and p_i are the dual variables. Consequently, the robust counterpart of problem (RO) is written as follows:

$$\begin{aligned} & \min z = \sum_j c_j x_j \\ & \sum_j a_{ij} x_j + \Gamma_i p_i + \sum_{j \in J_i} q_{ij} \leq b_i \\ & p_i + q_{ij} \geq \hat{a}_{ij} u_j \\ & -u_j \leq x_j \leq u_j \\ & p_i, q_{ij}, u_j \geq 0 \end{aligned} \tag{70}$$

Finally, the robust counterpart of the multi-objective relief problem with box and polyhedral uncertainty sets (RCRBPS) is shown as follows:

RCRBPS:

$$\text{Min } z1 \tag{71}$$

$$\text{Min } z2 \tag{72}$$

Subject to

Constrains (3–5) (8–13) and (31–34)

$$\begin{aligned} & \sum_h \sum_i \left(\bar{d}i_{hi} - \sum_s \sum_t Ni_{hist} \right) + \sum_{h \in H_o} \sum_{i \in I_o} q_i^{di} + P_o^{di} \Gamma_o^{di} \\ & + \sum_s \left(\bar{d}h_i - \sum_p \sum_t Nh_{ipt} \right) + \sum_{i \in I_o} q_i^{dh} + P_o^{dh} \Gamma_o^{dh} \leq z1 \end{aligned} \tag{73}$$

$$\begin{aligned} & \sum_k \sum_i \sum_t \left(\bar{d}k_{kit} - \sum_j x_{kjit} \right) + \sum_{k \in K_o} \sum_{i \in I_o} \sum_{t \in T_o} q_{kit}^{dk} + P_o^{dk} \Gamma_o^{dk} \\ & + \sum_k \sum_s \sum_t \left(\bar{d}s_{kst} - \sum_j x_{kjst} \right) + \sum_{k \in K_o} \sum_{s \in S_o} \sum_{t \in T_o} q_{kst}^{ds} \\ & + P_o^{ds} \Gamma_o^{ds} + \sum_k \sum_p \sum_t \left(\bar{d}v_{kpt} - \sum_j x_{kjpt} \right) \leq z \end{aligned} \tag{74}$$

$$\sum_p \sum_t Nh_{ipt} \leq \bar{d}h_i - \Gamma_i^{dh} \hat{d}h_i \quad \forall i \in I \tag{75}$$

$$\sum_s \sum_t Ni_{hist} \leq \bar{d}i_{hi} - \Gamma_{hi}^{dh} \hat{d}i_{hi} \quad \forall h \in H, i \in I \tag{76}$$

$$\sum_j x_{kjit} \leq \bar{d}k_{kit} - \Gamma_{kit}^{dk} \hat{d}k_{kit} \quad \forall k \in K, i \in I, t \in T \tag{77}$$

$$\sum_j x_{kjst} \leq \bar{d}s_{kst} + \Gamma_{kst}^{ds} \hat{d}s_{kst} \quad \forall k \in K, s \in S, t \in T \tag{78}$$

$$\bar{d}t_{vis} Nv_{isvt} + q_{vis}^{tvis} + P_{vis}^{tvis} \Gamma_{vis}^{tvis} \leq tc \quad \forall i \in I, s \in S, v \in V, t \in T \tag{79}$$

$$\bar{d}t_{vip} Nvh_{ipvt} + q_{vip}^{tvip} + P_{vip}^{tvip} \Gamma_{vip}^{tvip} \leq tc \quad \forall i \in I, p \in P, v \in V, t \in T \tag{80}$$

$$\bar{d}t_{vjs} Nvjs_{jsvt} + q_{vjs}^{tvjs} + P_{vjs}^{tvjs} \Gamma_{vjs}^{tvjs} \leq tc \quad \forall j \in J, s \in S, v \in V, t \in T \tag{81}$$

$$\bar{d}t_{vji} Nvj_{jivt} + q_{vji}^{tvji} + P_{vji}^{tvji} \Gamma_{vji}^{tvji} \leq tc \quad \forall j \in J, i \in I, v \in V, t \in T \tag{82}$$

$$q_{hi}^{di} + P_o^{di} \geq \hat{d}i_{hi} \quad \forall h \in H, i \in I \tag{83}$$

$$q_i^{dh} + P_o^{dh} \geq \hat{d}h_i \quad \forall i \in I \tag{84}$$

$$q_{kit}^{dk} + P_o^{dk} \geq \hat{d}k_{kit} \quad \forall k \in K, i \in I, t \in T \tag{85}$$

$$q_{kst}^{ds} + P_o^{ds} \geq \hat{d}k_{kst} \quad \forall k \in K, s \in S, t \in T \tag{86}$$

$$q_{vis}^{tvis} + P_{vis}^{tvis} \geq \hat{d}t_{vis} Nv_{isvt} \quad \forall v \in v, i \in I, s \in S, t \in T \tag{87}$$

$$q_{vip}^{tvip} + P_{vip}^{tvip} \geq \hat{d}t_{vip} Nvh_{ipvt} \quad \forall i \in I, p \in P, v \in V, t \in T \tag{88}$$

$$q_{vjs}^{tvjs} + P_{vjs}^{tvjs} \geq \hat{d}t_{vjs} Nvjs_{jsvt} \quad \forall j \in J, s \in S, v \in V, t \in T \tag{89}$$

$$q_{vjp}^{tvjp} + P_{vjp}^{tvjp} \geq \hat{d}t_{vjp} Nvk_{jpvt} \quad \forall j \in J, p \in P, v \in V, t \in T \tag{90}$$

$$q_{vji}^{tvji} + P_{vji}^{tvji} \geq \hat{d}t_{vji} Nvj_{jivt} \quad \forall j \in J, i \in I, v \in V, t \in T \tag{91}$$

$$\begin{aligned}
 & q_{hi}^{di}, P_o^{di}, q_i^{dh}, P_o^{dh}, q_{kit}^{dk}, P_o^{dk}, q_{kst}^{ds}, P_o^{ds}, q_{kst}^{ds}, P_o^{ds}, q_{vis}^{tvis}, \\
 & P_{vis}^{tvis}, q_{vip}^{tvip}, P_{vip}^{tvip}, q_{vjs}^{tvjs}, P_{vjs}^{tvjs}, q_{vjp}^{tvjp}, P_{vjp}^{tvjp}, q_{vji}^{tvji}, P_{vji}^{tvji} \geq 0
 \end{aligned} \tag{92}$$

