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ORIGINAL ARTICLE

Multiple-attribute group decision making for interval-valued intuitionistic fuzzy sets based on expert reliability and the evidential reasoning rule

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Abstract
This study proposes a novel fuzzy multiple-attribute group decision-making approach based on expert reliability and the evidential reasoning (ER) rule in an interval-valued intuitionistic fuzzy environment. First, to determine the reliabilities of experts, an objective method is developed by combining the similarity between the assessments provided before and after group discussion. Second, the proposed approach extends the ER rule to the case where belief degrees are intervals and employs it to combine experts' assessments. Hereinto, several optimization models are established to produce the aggregated assessments of the alternatives. Then, the overall priority degree of each alternative can be obtained according to the aggregated assessments and further utilized to yield a ranking of alternatives. Finally, a shopping center site selection problem is analyzed by the proposed approach to demonstrate its validity and applicability.

Keywords Interval-valued intuitionistic fuzzy sets · Expert reliability · Evidential reasoning rule · Multiple-attribute group decision making

1 Introduction

Decision making is usually considered a process in which human beings and enterprises make choices among several alternatives [1]. In real life, it is becoming impossible for a single decision maker or expert to take into account every related aspect of a decision-making problem without any difficulty due to increasingly complex socioeconomic circumstances [2–4]. Hence, many practical decisions are usually made by multiple decision makers or experts, which give rise to abundant research concerning the topic of multiple-attribute group decision-making problems (MAGDMs).

In group decision making, experts should provide their preferences for alternative attributes to achieve a collective decision. Because of various uncertainties in real world, the information received or provided by experts may be indeterminate and inconsistent. To address indeterminate information and inconsistent information, Smarandache [5] introduced the concept of neutrosophic set, in which each element of the universe has a degree of truth, falsity, and indeterminacy. Subsequently, the neutrosophic sets have been successfully used in decision-making field. For example, many researchers have succeeded in combining neutrosophic sets with some classical decision-making techniques, such as the TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method [6], the VIKOR (VIsekriterijumska optimizacija i

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KOmpromisno Resenje) method [7], the MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus Full Multiplicative Form) method [8], and the WASPAS (weighted aggregated sum product assessment) method [9], to solve various decision-making problems with neutrosophic information [10–13]. Broumi and Smarandache [14] further investigated the correlation measures of interval neutrosophic sets. On the other hand, due to the uncertainty and complexity of decision-making problems, as well as the ambiguity of human thinking, the preferences provided by experts may be imprecise. In this sense, the theory of fuzzy sets [15] has been used to model uncertainty or vagueness and applied to different kinds of decision-making problems. To better handle uncertain and vague information, fuzzy set theory has been extended to many higher-order extensions, including intuitionistic fuzzy sets [16] and interval-valued intuitionistic fuzzy (IVIF) sets [17]. Compared with the theory of intuitionistic fuzzy sets, the theory of IVIF sets more effectively applies to the situations where decision makers have limited attention, and information sources are insufficient for dealing with all possibilities. Thus, it is more suitable and convenient to represent the experts' assessments based on IVIF sets [18, 19].

Since its appearance in the literature, the IVIF set has attracted increasing attention, and many fuzzy MAGDM approaches based on IVIF sets have been presented [18–30]. For instance, Atanassov [22] and Xu [23] constructed the arithmetic operations of IVIF values. Later, with the aid of these operational rules, Xu and Chen [19] developed some IVIF aggregation operators, such as the IVIF weighted geometric and arithmetic aggregation operators, and developed an approach for group decision making with interval-valued intuitionistic information. Li et al. [20] presented an improved group decision-making method based on IVIF prioritized operators. By means of Hamacher operations, Liu [24] developed a few Hamacher IVIF aggregation operators and applied them to group decision making. Makui et al. [17] proposed a fuzzy multi-criteria group decision-making approach based on the IVIF preference relation and the IVIF decision matrix. With the use of Einstein operations, Wang and Liu [27] proposed several Einstein IVIF aggregation operators to aggregate IVIF information in multi-criteria group decision making. Although these fuzzy MAGDM approaches have some merits, there still exist two issues to be resolved with regard to IVIF set-based MAGDM problems.

The first issue is that expert reliability is rarely taken into account in these MAGDM approaches. In fact, experts in MAGDM are not necessarily reliable. They have bounded rationality in decision making because of their selective memory and perception, as well as limited computational ability, as analyzed in [31]. Expert reliability, as an important concept in MAGDM, is usually applied to evaluate the proficiency of specialists [32]. It is the inherent property of the specialists [33–36]. Hence, expert reliability should be considered and effectively measured in the process of MAGDM. To our knowledge, none of the above fuzzy MAGDM approaches has considered the reliabilities of experts, which has significant influence on the validity and rationality of decision results. Therefore, one aim of this study is to develop an objective method to determine expert reliability in the context of IVIF sets in MAGDM.

The second issue is that the IVIF aggregation operator-based MAGDM approaches (e.g., [19, 23, 24, 27]) will generate unreasonable preference orders of alternatives in some situations because of the deficiencies of the aforementioned IVIF operational rules. In fact, these operations are limited by their failures to take into account all the interval-valued membership degrees and the interval-valued non-membership degrees of elements that belong to IVIF sets, since they only consider the maximal membership degree and the minimal non-membership degree. Moreover, according to the operational rules, the interval-valued membership degrees and the interval-valued non-membership degrees are computed independently. In the situations, when there is an IVIF assessment whose membership degree equals [1] or non-membership degree equals [0,0], the aggregated IVIF assessment will be ([1,1], [0,0]) or ([0,0], [1,1]), which is biased and invalid and thus will yield unreasonable preference orders of alternatives in MAGDM problems [37, 38]. Dymova and Sevastjanov [39, 40] recently conducted a detailed analysis of the limitations of the operations on IVIF values and noted that the operations of "addition" and "multiplication" exhibit undesirable properties and are not always rational. However, all the IVIF aggregation operator-based MAGDM approaches mentioned above are developed in accordance with the operations that have some similar deficiencies, which make these approaches less reasonable. To address this matter, Mohammadi and Makui [38] developed a fuzzy MAGDM method on the basis of IVIF sets and the original ER approach [41, 42]. The method developed in [38] overcomes the second issue of the existing IVIF aggregation operator-based methods for fuzzy MAGDM in IVIF environments. However, the original ER algorithm [41, 42] only takes attribute weights into account when it is used to combine attribute values. In other words, expert reliability is still not considered in [38].

All the above analyses indicate that MAGDM with IVIF information is still an active field of research. Many contributions to this field have been made, but there are two issues, as discussed above. In light of this, it is necessary to develop a novel approach for MAGDM with IVIF sets that can overcome the above two issues, which serves as the main motivation of this study.

Yang and Xu [35] recently extended the original ER approach and established a unique ER rule to combine multiple pieces of evidence with their associated reliabilities and weights in the case where the degrees of belief in the evidence are precise values. The consideration of reliabilities follows Simon's theory of bounded rationality. Therefore, taking into account the above motivations, the focus of this study is to put forth a new fuzzy MAGDM approach. First, in order to effectively determine the reliabilities of experts, an objective method is developed through combining the similarities between the assessments provided before and after group discussion (GD), respectively. Then, we extend the ER rule to the case where the belief degrees are intervals and employ the extended ER rule to combine experts' assessments. Several optimization models are established and solved herein to generate aggregated assessments of the alternatives. The combination process of assessments considers expert weights and expert reliabilities simultaneously. Finally, the overall priority degrees of alternatives are calculated based on their aggregated assessments. The novelty of the developed approach can resolve the two aforementioned issues for fuzzy MAGDM problems in IVIF environments.

Due to the emergence of modern lifestyles, all kinds of businesses, especially service-oriented businesses, are witnessing rapid transformations in many aspects. Service firms are exploring new business models and taking actions to meet the needs of modern lifestyles. One of these basic actions is to select a suitable business location to expand the scope of operation. Proper site/location selection is a MAGDM problem where the decision-making process will be conducted by multiple experts based on different quantitative and qualitative attributes [43]. Many decision-making models have been developed for this application over the past few decades. Cheng et al. [44] applied the analytic network process approach to handle a shopping mall location selection problem. Kuo et al. [45] constructed a decision support system for store location selection by integrating fuzzy AHP and artificial neural network. Based on a perspective with foresight, Zolfani et al. [43] proposed a hybrid decision model to solve shopping mall location problems. Liu et al. [46] developed an MAGDM method with interval 2-tuple linguistic information to select the optimal disposal site for municipal solid waste. From a sustainability perspective, Rao et al. [47] presented a fuzzy MAGDM model for location selection of city logistics centers. In this study, we will also focus on this topic and apply the proposed approach to analyze a shopping center site selection problem to illustrate its validity and applicability.

The rest of this study is organized as follows. Section 2 reviews some basic concepts of IVIF sets and the ER rule. In Sect. 3, we present a detailed discussion of the proposed approach. In Sect. 4, the proposal is applied to solve a shopping center site selection problem to demonstrate its applicability and validity. In Sect. 5, comparisons with existing MAGDM approaches are made to highlight the effectiveness and feasibility of the proposed approach. Finally, we conclude this study in Sect. 6.

2 Preliminaries

To facilitate the introduction of the proposed approach, this section reviews IVIF sets and some basic theorems which will be used in the discussion that follows.

2.1 IVIF sets

Atanassov and Gargov [17] introduced the concept of IVIF sets as follows.

Definition 1 [17]. Given a universe of discourse X, an IVIF set $\tilde{A}$ on X is represented in the following manner:

$$\begin{aligned}\tilde{A} &= \left\{\left\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{v}_{\tilde{A}}(x)\right\rangle | x \in X\right\} \\ &= \left\{\left\langle x, \left[\mu_{\tilde{A}}^{L}(x), \mu_{\tilde{A}}^{U}(x)\right], \left[v_{\tilde{A}}^{L}(x), v_{\tilde{A}}^{U}(x)\right]\right\rangle | x \in X\right\},\end{aligned} \tag{1}$$

where intervals $\tilde{\mu}_{\tilde{A}}(x)$ and $\tilde{v}_{\tilde{A}}(x)$ symbolize the membership and the non-membership degrees of x to $\tilde{A}$, respectively, such that $\tilde{\mu}_{\tilde{A}}(x) = [\mu_{\tilde{A}}^{L}(x), \mu_{\tilde{A}}^{U}(x)] \in [0, 1]$, $\tilde{v}_{\tilde{A}}(x) = [v_{\tilde{A}}^{L}(x), v_{\tilde{A}}^{U}(x)] \in [0, 1]$, and $0 \le \mu_{\tilde{A}}^{U}(x) + v_{\tilde{A}}^{U}(x) \le 1$ for all $x \in X$. Based on $\tilde{\mu}_{\tilde{A}}(x)$ and $\tilde{v}_{\tilde{A}}(x)$, the corresponding interval-valued hesitation degree of x to $\tilde{A}$ is represented in the form:

$$\begin{aligned}\tilde{\pi}_{\tilde{A}}(x) &= \left[\pi_{\tilde{A}}^{L}(x), \pi_{\tilde{A}}^{U}(x)\right] \\ &= \left[1 - \mu_{\tilde{A}}^{U}(x) - v_{\tilde{A}}^{U}(x), 1 - \mu_{\tilde{A}}^{L}(x) - v_{\tilde{A}}^{L}(x)\right].\end{aligned} \tag{2}$$

Consider the case where each of $\tilde{\mu}_{\tilde{A}}(x)$ and $\tilde{v}_{\tilde{A}}(x)$ contains only one value for each $x \in X$, we have $\mu_{\tilde{A}}^{L}(x) = \mu_{\tilde{A}}^{U}(x)$ and $v_{\tilde{A}}^{L}(x) = v_{\tilde{A}}^{U}(x)$, and then the given IVIF set $\tilde{A}$ will be degraded to an intuitionistic fuzzy set [15]. The pair $(\tilde{\mu}_{\tilde{A}}(x), \tilde{v}_{\tilde{A}}(x))$ in the IVIF set $\tilde{A}$ is called an IVIF value [22], which is usually denoted by $\tilde{a} = ([\mu_{\tilde{a}}^{L}, \mu_{\tilde{a}}^{U}], [v_{\tilde{a}}^{L}, v_{\tilde{a}}^{U}])$ for simplicity, where $0 \le \mu_{\tilde{a}}^{L} \le \mu_{\tilde{a}}^{U} \le 1$, $0 \le v_{\tilde{a}}^{L} \le v_{\tilde{a}}^{U} \le 1$, and $0 \le \mu_{\tilde{a}}^{U}(x) + v_{\tilde{a}}^{U}(x) \le 1$.

To compare different IVIF values, the scholars in [48, 49] developed a few comparison mechanisms based on the real-valued score, accuracy, and hesitation functions of IVIF values. These mechanisms for comparison of IVIF values possess undeniable merit. Subsequently, to avoid information loss, Dymova et al. [50] extended the above real-valued functions to their associated interval forms and designed the interval-valued score and accuracy functions of an IVIF value as follows.

Definition 2 [50]. Given an IVIF value $\tilde{a} = ([\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U])$, the interval-valued score function of $\tilde{a}$ is calculated in the following manner:

$$\tilde{S}(\tilde{a}) = \left[\mu_{\tilde{a}}^L - v_{\tilde{a}}^U, \mu_{\tilde{a}}^U - v_{\tilde{a}}^L\right], \tag{3}$$

and the interval-valued accuracy function of a is calculated as follows:

$$\tilde{H}(\tilde{a}) = \left[\mu_{\tilde{a}}^L + v_{\tilde{a}}^L, \mu_{\tilde{a}}^U + v_{\tilde{a}}^U\right]. \tag{4}$$

With the use of such interval-valued score and accuracy functions, Dymova et al. [50] developed a two-criterion method to compare IVIF values, which will be detailed in the proposed MAGDM approach in Sect. 3.

In [51], Xu and Yager defined the normalized Hamming distance between two IVIF values, as shown in the following.

