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ORIGINAL ARTICLE

Robust exponential stabilization for uncertain neutral neural networks with interval time-varying delays by periodically intermittent control

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Abstract
This paper studies the robust exponential stabilization for a class of uncertain neutral neural networks with mixed interval time-varying delays. The aim of the paper is to design periodically intermittent control such that the closed-loop system is exponentially stable. By constructing a suitable Lyapunov–Krasovskii functional and by using some useful lemmas and some new analysis techniques, the researchers generate novel exponential stabilization criteria to ensure the robust exponential stabilization of considered uncertain neutral neural networks in terms of linear matrix inequalities. Based on the proposed criteria, an intermittent state-feedback controller design approach is introduced. Some numerical examples are given to show the effectiveness and benefits of the theoretical results.

Keywords Robust exponential stabilization · Uncertain neutral neural networks · Mixed time-varying delays · Periodically intermittent control

1 Introduction

In the past decades, neural networks have been extensively investigated as they have developed rapidly for their wide application in a variety of fields, such as secure communication, quantum devices, pattern classification, associative memory, image processing, mathematics, ecological system, and controlled constrained manipulators [1–4]. For many of these applications depending on dynamical behaviors of the network and requiring the equilibrium point of the model to be globally stable, it is important to consider stability problems of such kind of neural networks. Nowadays, the study of the stability analysis of neural networks has gained popularity among researchers, and some remarkable results have been reported in the literature [5–9]. Liu et al. [5] gave improved exponential stability criterion for neural networks with time-varying delay. Pahnehkolaei et al. [6] investigated uniform stability for fractional-order leaky integrator echo state neural network with multiple time delays, and the existence, uniqueness, and stability of the equilibrium point were provided. In [7], dynamic stability analysis of fractional-order leaky integrator echo state neural networks was given. Elahi et al. [8] considered the problem of finite-time H_∞ stability analysis of uncertain discrete-time network control systems with varying communication delays in a random fashion. In [9], robustly exponential stability analysis for discrete-time stochastic neural networks with interval time-varying delays was given.

A special kind of neural network is neutral neural network. It contains delays in both the state and the derivatives of the state. It is generally known that neutral neural network has more complicated characteristics. Many real-world systems can be fitly described by neutral-type neural networks. Many real-world systems can be adequately described by neutral-type neural networks, which interests scholars to study the neutral neural networks, especially the stability and stabilization analysis for the neutral-type neural networks, see [10–12]. Dharani et al. [10] researched on the delay-dependent stability for switched Hopfield neural networks of neutral type with additive time-varying delay components. In [11], a class of delayed neural networks described by nonlinear delay differential equations of the neutral type was studied and a sufficient condition for the existence, uniqueness and global

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exponential stability of an equilibrium point was derived. Park et al. [12] gave a new criterion for the global asymptotic stability of bidirectional associative memory neural networks of neutral type.

Intermittent control was proposed in the seminal paper of Craik (see [13]) and has aroused a wide range of interests because of its extensive applications [13–15]. Measuring the output of a system intermittently, rather than continuously, constitutes an effective control approach. Intermittent control has been extensively studied, which has generated an expanding body of literature in the field of chaotic systems and the field of neural networks [14–18]. In [14], exponential stabilization of neural networks with time-varying delay was investigated by periodically intermittent control. In [16], exponential stabilization of chaotic systems with delay was considered by periodically intermittent control. Zhang et al. [17] investigated the exponential stabilization for neutral-type neural networks with mixed interval time-varying delays by using intermittent control.

As we all know, uncertain factors such as environmental noise, uncertain parameters, and disturbance can be very often found in various practical systems [19]. The existence of uncertainty causes modeling errors, parameter variations, and measurement errors, making it rather difficult to develop an exact mathematical model [19–21]. Besides, the existence of uncertainty always leads to poor performance and even instability of control systems. In recent years, there has been a growing research interest in stability and stabilization of uncertain systems, yielding some results on this topic in the literature [2, 19, 20]. To the best of the authors' knowledge, no result has been produced through the investigation for the stabilization of uncertain neutral-type neural networks with mixed time-varying delays via intermittent control, which motivates our research on this topic.