In mathematical model (RCRBPS), there is only one uncertain parameter in the set of constraints (75–82). The maximum value of the robustness budget of these constraints is 1 and its minimum is 0. We determine a total robustness budget for a specified datum (e.g., dk_{kit}) and consider the share of each constraint in the total budget equal. In other words, we consider the parameter uncertainty of these constraints column-wise instead of row-wise [25]; this will facilitate the work and simplify the computations. Therefore, constraints (75–82) of model (RCRBPS) are extended as follows:

$$\sum_P \sum_t Nh_{ipt} \leq dh_i - \frac{\Gamma_o^{dh}}{I} \widehat{dh}_i \quad \forall i \in I \tag{93}$$

$$\sum_s \sum_t Ni_{hist} \leq di_{hi} - \frac{\Gamma_o^{dh}}{H.I} \widehat{di}_{hi} \quad \forall h \in H, i \in I \tag{94}$$

$$\sum_j x_{kjit} \leq dk_{kit} - \frac{\Gamma_o^{dk}}{K.I.T} \widehat{dk}_{kit} \quad \forall k \in K, i \in I, t \in T \tag{95}$$

$$\begin{aligned}
 dt_{vis} Nv_{isvt} + q_{vis}^{tvis} + P_{vis}^{tvis} \frac{\Gamma^{tvis}}{V.I.S} &\leq tc \\
 \forall i \in I, s \in S, v \in V, t \in T
 \end{aligned} \tag{96}$$

$$\begin{aligned}
 dt_{vip} Nv_{ipvt} + q_{vip}^{tvip} + P_{vip}^{tvip} \frac{\Gamma^{tvip}}{V.I.P} &\leq tc \\
 \forall i \in I, p \in P, v \in V, t \in T
 \end{aligned} \tag{97}$$

$$\begin{aligned}
 dt_{vjs} Nv_{jsvt} + q_{vjs}^{tvjs} + P_{vjs}^{tvjs} \frac{\Gamma^{tvjs}}{V.J.S} &\leq tc \\
 \forall j \in J, s \in S, v \in V, t \in T
 \end{aligned} \tag{98}$$

$$\begin{aligned}
 dt_{vjp} Nv_{jpvt} + q_{vjp}^{tvjp} + P_{vjp}^{tvjp} \frac{\Gamma^{tvjp}}{V.J.P} &\leq tc \\
 \forall j \in J, p \in P, v \in V, t \in T
 \end{aligned} \tag{99}$$

4.3 Maximum probability of constraint violation in the robust optimization

Consider the following uncertain constraint:

$$\sum_j \tilde{a}_{ij} x_j^* \leq b_i \quad \forall i \in I \tag{100}$$

As we know, the left-side coefficients (\tilde{a}_{ij}) are uncertain. RO is more after the solution robustness of constraint (100) under realization; therefore, it is possible to establish a relationship between the degree of conservatism and the probability of constraint violation. Ben-Tal and

Nemirovski [34] have shown the maximum probability of constraint (100) violation as follows:

$$\Pr \left(\sum_j \tilde{a}_{ij} x_j^* \geq b_i \right) \leq e^{(-\Omega_i^2)} \quad \forall i \in I \tag{101}$$

Although the above relation is to find the maximum probability of the constraint violation of the RO with box and polyhedral uncertainty sets, it is true for the RO with ellipsoidal uncertainty set as well [35].

Bertsimas and Sim [26], have introduced the maximum probability of constraint (100) violation with box and polyhedral uncertainty sets as follows:

$$\Pr \left(\sum_j \tilde{a}_{ij} x_j^* \geq b_i \right) \leq 1 - \Phi \left(\frac{\Gamma_i - 1}{\sqrt{|J_i|}} \right) \tag{102}$$

In constraint (102), $\Phi \left(\frac{\Gamma_i - 1}{\sqrt{|J_i|}} \right)$ is the cumulative probability of the normal distribution function.

To compare the performances of models (RCRES) and (RCRBPS) against uncertainty, the probabilities of the constraint violation are assumed equal and the conservatism degree is calculated for each model and compared under similar conditions.

5 Solution procedure

In the related literature, the extensive use of the multi-objective optimization problems has been made [36–38]. In this paper, we have used the Lexicographic Weighted Tchebycheff (LWT) method where every objective is first optimized individually and then a new objective is defined which is after minimizing the maximum weight deviation of each objective function from its optimum value [39]. Consider a multi-objective model with n variables and m objective functions as follows:

$$\begin{aligned}
 \text{LWT : Min } H(x) &= \{h_1(x), h_2(x), h_3(x), \dots, h_m(x)\} \\
 \text{Subject to} & \\
 x &\in X_h
 \end{aligned} \tag{103}$$

where $(x_f \subseteq x)$ is the set of the feasible solutions, $H(x)$ is the set of the decision variables in the decision space $x : [x = x_1, x_2, x_3, \dots, x_n]$ and the solution space $z : [z = z_1, z_2, z_3, \dots, z_m]$.

First, the optimum value of each objective function is found as follows:

$$z_i^* = \text{Min}\{h_i(x|x \in x_h)\} - \varepsilon_i \tag{104}$$

where ε_i is a very small positive value. The multi-objective model LWT is turned into a single objective model (namely LWTS) as follows:

LWTS : Min α

$$\begin{aligned}
 &S.t \\
 &\alpha \geq \lambda_i (h_i(x) - z_i^*) \\
 &\sum_{i=1}^m \lambda_i = 1 \quad 0 \leq \lambda_i \leq 1, x \in x_h
 \end{aligned}
 \tag{105}$$

where (λ_i) is the importance of each objective function.

Problem LWTS minimizes the maximum weight deviation of each objective function from its optimum value and may produce weak Pareto optimum solutions. Therefore, some of the solutions may dominate the weak Pareto solutions for the elimination of which we can use the model as follows if we define x_w as the Pareto optimum solution of model LWTS:

$$\begin{aligned}
 &Min \sum_{i=1}^m (h_i(x) - z_i^*) \\
 &Subject to \\
 &x \in x_w
 \end{aligned}
 \tag{106}$$

6 Case study

Tehran, with a population of more than 8 million, is the largest metropolitan city in Iran. With this population, many people may face irrecoverable difficulties in case of an earthquake. Tehran is an earthquake-prone city because of its famous faults (Rey, Mosha, North Tehran, etc.).