Definition 3 [51]. Given any two IVIF values $\tilde{a}_i = ([\mu_i^L, \mu_i^U], [v_i^L, v_i^U])$ and $\tilde{a}_j = ([\mu_j^L, \mu_j^U], [v_j^L, v_j^U])$, the normalized Hamming distance measure between the IVIF values $\tilde{a}_i$ and $\tilde{a}_j$ is defined as follows:

$$\begin{aligned} d(\tilde{a}_i, \tilde{a}_j) &= \frac{1}{4}\Big(\left|\mu_i^L - \mu_j^L\right| + \left|\mu_i^U - \mu_j^U\right| + \left|v_i^L - v_j^L\right| + \left|v_i^U \right.\\ &\quad \left. - v_j^U\right| + \left|\pi_i^L - \pi_j^L\right| + \left|\pi_i^U - \pi_j^U\right|\Big) \\ &= \frac{1}{4}\Big(\left|\mu_i^L - \mu_j^L\right| + \left|\mu_i^U - \mu_j^U\right| + \left|v_i^L - v_j^L\right| \\ &\quad + \left|v_i^U - v_j^U\right| + \left|\mu_j^L + v_j^L - \mu_i^L - v_i^L\right| \\ &\quad + \left|\mu_j^U + v_j^U - \mu_i^U - v_i^U\right|\Big). \end{aligned} \tag{5}$$

One can easily verify that $0 \le d(\tilde{a}_i, \tilde{a}_j) \le 1$.

Definition 4 [23]. Given any two IVIF values $\tilde{a}_i = ([\mu_i^L, \mu_i^U], [v_i^L, v_i^U])$ and $\tilde{a}_j = ([\mu_j^L, \mu_j^U], [v_j^L, v_j^U])$, the operational rules of IVIF values are defined in the following manner:

1. $\tilde{a}_i \oplus \tilde{a}_j = \Big(\left[\mu_i^L + \mu_j^L - \mu_i^L \cdot \mu_j^L, \mu_i^U + \mu_j^U - \mu_i^U \cdot \mu_j^U\right], \left[v_i^L \cdot v_j^L, v_i^U \cdot v_j^U\right]\Big)$,
2. $\tilde{a}_i \otimes \tilde{a}_j = \Big(\left[\mu_i^L \cdot \mu_j^L, \mu_i^U \cdot \mu_j^U\right], \left[v_i^L + v_j^L - v_i^L \cdot v_j^L, v_i^U + v_j^U - v_i^U \cdot v_j^U\right],\Big)$
3. $\kappa\tilde{a}_i = \left(\left[1 - (1 - \mu_i^L)^\kappa, 1 - (1 - \mu_i^U)^\kappa\right], \left[v_i^{L\kappa}, v_i^{U\kappa}\right]\right)$, $\kappa > 0$,
4. $\tilde{a}_i^\kappa = \left(\left[\mu_i^{L\kappa}, \mu_i^{U\kappa}\right], \left[1 - (1 - v_i^L)^\kappa, 1 - (1 - v_i^U)^\kappa\right]\right)$, $\kappa > 0$.

Based on the above operations of IVIF values, Xu and Chen [19] developed a family of IVIF aggregation operators, including the IVIF arithmetic and geometric weighted averaging operators. Here, the IVIF arithmetic weighted averaging (IVIFAWA) operator is presented as an example.

Definition 5 [19]. Given a collection of n IVIF values $\tilde{a}_i = ([\mu_i^L, \mu_i^U], [v_i^L, v_i^U])$ $(i = 1,2,\ldots, n)$, the IVIFAWA operator is defined as follows:

$$\begin{aligned} &\mathrm{IVIFAWA}(\tilde{a}_1, \tilde{a}_2, \ldots \tilde{a}_n) \\ &= \left(\left[1 - \prod_{i=1}^n \left(1 - \mu_i^L\right)^{\omega_i}, 1 - \prod_{i=1}^n \left(1 - \mu_i^U\right)^{\omega_i}\right], \right. \\ &\qquad \left. \left[\prod_{i=1}^n \left(v_i^L\right)^{\omega_1}, \prod_{i=1}^n \left(v_i^U\right)^{\omega_i}\right]\right), \end{aligned} \tag{6}$$

where ω_i is the weight of $\tilde{a}_i$, satisfying that $0 \le \omega_i \le 1$ and $\sum_{i=1}^n \omega_i = 1$.

The aggregation operators presented in [19] are based on the algebraic operations of IVIF values. With the use of the Einstein operations of IVIF values, Wang and Liu [27] developed several IVIF Einstein aggregation operators. Recently, Liu [24] extended the Hamacher operations to IVIF environments and presented a series of IVIF Hamacher aggregation operators for aggregating IVIF information in decision making. In particular, the IVIF Hamacher aggregation operators can be reduced to the algebraic and the Einstein aggregation operators when the parameter in the Hamacher operations is equal to 1 and 2, respectively.

Note that the IVIF aggregation operators in [19, 24, 27] have the drawback that when there is only one IVIF value whose membership degree equals [1,1] or non-membership degree equals [0,0], the aggregated IVIF value will be ([1,1], [0,0]) or ([0,0], [1,1]) even if the membership degrees of the other IVIF values are not equal to [1,1] or the non-membership degrees are not equal to [0,0]. That is to say, these operators do not consider all the interval-valued membership degrees and the interval-valued non-membership degrees of elements that belong to IVIF sets.

In such situation, the aggregated results are invalid and further will yield unreasonable preference orders of alternatives in decision-making problems. Consider the following example, which illustrates the drawback of the IVIFAWA operator. Obviously, the other aggregation operators mentioned above can be tested similarly.

Example 1 Suppose that there is one collection of four IVIF values $\{\tilde{a}_1 = ([0.8, 0.9], [0.1, 0.2]),\ \tilde{a}_2 = ([1, 1], [0, 0]),\ \tilde{a}_3 = ([0.7, 0.8], [0.1, 0.2]),\ \tilde{a}_4 = ([0.6, 0.8], [0.1, 0.2])\}$ and their corresponding weights are 0.25, 0.25, 0.35, and 0.15. Then, using the IVIFAWA operator, one can obtain the aggregated IVIF value:

$$\mathrm{IVIFAWA}(\tilde{a}_1, \tilde{a}_2, \ldots \tilde{a}_4) = ([1, 1], [0, 0]).$$

Given another collection of four IVIF values $\{\tilde{a}_1 = ([0.1, 0.2], [0.8, 0.9]),\ \tilde{a}_2 = ([1, 1], [0, 0]),\ \tilde{a}_3 = ([0.1, 0.2], [0.7, 0.8]),\ \tilde{a}_4 = ([0.1, 0.2], [0.6, 0.8])\}$ and the same weights above, then the aggregated IVIF value obtained by the IVIFAWA operator is

$$\mathrm{IVIFAWA}(\tilde{a}_1, \tilde{a}_2, \ldots \tilde{a}_4) = ([1, 1], [0, 0]).$$

From the above calculations, one can find that except the second IVIF value, the other three IVIF values in the first collection are completely different from those in the second collection. However, the aggregated results are the same, namely ([1, 1],[0,0]), which is counterintuitive. More details of the drawbacks of the aggregation operator-based methods [19, 24, 27] will be discussed by a comparative analysis in Sect. 5. As such, this paper will employ the ER rule instead of the IVIF aggregation operators to aggregate IVIF information in the following proposed approach.

2.2 ER rule with weight and reliability

The ER rule has been proven to be an effective technique for aggregating information from multiple experts. In the ER rule, different pieces of evidence are combined with their associated reliabilities and weights. To facilitate the presentation of the ER rule, the evidence representation is introduced first.

Suppose that $\Omega = \{H_1, \ldots, H_N\}$ is a set of propositions or grades and $P(\Omega) = 2\Omega = \{\emptyset, \{H_1\}, \ldots, \{H_N\}, \{H_1, H_2\}, \ldots, \{H_1, H_N\}, \ldots, \{H_1, \ldots, H_{N-1}\}, \Omega\}$ is the power set of Ω. Then, L pieces of independent evidence can be profiled on $P(\Omega)$ by the following belief distribution:

$$E_i = \{(H, \beta_{H,i}), \quad \forall H \subseteq \Omega, \sum_{H \subseteq \Omega} \beta_{H,i} = 1\},\ i = 1, \ldots, L, \tag{7}$$

where $(H, \beta_{H,i})$ is an element of evidence E_i, meaning that the evidence supports an element H with a belief degree of $\beta_{H,i}$. Let $\bar{w}_i$ ($i = 1, \ldots, L$) with $0 \le \bar{w}_i \le 1$, and R_i ($i = 1, \ldots, L$) with $0 \le R_i \le 1$ denote the weights and reliabilities of E_i ($i = 1, \ldots, L$), respectively. When the weights and reliabilities of E_i are simultaneously considered in the combination of E_i, hybrid weights can be obtained by the following formula:

$$\tilde{w}_i = \bar{w}_i / (1 + \bar{w}_i - R_i) \quad (i = 1, \ldots, L). \tag{8}$$

In these circumstances, the ER rule with weight and reliability is presented in the following manner.

Theorem 1 [35]. Suppose that L pieces of evidence E_i ($i = 1, \ldots, L$) are described by Eq. (7) with the hybrid weights $\tilde{w}_i$ of Eq. (8). Then, the combined result of the first i pieces of evidence can be represented by the following belief distribution: $E(i) = \{(H, \beta_{H,E(i)}), \quad \forall H \subseteq \Omega, \sum_{H \subseteq \Omega} \beta_{H,E(i)} = 1\}$ ($i = 2, \ldots, L$), with

$$\beta_{H,E(i)} = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_{H,E(i)}}{\sum_{D \subseteq \Omega} \hat{m}_{D,E(i)}} & H \subseteq \Omega, H \ne \emptyset \end{cases}, \tag{9}$$

$$m_{H,E(i)} = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_{H,E(i)}}{\sum_{D \subseteq \Omega} \hat{m}_{D,E(i)} + \hat{m}_{p(\Omega),E(i)}} & H \subseteq \Omega, H \ne \emptyset \end{cases}, \tag{10}$$

$$m_{p(\Omega),E(i)} = \frac{\hat{m}_{p(\Omega),E(i)}}{\sum_{D \subseteq \Omega} \hat{m}_{D,E(i)} + \hat{m}_{p(\Omega),E(i)}}, \tag{11}$$

$$\hat{m}_{H,E(i)} = \left[(1 - \tilde{w}_i) \cdot m_{H,E(i-1)} + m_{p(\Omega),E(i-1)} \cdot \tilde{w}_i \beta_{H,i}\right] + \sum_{B \cap C = H} m_{B,E(i-1)} \cdot \tilde{w}_i \beta_{C,i}, \quad \forall H \subseteq \Omega, \text{ and} \tag{12}$$

$$\hat{m}_{p(\Omega),E(i)} = (1 - \tilde{w}_i) \cdot m_{p(\Omega),E(i-1)}, \tag{13}$$

where $0 \le \beta_{H,E(i)}, m_{H,E(i)} \le 1, \quad \forall H \subseteq \Omega, 0 \le m_{p(\Omega),E(i)} \le 1$, and $\sum_{H \subseteq \Omega} m_{H,E(i)} + m_{p(\Omega),E(i)} = 1$ for $i = 2, \ldots, L$ recursively. The detailed proof for Theorem 1 can be found in [35].

3 The proposed approach

This section will explore a novel MAGDM approach based on expert reliability and the ER rule for fuzzy MAGDM in which the experts' assessments are expressed as IVIF sets according to their preferences. To facilitate the introduction of this approach, MAGDM problems with IVIF assessments are first modeled. In the model, we explore how to measure expert reliabilities in fuzzy MAGDM problems. Then, we describe how to combine experts' IVIF assessments with their weights and reliabilities, which is followed by an introduction of how to derive solutions to MAGDM problems. As a whole, an integrated procedure of the proposed MAGDM approach is presented.

3.1 Modeling of MAGDM problems with IVIF assessments

Assume that a MAGDM problem involves an expert team consisting of T experts e_j (j = 1, …, T) and one moderator who organizes this decision-making activity. The set of alternatives is denoted as $A = \{a_1,\ldots, a_l, \ldots, a_M\}$ and needs to be evaluated with respect to L attributes, denoted c_i (i = 1, …, L). Suppose that the weight of attribute c_i is symbolized by w_i with $0 \le w_i \le 1$ for i = 1, …, L and $\sum_{i=1}^{L} w_i = 1$, and the reliability vector of these attributes is $\{r_1, \ldots, r_L\}$. For attribute c_i, the relative weight of expert e_j is denoted by λ^j (c_i), such that $\sum_{j=1}^{T} \lambda^j(c_i) = 1$. Let $\tilde{A}^j_{M\times L}$ be the decision matrix provided by expert e_j given as follows:

$$\tilde{A}^j_{M\times L} = \begin{bmatrix} (\tilde{u}^j_{11}, \tilde{v}^j_{11}) & (\tilde{u}^j_{12},\tilde{v}^j_{12}) & \cdots & (\tilde{u}^j_{1L},\tilde{v}^j_{1L}) \\ (\tilde{u}^j_{21}, \tilde{v}^j_{21}) & (\tilde{u}^j_{22}, \tilde{v}^j_{22}) & \cdots & (\tilde{u}^j_{2L}, \tilde{v}^j_{2L}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{u}^j_{M1}, \tilde{v}^j_{M1}) & (\tilde{u}^j_{M2}, \tilde{v}^j_{M2}) & \cdots & (\tilde{u}^j_{ML}, \tilde{v}^j_{ML}) \end{bmatrix}, \tag{14}$$

where $(\tilde{\mu}^j_{li}, \tilde{v}^j_{li})$ is the IVIF information denoting the assessment of expert e_j with respect to attribute c_i of alternative a_l.

3.2 Measurement of expert reliabilities

In the process of group decision making, group discussion (GD) is usually needed for experts to clarify the decision problem under consideration. With the development of GD, experts may modify their own opinions or assessments due to the influence of other experts' views. After GD, experts with a better understanding of the decision problem will be less willing to modify their assessments. They will be more reliable than those who are less familiar with the same problem. In this sense, the reliability of an expert can be objectively measured in accordance with the degree that he/she is willing to modify his/her assessments based on other experts' assessments. In [32], Fu et al. first provided the qualitative definition of expert reliability, presented as follows:

Definition 6 [32]. In a group, the reliability of an expert is defined as a combination of the similarities between the assessment provided by the expert before GD and that provided by any other expert after GD.

Experts will become more famous with the decision problem through GD, which indicates that the assessments provided after GD will be more credible than those provided before GD for other experts. For this reason, the assessments of other experts after GD, instead of those before GD, are used to evaluate expert reliability in Definition 6. The process of GD is shown in Fig. 1.

One can easily find that the definition of expert reliability is based on the majority rule, meaning that the truth generally falls in the hands of the majority. According to Definition 6, the closer an expert's assessment is to the assessments provided by the rest of experts after GD, the higher the reliability of that expert is. If one expert does not alter his/her assessment after GD, three cases will occur: (1) the assessments of other experts may move toward that of the given expert; (2) the assessments of other experts may move in the opposite direction to that of the given expert; and (3) the assessments of other experts may remain unchanged. In the first case, the reliability of the expert will increase; in the second case, the reliability of the expert will decline; and in the third case, the reliability of the expert will stay the same. In other words, the reliability of one expert is subject to the movements of the assessments provided by the rest of experts after GD.