This paper studies the problem of exponential stabilization for uncertain neutral neural networks with time-varying delay via periodically intermittent control. By employing new Lyapunov–Krasovskii functional and introducing free-weighting matrices, we have established new sufficient conditions of robustly exponential stabilization for a class of uncertain neutral neural networks by intermittent control. The developed stabilization criteria are delay dependent and characterized by linear matrix inequalities (LMIs), which can be easily calculated by MATLAB LMI control toolbox. We also provide two numerical examples to demonstrate the effectiveness of the proposed stability results.

The rest of the paper is organized as follows. Section 2 first provides some preliminary details, necessary definitions, and useful lemmas. Then, some new criteria are established to guarantee the exponential stabilization of neutral neural networks in Sect. 3. Section 4 gives two numerical examples, illustrating the effectiveness of the obtained results. And finally, Sect. 5 concludes the paper with a summary of the findings.

Notations: throughout this paper, R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $n \times m$ real matrices; * represents the elements below the main diagonal of a symmetric matrix. M^{T} means the transpose of M; $\|\cdot\|$ is the Euclidean norm of a vector; $M > 0(<0, \leq 0, \geq 0)$ means that the matrix is symmetric positive (negative, semi-negative, semi-positive) definite matrix; I is an appropriately dimensioned identify matrix; $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ stand for the minimum and maximal eigenvalue of a matrix M, respectively.

2 Problem statement and preliminaries

Consider the following uncertain neutral neural networks with mixed interval time-varying delays

$$\begin{aligned}
&\dot{x}(t) - (C + \Delta C(t))\dot{x}(t-\tau(t)) = -(A + \Delta A(t))x(t) \\
&\quad + (B + \Delta B(t))f(x(t)) + (D + \Delta D(t))f(x(t-h(t))) \\
&\quad + Eu(t), \quad t \geq 0, \\
&x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0],
\end{aligned} \tag{1}$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^T \in R^n$ is the state vector of the neural networks associated with n neurons at time t, $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ denotes the neuron activation function with $f(0) = 0$, and $u(t) \in R^n$ is the control input vector. The matrix $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries $a_i > 0, i = 1, 2, \ldots, n$, and C is a known constant matrix with appropriate dimension. B and D are the connection weight matrix and delayed connection weight matrix, respectively. E is a reversible matrix. $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$, and $\Delta D(t)$ are all unknown time-varying matrices with appropriate dimensions which represent the system uncertainty and stochastic perturbation uncertainty, respectively, which satisfy:

$$[\Delta A(t)\ \Delta B(t)\ \Delta C(t)\ \Delta D(t)] = HF(t)[X_1\ X_2\ X_3\ X_4],$$

where H, X_1, X_2, X_3, X_4 are known real constant matrices with appropriate dimensions, and $F(t)$ is unknown real time-varying matrix with Lebesgue measurable elements bounded by

$$F^{\mathrm{T}}(t)F(t) \leq I.$$

The initial condition $\phi(t)$ denotes a continuous vector-valued initial function on the interval $[-\bar{h}, 0]$.

Assumption 1 The time delays, $h(t)$ and $\tau(t)$ are time-varying differentiable function that satisfies

$$\begin{aligned} &0 \leq h_1 \leq h(t) \leq h_2 < \infty, \quad \dot{h}(t) \leq h_d < 1, \\ &0 \leq \tau_1 \leq \tau(t) \leq \tau_2 < \infty, \quad \dot{\tau}(t) \leq \tau_d < 1, \\ &\qquad \bar{h} = \max\{h_2, \tau_2\}, \end{aligned} \tag{2}$$

where h_1 and τ_1 are the lower bound of $h(t)$ and $\tau(t)$, h_2 and τ_2 are the upper bound of $h(t)$ and $\tau(t)$, respectively, and h_d and τ_d are the real constants.