6.1 Case description

Figure 2 shows that there are relatively many faults in the northern part of Tehran meaning that the first five districts (out of 22) are among the high-risk places. Therefore, the centers of these five districts have been considered as the affected areas. As shown in Fig. 3, the hospitals and the

candidate locations to set up shelters and distribution centers are outside the affected areas.

The main sources of data for this case study are the reports provided by this region’s disaster management experts, the Red Crescent Society, Japan International Cooperation Agency, and online available data on population and available resources. The data helped us calculate the location of the affected area, the number of injured people, the number of relief commodities, transportation times between all nodes of the network and the costs parameters, and the number and capacities of distribution centers, shelters, hospitals, and vehicles.

Each distribution center can be large, medium, or small, and the relief commodities stored in them include water, food, and pharmaceuticals. Capacities and setup costs of each type are shown in Table 1.

In this issue, there are five candidate locations to set up the distribution centers; the districts addresses and the candidate locations are shown in Table 2.

There are six different capacity hospitals at different locations, the names and capacities of which are shown in Table 3.

There are five candidate locations to set up shelters with different capacities; their brief information is provided in Table 4.

Tables 5 and 6 show the transportation vehicles’ features and the weight/volume of every 1000 units of the relief commodities, respectively.

And, finally, Table 7 shows the uncertainty interval for every stochastic parameter.

As mentioned before, hospitals, shelters, and distribution centers are located outside the affected areas to make most of their capacities and capabilities because if they are inside, some of them may get totally destroyed and some

Fig. 2 Map of Tehran faults

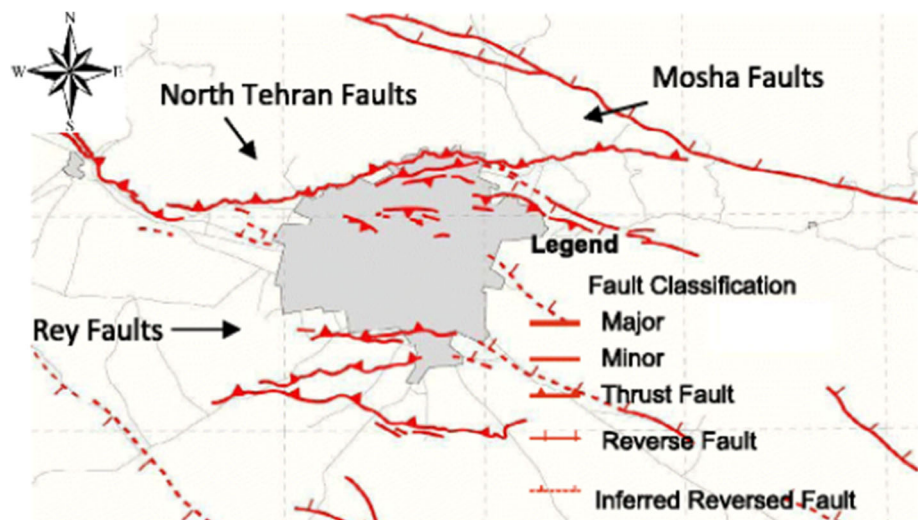


Fig. 3 Affected areas, hospitals, candidate locations to set up shelters and distribution centers

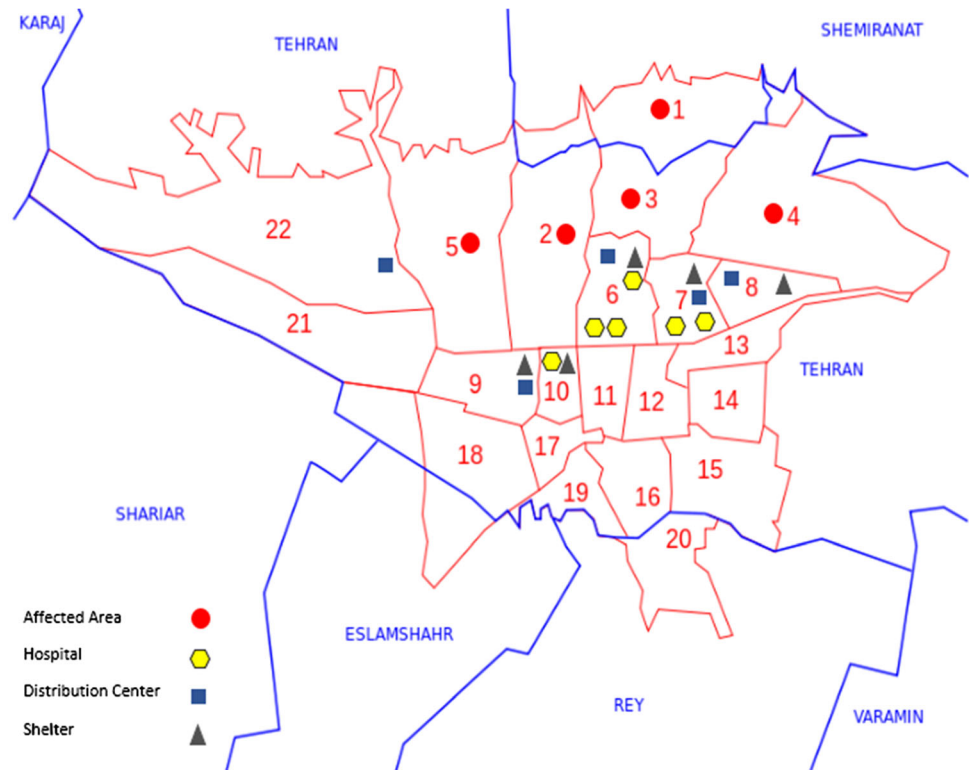


Table 1 The capacity level of distribution centers and their setup costs

Size	Fix cost (10 ³ \$)	Capacity (10 ³ unit)
Small	50	10
Medium	80	16
Large	120	24

may lose their efficiencies in case an earthquake or other disasters occur.