In the proposed MAGDM approach, the reliability of an expert will be measured with the use of the original IVIF assessments provided before GD and the updated IVIF assessments after GD. To do this, we first investigate a similarity measure between two IVIF values $(\tilde{\mu}^j_{li}, \tilde{v}^j_{li})$ and $(\tilde{\mu}^k_{li}, \tilde{v}^k_{li})$ in the discussion that follows.

A similarity measure reflects the degree of similarity between two objects, whereas a distance measure is used to distinguish the difference between them. The two measures are usually considered complementary concepts. Thus, based on their relationship, a similarity measure can be deduced from its associated distance measure, and vice versa. Then, by means of the distance measure between two IVIF values defined in Definition 3, a similarity measure between two IVIF values can be deduced in the following manner:

Definition 7 Given any two IVIF values $\tilde{a}^j_{li} = (\tilde{\mu}^j_{li}, \tilde{v}^j_{li})$ and $\tilde{a}^k_{li} = (\tilde{\mu}^k_{li}, \tilde{v}^k_{li})$, then the similarity measure between the IVIF values $\tilde{a}^j_{li}$ and $\tilde{a}^k_{li}$ is defined as:

$$S^{jk}(li) = 1 - d\left(\tilde{a}^j_{li}, \tilde{a}^k_{li}\right) \tag{15}$$

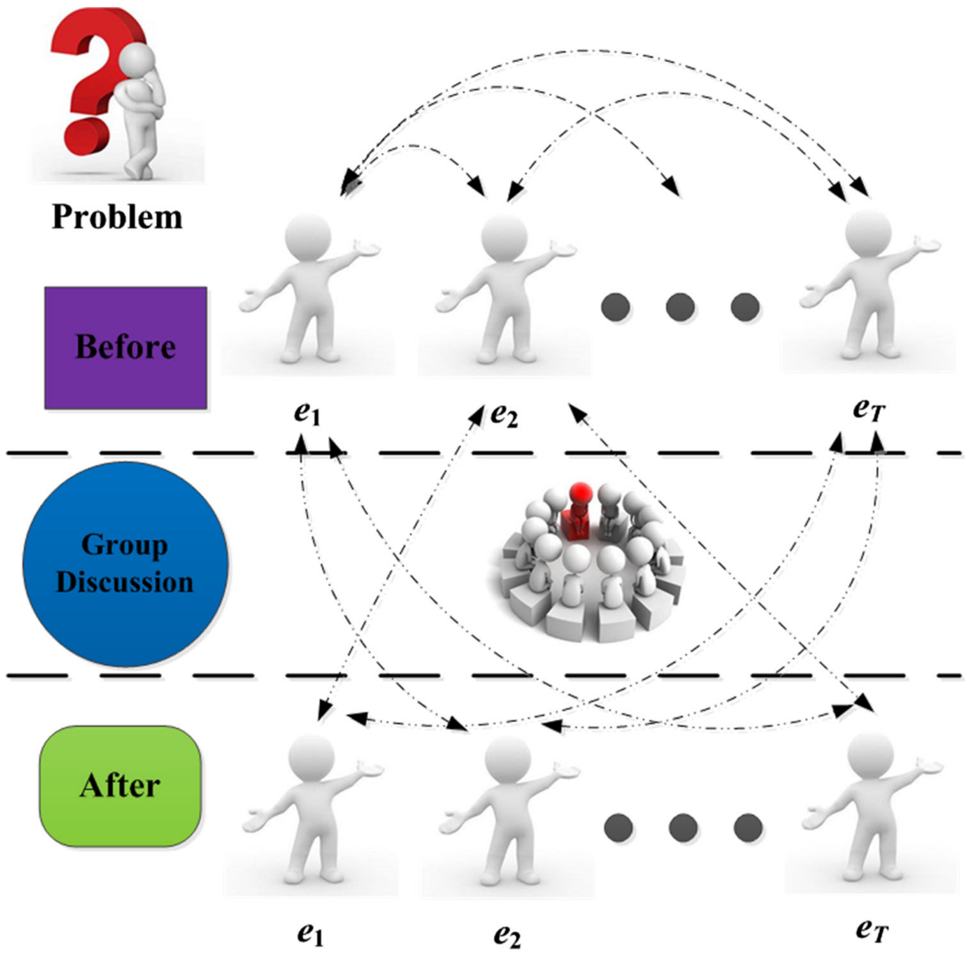

Fig. 1 Graph representation of GD

where $d(\tilde{a}_{li}^j, \tilde{a}_{li}^k)$ represents the normalized Hamming distance measure between the IVIF values defined in Definition 3. Note that the similarity measure $S^{jk}(li)$ satisfies the properties as follows:

1. $0 \leq S^{jk}(li) \leq 1$;
2. $S^{jk}(li) = 1$ if and only if $\tilde{a}_{li}^j = \tilde{a}_{li}^k$;
3. $S^{jk}(li) = S^{kj}(li)$.

The proofs for properties (1)–(3) can be directly deduced from the properties of the normalized Hamming distance measure in Definition 3; thus, we omit them here.

Then, based on the qualitative definition of expert reliability in Definition 6, the reliability of expert e_j can be quantified by using the similarity measure in Definition 7, defined as follows:

Definition 8 Let $(\tilde{\mu}_{li}^j, \tilde{v}_{li}^j)_{(0)}$ and $(\tilde{\mu}_{li}^j, \tilde{v}_{li}^j)_{(1)}$ ($i = 1,\ldots,L$; $l = 1,\ldots,M$) be the IVIF assessments provided by expert e_j ($j = 1,\ldots,T$) before and after GD for a MAGDM problem, $S_{(0)(1)}^{jk}(li)$ represent the similarity between $(\tilde{\mu}_{li}^j, \tilde{v}_{li}^j)_{(0)}$ and $(\tilde{\mu}_{li}^k, \tilde{v}_{li}^k)_{(1)}$, and $S_{(0)(0)}^{kh}(li)$ represent the similarity between $(\tilde{\mu}_{li}^k, \tilde{v}_{li}^k)_{(0)}$ and $(\tilde{\mu}_{li}^h, \tilde{v}_{li}^h)_{(0)}$. Then, after GD the expert reliability denoted by $R^j(li)$ is calculated in the following way:

$$R^j(li) = \frac{\sum_{k=1,k\neq j}^{T} \sqrt{S_{(0)(1)}^{jk}(li) \cdot R_{(0)}^k(li)}}{T-1} \tag{16}$$

with

$$R_{(0)}^k(li) = \frac{\sum_{h=1,h\neq k}^{T} S_{(0)(0)}^{kh}(li)}{T-1}, \tag{17}$$

where $R_{(0)}^k(li)$ stands for the initial reliability of expert e_k before GD. Consider the following example, which illustrates the calculation process for expert reliability.

Example 2 Suppose that four different experts $\{e_1, e_2, e_3, e_4\}$ anonymously provide the following IVIF assessments with respect to attribute c_i of alternative a_l before GD:

$(\tilde{\mu}_{li}^1, \tilde{v}_{li}^1)_{(0)} = ([0.5, 0.7], [0.2, 0.3]);$
$(\tilde{\mu}_{li}^2, \tilde{v}_{li}^2)_{(0)} = ([0.4, 0.6], [0.1, 0.3]);$
$(\tilde{\mu}_{li}^3, \tilde{v}_{li}^3)_{(0)} = ([0.6, 0.9], [0, 0.1]);$
$(\tilde{\mu}_{li}^4, \tilde{v}_{li}^4)_{(0)} = ([0.3, 0.4], [0.2, 0.4]).$

Then, the four experts have a discussion about the decision problem under consideration under the guidance of the moderator. After the discussion, the experts anonymously provide their updated assessments as follows:

$(\tilde{\mu}_{li}^1, \tilde{v}_{li}^1)_{(1)} = ([0.5, 0.7], [0.2, 0.3]);$
$(\tilde{\mu}_{li}^2, \tilde{v}_{li}^2)_{(1)} = ([0.5, 0.7], [0.2, 0.3]);$
$(\tilde{\mu}_{li}^3, \tilde{v}_{li}^3)_{(1)} = ([0.5, 0.8], [0.1, 0.2]);$
$(\tilde{\mu}_{li}^4, \tilde{v}_{li}^4)_{(1)} = ([0.4, 0.5], [0.2, 0.3]).$

Using the similarity measure defined in Definition 7, we have:

$$S_{(0)(0)}(li) = \begin{pmatrix} - & 0.85 & 0.8 & 0.75 \\ 0.85 & - & 0.75 & 0.85 \\ 0.8 & 0.75 & - & 0.6 \\ 0.75 & 0.85 & 0.6 & - \end{pmatrix},$$

$$S_{(0)(1)}(li) = \begin{pmatrix} - & 1 & 0.9 & 0.85 \\ 0.85 & - & 0.85 & 0.9 \\ 0.8 & 0.8 & - & 0.7 \\ 0.75 & 0.75 & 0.7 & - \end{pmatrix}.$$

From Eq. (17), the initial reliabilities of the four experts before GD can be calculated: $R_{(0)}^1(li) = 0.8$; $R_{(0)}^2(li) = 0.8167$; $R_{(0)}^3(li) = 0.7167$; $R_{(0)}^4(li) = 0.7333$.

From Eq. (16), the reliabilities of the four experts after GD can be obtained:

$R^1(li) = 0.8321$; $R^2(li) = 0.8058$; $R^3(li) = 0.7749$; $R^4(li) = 0.7552$. The calculation results reflect that expert e_1 is more reliable than the other three experts. Note that in the above example, the IVIF assessments provided by other three experts after GD simultaneously get close to that provided by expert e_1, and the IVIF assessment provided by expert e_1 after GD is the same as that provided before GD. This indicates that expert e_1 has a better understanding of the decision problem compared with the other three experts. In other words, the reliability of expert e_1 is subject to the movements of the IVIF assessments provided by other three experts after GD.

This section discusses the method for determining expert reliability in the context of MAGDM with IVIF information. In the next section, we will discuss how to combine experts' assessments with weights and reliabilities for an MAGDM problem.

3.3 Combination of IVIF assessments with reliabilities and weights based on ER rule

To circumvent the flaws of the operator-based IVIF information aggregation MAGDM approaches mentioned in Introduction, in the following, we apply the ER rule to combine the assessments of experts with reliabilities and weights for the MAGDM problem modeled in Sect. 3.1. Suppose that alternatives are assessed with respect to each attribute using two evaluation grades denoted by $\Omega = \{H_1, H_2\}$, where H_1 and H_2 stand for completely matching and not matching the fuzzy concept of "excellence," respectively. We then can transform the IVIF assessment $(\tilde{\mu}_{li}^j, \tilde{v}_{li}^j)_{(1)}$ into an interval-valued distribution assessment denoted by $\tilde{B}^j(li) = \{(H_1, [\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)]), (H_2, [\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)]), (\Omega, [\beta_{\Omega}^{jL}(li), \beta_{\Omega}^{jU}(li)])\}$, where $[\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)] = \tilde{\mu}_{li}^j$ and $[\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)] = \tilde{v}_{li}^j$ stand for the interval belief degrees of expert e_j on attribute c_i of alternative a_l with regard to the grades H_1 and H_2, respectively, and $[\beta_{\Omega}^{jL}(li), \beta_{\Omega}^{jU}(li)] = \tilde{\pi}_{li}^j$ stands for the degree of global ignorance [35, 42]. Here, Ω can be interpreted as the grade of "indeterminacy." In the ER context, the assessment $\tilde{B}^j(li)$ is considered as a piece of evidence, and now the ER rule can be utilized to combine these assessments.

Step 1 Combine the individual assessments $\tilde{B}^j(li)$ ($j = 1,\ldots,T$) with $R^j(li)$ and $\lambda^j(c_i)$ by using the ER rule to calculate the aggregated group assessment $\tilde{B}(li) = \{(H_1, [\beta_{H_1}^{L}(li), \beta_{H_1}^{U}(li)]), (H_2, [\beta_{H_2}^{L}(li), \beta_{H_2}^{U}(li)]), (\Omega, [\beta_{\Omega}^{L}(li), \beta_{\Omega}^{U}(li)])\}$. To complete this step, the following sub-steps should be done first:

Step 1.1 Convert the interval-valued distribution assessment $\tilde{B}^j(li)$ into the weighted interval-valued distribution assessment with reliability denoted by $m^j(li) = \{(H, [\tilde{m}_H^{jL}(li), \tilde{m}_H^{jU}(li)]), \forall H \subseteq \Omega; ((P(\Omega), [\tilde{m}_{p(\Omega)}^{jL}(li), \tilde{m}_{p(\Omega)}^{jU}(li)])\}$, presented as follows:

$$\tilde{m}_H^j(li) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\lambda^j(c_i)\beta_H^j(li)}{(1 + \lambda^j(c_i) - R^j(li))} & H \subseteq \Omega, H \neq \emptyset \\ \dfrac{1 - R^j(li)}{(1 + \lambda^j(c_i) - R^j(li))} & H = P(\Omega) \end{cases}. \quad (18)$$

In Eq. (18), $\tilde{m}_H^j(li)$ is the basic probability mass for $\tilde{B}^j(li)$ with both the reliability and weight of expert e_j taken into account. From Eq. (8), we can obtain the hybrid weight of expert e_j on attribute c_i, i.e., $\tilde{\lambda}^j(c_i) = \lambda^j(c_i)/(1 + \lambda^j(c_i) - R^j(li))$. Thus, Eq. (18) can be equivalently rewritten as follows:

$$\tilde{m}_H^j(li) = \begin{cases} 0 & H = \emptyset \\ \tilde{\lambda}^j(c_i)\beta_H^j(li) & H \subseteq \Omega, H \neq \emptyset \\ 1 - \tilde{\lambda}^j(c_i) & H = P(\Omega) \end{cases}. \tag{19}$$

Theorem 1 presents the combined result of assessments in the situation where belief degrees are precise values. Hereafter, we extend the ER rule to the situation where belief degrees are intervals by constructing two nonlinear optimization problems.

Step 1.2 Combine the first j-independent interval-valued distribution assessments $\tilde{B}^j(li)$ (j = 2, …, T) with their hybrid weights $\tilde{\lambda}^j(c_i)$ (j = 2, …, T) by using the recursive ER rule. The combined interval-valued distribution assessment denoted by $\tilde{B}^{(j)}(li) = \{(H, [\beta_H^{(j)L}(li), \beta_H^{(j)U}(li)]), \forall H \subseteq \Omega$ and $\beta_H^{(j)L}(li) \leq \beta_H^{(j)}(li) \leq \beta_H^{(j)U}(li), \sum_{H\subseteq\Omega} \beta_H^{(j)}(li) = 1)\}$ is calculated through resolving the following nonlinear optimization problems with variables $\beta_H^{j*}(li)$ developed by using Theorem 1.