Assumption 2 [20] The nonlinear activation function $f(\cdot)$ satisfies the following condition, for any $i = 1, 2, \ldots, n$ there exist constants l_i^- and l_i^+ such that

$$l_i^- \leq \frac{f_i(x) - f_i(y)}{x - y} \leq l_i^+, \quad i = 1, 2, \ldots, n, \tag{3}$$

where $x, y \in R$, $x \neq y$.

For expression convenience, we denote

$$\begin{aligned} L_1 &= \mathrm{diag}\left(l_1^- l_1^+, l_2^- l_2^+, \ldots, l_n^- l_n^+\right), \\ L_2 &= \mathrm{diag}\left(\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right), \\ L_3 &= \mathrm{diag}(\max\{|l_1^-|, |l_1^+|\}, \max\{|l_2^-|, |l_2^+|\}, \ldots, \\ &\qquad \max\{|l_n^-|, |l_n^+|\}). \end{aligned}$$

For system (1) with initial value, we consider an intermittent state-feedback controller expressed as follows:

$$u(t) = \begin{cases} Kx(t), & t \in [kT, kT + \delta), \\ 0, & t \in [kT + \delta, (k+1)T), \end{cases} \tag{4}$$

for any nonnegative integer k, where K is a constant control gain, T is the control period, $0 < \delta \leq T$, and δ is the so-called control width.

When the intermittent state-feedback control (4) is applied to (1), system (1) can be rewritten as follows:

$$\begin{aligned} &\dot{x}(t) - \bar{C}(t)\dot{x}(t - \tau(t)) = -(\bar{A}(t) - EK)x(t) \\ &\quad + \bar{B}(t)f(x(t)) + \bar{D}(t)f(x(t - h(t))), \ t \in [kT, kT + \delta), \\ &\dot{x}(t) - \bar{C}(t)\dot{x}(t - \tau(t)) = -\bar{A}(t)x(t) + \bar{B}(t)f(x(t)) \\ &\quad + \bar{D}(t)f(x(t - h(t))), \quad t \in [kT + \delta, (k+1)T), \\ &x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0], \end{aligned} \tag{5}$$

where $\bar{A}(t) = A + \Delta A(t), \bar{B}(t) = B + \Delta B(t), \bar{C}(t) = C + \Delta C(t), \bar{D}(t) = D + \Delta D(t)$.

Definition 1 System (1) is said to be robustly α-exponentially stabilizable via intermittent state-feedback control (4), if there exist $\alpha > 0$ and $\varrho > 0$ such that the solution $x(t)$ of system (5) satisfies

$$\|x(t)\| \leq \varrho \mathrm{e}^{-\alpha t}\|\phi\|, \quad \forall t \geq 0,$$

for all admissible uncertainties $F^{\mathrm{T}}(t)F(t) \leq I$, where

$$\|\phi\| = \sup_{\theta \in [-\bar{h}, 0]}\{\|x(\theta)\|, \|\dot{x}(\theta)\|\}.$$

Lemma 1 [21] *For any matrix* $R \in R^{n \times n}$, $R = R^T > 0$, *scalars* α *and* $\beta : \beta < \alpha$, *vector* $x : [\beta, \alpha] \mapsto R^n$ *such that the integration concerned are well defined, then:*

$$\begin{aligned} -\int_\beta^\alpha x^T(s)Rx(s)\mathrm{d}s &\leq -\frac{1}{\alpha - \beta}\chi_1^T R \chi_1, \\ -\int_\beta^\alpha \int_\theta^\alpha x^T(s)Rx(s)\mathrm{d}s\mathrm{d}\theta &\leq -\frac{2}{(\alpha - \beta)^2}\chi_2^T R \chi_2, \end{aligned}$$

where

$$\chi_1 = \int_\beta^\alpha x(s)\mathrm{d}s, \quad \chi_2 = \int_\beta^\alpha \int_\theta^\alpha x(s)\mathrm{d}s\mathrm{d}\theta.$$