7 Results

As discussed in the earlier sections, the goal is a comparison between the optimization models (RCRES) and (RCRBPS) to see which one performs better in facing

uncertainties. But, establishing a direct relationship between these models’ conservatism degree is not an easy task, and for us in order to do it, we use another trick. If we take the probabilities of all the constraints violations of both models equal, we will be able to calculate their robustness budgets. For this purpose and to do the sensitivity analyses and models tests, we have considered five levels (0.1, 0.15, 0.2, 0.25, and 0.3) for the maximum probabilities of constraints violations and three levels (5, 10, and 15%) for the deviation of the uncertainty data from the nominal data. We have already pointed out that there are five affected areas, six hospitals, five candidate locations (to set up shelters), and five candidate locations (to set up distribution centers). Also, there are three types of transportation vehicles, two types of injured people, and three periods in the planning horizon. Problems (RCRES) and (RCRBPS) and also the deterministic problem (HRCND) have been solved using nominal data, DELL Laptop (core i5 and RAM4), GAMS Optimization

Table 2 Candidate locations for distribution centers

Number	Address	Region of Tehran
1	Highway intersection of Ayatollah Hakim and Kordestan	6
2	Shahid Sayyad Shirazi Highway	7
3	Highway intersection of Imam Ali and Western Janbazan	8
4	Ayatollah Saeedi Highway	9
5	Allameh Jafari Highway	22

Table 3 Locations and capacities of hospitals

Number	Name of hospitals	Capacity (unit)	Region of Tehran
1	Kasra	300	6
2	Imam Khomeini	1200	6
3	Mostafa Khomeini	300	7
4	Imam Hossein	614	7
5	Arad	145	7
6	Azadi	300	10

Software (Ver. 24.0.1), and CPLEX Powerful Solver. Five different weight sets (λ_1 and λ_2) have been considered for the objective functions.

By reducing the maximum probabilities of the constraints violations, we make the conditions more severe for the mathematical model; it means that if this maximum is reduced in each step, the conservatism budgets (Ω and Γ) will increase and, as we know, the optimum solution can suffer with this increase. Again, if the maximum deviation from the nominal data is increased, the conditions will get more difficult for modeling and, under such circumstances, the optimum solution may get worse. The summary of the results found from the solution of models (HRCND), (RCRES), and (RCRBPS) is provided in Tables 8, 9, 10, 11, 12, 13. As shown, the objective function has improved in each step by increasing the maximum probabilities of the constraints violations (which reduces the conservatism degree) and keeping the deviations from the nominal data constant.

Table 6 Weight and volume of the relief commodities

Type	Weight (ton)	Volume (m3)
Water (10^3 unit)	5	4.5
Food (10^3 unit)	3	2.5
Drug (10^3 unit)	2	6

Table 7 The sources of random generation of the nominal data

Parameter	Corresponding random distribution
dh_i (person)	Uniform (17,000,17,600)
dl_{hi} (person)	Uniform (5500,6000)
dk_{kit} (unit)	Uniform (7000,8000)
ds_{kst} (unit)	Uniform (800,900)
dt_{vis} (minute)	Uniform (16,20)
dt_{vjp} (minute)	Uniform (16,20)
dt_{vjs} (minute)	Uniform (16,20)
dt_{vjp} (minute)	Uniform (16,20)
dt_{vji} (minute)	Uniform (16,20)

In Tables 8, 9, 10, 11, 12, 13, z_i^* is the optimum value of the i th objective function when the problem is single objective and (z_i^-) is the worst value of the i th objective function when the problem is single objective (with another objective function). By “weight,” we mean the importance given to the first and second objective functions [(λ_1) and (λ_2) , respectively] in multi-objective cases ($\lambda_1, \lambda_2 \leq 1$, and $\lambda_1 + \lambda_2 = 1$). Also, (α) is the value of the objective function of the multi-objective programming when importance λ_1 is

Table 4 Candidate locations for shelters

Number	Address	Capacity (10^3 person)	Fix cost (10^3 \$)	Region of Tehran
1	Highway intersection of Resalat and Africa	60	1200	6
2	Highway intersection of Ayatollah Ghodousi and Shahid Sayyad Shirazi Highway	70	1350	7
3	Shahid Hasan Bagheri Highway	74	1600	8
4	Azadi Square	73	1500	9
5	Yadegar Imam	72	1400	10

Table 5 Capacities of vehicles

Type	Weight capacity (ton)	Volume capacity (m^3)	Injure capacity (person)	Homeless capacity (person)
V_1	28	38	70	80
V_2	18	12	20	26
V_3	3.8	6	10	17

Table 8 Results from the deterministic model tests with the nominal data

$Z_1^* = 3386$	α	$Z_2^- = 496,588$	Weights
2459	7221	269,980	$\lambda_1 = 0.3, \lambda_2 = 0.7$
25,310	8769	274,280	$\lambda_1 = 0.4, \lambda_2 = 0.6$
22,873	9744	279,152	$\lambda_1 = 0.5, \lambda_2 = 0.5$
20,089	10,022	284,720	$\lambda_1 = 0.6, \lambda_2 = 0.4$
16,877	9444	291,144	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 35,476$		$z_2^* = 259,664$	

given to the first objective function (z_1) and λ_2 is given to the second objective function (z_2). MPCV and DFNV stand, respectively, for “Maximum Probability of Constraint Violation” and “Deviation from Nominal Value.”

For instance, Fig. 4 is drawn for the first objective function (z_1) of the deterministic model, robust ellipsoidal model, and robust box and polyhedral models and Fig. 5 is prepared for the second objective function (z_2) for 5%

deviation from the nominal data for all levels of the maximum probabilities of constraints violations. We can see that an increase in the maximum probability has improved the first and second objective functions in each step. It is worth mentioning that the worst case for robust models has occurred at 0.1 and 15%, respectively, for maximum probability of constraint violation and deviation of the stochastic data from the nominal data; this is the highest conservatism degree among all degrees introduced. Both robust models have been worse, in all cases, than the deterministic model. To achieve solution robustness, the robust modeling is after optimization under bad conditions (considering a specified conservatism degree) in such a way that the mathematical model will stay feasible under realization.

Objective functions values have been worse compared with those of the deterministic model.

Moreover, with an increase in the deviation from the nominal data, the optimum values of the objective functions of the robust models have become worse in each step.