Model 1

$$\text{MIN/MAX} \quad \beta_H^{(j)}(li) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H^{(j)}(li)}{\sum_{D\subseteq\Omega} \hat{m}_D^{(j)}(li)} & H \subseteq \Omega, H \neq \emptyset \end{cases} \tag{20}$$

s.t.

$$m_H^{(j)}(li) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H^{(j)}(li)}{\sum_{D\subseteq\Omega} \hat{m}_D^{(j)}(li) + \hat{m}_{p(\Omega)}^{(j)}(li)} & H \subseteq \Omega, H \neq \emptyset \end{cases}, \tag{21}$$

$$m_{p(\Omega)}^{(j)}(li) = \frac{\hat{m}_{p(\Omega)}^{(j)}(li)}{\sum_{D\subseteq\Omega} \hat{m}_D^{(j)}(li) + \hat{m}_{p(\Omega)}^{(j)}(li)}, \tag{22}$$

$$\hat{m}_H^{(j)}(li) = \left[\left(1 - \tilde{\lambda}^j(c_i)\right) \cdot m_H^{(j-1)}(li) + m_{p(\Omega)}^{(j-1)}(li) \cdot \tilde{m}_H^j(li)\right] + \sum_{B\cap C=H} m_B^{(j-1)}(li) \cdot \tilde{\lambda}^j(c_i)\beta_C^j(li), \quad \forall H \subseteq \Omega, \tag{23}$$

$$\hat{m}_{p(\Omega)}^{(j)}(li) = \left(1 - \tilde{\lambda}^j(c_i)\right) \cdot m_{p(\Omega)}^{(j-1)}(li), \tag{24}$$

$$\beta_H^{jL}(li) \leq \beta_H^{j*}(li) \leq \beta_H^{jU}(li), \tag{25}$$

$$\sum_{H\subseteq\Omega} \beta_H^{j*}(li) = 1. \tag{26}$$

In the problems above, $\hat{m}_H^{(j)}(li)$ is the combined probability mass for H, $0 \leq \beta_H^{(j)}(li)$, $m_H^{(j)}(li) \leq 1$, $\forall H \subseteq \Omega$, $0 \leq m_{p(\Omega)}^{(j)}(li) \leq 1$, and $\sum_{H\subseteq\Omega} m_H^{(j)}(li) + m_{p(\Omega)}^{(j)}(li) = 1$ for j = 2, …, T recursively.

When all the T-independent interval-valued distribution assessments are aggregated recursively, we can obtain the combined interval-valued distribution assessment (or the aggregated group assessment) denoted by $\tilde{B}^{(T)}(li) = \{(H, [\beta_H^{(T)L}(li), \beta_H^{(T)U}(li)]), \forall H \subseteq \Omega$ and $\beta_H^{(T)L}(li) \leq \beta_H^{(T)}(li) \leq \beta_H^{(T)U}(li), \sum_{H\subseteq\Omega} \beta_H^{(T)}(li) = 1)\}$. To facilitate the discussion below, we simplify $\tilde{B}^{(T)}(li)$ as $\tilde{B}(li) = \{(H_1, [\beta_{H_1}^L(li), \beta_{H_1}^U(li)]), (H_2, [\beta_{H_2}^L(li), \beta_{H_2}^U(li)]), (\Omega, [\beta_\Omega^L(li), \beta_\Omega^U(li)])\}$. Let $R^{(T)}(li)$ stand for the combined reliability; $\bar{\lambda}^{(T)}(li)$ for the combined weight; and $\tilde{\lambda}^{(T)}(li)$ for the combined hybrid weight of $\tilde{B}^j(li)$. In the following, $R^{(T)}(li)$ will be utilized to combine $\tilde{B}(li)$ (i = 1, …, L). As $m_{p(\Omega)}^{(T)}(li) = 1 - \tilde{\lambda}^{(T)}(li) = \frac{1-R^{(T)}(li)}{1+\bar{\lambda}^{(T)}(li)-R^{(T)}(li)}$, and $R^{(T)}(li)$ can be obtained with a precise $\bar{\lambda}^{(T)}(li)$ judged by the moderator, according to the following formula:

$$R^{(T)}(li) = \frac{1 - m_{p(\Omega)}^{(T)}(li) \cdot (1 + \bar{\lambda}^{(T)}(li))}{1 - m_{p(\Omega)}^{(T)}(li)}. \tag{27}$$

If a precise $\bar{\lambda}^{(T)}(li)$ cannot be provided by the moderator, $R^{(T)}(li)$ will lie in the interval $\left[R^{(T)}(li)^-, R^{(T)}(li)^+\right] = \left[\frac{1-2\cdot m_{p(\Omega)}^{(T)}(li)}{1-m_{p(\Omega)}^{(T)}(li)}, \frac{1-m_{p(\Omega)}^{(T)}(li)\cdot(1+\max_{j\in\{1,\ldots,T\}}\{\lambda^j(c_i)\})}{1-m_{p(\Omega)}^{(T)}(li)}\right]$, due to the fact that $\max_{j\in\{1,\ldots,T\}}\{\lambda^j(c_i)\} \leq \bar{\lambda}^{(T)}(li) \leq 1$ [36].

Step 2 Combine the aggregated group assessments $\tilde{B}(li)$ (i = 2, …, L) with the reliabilities and weights of attributes to compute the aggregated assessment $\tilde{B}(l) = \{(H_1, [\beta_{H_1}^L(l), \beta_{H_1}^U(l)]), (H_2, [\beta_{H_2}^L(l), \beta_{H_2}^U(l)]), (\Omega, [\beta_\Omega^L(l), \beta_\Omega^U(l)])\}$.

With the use of the fundamental reliability r_i and weight w_i of attribute c_i for alternative a_l, the overall reliability and weight of $\tilde{B}(li)$ are, respectively, computed as $\widehat{w}_i = \bar{\lambda}^{(T)}(li) \times w_i$ and $\widehat{r}_i = R^{(T)}(li) \times r_i$. From the above analysis, we can obtain $\tilde{B}(l)$ by applying the ER rule to combine $\tilde{B}(li)$ with $\widehat{r}_i$ and $\widehat{w}_i$.

Step 2.1 Convert the aggregated group assessment $\tilde{B}(li)$ into the weighted interval-valued distribution assessment with reliability denoted by $m(li) = \{(H, [\tilde{m}_H^L(li), \tilde{m}_H^U(li)]), \forall H \subseteq \Omega; ((P(\Omega), [\tilde{m}_{p(\Omega)}^L(li), \tilde{m}_{p(\Omega)}^U(li)])\}$, according to the following formula:

$$\tilde{m}_H(li) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\widehat{w}_i}{(1 + \widehat{w}_i - \widehat{r}_i)} & H \subseteq \Omega, H \neq \emptyset \\ \dfrac{1 - r_i}{(1 + \widehat{w}_i - \widehat{r}_i)} & H = P(\Omega) \end{cases}. \tag{28}$$

In Eq. (28), $\tilde{m}_H(li)$ is the basic probability mass for $\tilde{B}(li)$ with both the reliability and weight of attribute c_i taken into account. Similarly, Eq. (28) can be rewritten as

$$\tilde{m}_H(li) = \begin{cases} 0 & H = \emptyset \\ \tilde{w}_i \beta_H(li) & H \subseteq \Omega,\ H \neq \emptyset \\ 1 - \tilde{w}_i & H = P(\Omega) \end{cases}, \tag{29}$$

where $\tilde{w}_i = \widehat{w}_i/(1 + \widehat{w}_i - \widehat{r}_i)$.

Step 2.2 Combine the first i-independent interval-valued distribution assessments $\tilde{B}(li)$ (i = 2, …, L) with their hybrid weights $\tilde{w}_i$ (i = 2, …, L) by using the recursive ER rule. In the following, we discuss two situations:

(1) Consider the case where a precise $\bar{\lambda}^{(T)}(li)$ is provided by the moderator, and the combined interval-valued distribution assessment $\tilde{B}(l(i)) = \{(H, [\beta_H^L(l(i)), \beta_H^U(l(i))]),$ $\forall H \subseteq \Omega$ and $\beta_H^L(l(i)) \leq \beta_H(l(i)) \leq \beta_H^U(l(i)), \sum_{H \subseteq \Omega} \beta_H(l(i)) = 1)\}$ is computed by solving the following nonlinear optimization problems with boundary constraints on the variables $\beta_H^*(li)$.

Model 2

$$\text{MIN/MAX} \quad \beta_H(l(i)) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i))} & H \subseteq \Omega, H \neq \emptyset \end{cases}, \tag{30}$$

$$\text{s.t.} \quad m_H(l(i)) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i)) + \hat{m}_{p(\Omega)}(l(i))} & H \subseteq \Omega, H \neq \emptyset \end{cases}, \tag{31}$$

$$m_{p(\Omega)}(l(i)) = \frac{\hat{m}_{p(\Omega)}(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i)) + \hat{m}_{p(\Omega)}(l(i))}, \tag{32}$$

$$\begin{aligned} \hat{m}_H(l(i)) = {} & \left[(1 - \tilde{w}_i) \cdot m_H(l(i-1)) + m_{p(\Omega)}(l(i-1)) \cdot \tilde{m}_H(li)\right] \\ & + \sum\nolimits_{B \cap C = H} m_B(l(i-1)) \cdot \tilde{w}_i \beta_C(li), \quad \forall H \subseteq \Omega, \end{aligned} \tag{33}$$

$$\hat{m}_{p(\Omega)}(l(i)) = (1 - \tilde{w}_i) \cdot m_{p(\Omega)}(l(i-1)), \tag{34}$$

$$\beta_H^L(li) \leq \beta_H^*(li) \leq \beta_H^U(li), \tag{35}$$

$$\sum_{H \subseteq \Omega} \beta_H^*(li) = 1. \tag{36}$$

In these optimization problems, $0 \leq \beta_H(l(i)),$ $m_H(l(i)) \leq 1,$ $\forall H \subseteq \Omega,$ $0 \leq m_{p(\Omega)}(l(i)) \leq 1,$ and $\sum_{H \subseteq \Omega} m_H(l(i)) + m_{p(\Omega)}(l(i)) = 1$ for i = 2, …, L recursively.

(2) In the case where a precise $\bar{\lambda}^{(T)}(li)$ cannot be provided by the moderator, the constraint $\max_{j \in \{1,\ldots,T\}} \{\lambda^j(c_i)\} \leq \bar{\lambda}_*^{(T)}(li) \leq 1$ is added to the aforementioned optimization problems to calculate $\tilde{B}(l(i))$, shown as follows:

Model 3

MIN/MAX

$$\beta_H(l(i)) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i))} & H \subseteq \Omega, H \neq \emptyset \end{cases}, \tag{37}$$

$$\text{s.t.} \quad m_H(l(i)) = \begin{cases} 0 & H = \emptyset \\ \dfrac{\hat{m}_H(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i)) + \hat{m}_{p(\Omega)}(l(i))} & H \subseteq \Omega, H \neq \emptyset \end{cases}, \tag{38}$$

$$m_{p(\Omega)}(l(i)) = \frac{\hat{m}_{p(\Omega)}(l(i))}{\sum_{D \subseteq \Omega} \hat{m}_D(l(i)) + \hat{m}_{p(\Omega)}(l(i))}, \tag{39}$$

$$\begin{aligned} \hat{m}_H(l(i)) = {} & \left[(1 - \tilde{w}_i) \cdot m_H(l(i-1)) + m_{p(\Omega)}(l(i-1)) \cdot \tilde{m}_H(li)\right] \\ & + \sum\nolimits_{B \cap C = H} m_B(l(i-1)) \cdot \tilde{w}_i \beta_C(li), \quad \forall H \subseteq \Omega, \end{aligned} \tag{40}$$

$$\hat{m}_{p(\Omega)}(l(i)) = (1 - \tilde{w}_i) \cdot m_{p(\Omega)}(l(i-1)), \tag{41}$$

$$\beta_H^L(li) \leq \beta_H^*(li) \leq \beta_H^U(li), \tag{42}$$

$$\max_{j \in \{1,\ldots,T\}} \{\lambda^j(c_i)\} \leq \bar{\lambda}_*^{(T)}(li) \leq 1, \tag{43}$$

$$\sum_{H \subseteq \Omega} \beta_H^*(li) = 1. \tag{44}$$

In the problems above, $0 \leq \beta_H(l(i)),$ $m_H(l(i)) \leq 1,$ $\forall H \subseteq \Omega,$ $0 \leq m_{p(\Omega)}(l(i)) \leq 1,$ and $\sum_{H \subseteq \Omega} m_H(l(i)) + m_{p(\Omega)}(l(i)) = 1$ for i = 2, …, L recursively.

Until all the L-independent interval-valued distribution assessments are combined recursively, the aggregated assessment $\tilde{B}(l)$ of a_l such that $\tilde{B}(l) = \{(H_1, [\beta_{H_1}^L(l),$ $\beta_{H_1}^U(l)]), (H_2, [\beta_{H_2}^L(l),$ $\beta_{H_2}^U(l)]), (\Omega, [\beta_\Omega^L(l), \beta_\Omega^U(l)])\},$ where $(H, [\beta_H^L(l), \beta_H^U(l)]) = (H, [\beta_H^L(l(L)), \beta_H^U(l(L))]),$ $\forall H \subseteq \Omega,$ $\beta_H^L(l) \leq \beta_H(l) \leq \beta_H^U(l),$ and $\sum_{H \subseteq \Omega} \beta_H(l) = 1),$ can be obtained.

3.4 Generation of solutions

Based on the aggregated assessment $\tilde{B}(l)$, we can obtain its associated IVIF assessment such that $\tilde{a}_l = ([\mu_l^L, \mu_l^U],$

$[\nu_l^L, \nu_l^U])$, where $[\mu_l^L, \mu_l^U] = [\beta_{H_1}^L(l), \beta_{H_1}^U(l)]$ and $[\nu_l^L, \nu_l^U] = [\beta_{H_2}^L(l),\ \beta_{H_2}^U(l)]$, and $1 \le l \le M$. To compare alternatives represented by the obtained IVIF assessments $\tilde{a}_l$ (l = 1, …, M), the two-criterion method for IVIF value comparison proposed in [50] is used in this paper.