Lemma 2 [20] *For a given matrix* $M > 0$, *the following inequality holds for all continuously differentiable function* ω *in* $[a, b] \longrightarrow R^n$:

$$\begin{aligned} \int_a^b \dot{\omega}^T(u)M\dot{\omega}(u)\mathrm{d}u \geq &\frac{1}{b - a}(\omega(b) - \omega(a))^T M(\omega(b) - \omega(a)) \\ &+ \frac{3}{b - a}\tilde{\Omega}^T M \tilde{\Omega}, \end{aligned}$$

where

$$\tilde{\Omega} = \omega(b) + \omega(a) - \frac{2}{b - a}\int_a^b \omega(u)\mathrm{d}u.$$

And that could be turned into the following matrix inequality,

$$\int_a^b \dot{\omega}^T(u)M\dot{\omega}(u)\mathrm{d}u \geq \frac{1}{b - a}\varpi^T \hat{\Omega}\varpi,$$

where

$$\hat{\Omega} = \begin{bmatrix} 4M & 2M & -6M \\ * & 4M & -6M \\ * & * & 12M \end{bmatrix}^T,$$

$$\varpi = \begin{bmatrix} \omega^T(b) & \omega^T(a) & \frac{1}{b-a}\int_a^b \omega^T(u)\mathrm{d}u \end{bmatrix}^T.$$

Lemma 3 [20] *For matrices Y, D and E of appropriate dimensions, where Y is a symmetric matrix, then*

$$Y + DFE + E^T F^T D^T < 0,$$

holds for all matrix F satisfying $F^T F \le I$, if and only if there exist a constant $\varepsilon > 0$, such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0,$$

holds.

3 Main results

In this section, robustly the exponential stabilization by periodically intermittent control is investigated. The main result is stated as follows.

Theorem 1 *Suppose that Assumptions 1 and 2 are satisfied. For given constants $\alpha > 0$ and γ, system (1) is robustly α-exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0$ $(i = 1,2,3,4,5,6)$, $R_j > 0$, $S_j > 0$, $V_j > 0$, $W_j > 0$ $(j = 1,2)$, $U_1 > 0$, Z, K and positive scalars $\mu > 0$ and $\epsilon > 0$ such that*

$$\Pi_j = \begin{bmatrix} \Xi_{11,j} & \Xi_{12,j} \\ * & \Xi_{22} \end{bmatrix} < 0, \quad j = 1,2, \tag{6}$$

$$\alpha\delta - \rho(T - \delta) > 0, \tag{7}$$

where

$$\Xi_{11,j} = \begin{bmatrix} \Psi_j & O_{12} & 0 & O_{14} & 0 & 0 & 0 \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & O_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & O_{55} & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 \\ * & * & * & * & * & * & \Pi_{77} \end{bmatrix},$$

$$\Xi_{12,j} = \begin{bmatrix} \Pi_{18} & \bar{O}_{12} & \Phi_j & 0 & \bar{O}_{15} & \bar{O}_{16} & \Lambda & PH \\ 0 & 0 & 0 & 0 & 0 & \bar{O}_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2W_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{22} = \begin{bmatrix} \Xi_{221} & \Xi_{222} \\ * & \Xi_{223} \end{bmatrix},$$

$$\Xi_{221} = \begin{bmatrix} \Pi_{88} & \epsilon X_2^T X_4 & \mu B^T P & 0 & \epsilon X_2^T X_3 \\ * & \Pi_{99} & \mu D^T P & 0 & \epsilon X_4^T X_3 \\ * & * & \Pi_{10,10} & 0 & \mu PC \\ * & * & * & \Pi_{11,11} & 0 \\ * & * & * & * & \Pi_{12,12} \end{bmatrix},$$

$$\Xi_{221} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu PH \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Xi_{223} = \begin{bmatrix} \Pi_{13,13} & 0 & 0 \\ * & \Pi_{14,14} & 0 \\ * & * & -\epsilon I \end{bmatrix},$$

$$\begin{aligned} \Psi_1 = {} & 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + h_1 R_1 \\ & + \tau_1 R_2 - PA - A^T P + Z + Z^T - L_1 W_1 \\ & - 4e^{-2\alpha h_1}\frac{1}{h_1}V_1 - 4e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 + \epsilon X_1^T X_1, \end{aligned}$$