Table 9 Results from two uncertainty sets tests with a maximum constraint violation probability of (0.1)

Ellipsoidal uncertainty set			Box and polyhedral uncertainty sets			Weights
$Z_1^* = 4998$	α	$Z_2^- = 502,584$	$Z_1^* = 6313$	α	$Z_2^- = 508,039$	
<i>MPCV = 0.1, DFNV = 5%</i>						
29,071	7221	275,977	28,836	6756	271,600	$\lambda_1 = 0.3, \lambda_2 = 0.7$
26,921	8769	280,276	26,824	8204	275,622	$\lambda_1 = 0.4, \lambda_2 = 0.6$
24,485	9744	285,148	24,545	9116	280,180	$\lambda_1 = 0.5, \lambda_2 = 0.5$
21,702	10,022	290,715	21,941	9376	285,390	$\lambda_1 = 0.6, \lambda_2 = 0.4$
18,489	9444	297,140	18,935	8835	291,400	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 37,088$		$z_2^* = 265,660$	$z_1^- = 36,535$		$z_2^* = 261,948$	
<hr/>						
$Z_1^* = 6610$	α	$Z_2^- = 508,580$	$Z_1^* = 9237$	α	$Z_2^- = 519,502$	Weights
<i>MPCV = 0.1, DFNV = 10%</i>						
30,683	7222	281,972	30,211	6292	273,220	$\lambda_1 = 0.3, \lambda_2 = 0.7$
28,534	8769	286,270	28,338	7640	276,966	$\lambda_1 = 0.4, \lambda_2 = 0.6$
26,097	9744	291,144	26,216	8489	281,211	$\lambda_1 = 0.5, \lambda_2 = 0.5$
23,313	10,022	296,712	23,791	8732	286,062	$\lambda_1 = 0.6, \lambda_2 = 0.4$
20,101	9444	303,136	20,992	8228	291,660	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 38,700$		$z_2^* = 271,656$	$z_1^- = 37,594$		$z_2^* = 264,232$	
<hr/>						
$Z_1^* = 8222$	α	$Z_2^- = 514,576$	$Z_1^* = 12,164$	α	$Z_2^- = 530,957$	Weights
<i>MPCV = 0.1, DFNV = 15%</i>						
32,295	7222	287,968	31,588	5827	274,840	$\lambda_1 = 0.3, \lambda_2 = 0.7$
30,146	8769	292,266	29,855	7076	278,310	$\lambda_1 = 0.4, \lambda_2 = 0.6$
27,709	9744	297,140	27,888	7862	282,240	$\lambda_1 = 0.5, \lambda_2 = 0.5$
24,925	10,022	302,708	25,643	8087	286,734	$\lambda_1 = 0.6, \lambda_2 = 0.4$
21,713	9444	309,132	23,050	7620	291,916	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 40,312$		$z_2^* = 277,652$	$z_1^- = 38,653$		$z_2^* = 266,516$	

Table 10 Results from two uncertainty sets tests with a maximum constraint violation probability of (0.15)

Ellipsoidal uncertainty set			Box and polyhedral uncertainty sets			Weights
$Z_1^* = 4847$	α	$Z_2^- = 502,023$	$Z_1^* = 5906$	α	$Z_2^- = 509,095$	
<i>MPCV = 0.15, DFNV = 5%</i>						
28,920	7222	275,415	28,647	6822	271,350	$\lambda_1 = 0.3, \lambda_2 = 0.7$
26,771	8769	279,713	26,615	8283	275,410	$\lambda_1 = 0.4, \lambda_2 = 0.6$
24,335	9744	284,585	24,314	9204	280,012	$\lambda_1 = 0.5, \lambda_2 = 0.5$
21,551	10,022	290,153	21,685	9467	285,272	$\lambda_1 = 0.6, \lambda_2 = 0.4$
18,338	9444	296,579	18,650	8920	291,340	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 36,937$		$z_2^* = 265,099$	$z_1^- = 36,386$		$z_2^* = 261,604$	
<hr/>						
$Z_1^* = 6309$	α	$Z_2^- = 507,459$	$Z_1^* = 8424$	α	$Z_2^- = 521,610$	
<i>MPCV = 0.15, DFNV = 10%</i>						
30,382	7221	280,851	29,832	6422	272,720	$\lambda_1 = 0.3, \lambda_2 = 0.7$
28,233	8769	285,149	27,921	7798	276,543	$\lambda_1 = 0.4, \lambda_2 = 0.6$
25,797	9744	290,021	25,754	8665	280,875	$\lambda_1 = 0.5, \lambda_2 = 0.5$
23,013	10,022	295,589	23,279	8912	285,827	$\lambda_1 = 0.6, \lambda_2 = 0.4$
19,800	9444	302,015	20,422	8398	291,540	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^* = 6309$	α	$z_2^- = 507,459$	$z_1^* = 8424$	α	$z_2^- = 521,610$	
<hr/>						
$Z_1^* = 7770$	α	$Z_2^- = 512,895$	$Z_1^* = 10,944$	α	$Z_2^- = 534,121$	
<i>MPCV = 0.15, DFNV = 15%</i>						
31,843	7222	286,287	31,020	6022	274,089	$\lambda_1 = 0.3, \lambda_2 = 0.7$
29,693	8769	290,587	29,227	7313	277,674	$\lambda_1 = 0.4, \lambda_2 = 0.6$
27,257	9744	295,459	27,196	8126	281,737	$\lambda_1 = 0.5, \lambda_2 = 0.5$
24,473	10,022	301,027	24,874	8358	286,380	$\lambda_1 = 0.6, \lambda_2 = 0.4$
21,261	9444	307,451	22,195	7875	291,738	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 39,860$		$z_2^* = 275,971$	$z_1^- = 38,207$		$z_2^* = 265,485$	

For example, the values of the first and the second objective functions (z_1 and z_2 , respectively) are drawn in Figs. 6 and 7, respectively, considering 0.1 for the maximum probability of constraint violation with deviations of 5, 10, and 15% from the nominal data.

We can see that with an increase in the deviation from the nominal data, the optimum values of the objective functions have obtained worse in each step. If the uncertainty interval is increased, we expect that the optimum value of the objective function has obtained worse in each step. In other levels of the maximum probability of constraint violation and deviation of the uncertainty data from the nominal data, the trend has been the same and the results have conformed to our expectations.