For any two IVIF assessments $\tilde{a}_l$ and $\tilde{a}_m$, the interval local "net profit" is calculated as

$$U_{\Delta\tilde{S}}\left(\tilde{S}(\tilde{a}_l) - \tilde{S}(\tilde{a}_m)\right) = \frac{(\tilde{S}(\tilde{a}_l) - \tilde{S}(\tilde{a}_m)) + 2}{4}, \tag{45}$$

and the "risk" criterion is computed as

$$U_{\Delta\tilde{H}}(\tilde{H}(\tilde{a}_l) - \tilde{H}(\tilde{a}_m)) = \frac{(\tilde{H}(\tilde{a}_l) - \tilde{H}(\tilde{a}_m)) + 2}{4}. \tag{46}$$

With the aid of $U_{\Delta\tilde{S}}(\tilde{S}(\tilde{a}_l) - \tilde{S}(\tilde{a}_m))$ and $U_{\Delta\tilde{H}}(\tilde{H}(\tilde{a}_l) - \tilde{H}(\tilde{a}_m))$, the interval possibilities $[P(\tilde{a}_l > \tilde{a}_m)]$ and $[P(\tilde{a}_l < \tilde{a}_m)]$ can be obtained:

$$\begin{aligned}[P(\tilde{a}_l > \tilde{a}_m)] = {} & \theta U_{\Delta\tilde{S}}\left(\tilde{S}(\tilde{a}_l) - \tilde{S}(\tilde{a}_m)\right) \\ & + (1-\theta) U_{\Delta\tilde{H}}\left(\tilde{H}(\tilde{a}_l) - \tilde{H}(\tilde{a}_m)\right), \text{ and}\end{aligned} \tag{47}$$

$$\begin{aligned}[P(\tilde{a}_l < \tilde{a}_m)] = {} & \theta U_{\Delta\tilde{S}}\left(\tilde{S}(\tilde{a}_m) - \tilde{S}(\tilde{a}_l)\right) \\ & + (1-\theta) U_{\Delta\tilde{H}}\left(\tilde{H}(\tilde{a}_m) - \tilde{H}(\tilde{a}_l)\right),\end{aligned} \tag{48}$$

in which $0 \le \theta \le 1$ is a balance factor measuring the risk attitude of the moderator.

From the resulting $[P(\tilde{a}_l > \tilde{a}_m)]$ and $[P(\tilde{a}_l < \tilde{a}_m)]$, an interval overall priority degree of alternative a_l with respect to other alternatives can be designed as

$$\begin{aligned}[ST(a_l)] = {} & \sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l > \tilde{a}_m)] \\ & - \sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l < \tilde{a}_m)],\end{aligned} \tag{49}$$

which represents the interval-valued overall strength of the inequality of alternative a_l with respect to other alternatives. In what follows, the real-valued overall priority degree of alternative a_l can be obtained:

$$\begin{aligned}ST(a_l) = \frac{1}{2}\Bigg(& \left(\sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l > \tilde{a}_m)]\right)^L - \left(\sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l < \tilde{a}_m)]\right)^U \\ & + \left(\sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l > \tilde{a}_m)]\right)^U - \left(\sum\nolimits_{m=1, m\neq l}^{M} [P(\tilde{a}_l < \tilde{a}_m)]\right)^L \Bigg).\end{aligned} \tag{50}$$

Thus, we can rank alternatives to produce an order in accordance with the value of $ST(a_l)$.

3.5 Procedure of the proposed MAGDM approach

As a whole, the decision flowchart of solving an MAGDM is shown in Fig. 2, and the procedure of the proposal is presented as follows:

Step 1 Identify an MAGDM problem. A moderator invites T experts to form an expert team and identifies the set of alternatives $A = \{a_1, \ldots, a_l, \ldots, a_M\}$ as well as the set of attributes c_i (i = 1, …, L).

Step 2 Prepare for the developed approach. The moderator assigns the values of $\lambda(c_i)$, w, r_i, and $\bar{\lambda}^{(T)}(li)$ (i = 1, …, L, l = 1, …, M).

Step 3 Organize GD and collect IVIF preferences provided by the expert team before and after GD. Experts express their preferences and provide their original IVIF assessments $(\tilde{\mu}_{li}^j, \tilde{\nu}_{li}^j)_{(0)}$ (i = 1,…,L, l = 1,…,M, j = 1,…,T). Then, the moderator organizes GD which improves the experts' understanding of the problem. After that, they modify their preferences and provide their updated IVIF assessments $(\tilde{\mu}_{li}^j, \tilde{\nu}_{li}^j)_{(1)}$ (i = 1,…,L, l = 1,…,M, j = 1,…,T).

Step 4 Compute the reliability of each expert. Based on $(\tilde{\mu}_{li}^j, \tilde{\nu}_{li}^j)_{(0)}$ and $(\tilde{\mu}_{li}^j, \tilde{\nu}_{li}^j)_{(1)}$ (i = 1,…,L, l = 1,…,M, j = 1,…,T), the reliabilities $R^j(li)$ (i = 1,…,L, l = 1,…,M, j = 1,…,T) of experts on each attribute can be obtained by using Eq. (16).

Step 5 Transform the updated IVIF assessments into the ER context. Let $[\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)] = \tilde{\mu}_{li}^j$ and $[\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)] = \tilde{\nu}_{li}^j$, then $(\tilde{\mu}_{li}^j, \tilde{\nu}_{li}^j)_{(1)}$ can be transformed into ER belief distribution assessments profiled by $\tilde{B}^j(li) = \{(H_1, [\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)]), (H_2, [\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)]), (\Omega,\ [\beta_{\Omega}^{jL}(li), \beta_{\Omega}^{jU}(li)])\}$, where $[\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)]$ and $[\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)]$ denote the interval belief degrees of expert e_j on attribute c_i of alternative a_l with regard to the grades H_1 and H_2, respectively, and $[\beta_{\Omega}^{jL}(li), \beta_{\Omega}^{jU}(li)]$ is the degree of global ignorance.

Step 6 Generate the aggregated assessment of each alternative

Step 6.1 Based on Eq. (18), get the basic probability mass $\tilde{m}_H^j(li)$ for $\tilde{B}^j(li)$ with both the reliability $R^j(li)$ and weight $\lambda^j(c_i)$ of expert e_j taken into account

Step 6.2 Using Model 1, aggregate $\tilde{B}^j(li)$ ($j = 2, \ldots, T$) and get the aggregated group assessment $\tilde{B}(li) = \{(H_1, [\beta_{H_1}^L(li), \beta_{H_1}^U(li)]), (H_2, [\beta_{H_2}^L(li), \beta_{H_2}^U(li)]), (\Omega, [\beta_\Omega^L(li), \beta_\Omega^U(li)])\}$ of alternative a_l on attribute c_i

Step 6.3 Based on Eq. (28), get the basic probability mass $\tilde{m}_H(li)$ for $\tilde{B}(li)$ with both the reliability r_i and weight w_i of attribute c_i taken into account

Step 6.4 Using Model 2, if $\bar{\lambda}^{(T)}(li)$ is provided in Step 2, otherwise using Model 3, aggregate $\tilde{B}(li)$ ($i = 2, \ldots, L$) and get the aggregated assessment $\tilde{B}(l) = \{(H_1, [\beta_{H_1}^L(l), \beta_{H_1}^U(l)]), (H_2, [\beta_{H_2}^L(l), \beta_{H_2}^U(l)]), (\Omega, [\beta_\Omega^L(l), \beta_\Omega^U(l)])\}$ of alternative a_l

Step 7 Produce a ranking of the M alternatives. As per Eq. (50), calculate the overall priority degree $ST(a_l)$ of alterative a_l and then obtain a ranking of the M alternatives in accordance with the values of $ST(a_l)$ ($l = 1,\ldots,M$).

Step 8 Finish the decision.

4 A numerical example

In this section, we apply the proposal to analyze a shopping center site selection problem in order to demonstrate its validity and applicability.

4.1 Introduction of the shopping center site selection problem

In this example, we investigate the decision of one service firm in Anhui Province of China to select the appropriate site for a new shopping center. First, an expert committee of four experts, including a manager of the firm, a professional consultant from a consulting company, a specialist in development strategy research in our research institute, and a staff representative of the firm, was formed to help the moderator evaluate the most suitable location alternatives. In this study, the business development department of the firm, together with our research institute, is responsible for establishing the strategies, and the moderator is the general manager of the firm. Then, four potential locations in Anhui Province are identified to form the set of alternatives for this problem. The potential locations are Baohe, Yaohai, Shushan, and Luyang, which are four urban districts in Hefei (the capital of Anhui Province) and shown in Fig. 3. Finally, after discussing with the expert committee and consulting various studies [43, 47], six attributes, including *total cost*, *population characteristics*, *degree of competition*, *environmental considerations*, *accessibility*, and *flexibility*, are selected to carry out the analysis. Assume that the four experts, the four potential locations, and the six attributes are denoted by e_j ($j = 1,\ldots,4$), a_l ($l = 1,\ldots,4$), and c_i ($i = 1,\ldots,6$), respectively. The experts express their preferences of locations for each attribute by using IVIF sets. Step 1 has been completed.

Supported by the documentations about the six attributes, the moderator utilizes the method of [52] to calculate their weights. In detail, the moderator first identifies the most important attribute, i.e., the first attribute, and then compares other attributes with the first one to analyze the importance of these attributes. Finally, by normalizing these relative weights, we can find that ($w_1, \ldots, w_6$) = (0.23, 0.19, 0.15, 0.14, 0.14, 0.15). Based on the positive correlation between w_i and r_i, the reliabilities of these attributes, i.e., r_i ($i = 1, \ldots, 6$) = (0.7, 0.58, 0.46, 0.43, 0.43, 0.46), can also be obtained. In the same manner, the moderator can obtain the relative weights of the experts with the aid of their knowledge and different backgrounds, which are presented in Table 1. Furthermore, the combined weights are specified as 1, i.e., $\bar{\lambda}^{(4)}(li) = 1$ ($i = 1, \ldots, 6$, $l = 1,\ldots,4$). Step 2 has been completed.

4.2 Generation of the aggregated assessments of potential locations

To find the solution to the shopping center site selection problem, the aggregated assessment of each potential location should be first produced by aggregating the experts' assessments. The aggregated assessments of the four potential locations are then utilized to compute the overall priority degree of each potential location.

Each expert expresses his/her initial preference of the four potential locations on the six attributes in the form of an IVIF value, as presented in Table 2. Then, the moderator organizes the experts to have a GD, and after that, they independently update their preferences, which are given in Table 3. Step 3 has been completed.

With the use of the two sets of IVIF assessments presented in Tables 2 and 3, we can obtain the reliability of

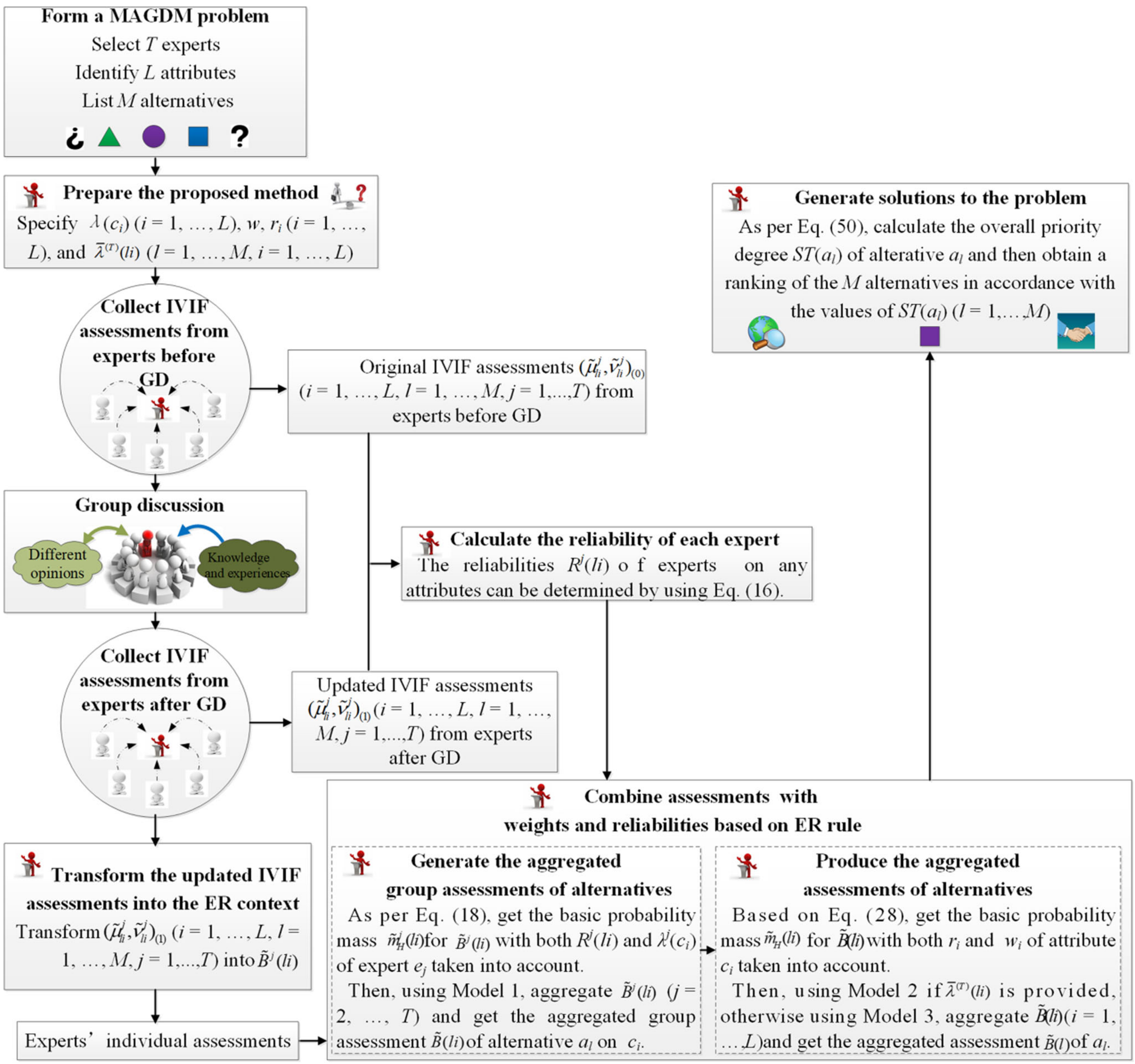

Fig. 2 The decision flowchart of the proposed MAGDM approach

each expert by using Eq. (16). The resulting reliabilities are presented in Table 4. Step 4 has been completed.