$$\begin{aligned} \Psi_2 = {} & 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 \\ & + h_1 R_1 + \tau_1 R_2 - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1 - L_1 W_1 \\ & - PA - A^T P - 4\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 + \epsilon X_1^T X_1, \end{aligned}$$

$$\Pi_{18} = PB + L_2 W_1 - \epsilon X_1^T X_2,$$

$$\Phi_1 = -\mu A^T P + \mu Z^T, \quad \Phi_2 = -\mu A^T P,$$

$$\Pi_{22} = -\mathrm{e}^{-2\alpha h_1} Q_1 - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1,$$

$$\Pi_{44} = -\mathrm{e}^{-2\alpha\tau_1} Q_3 - 4e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2,$$

$$\Pi_{66} = -(1 - h_d)\mathrm{e}^{-2\alpha h_2} Q_5 - L_1 W_2,$$

$$\Pi_{77} = -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2} Q_6,$$

$$\Pi_{88} = U_1 - W_1 + \epsilon X_2^T X_2,$$

$$\Pi_{99} = -(1 - h_d)e^{-2\alpha h_2} U_1 - W_2 + \epsilon X_4^T X_4,$$

$$\Pi_{10,10} = S_1 + S_2 + h_1 V_1 + \tau_1 V_2 - 2\mu P,$$

$$\Pi_{11,11} = -(1 - h_d)e^{-2\alpha h_2} S_1,$$

$$\Pi_{12,12} = -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2} S_2 + \epsilon X_3^T X_3,$$

$$\Lambda = 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2,$$

$$\Pi_{13,13} = -h_1\mathrm{e}^{-2\alpha h_1} R_1 - 12\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1,$$

$$\Pi_{14,14} = -\tau_1\mathrm{e}^{-2\alpha\tau_1} R_2 - 12\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2, \quad \rho = \gamma - \alpha,$$

$$O_{12} = -2\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1, O_{14} = -2\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2,$$

$$O_{33} = -\mathrm{e}^{-2\alpha h_2} Q_2, \quad O_{55} = -\mathrm{e}^{-2\alpha\tau_2} Q_4,$$

$$\bar{O}_{12} = PD - \epsilon X_1^T X_4, \quad \bar{O}_{15} = PC - \epsilon X_1^T X_3,$$

$$\bar{O}_{16} = 6\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1.$$

Moreover, the gain matrix in the periodically intermittent controller (4) *is* $K = (PE)^{-1}Z$.

Proof Choose a Lyapunov–Krasovskii functional candidate as:

$$V(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t) + \mathcal{V}_3(t) + \mathcal{V}_4(t) + \mathcal{V}_5(t) + \mathcal{V}_6(t), \tag{8}$$

where

$$\begin{aligned}
\mathcal{V}_1(t) &= x^T(t)Px(t),\\
\mathcal{V}_2(t) &= \int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)Q_1x(s)ds\\
&\quad + \int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Q_2x(s)ds\\
&\quad + \int_{t-\tau_1}^{t} e^{2\alpha(s-t)}x^T(s)Q_3x(s)ds\\
&\quad + \int_{t-\tau_2}^{t} e^{2\alpha(s-t)}x^T(s)Q_4x(s)ds\\
&\quad + \int_{t-h(t)}^{t} e^{2\alpha(s-t)}x^T(s)Q_5x(s)ds\\
&\quad + \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)}x^T(s)Q_6x(s)ds,\\
\mathcal{V}_3(t) &= \int_{-h_1}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)dsd\theta\\
&\quad + \int_{-\tau_1}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)dsd\theta,\\
\mathcal{V}_4(t) &= \int_{t-h(t)}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)S_1\dot{x}(s)ds\\
&\quad + \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)S_2\dot{x}(s)ds,\\
\mathcal{V}_5(t) &= \int_{-h_1}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)V_1\dot{x}(s)dsd\theta\\
&\quad + \int_{-\tau_1}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)V_2\dot{x}(s)dsd\theta,\\
\mathcal{V}_6(t) &= \int_{t-h(t)}^{t} e^{2\alpha(s-t)}f^T(x(s))U_1f(x(s))ds.
\end{aligned}$$