In optimization, if the first objective function (z_1) gets a 100% importance ($\lambda_1 = 1$), the optimum value of the objective function of the robust model will perform better with ellipsoidal set than box and polyhedral model, but if

$\lambda_1 < 1$, sometimes the solution optimality is displaced among the ellipsoidal, box, and polyhedral models. If the 100% importance ($\lambda_2 = 1$) is given to the second objective function (z_2), the robust model with the ellipsoidal set will perform worse than the box and polyhedral models. Solutions of the robust models have been calculated in (75) cases (5 levels of maximum probability of constraint violation, 3 levels of deviation from nominal data, and 5 weight sets) without giving the 100% importance to one objective function ($\lambda_1 < 1, \lambda_2 < 1$, and $\lambda_1 + \lambda_2 = 1$). If we count the number of the cases where the value of (z_1) in the robust model with box and polyhedral uncertainty sets has become better than that with the ellipsoidal set, in (56) cases the robust model with box and polyhedral uncertainty sets has performed better. If the same comparison is made for (z_2), again the robust model with box and polyhedral uncertainty sets has outperformed in (74) cases. Therefore, the point average of the robust model with box and poly-

Table 11 Results from two uncertainty sets tests with a maximum constraint violation probability of (0.2)

Ellipsoidal uncertainty set			Box and polyhedral uncertainty sets			Weights
$Z_1^* = 4734$	α	$Z_2^- = 501,603$	$Z_1^* = 5567$	α	$Z_2^- = 509,971$	
<i>MPCV = 0.2, DFNV 5%</i>						
28,807	7222	274,995	28,487	6876	271,139	$\lambda_1 = 0.3, \lambda_2 = 0.7$
26,658	8769	279,293	26,441	8349	275,232	$\lambda_1 = 0.4, \lambda_2 = 0.6$
24,221	9744	284,167	24,121	9277	279,870	$\lambda_1 = 0.5, \lambda_2 = 0.5$
21,437	10,022	289,735	21,471	9542	285,172	$\lambda_1 = 0.6, \lambda_2 = 0.4$
18,225	9444	296,159	18,412	8991	291,288	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 36,824$		$z_2^* = 264,679$	$z_1^- = 36,262$		$z_2^* = 261,316$	
<hr/>						
$Z_1^* = 6083$	α	$Z_2^- = 506,618$	$Z_1^* = 7746$	α	$Z_2^- = 523,362$	
<i>MPCV = 0.2, DFNV = 10%</i>						
30,156	7221	280,010	29,516	6531	272,299	$\lambda_1 = 0.3, \lambda_2 = 0.7$
28,006	8769	284,310	27,571	7930	276,186	$\lambda_1 = 0.4, \lambda_2 = 0.6$
25,570	9744	289,182	25,368	8811	280,591	$\lambda_1 = 0.5, \lambda_2 = 0.5$
23,600	10,510	295,574	22,851	9063	285,627	$\lambda_1 = 0.6, \lambda_2 = 0.4$
19,574	9444	301,174	19,947	8540	291,436	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 38,173$		$z_2^* = 269,694$	$z_1^- = 37,048$		$z_2^* = 262,969$	
<hr/>						
$Z_1^* = 7431$	α	$Z_2^- = 511,634$	$Z_1^* = 9926$	α	$Z_2^- = 536,753$	
<i>MPCV = 0.2, DFNV = 15%</i>						
31,504	7221	285,026	30,543	6185	273,458	$\lambda_1 = 0.3, \lambda_2 = 0.7$
29,354	8769	289,326	28,703	7510	277,140	$\lambda_1 = 0.4, \lambda_2 = 0.6$
26,918	9744	294,198	26,616	8345	281,312	$\lambda_1 = 0.5, \lambda_2 = 0.5$
25,101	10,775	301,649	24,232	8583	286,081	$\lambda_1 = 0.6, \lambda_2 = 0.4$
20,922	9444	306,190	21,481	8088	291,584	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 39,521$		$z_2^* = 274,710$	$z_1^- = 37,834$		$z_2^* = 264,622$	

hedral uncertainty sets has been better compared to that with the ellipsoidal set.

Since RO approaches based on the box and polyhedral and ellipsoidal uncertainty sets are quite different, their direct comparison is very difficult. If the MPCV is considered equal in both methods and appropriate robustness budgets are calculated from relations (101) and (102), their comparison will be possible under such similar conditions; this comparison method has been used in the proposed model. As explained in the paper (page 25 Paragraph 1), if robust models are solved with a single objective, the ellipsoidal uncertainty set will perform better than the box and polyhedral uncertainty set (as regards the first objective function), but as regards the second objective function, it is vice versa. If robust models are solved with multi-objective approaches considering weight (importance) for the objective functions, the box and polyhedral uncertainty set will perform better than the ellipsoidal uncertainty set.

Therefore, on average, the robust method based on the box uncertainty set performs better than the ellipsoidal uncertainty set.

Strong Pareto solutions have been produced at all levels of the maximum probability of constraint violation and different percentages of deviation from the nominal data. For instance, Pareto (Fig. 8) has been drawn for models (RCRES), (RCRBPS), and (HRCND) at the maximum probability of constraint violation of 0.2 and 10% deviation from the nominal data. The vertical axis of the Pareto figure shows the value of (z_1), and the horizontal axis shows that of (z_2). When Pareto solutions are reported, one solution set should not dominate another one at the same level considering a known conservatism degree. Decision makers decide based on the importance they attach to each objective function. With a decrease in (z_1), (z_2) increases in each step; the same is true for (z_2) as well. When the Pareto figure gets closer to the horizontal axis, it means that (z_1)

Table 12 Results from two uncertainty sets tests with a maximum constraint violation probability of (0.25)

Ellipsoidal uncertainty set			Box and polyhedral uncertainty sets			Weights
$Z_1^* = 4636$	α	$Z_2^- = 501,239$	$Z_1^* = 5288$	α	$Z_2^- = 510,691$	
<i>MPCV = 0.25, DFNV = 5%</i>						
28,709	7221	274,631	28,356	6920	270,959	$\lambda_1 = 0.3, \lambda_2 = 0.7$
26,559	8769	278,931	26,297	8403	275,079	$\lambda_1 = 0.4, \lambda_2 = 0.6$
24,123	9744	283,803	23,962	9337	279,747	$\lambda_1 = 0.5, \lambda_2 = 0.5$
22,302	10,599	290,814	21,295	9604	285,083	$\lambda_1 = 0.6, \lambda_2 = 0.4$
18,127	9444	295,795	18,216	9049	291,239	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 36,726$		$z_2^* = 264,315$	$z_1^- = 36,159$		$z_2^* = 261,073$	
$Z_1^* = 5887$	α	$Z_2^- = 505,890$	$Z_1^* = 7188$	α	$Z_2^- = 524,802$	
<i>MPCV = 0.25, DFNV = 10%</i>						
29,960	7221	279,282	29,253	6619	271,938	$\lambda_1 = 0.3, \lambda_2 = 0.7$
27,810	8769	283,582	27,283	8038	275,879	$\lambda_1 = 0.4, \lambda_2 = 0.6$
25,374	9744	288,454	25,050	8931	280,344	$\lambda_1 = 0.5, \lambda_2 = 0.5$
22,590	10,022	294,022	22,499	9186	285,448	$\lambda_1 = 0.6, \lambda_2 = 0.4$
19,378	9444	300,446	19,554	8656	291,336	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 37,977$		$z_2^* = 268,966$	$z_1^- = 36,843$		$z_2^* = 262,482$	
$Z_1^* = 7137$	α	$Z_2^- = 510,541$	$Z_1^* = 9089$	α	$Z_2^- = 538,913$	
<i>MPCV = 0.25, DFNV = 15%</i>						
31,210	7221	283,933	30,151	6318	272,918	$\lambda_1 = 0.3, \lambda_2 = 0.7$
29,060	8769	288,233	28,271	7672	276,679	$\lambda_1 = 0.4, \lambda_2 = 0.6$
26,624	9744	293,105	26,139	8525	280,941	$\lambda_1 = 0.5, \lambda_2 = 0.5$
23,840	10,022	298,673	23,704	8768	285,813	$\lambda_1 = 0.6, \lambda_2 = 0.4$
20,628	9444	305,097	20,893	8262	291,434	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 39,227$		$z_2^* = 273,617$	$z_1^- = 37,527$		$z_2^* = 263,891$	