Suppose that the locations are evaluated by using the evaluation grades H_1 and H_2, as described in Sect. 3.3. We can transform the updated IVIF assessments $(\tilde{\mu}_{li}^{j}, \tilde{v}_{li}^{j})_{(1)}$ given in Table 3 into the interval-valued belief distribution assessments $\tilde{B}^{j}(li) = \{(H_1, [\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)]), (H_2, [\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)]), (\Omega, [\beta_{\Omega}^{jL}(li), \beta_{\Omega}^{jU}(li)])\}$, where $[\beta_{H_1}^{jL}(li), \beta_{H_1}^{jU}(li)] = \tilde{\mu}_{li}^{j}$ and $[\beta_{H_2}^{jL}(li), \beta_{H_2}^{jU}(li)] = \tilde{v}_{li}^{j}$. Taking attributes c_1 and c_2 as examples, the transformed interval-valued belief distribution assessments of the four potential locations are given in Table 5. Step 5 has been completed.

To generate the solution to the shopping center site selection problem, the transformed interval-valued distribution assessments $\tilde{B}^{j}(li)$ ($i = 1,\ldots,6$; $l = 1,\ldots,4$; $j = 1, \ldots, 4$) are aggregated to produce the aggregated assessment $\tilde{B}(l)$ ($l = 1,\ldots,4$). Following Steps 1 and 2 discussed in Sect. 3.3, $\tilde{B}(l)$ ($l = 1,\ldots,4$) can be obtained. The results are presented in Table 6. Step 6 has been completed.

The aggregated assessments in Table 6 can effectively reflect the real situations of the four locations in Hefei. Let us take Yaohai District (a_2) and Shushan District (a_3) as examples to demonstrate this. With strong support from Anhui government, Hefei has undergone unprecedented development during the last 10 years. Particularly, the

Fig. 3 The four potential locations

Table 1 Relative weights of the experts on each attribute

Attribute	λ^1 (c_i)	λ^2 (c_i)	λ^3 (c_i)	λ^4 (c_i)
c_1	0.35	0.2	0.25	0.2
c_2	0.25	0.25	0.35	0.15
c_3	0.3	0.2	0.2	0.3
c_4	0.25	0.35	0.2	0.2
c_5	0.35	0.2	0.2	0.25
c_6	0.3	0.3	0.25	0.15

districts closer to the city center compared to other districts develop more rapidly. Figure 3 shows that among the four districts, Yaohai District is the farthest one from the city center, which implies that its development is the slowest. As such, for the service firm, the cost of building a shopping center in Yaohai District is lower than that in other districts. However, Yaohai District is relatively undeveloped. Its infrastructure is poor, which makes it perform badly on the attributes *degree of competition* and *accessibility*. In comparison with Yaohai District, Shushan District is closer to the city center of Hefei. Its development has always been valued by Hefei government. So, Shushan District with good infrastructure owns outstanding performances in the aspects of competition and accessibility. Although the total cost of building a shopping center in this district is not as low as that in Yaohai District, it is mostly at an acceptable level. Meanwhile, in the other three aspects, the performances of Shushan District are not poorer than those of Yaohai District. Overall, it is rational that $[\beta^L_{H_1}(3), \beta^U_{H_1}(3)] > [\beta^L_{H_1}(2), \beta^U_{H_1}(2)]$ and $[\beta^L_{H_2}(3), \beta^U_{H_2}(3)] < [\beta^L_{H_2}(2), \beta^U_{H_2}(2)]$.

4.3 Generation of the solution to the shopping center site selection problem

Based on the aggregated assessment $\tilde{B}(l)$ of location a_l, we can find its IVIF assessment $\tilde{a}_l = ([\mu^L_l, \mu^U_l], [v^L_l, v^U_l])$, where $[\mu^L_l, \mu^U_l] = [\beta^L_{H_1}(l), \beta^U_{H_1}(l)]$ and $[v^L_l, v^U_l] = [\beta^L_{H_2}(l), \beta^U_{H_2}(l)]$, and $1 \leq l \leq 4$. The overall priority degrees of the four potential locations are then calculated using Eq. (50) given $\theta = 0.5$, as decided by the moderator. The results are presented in Table 7. Consequently, we can obtain a ranking of the four potential locations as $a_3 \succ a_1 \succ a_4 \succ a_2$. Step 7 has been completed.

Finally, the resulting ranking which is the solution to the shopping center site selection problem indicates that the optimal location is alternative a_3, i.e., the Shushan District can be selected to construct a shopping center of the firm. Step 8 has been completed.

4.4 Sensitivity analysis

From the above decision process, one can observe that the resulting ranking is relative not only to the attribute reliability r_i $(i = 1, \ldots, 6)$ but also to the risk attitude of the moderator as well. In view of this, the sensitivity analyses for r_i and parameter θ are performed to determine their effects on the solutions.

Table 2 Original IVIF assessments from the expert committee for each location before GD

Attribute	a_1	a_2	a_3	a_4
c_1	e_1: ([0.3, 0.6], [0.1, 0.2]);	e_1: ([0.4, 0.7], [0, 0.1]);	e_1: ([0.3, 0.7], [0.2, 0.3]);	e_1: ([0.5, 0.6], [0.3, 0.4]);
	e_2: ([0.2, 0.4], [0.4, 0.5]);	e_2: ([0.1, 0.4], [0.4, 0.5]);	e_2: ([0.6, 0.8], [0, 0.2]);	e_2: ([0.2, 0.4], [0.5, 0.6]);
	e_3: ([0.2, 0.4], [0.4, 0.5]);	e_3: ([0.7, 0.9], [0, 0.1]);	e_3: ([0.7, 0.8], [0, 0.2]);	e_3: ([0.2, 0.3], [0.4, 0.6]);
	e_4: ([0.4, 0.5], [0.3, 0.4])	e_4: ([0.2, 0.3], [0.5, 0.6])	e_4: ([0, 0.2], [0.5, 0.7])	e_4: ([0.4, 0.6], [0.2, 0.4])
c_2	e_1: ([0.5, 0.7], [0.1, 0.2]);	e_1: ([0.2, 0.3], [0.4, 0.6]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.7, 0.8], [0, 0.1]);
	e_2: ([0.6, 0.7], [0.1, 0.2]);	e_2: ([0.8, 0.9], [0, 0.1]);	e_2: ([0.3, 0.7], [0, 0.1]);	e_2: ([0.5, 0.6], [0.3, 0.4]);
	e_3: ([0.2, 0.4], [0.4, 0.5]);	e_3: ([0.3, 0.4], [0.4, 0.5]);	e_3: ([0.3, 0.8], [0, 0.1]);	e_3: ([0.2, 0.3], [0.4, 0.6]);
	e_4: ([0.3, 0.4], [0.4, 0.5])	e_4: ([0.5, 0.6], [0.3, 0.4])	e_4: ([0.4, 0.6], [0.2, 0.3])	e_4: ([0.1, 0.2], [0.4, 0.6])
c_3	e_1: ([0.6, 0.7], [0.2, 0.3]);	e_1: ([0.1, 0.4], [0.4, 0.5]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.6, 0.8], [0, 0.2]);
	e_2: ([0.5, 0.6], [0.2, 0.3]);	e_2: ([0.4, 0.7], [0.2, 0.3]);	e_2: ([0.6, 0.7], [0.1, 0.2]);	e_2: ([0.1, 0.3], [0.4, 0.6]);
	e_3: ([0.4, 0.7], [0, 0.1]);	e_3: ([0.1, 0.3], [0.3, 0.5]);	e_3: ([0.4, 0.7], [0.2, 0.3]);	e_3: ([0.6, 0.7], [0.2, 0.3]);
	e_4: ([0.3, 0.5], [0.3, 0.4])	e_4: ([0.6, 0.7], [0.2, 0.3])	e_4: ([0.3, 0.4], [0, 0.2])	e_4: ([0.5, 0.6], [0.1, 0.2])
c_4	e_1: ([0.4, 0.5], [0.2, 0.4]);	e_1: ([0.3, 0.4], [0.4, 0.6]);	e_1: ([0.1, 0.3], [0.4, 0.6]);	e_1: ([0.6, 0.8], [0, 0.2]);
	e_2: ([0.5, 0.7], [0.1, 0.2]);	e_2: ([0.1, 0.4], [0.2, 0.5]);	e_2: ([0.2, 0.3], [0.4, 0.6]);	e_2: ([0.7, 0.8],[0.1, 0.2]);
	e_3: ([0.7, 0.9], [0, 0.1]);	e_3: ([0.2, 0.4], [0.4, 0.5]);	e_3: ([0.6, 0.7], [0, 0.2]);	e_3: ([0.5, 0.7], [0.1, 0.2]);
	e_4: ([0.3, 0.5], [0.4, 0.5])	e_4: ([0.6, 0.7], [0.1, 0.3])	e_4: ([0.5, 0.6], [0.2, 0.4])	e_4: ([0.4, 0.6], [0.2, 0.3])
c_5	e_1: ([0.6, 0.8], [0.1, 0.2]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.7, 0.8], [0.1, 0.2]);	e_1: ([0.5, 0.7], [0.2, 0.3]);
	e_2: ([0.6,0.8], [0, 0.2]);	e_2: ([0.2, 0.3], [0.4, 0.6]);	e_2: ([0.6, 0.7], [0.2, 0.3]);	e_2: ([0.6, 0.8], [0.1, 0.2]);
	e_3: ([0.3, 0.8], [0.1, 0.2]);	e_3: ([0.3, 0.4], [0.4, 0.6]);	e_3: ([0.2, 0.3], [0.5, 0.6]);	e_3: ([0.1, 0.2], [0.4, 0.6]);
	e_4: ([0.4, 0.5], [0.2, 0.3])	e_4: ([0.7, 0.8], [0, 0.1])	e_4: ([0.3, 0.4], [0.4, 0.6])	e_4: ([0.3, 0.5], [0.4, 0.5])
c_6	e_1: ([0.7, 0.8], [0.1, 0.2]);	e_1: ([0.3, 0.6], [0.1, 0.2]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.5, 0.7], [0.1, 0.2]);
	e_2: ([0.1, 0.4], [0.2, 0.5]);	e_2: ([0.6, 0.7], [0.2, 0.3]);	e_2: ([0.7, 0.8], [0.1, 0.2]);	e_2: ([0.6, 0.8], [0, 0.2]);
	e_3: ([0.2, 0.4], [0.4, 0.5]);	e_3: ([0.1, 0.2], [0.4, 0.6]);	e_3: ([0.4, 0.7], [0, 0.1]);	e_3: ([0.1, 0.3], [0.3, 0.5]);
	e_4: ([0.5, 0.6], [0.1,0.2])	e_4: ([0.6, 0.7], [0.1, 0.3])	e_4: ([0.3, 0.4], [0, 0.2])	e_4: ([0.4, 0.6], [0.2, 0.4])

To perform sensitivity analysis for r_i ($i = 1, \ldots, 6$), we suppose that the ratios of r_i ($i = 2, \ldots, 6$) to r_1 equal to (0.83, 0.66, 0.61, 0.61, 0.66), while we keep the previous assumptions for the ratios of w_i ($i = 2, \ldots, 6$) to w_1 and assume that the moderator is risk neutral, namely $\theta = 0.5$. Under such conditions, ten different values within the interval [0,1] are assigned to r_1, and then the overall priority degrees of the four alternatives are obtained (Table 8). The results in Table 8 show that the ranking orders of the alternatives are stable when the value of r_i changes between 0.1 and 1. The attribute reliability r_i ($i = 1, \ldots, 6$) has a great influence on the overall priority degree of each alternative (Fig. 4). Figure 4 shows that the priority degrees of alternatives a_2 and a_3 are positively related to the attribute reliability r_1. In contrast, the priority degrees of alternatives a_1 and a_4 are negatively related to r_1.

To perform sensitivity analysis for the parameter θ, 21 different values within the interval [0,1] are assigned to this parameter. The overall priority degrees of the alternatives are presented in Table 9, and the variation trend of the overall priority degree for each alternative is shown in Fig. 5. The results in Table 9 and Fig. 5 indicate that the risk attitude of the moderator has a significant impact on the final ranking of the alternatives. For example, if the moderator is risk-averse, the optimal location for the considered selection problem is the alternative a_4. The ranking of a_3 gradually increased with the increase in θ. In particular, a_3 becomes the third optimal location when θ lies in [0.05,0.2] and then becomes the optimal location when θ increases to [0.25,1]. There exist two stable intervals of θ from 0.05 to 0.2 and from 0.3 to 1, in which the ranking orders of the alternatives remain as $a_4 \succ a_1 \succ a_3 \succ a_2$ and $a_3 \succ a_1 \succ a_4 \succ a_2$, respectively. More importantly, the distinctions between the alternatives become increasingly apparent with the increase in θ. One can see that the proposed approach meets the different risk attitudes of moderator.