It is clear that

$$V(t) \geq \lambda_{\min}(P)\|x(t)\|^2. \tag{9}$$

Calculating the time derivatives of $\mathcal{V}_i(t)$, $i = 1, 2, \ldots, 6$, along the trajectory of system (1) yields

$$\begin{aligned}
\dot{\mathcal{V}}_1(t) &= 2x^T(t)P\dot{x}(t)\\
&= -2x^T(t)P(A + \Delta A(t))x(t) + 2x^T(t)PEu(t)\\
&\quad + 2x^T(t)P(C + \Delta C(t))\dot{x}(t-\tau(t))\\
&\quad + 2x^T(t)P(B + \Delta B(t))f(x(t))\\
&\quad + 2x^T(t)P(D + \Delta D(t))f(x(t-h(t))),\\
\dot{\mathcal{V}}_2(t) &= -2\alpha\mathcal{V}_2(t) + x^T(t)Q_1x(t) + x^T(t)Q_2x(t)\\
&\quad - e^{-2\alpha h_1}x^T(t-h_1)Q_1x(t-h_1)\\
&\quad - e^{-2\alpha h_2}x^T(t-h_2)Q_2x(t-h_2)\\
&\quad - e^{-2\alpha\tau_1}x^T(t-\tau_1)Q_3x(t-\tau_1) + x^T(t)Q_3x(t)\\
&\quad + x^T(t)Q_4x(t) + x^T(t)Q_5x(t) + x^T(t)Q_6x(t)\\
&\quad - e^{-2\alpha\tau_2}x^T(t-\tau_2)Q_4x(t-\tau_2)\\
&\quad - (1-\dot{h}(t))e^{-2\alpha h(t)}x^T(t-h(t))Q_5x(t-h(t))\\
&\quad - (1-\dot{\tau}(t))e^{-2\alpha\tau(t)}x^T(t-\tau(t))Q_6x(t-\tau(t))\\
&\leq -2\alpha\mathcal{V}_2(t) + x^T(t)Q_1x(t) + x^T(t)Q_2x(t)\\
&\quad - e^{-2\alpha h_1}x^T(t-h_1)Q_1x(t-h_1) + x^T(t)Q_3x(t)\\
&\quad - e^{-2\alpha h_2}x^T(t-h_2)Q_2x(t-h_2) + x^T(t)Q_4x(t)\\
&\quad - e^{-2\alpha\tau_1}x^T(t-\tau_1)Q_3x(t-\tau_1) + x^T(t)Q_5x(t)\\
&\quad - e^{-2\alpha\tau_2}x^T(t-\tau_2)Q_4x(t-\tau_2) + x^T(t)Q_6x(t)\\
&\quad - (1-h_d)e^{-2\alpha h_2}x^T(t-h(t))Q_5x(t-h(t))\\
&\quad - (1-\tau_d)e^{-2\alpha\tau_2}x^T(t-\tau(t))Q_6x(t-\tau(t)),\\
\dot{\mathcal{V}}_3(t) &= -2\alpha\mathcal{V}_3(t) + h_1x^T(t)R_1x(t) + \tau_1x^T(t)R_2x(t)\\
&\quad - \int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)ds\\
&\quad - \int_{t-\tau_1}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)ds\\
&\leq -2\alpha\mathcal{V}_3(t) + h_1x^T(t)R_1x(t) + \tau_1x^T(t)R_2x(t)\\
&\quad - e^{-2\alpha h_1}\int_{t-h_1}^{t} x^T(s)R_1x(s)ds\\
&\quad - e^{-2\alpha\tau_1}\int_{t-\tau_1}^{t} x^T(s)R_2x(s)ds,\\
\dot{\mathcal{V}}_4(t) &= -2\alpha\mathcal{V}_4(t) + \dot{x}^T(t)S_1\dot{x}(t) + \dot{x}^T(t)S_2\dot{x}(t)\\
&\quad - (1-\dot{h}(t))e^{-2\alpha h(t)}\dot{x}^T(t-h(t))S_1\dot{x}(t-h(t))\\
&\quad - (1-\dot{\tau}(t))e^{-2\alpha\tau(t)}\dot{x}^T(t-\tau(t))S_2\dot{x}(t-\tau(t))\\
&\leq -2\alpha\mathcal{V}_4(t) + \dot{x}^T(t)S_1\dot{x}(t) + \dot{x}^T(t)S_2\dot{x}(t)\\
&\quad - (1-h_d)e^{-2\alpha h_2}\dot{x}^T(t-h(t))S_1\dot{x}(t-h(t))\\
&\quad - (1-\tau_d)e^{-2\alpha\tau_2}\dot{x}^T(t-\tau(t))S_2\dot{x}(t-\tau(t)),\\
\dot{\mathcal{V}}_5(t) &= -2\alpha\mathcal{V}_5(t) + h_1\dot{x}^T(t)V_1\dot{x}(t) + \tau_1\dot{x}^T(t)V_2\dot{x}(t)\\
&\quad - \int_{t-h_1}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)V_1\dot{x}(s)ds\\
&\quad - \int_{t-\tau_1}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)V_2\dot{x}(s)ds,\\
\dot{\mathcal{V}}_6(t) &= -2\alpha\mathcal{V}_6(t) + f^T(x(t))U_1f(x(t)) - (1-\dot{h}(t))\\
&\quad \times e^{-2\alpha h(t)}f^T(x(t-h(t)))U_1f(x(t-h(t)))\\
&\leq -2\alpha\mathcal{V}_6(t) + f^T(x(t))U_1f(x(t)) - (1-h_d)\\
&\quad \times e^{-2\alpha h_2}f^T(x(t-h(t)))U_1f(x(t-h(t))).
\end{aligned} \tag{10}$$