has been more important than (z_2). Similarly, when it gets nearer to the vertical axis, (z_2) has been more important. Uniform slope of Fig. 8 is an indication that strong Pareto solutions have been produced.

Figure 8 shows Pareto diagrams of deterministic and robust models at a maximum constraint violation probability of 0.2 and a 10% deviation from the nominal data.

8 Conclusions

Considering the importance of transferring the injured/homeless people and distributing relief commodities in earthquakes, a bi-objective rescue-relief model under uncertainty has been developed. The first objective minimizes the total number of the injured/homeless people who have not been transferred from the affected area, and the second objective is after minimizing the total shortages of

the relief commodities. To model uncertainty, two approaches have been developed: (1) RO with ellipsoidal uncertainty set and (2) RO with box and polyhedral uncertainty sets. To do sensitivity analyses and compare deterministic and robust models, the use of Tehran Case Study has been made. The results obtained from solving the deterministic and robust models show that the robust models resulted in more conservative solutions in all cases. Also, by comparing the two proposed robust models, it can be concluded that using the box and polyhedral uncertainty sets results in a better performance than the ellipsoidal uncertainty set.

Finally, suggestions for future studies are: (1) Routing can be added to commodity distribution and injured/homeless transfer, (2) using exact and meta-heuristic algorithms for large-scale problems, and (3) using other uncertainty sets such as box-ellipsoidal and ellipsoidal-

Table 13 Results from two uncertainty sets tests with a maximum constraint violation probability of (0.3)

Ellipsoidal uncertainty set			Box and polyhedral uncertainty sets			Weights
$Z_1^* = 4553$	α	$Z_2^- = 500,931$	$Z_1^* = 5029$	α	$Z_2^- = 511,355$	
<i>MPCV = 0.3, DFNV = 5%</i>						
28,626	7221	274,324	28,234	6961	270,800	$\lambda_1 = 0.3, \lambda_2 = 0.7$
26,476	8769	278,623	26,162	8453	274,944	$\lambda_1 = 0.4, \lambda_2 = 0.6$
24,040	9744	283,495	23,814	9392	279,640	$\lambda_1 = 0.5, \lambda_2 = 0.5$
21,256	10,022	289,063	21,131	9660	285,007	$\lambda_1 = 0.6, \lambda_2 = 0.4$
18,044	9444	295,487	18,034	9103	291,200	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^* = 4553$		$z_2^- = 264,114$	$z_1^- = 5029$	α	$z_2^* = 260,631$	
$Z_1^* = 5721$	α	$Z_2^- = 505,274$	$Z_1^* = 6671$	α	$Z_2^- = 36,656$	
<i>MPCV = 0.3, DFNV = 10%</i>						
29,794	7221	278,667	29,011	6701	271,620	$\lambda_1 = 0.3, \lambda_2 = 0.7$
27,644	8769	282,966	27,015	8137	275,609	$\lambda_1 = 0.4, \lambda_2 = 0.6$
25,208	9744	287,838	24,755	9042	280,130	$\lambda_1 = 0.5, \lambda_2 = 0.5$
22,424	10,022	293,406	22,171	9300	285,296	$\lambda_1 = 0.6, \lambda_2 = 0.4$
19,212	9444	299,830	19,191	8763	291,258	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 37,811$		$z_2^* = 268,350$	$z_1^- = 526,130$		$z_2^* = 262,046$	
$Z_1^* = 6889$	α	$Z_2^- = 509,617$	$Z_1^* = 8315$	α	$Z_2^- = 540,897$	
<i>MPCV = 0.3, DFNV = 15%</i>						
30,962	7221	283,009	29,786	6441	272,440	$\lambda_1 = 0.3, \lambda_2 = 0.7$
28,812	8769	287,309	27,869	7821	276,274	$\lambda_1 = 0.4, \lambda_2 = 0.6$
26,376	9744	292,181	25,696	8690	280,619	$\lambda_1 = 0.5, \lambda_2 = 0.5$
24,559	10,602	299,198	23,213	8938	285,585	$\lambda_1 = 0.6, \lambda_2 = 0.4$
20,380	9444	304,173	20,348	8423	291,315	$\lambda_1 = 0.7, \lambda_2 = 0.3$
$z_1^- = 38,979$		$z_2^* = 272,693$	$z_1^- = 37,246$		$z_2^* = 263,238$	

Fig. 4 First objective function with respect to the maximum constraint violation probability with 5% deviation from the nominal data

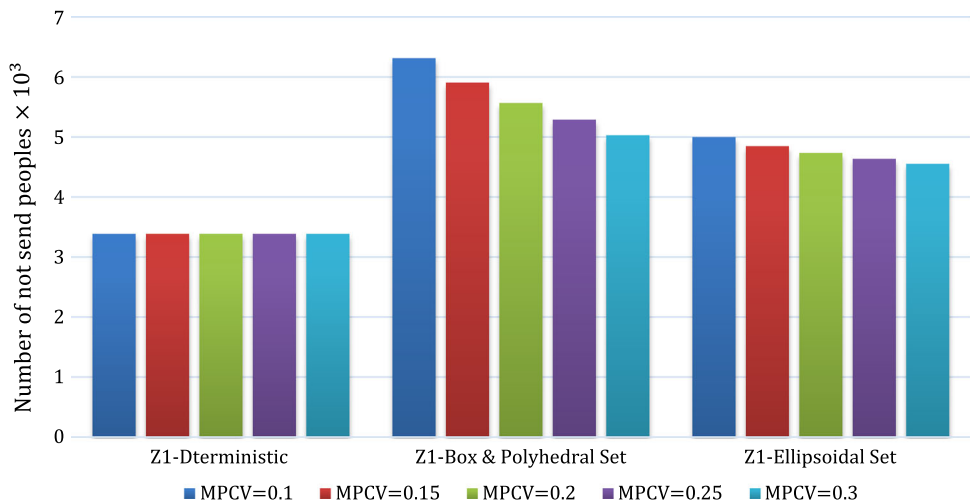


Fig. 5 Second objective function with respect to the maximum constraint violation probability with 5% deviation from the nominal data