Table 3 Updated IVIF assessments from the expert committee for each location after GD

Attribute	a_1	a_2	a_3	a_4
c_1	e_1: ([0.3, 0.5], [0.1, 0.2]);	e_1: ([0.4, 0.6], [0, 0.1]);	e_1: ([0.4, 0.7], [0.2, 0.3]);	e_1: ([0.5, 0.6], [0.3, 0.4]);
	e_2: ([0.3, 0.4], [0.4, 0.5]);	e_2: ([0.2, 0.4], [0.4, 0.6]);	e_2: ([0.6, 0.7], [0, 0.2]);	e_2: ([0.3, 0.4], [0.5, 0.6]);
	e_3: ([0.2, 0.5], [0.4, 0.5]);	e_3: ([0.8, 0.9], [0, 0.1]);	e_3: ([0.7, 0.8], [0.1, 0.2]);	e_3: ([0.2, 0.3], [0.4, 0.5]);
	e_4: ([0.4, 0.5], [0.3, 0.4])	e_4: ([0.2, 0.4], [0.5, 0.6])	e_4: ([0, 0.2], [0.6, 0.8])	e_4: ([0.4, 0.5], [0.3, 0.5])
c_2	e_1: ([0.6, 0.7], [0.1, 0.2]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.2, 0.4], [0.4, 0.5]);	e_1: ([0.7, 0.8], [0, 0.1]);
	e_2: ([0.6, 0.8], [0.1, 0.2]);	e_2: ([0.8, 0.9], [0, 0.1]);	e_2: ([0.3, 0.6], [0, 0.2]);	e_2: ([0.4, 0.5], [0.3, 0.5]);
	e_3: ([0.2, 0.3], [0.5, 0.7]);	e_3: ([0.3, 0.5], [0.4, 0.5]);	e_3: ([0.4, 0.8], [0.1, 0.2]);	e_3: ([0.2, 0.3], [0.4, 0.7]);
	e_4: ([0.3, 0.4], [0.4, 0.6])	e_4: ([0.5, 0.6], [0.3, 0.4])	e_4: ([0.4, 0.6], [0.2, 0.3])	e_4: ([0.1, 0.2], [0.4, 0.6])
c_3	e_1: ([0.6, 0.7], [0.1, 0.2]);	e_1: ([0.2, 0.4], [0.4, 0.6]);	e_1: ([0.2, 0.3], [0.4, 0.5]);	e_1: ([0.6, 0.8], [0, 0.2]);
	e_2: ([0.4, 0.6], [0.2, 0.3]);	e_2: ([0.4, 0.6], [0.2, 0.3]);	e_2: ([0.5, 0.7], [0.1, 0.2]);	e_2: ([0.2, 0.3], [0.4, 0.5]);
	e_3: ([0.5, 0.7], [0, 0.1]);	e_3: ([0.2, 0.3], [0.4, 0.5]);	e_3: ([0.4, 0.8], [0.1, 0.2]);	e_3: ([0.6, 0.7], [0.2, 0.3]);
	e_4: ([0.3, 0.5], [0.2, 0.4]);	e_4: ([0.5, 0.6], [0.2, 0.3]);	e_4: ([0.4, 0.5], [0, 0.2]);	e_4: ([0.5, 0.6], [0.1, 0.2]);
c_4	e_1: ([0.4, 0.5], [0.2, 0.4])	e_1: ([0.3, 0.4], [0.3, 0.5])	e_1: ([0.1, 0.3], [0.4, 0.6])	e_1: ([0.7, 0.8], [0, 0.2])
	e_2: ([0.6, 0.7], [0.1, 0.2]);	e_2: ([0.2, 0.4], [0.3, 0.5]);	e_2: ([0.2, 0.3], [0.4, 0.5]);	e_2: ([0.7, 0.9], [0, 0.1]);
	e_3: ([0.7, 0.8], [0, 0.2]);	e_3: ([0.1, 0.3], [0.4, 0.5]);	e_3: ([0.6, 0.8], [0, 0.2]);	e_3: ([0.6, 0.7], [0.1, 0.2]);
	e_4: ([0.2, 0.5], [0.4, 0.5])	e_4: ([0.6, 0.7], [0.1, 0.2])	e_4: ([0.5, 0.7], [0.2, 0.3])	e_4: ([0.4, 0.6], [0.2, 0.4])
c_5	e_1: ([0.5, 0.8], [0.1, 0.2]);	e_1: ([0.3, 0.5], [0.4, 0.5]);	e_1: ([0.6, 0.8], [0.1, 0.2]);	e_1: ([0.5, 0.7], [0.1, 0.2]);
	e_2: ([0.5, 0.6], [0.3, 0.4]);	e_2: ([0.1, 0.3], [0.4, 0.6]);	e_2: ([0.6, 0.7], [0.1, 0.2]);	e_2: ([0.6, 0.7], [0.1, 0.3]);
	e_3: ([0.4, 0.8], [0.1, 0.2]);	e_3: ([0.2, 0.4], [0.4, 0.6]);	e_3: ([0.3, 0.4], [0.5, 0.6]);	e_3: ([0.1, 0.2], [0.4, 0.6]);
	e_4: ([0.4, 0.5], [0.2, 0.3])	e_4: ([0.7, 0.8], [0, 0.1])	e_4: ([0.3, 0.5], [0.4, 0.5])	e_4: ([0.4, 0.5], [0.4, 0.5])
c_6	e_1: ([0.7, 0.8], [0, 0.2]);	e_1: ([0.4, 0.6], [0.2, 0.3]);	e_1: ([0.3, 0.4], [0.4, 0.6]);	e_1: ([0.6, 0.7], [0.2, 0.3]);
	e_2: ([0.1, 0.3], [0.2, 0.5]);	e_2: ([0.5, 0.6], [0.2, 0.3]);	e_2: ([0.7, 0.8], [0, 0.2]);	e_2: ([0.6, 0.8], [0.1, 0.2]);
	e_3: ([0.3, 0.4], [0.4, 0.5]);	e_3: ([0.2, 0.3], [0.4, 0.6]);	e_3: ([0.5, 0.7], [0, 0.1]);	e_3: ([0.2, 0.3], [0.4, 0.5]);
	e_4: ([0.4, 0.6], [0.1, 0.2])	e_4: ([0.7, 0.8], [0.1, 0.2])	e_4: ([0.3, 0.5], [0.1, 0.2])	e_4: ([0.5, 0.6], [0.2, 0.4])

5 Comparative analysis

In this section, the proposed method is compared with one representative ER-based IVIF MAGDM method [38] and three IVIF aggregation operator-based MAGDM methods [19, 23, 24, 27] to verify its effectiveness and feasibility.

5.1 Comparison with the ER-based IVIF MAGDM method

Based on IVIF sets and the ER methodology [41, 42], Mohammadi and Makui [38] developed an ER-based IVIF approach for addressing MAGDM problems. The key idea of the approach in [38] is briefly described as follows. In the approach of Mohammadi and Makui [38], the individual IVIF assessments on each attribute for each alternative are first transformed into their associated belief distribution assessments. Second, the original ER approach [41, 42] is utilized to aggregate the belief distribution assessments and the weights of attributes to obtain the aggregated assessments of individual. Then, the ER approach is employed again to aggregate the previously obtained aggregated assessments of individuals and their associated weights to produce an aggregated assessment of each alternative. After that, a positive ideal solution (PIS) and a negative ideal solution (NIS) are used as references to calculate the gray relational coefficients of each alternative from these baselines. Finally, in accordance with the degree of gray relational coefficients of each alternative from PIS and NIS, the rank order of alternatives can be generated. The superiority of their developed approach in dealing with MAGDM with IVIF information has also been demonstrated in [38]. In what follows, in order to compare this paper's developed approach with the approach of Mohammadi and Makui, the considered selection problem was solved a second time by applying the method.

Assume that both the reliabilities of experts and those of the attributes are equal to 1. On the basis of the weights of experts and attributes determined in Sect. 4.1, as well as the updated IVIF assessments in Table 3, the resulting aggregated assessments of the four potential locations by employing the approach of Mohammadi and Makui are given in Table 10.

Table 4 Expert reliability on each attribute for each location

Attribute	a_1	a_2	a_3	a_4
c_1	e_1: (0.7751); e_2: (0.8134); e_3: (0.8134); e_4: (0.8193)	e_1: (0.5861); e_2: (0.5972); e_3: (0.5892); e_4: (0.5931)	e_1: (0.6569); e_2: (0.6630); e_3: (0.6524); e_4: (0.5616)	e_1: (0.7963); e_2: (0.8024); e_3: (0.7884); e_4: (0.7936)
c_2	e_1: (0.7312); e_2: (0.7493); e_3: (0.7438); e_4: (0.7512)	e_1: (0.6560); e_2: (0.6331); e_3: (0.6927); e_4: (0.7025)	e_1: (0.7317); e_2: (0.7246); e_3: (0.7405); e_4: (0.7736)	e_1: (0.5743); e_2: (0.6425); e_3: (0.6672); e_4: (0.6388)
c_3	e_1: (0.7777); e_2: (0.8014); e_3: (0.7888); e_4: (0.7803)	e_1: (0.7451); e_2: (0.7419); e_3: (0.7190); e_4: (0.7123)	e_1: (0.6855); e_2: (0.7123); e_3: (0.7310); e_4: (0.6998)	e_1: (0.7127); e_2: (0.6371); e_3: (0.7129); e_4: (0.7253)
c_4	e_1: (0.7546); e_2: (0.7621); e_3: (0.7191); e_4: (0.7410)	e_1: (0.7465); e_2: (0.7550); e_3: (0.7824); e_4: (0.6981)	e_1: (0.6825); e_2: (0.7019); e_3: (0.6930); e_4: (0.7094)	e_1: (0.8553); e_2: (0.8605); e_3: (0.8360); e_4: (0.8122)
c_5	e_1: (0.8106); e_2: (0.8353); e_3: (0.7987); e_4: (0.8192);	e_1: (0.7480); e_2: (0.7346); e_3: (0.7431); e_4: (0.6375);	e_1: (0.7118); e_2: (0.7331); e_3: (0.6861); e_4: (0.7148);	e_1: (0.7156); e_2: (0.6862); e_3: (0.6505); e_4: (0.7006);
c_6	e_1: (0.6430); e_2: (0.6706); e_3: (0.6737); e_4: (0.6869)	e_1: (0.6905); e_2: (0.7194); e_3: (0.6240); e_4: (0.7163)	e_1: (0.6352); e_2: (0.6609); e_3: (0.6660); e_4: (0.6415)	e_1: (0.7510); e_2: (0.7274); e_3: (0.6585); e_4: (0.7509)

According to the aggregated assessments in Table 10, the PIS and NIS can be obtained as a^+ = ([0.7236, 0.9679], [0.0135, 0.0271], [0.0049, 0.2629]) and a^- = ([0.4140, 0.8418], [0.0473, 0.1342], [0.0240, 0.5387]), respectively. Then, the gray relational degree of each location from PIS and NIS is calculated.

$$\xi_1^+ = 0.7766, \xi_2^+ = 0.3434, \xi_3^+ = 0.5850, \xi_4^+ = 1;$$

$$\xi_1^- = 0.3851, \xi_2^- = 1, \xi_3^- = 0.4603, \xi_4^- = 0.3465.$$

After that the relative gray relational degree of each location from PIS is ζ_1 = 0.6685, ζ_2 = 0.2556, ζ_3 = 0.5597, ζ_4 = 0.7427. Finally, in accordance with the values of ζ_i (i = 1,…,4), a ranking order of the four locations is produced as $a_4 \succ a_1 \succ a_3 \succ a_2$. It is evident that the ranking result can be obtained by the proposed approach when $\theta \in$ [0.05, 0.2]. This reflects that the proposed approach is effective and more flexible compared to the approach of Mohammadi and Makui. Besides, the reliabilities of different experts are assumed to be the same and equal to 1 when the approach of Mohammadi and Makui is used to solve group decision-making problems. In other words, it

Table 5 Interval-valued belief distribution assessments transformed from the updated IVIF assessments

Attribute	a_1	a_2	a_3	a_4
c_1	e_1: {(H_1, [0.3, 0.5]), (H_2, [0.1, 0.2]), (Ω, [0.3, 0.6])};	e_1: {(H_1, [0.4, 0.6]), (H_2, [0, 0.1]), (Ω, [0.3, 0.6])};	e_1: {(H_1, [0.4, 0.7]), (H_2, [0.2, 0.3]), (Ω, [0, 0.4])};	e_1: {(H_1, [0.5, 0.6]), (H_2, [0.3, 0.4]), (Ω, [0, 0.2])};
	e_2: {(H_1, [0.3, 0.4]), (H_2, [0.4, 0.5]), (Ω, [0.1, 0.3])};	e_2: {(H_1, [0.2, 0.4]), (H_2,[0.4, 0.6]), (Ω, [0, 0.4])};	e_2: {(H_1, [0.6, 0.7]), (H_2, [0, 0.2]), (Ω, [0.1, 0.4])};	e_2: {(H_1, [0.3, 0.4]), (H_2, [0.5, 0.6]), (Ω, [0, 0.2])};
	e_3: {(H_1, [0.2, 0.5]), (H_2, [0.4, 0.5]), (Ω, [0, 0.4])};	e_3: {(H_1,[0.8, 0.9]), (H_2, [0, 0.1]), (Ω, [0, 0.2])};	e_3: {(H_1, [0.7, 0.8]), (H_2, [0.1, 0.2]), (Ω, [0, 0.2])};	e_3: {(H_1, [0.2, 0.3]), (H_2, [0.4, 0.5]), (Ω, [0.2, 0.4])};
	e_4: {(H_1, [0.4, 0.5]), (H_2, [0.3, 0.4]), (Ω, [0.1, 0.3])}	e_4: {(H_1, [0.2, 0.4]), (H_2, [0.5, 0.6]), (Ω, [0, 0.3])}	e_4: {(H_1, [0, 0.2]), (H_2, [0.6, 0.8]), (Ω, [0, 0.4])}	e_4: {(H_1, [0.4, 0.5]), (H_2, [0.3, 0.5]), (Ω, [0, 0.3])}
c_2	e_1: {(H_1, [0.6, 0.7]), (H_2, [0.1, 0.2]), (ΩΩ, [0.1, 0.3])};	e_1: {(H_1, [0.2, 0.4]), (H_2, [0.4, 0.5]), (Ω, [0.1, 0.4])};	e_1: {(H_1, [0.2, 0.4]), (H_2, [0.4, 0.5]), (Ω, [0.1, 0.4])};	e_1: {(H_1, [0.7, 0.8]), (H_2, [0, 0.1]), (Ω, [0.1, 0.3])};
	e_2: {(H_1, [0.6, 0.8]), (H_2, [0.1, 0.2]), (Ω, [0, 0.3])};	e_2: {(H_1, [0.8, 0.9]), (H_2, [0, 0.1]), (Ω, [0, 0.2])};	e_2: {(H_1, [0.3, 0.6]), (H_2, [0, 0.2]), (Ω, [0.2, 0.7])};	e_2: {(H_1, [0.4, 0.5]), (H_2, [0.3, 0.5]), (Ω, [0, 0.3])};
	e_3: {(H_1, [0.2, 0.3]), (H_2, [0.5, 0.7]), (Ω, [0, 0.3])};	e_3: {(H_1, [0.3, 0.5]), (H_2, [0.4, 0.5]), (Ω, [0, 0.3])};	e_3: {(H_1, [0.4, 0.8]), (H_2, [0.1, 0.2]), (Ω, [0, 0.5])};	e_3: {(H_1, [0.2, 0.3]), (H_2, [0.4, 0.7]), (Ω, [0, 0.4])};
	e_4: {(H_1, [0.4, 0.6]), (H_2, [0.3, 0.4]), (Ω, [0, 0.3])}	e_4: {(H_1, [0.5, 0.6]), (H_2, [0.3, 0.4]), (Ω, [0, 0.2])}	e_4: {(H_1, [0.4, 0.6]), (H_2, [0.2, 0.3]), (Ω, [0.1, 0.4])}	e_4: {(H_1, [0.1, 0.2]), (H_2, [0.4, 0.6]), (Ω, [0.2, 0.5])}

Table 6 Aggregated assessments of the four locations

Location	Aggregated assessments
a_1	$\{(H_1, [0.4996, 0.7897]), (H_2, [0.1422, 0.1474]), (\Omega, [0.0629, 0.3582])\}$
a_2	$\{(H_1, [0.4380, 0.7214]), (H_2, [0.1653, 0.2013]), (\Omega, [0.0773, 0.3967])\}$
a_3	$\{(H_1, [0.5169, 0.8152]), (H_2, [0.1085, 0.1161]), (\Omega, [0.0687, 0.3746])\}$
a_4	$\{(H_1, [0.5154, 0.7508]), (H_2, [0.1181, 0.1854]), (\Omega, [0.0638, 0.3665])\}$

Table 7 Overall priority degrees of the four locations

Location	Overall priority degrees	Rank order
a_1	0.0276	$a_3 \succ a_1 \succ a_4 \succ a_2$
a_2	− 0.1024	
a_3	0.0704	
a_4	0.0045	

Table 8 Overall priority degrees obtained by the proposed approach with variation in r_1

r_1	a_1	a_2	a	a_4	Rank order
0.1	0.0385	− 0.1148	0.0500	0.0262	$a_3 \succ a_1 \succ a_4 \succ a_2$
0.2	0.0374	− 0.1137	0.0520	0.0242	
0.3	0.0362	− 0.1123	0.0544	0.0218	
0.4	0.0347	− 0.1107	0.0572	0.0189	
0.5	0.0329	− 0.1086	0.0606	0.0152	
0.6	0.0306	− 0.1059	0.0648	0.0105	
0.7	0.0276	− 0.1023	0.0704	0.0044	
0.8	0.0236	− 0.0972	0.0778	− 0.0042	
0.9	0.0178	− 0.0895	0.0886	− 0.0169	
1	0.0085	− 0.0766	0.1060	− 0.0379	

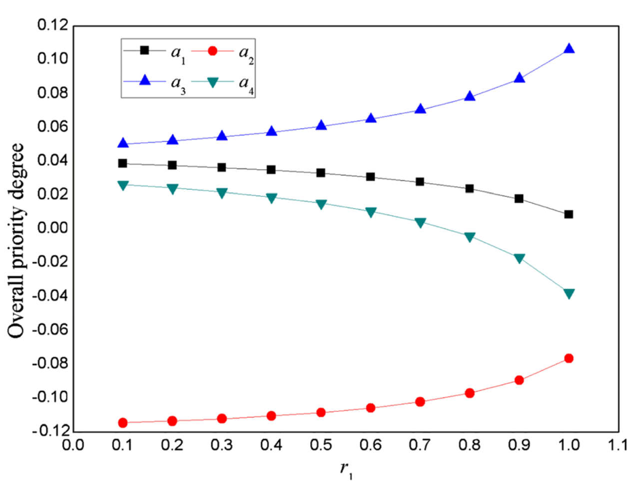

Fig. 4 Movement of overall priority degrees of the four locations with variation in r_1

cannot allow the experts to have different reliabilities as the proposed approach does.

5.2 Comparison with the IVIF aggregation operator-based MAGDM methods

With the use of IVIF aggregation operators such as the IVIFAWA operator [19, 23], the IVIF Einstein weighted averaging (IVIFEWA) operator [27], and the IVIF Hamacher weighted averaging (IVIFHWA) operator [24], three IVIF aggregation operator-based MAGDM approaches are presented in [19, 23, 24, 27]. In these approaches, the IVIF aggregation operators are used twice to implement attribute aggregation and the aggregation of individual assessments. Finally, based on the obtained aggregated assessments, a ranking order of the alternatives can be produced. In the following, in order to compare the approach developed in this paper with the approaches in [19, 23, 24, 27], the considered selection problem was solved by using the latter three approaches.

Under the same assumptions as stated in Sect. 5.1, the resulting aggregated assessments of the four potential locations by using the three different aggregation methods are presented in Table 11. As the IVIFAWA operator and the IVIFEWA operator are the special cases of the IVIFHWA operator when $\tau = 1$, 2, respectively, Table 11 shows that the aggregated assessments using the IVIFHWA operator in the setting of $\tau = 1$, 2 are the same as those, respectively, using the IVIFAWA operator and the IVIFEWA operator. More importantly, the lower bound of the aggregated interval-valued non-membership degree is equal to 0, even if the lower bounds of the most individual interval-valued non-membership degrees are not equal to 0 as listed in Table 3. This is due to the drawback of these operators that they only consider the individual interval-valued non-membership degrees whose lower bounds are equal to 0 but fail to consider all the other individual interval-valued non-membership degrees.

In order to eliminate the impact of different IVIF comparison rules, we use the two-criterion rule [50] to compare the aggregated IVIF assessments of the four locations. The results are presented in Table 12. As given in Table 12, the three aggregation operator-based MAGDM methods generate the same ranking order of the four locations: $a_3 \succ a_4 \succ a_1 \succ a_2$, where the rankings of

Table 9 Overall priority degrees obtained by the proposed approach with variation in θ

θ	a_1	a_2	a_3	a_4	Rank order
0	0.0082	− 0.0193	− 0.0246	0.0357	$a_4 \succ a_1 \succ a_2 \succ a_3$
0.05	0.0101	− 0.0276	− 0.0151	0.0325	$a_4 \succ a_1 \succ a_3 \succ a_2$
0.1	0.0091	− 0.0234	− 0.0198	0.0341	
0.15	0.0096	− 0.0255	− 0.0174	0.0333	
0.2	0.0094	− 0.0244	− 0.0186	0.0337	
0.25	0.0179	− 0.0608	0.0229	0.0201	$a_3 \succ a_4 \succ a_1 \succ a_2$
0.3	0.0198	− 0.0691	0.0324	0.0169	$a_3 \succ a_1 \succ a_4 \succ a_2$
0.35	0.0217	− 0.0774	0.0419	0.0138	
0.4	0.0237	− 0.0857	0.0514	0.0107	
0.45	0.0256	− 0.0940	0.0609	0.0076	
0.5	0.0276	− 0.1024	0.0704	0.0045	
0.55	0.0295	− 0.1107	0.0798	0.0013	
0.6	0.0314	− 0.1190	0.0893	− 0.0018	
0.65	0.0334	− 0.1273	0.0988	− 0.0049	
0.7	0.0353	− 0.1356	0.1083	− 0.0080	
0.75	0.0373	− 0.1439	0.1178	− 0.0112	
0.8	0.0392	− 0.1522	0.1273	− 0.0143	
0.85	0.0411	− 0.1605	0.1368	− 0.0174	
0.9	0.0431	− 0.1688	0.1463	− 0.0205	
0.95	0.0450	− 0.1771	0.1558	− 0.0236	
1	0.0470	− 0.1855	0.1653	− 0.0266	

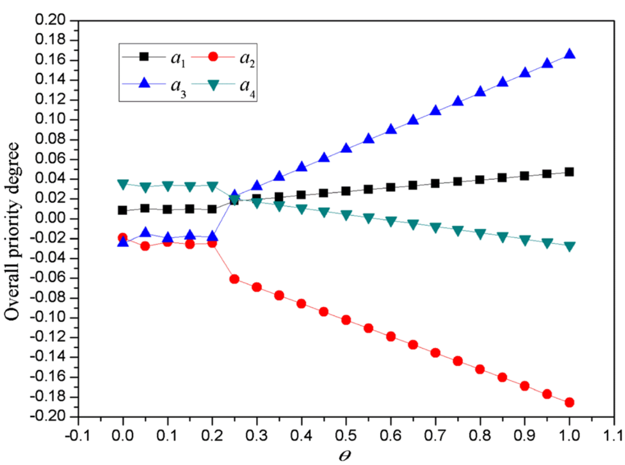

Fig. 5 Movement of overall priority degrees of the four locations with variation in θ

a_1 and a_4 differ from those generated by the proposed method, but the best and the worst choices are still a_3 and a_2, respectively. Similar to the method of Mohammadi and Makui, all the experts are assumed to be fully reliable when these aggregation operator-based MAGDM methods are used to solve group decision-making problems. Thus, they cannot allow the experts to have different reliabilities as the proposed method does.

In summary, the decision results generated by the methods [19, 23, 24, 27, 38] can be achieved by the proposed method. Meanwhile, relative to a static fixed decision result obtained by the method of Mohammadi and Makui [38], the dynamic decision result generated by the proposed method can better reflect the inherent variety rule. This indicates that the proposed method is effective and is more flexible than the existing one [38]. When aggregating IVIF information, the proposed method takes into account all the interval-valued membership degrees and the interval-valued non-membership degrees of elements that belong to IVIF sets instead of only considering the maximal membership degree and the minimal non-membership degree as the aggregation operator-based methods [19, 23, 24, 27] do. More importantly, different

Table 10 Aggregated assessments of the four locations

Location	Aggregated assessments
a_1	$\{(H_1, [0.6525, 0.9679]), (H_2, [0.0175, 0.0271]), (\Omega, [0.0049, 0.3300])\}$
a_2	$\{(H_1, [0.4140, 0.8418]), (H_2, [0.0389, 0.1342]), (\Omega, [0.0241, 0.5471])\}$
a_3	$\{(H_1, [0.5907, 0.9351]), (H_2, [0.0473, 0.0481]), (\Omega, [0.0168, 0.3620])\}$
a_4	$\{(H_1, [0.7236, 0.9614]), (H_2, [0.0135, 0.0325]), (\Omega, [0.0061, 0.2629])\}$

Table 11 Aggregated assessments of the four locations using different aggregation methods

Aggregation method	a_1	a_2	a_3	a_4
Using the IVIFHWA operator [24]				
$\tau = 0.1$	([0.4513, 0.6302], [**0.0000**, 0.2816])	([0.4695, 0.6373], [**0.0000**, 0.2762])	([0.4541, 0.6632], [**0.0000**, 0.2641])	([0.4994, 0.6592], [**0.0000**, 0.2908])
$\tau = 0.5$	([0.4388, 0.6130], [0.0000, 0.2981])	([0.4376, 0.6001], [0.0000, 0.3116])	([0.4390, 0.6445], [0.0000, 0.2803])	([0.4852, 0.6323], [0.0000, 0.3160])
$\tau = 1$	*([0.4289, 0.6034], [0.0000, 0.3065])*	*([0.4185, 0.5835], [0.0000, 0.3274])*	*([0.4280, 0.6333], [0.0000, 0.2891])*	*([0.4748, 0.6179], [0.0000, 0.3279])*
$\tau = 1.5$	([0.4229, 0.5982], [0.0000, 0.3110])	([0.4072, 0.5751], [0.0000, 0.3353])	([0.4207, 0.6268], [0.0000, 0.2940])	([0.4679, 0.6099], [0.0000, 0.3341])
$\tau = 2$	*([0.4187, 0.5948], [0.0000, 0.3139])*	*([0.3995, 0.5699], [0.0000, 0.3401])*	*([0.4154, 0.6225], [0.0000, 0.2972])*	*([0.4630, 0.6046], [0.0000, 0.3379])*
$\tau = 5$	([0.4065, 0.5867], [0.0000, 0.3205])	([0.3787, 0.5578], [0.0000, 0.3509])	([0.3989, 0.6114], [0.0000, 0.3051])	([0.4484, 0.5911], [0.0000, 0.3468])
$\tau = 100$	([0.3908, 0.5792], [0.0000, 0.3266])	([0.3539, 0.5471], [0.0000, 0.3600])	([0.3591, 0.6001], [0.0000, 0.3129])	([0.4285, 0.5773], [0.0000, 0.3547])
Using the IVIFAWA operator [19, 23]	([0.4289, 0.6034], [**0.0000**, 0.3065])	([0.4185, 0.5835], [**0.0000**, 0.3274])	([0.4280, 0.6333], [**0.0000**, 0.2891])	([0.4748, 0.6179], [**0.0000**, 0.3279])
Using the IVIFEWA operator [27]	([0.4187, 0.5948], [**0.0000**, 0.3139])	([0.3995, 0.5699], [**0.0000**, 0.3401])	([0.4154, 0.6225], [**0.0000**, 0.2972])	([0.4630, 0.6046], [**0.0000**, 0.3379])

Values in bold represent the unreasonable results obtained by aggregation methods

Values in italic represent the same results obtained by different aggregation methods

Table 12 Overall priority degrees of the four locations using different methods

Method	a_1	a_2	a_3	a_4	Rank order
Using the IVIFHWA operator [24]					
$\tau = 0.1$	− 0.0035	− 0.0558	0.0358	0.0235	$a_3 \succ a_4 \succ a_1 \succ a_2$
$\tau = 0.5$	0.0032	− 0.0686	0.0407	0.0247	
$\tau = 1$	0.0064	− 0.0737	0.0423	0.0251	
$\tau = 1.5$	0.0082	− 0.0762	0.0428	0.0251	
$\tau = 2$	0.0095	− 0.0776	0.0431	0.0250	
$\tau = 5$	0.0129	− 0.0804	0.0429	0.0245	
$\tau = 100$	0.0193	− 0.0802	0.0353	0.0256	
Using the IVIFAWA operator [19, 23]	0.0064	− 0.0737	0.0423	0.0251	$a_3 \succ a_4 \succ a_1 \succ a_2$
Using the IVIFEWA operator [27]	0.0095	− 0.0776	0.0431	0.0250	$a_3 \succ a_4 \succ a_1 \succ a_2$

from the methods [19, 23, 24, 27, 38], the proposed method allows experts to have different reliabilities when it is employed to address group decision-making problems. The above comparisons verify the effectiveness and feasibility of the proposed method.

6 Conclusion and future study

This study proposes a novel fuzzy approach for MAGDM with IVIF information. For the purpose of resolving the issues with the operator-based IVIF aggregation MAGDM methods [19, 23, 24, 27, 38], we first transform the IVIF assessments into the ER context and then use the ER rule twice to combine experts' assessments. Several optimization models are established and solved in order to produce the interval-valued aggregated assessments of alternatives. More importantly, expert reliabilities and expert weights are taken into account simultaneously, which has rarely been considered in most of the existing IVIF set-based MAGDM methods. In other words, the proposed approach explores a new way to address expert reliability in fuzzy MAGDM. Finally, the proposed approach is utilized to solve a service firm's shopping center site selection problem to demonstrate its applicability and validity. By solving the practical example, we find that the proposal of this paper puts forth an effective tool for us to handle MAGDM with IVIF information.

In this study, we discuss the reliabilities of experts in MAGDM with IVIF information. However, there exist few works that consider this topic in fuzzy circumstances. In the future, we will explore new ways to measure expert reliability in other circumstances, including the neutrosophic set [5, 9–12], the hesitant fuzzy environment [53–55], the interval-valued hesitant fuzzy context [56], the probabilistic soft circumstance [57], and other contexts.

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Compliance with ethical standards

Conflict of interest The authors declare that there are no conflicts of interest regarding the publication of this paper.

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