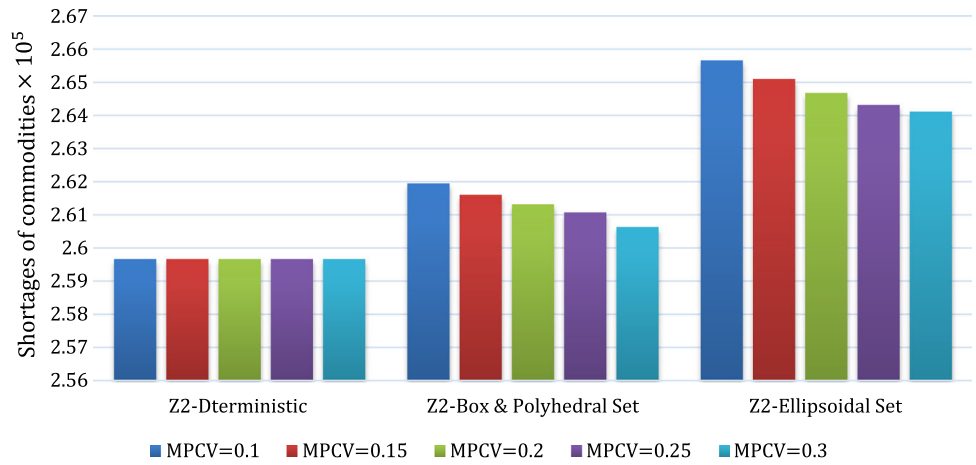


Fig. 6 First objective function with respect to the uncertainty data deviation from the nominal data and the maximum constraint violation probability of 0.1

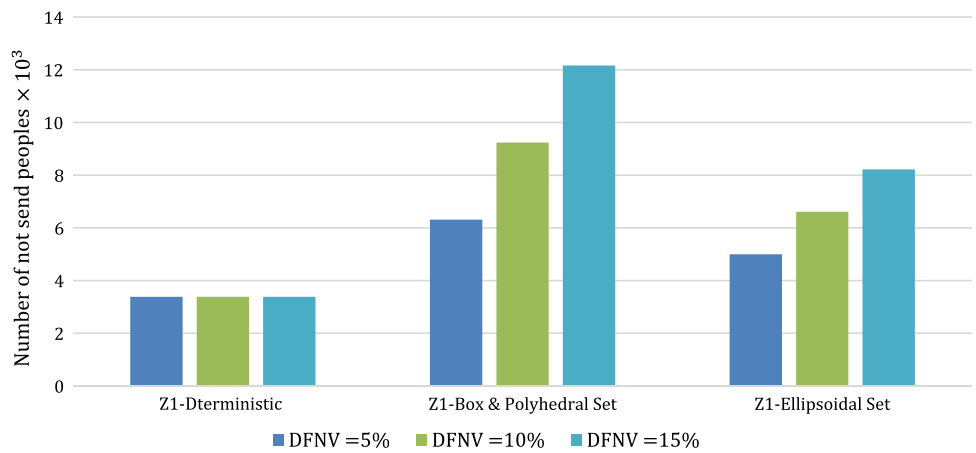
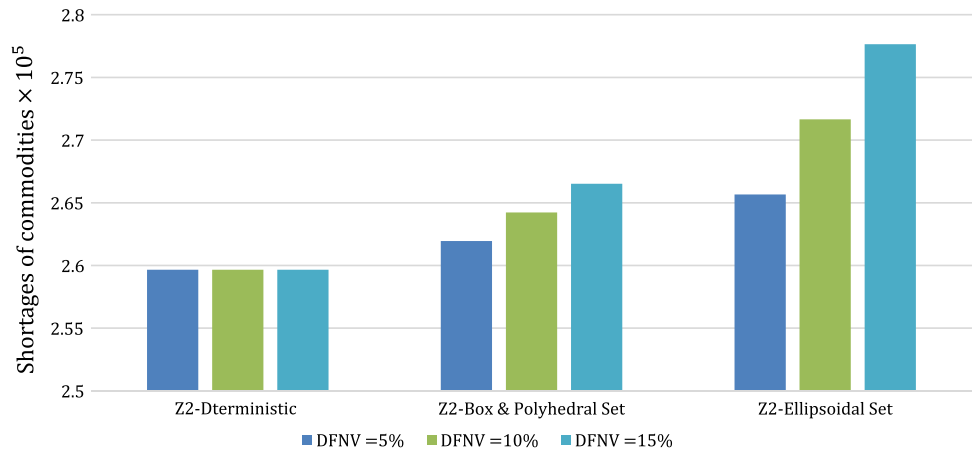


Fig. 7 Second objective function with respect to the stochastic data deviation from the nominal data and the maximum constraint violation probability of 0.1



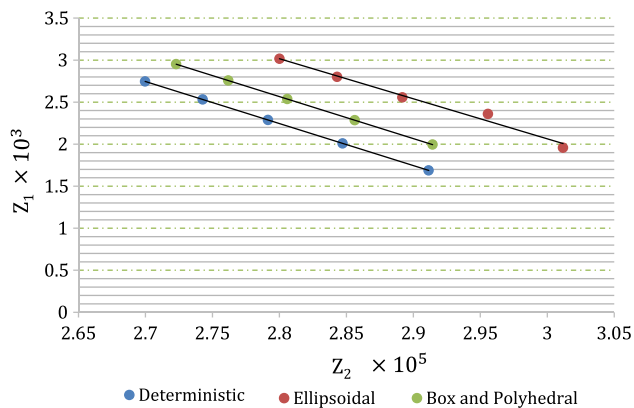


Fig. 8 Pareto diagrams of deterministic and robust models at a maximum constraint violation probability of 0.2 and a 10% deviation from the nominal data

polyhedral to see which RO model with uncertainty sets outperforms the other alternatives.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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