□

By using Lemma 1, it can be seen that

$$
\begin{aligned}
&-\int_{t-h_1}^{t} \mathrm{e}^{2\alpha(s-t)} x^T(s) R_1 x(s)\mathrm{d}s \\
&\quad \le -\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}\left(\int_{t-h_1}^{t} x(s)\mathrm{d}s\right)^T R_1 \left(\int_{t-h_1}^{t} x(s)\mathrm{d}s\right),
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&-\int_{t-\tau_1}^{t} \mathrm{e}^{2\alpha(s-t)} x^T(s) R_2 x(s)\mathrm{d}s \\
&\quad \le -\mathrm{e}^{-2\alpha \tau_1}\frac{1}{\tau_1}\left(\int_{t-\tau_1}^{t} x(s)\mathrm{d}s\right)^T R_2 \left(\int_{t-\tau_1}^{t} x(s)\mathrm{d}s\right).
\end{aligned} \tag{12}
$$

Based on Lemma 2, it holds that

$$
-\int_{t-h_1}^{t} \mathrm{e}^{2\alpha(s-t)} \dot{x}^T(s) V_1 \dot{x}(s)\mathrm{d}s \le \mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}\varpi_1^T \Omega_1 \varpi_1, \tag{13}
$$

$$
-\int_{t-\tau_1}^{t} \mathrm{e}^{2\alpha(s-t)} \dot{x}^T(s) V_2 \dot{x}(s)\mathrm{d}s \le \mathrm{e}^{-2\alpha \tau_1}\frac{1}{\tau_1}\varpi_2^T \Omega_2 \varpi_2, \tag{14}
$$

where

$$
\varpi_1 = \begin{bmatrix} x^T(t) & x^T(t-h_1) & \frac{1}{h_1}\left(\int_{t-h_1}^{t} x(s)\mathrm{d}s\right)^T \end{bmatrix}^{\mathrm{T}},
$$

$$
\varpi_2 = \begin{bmatrix} x^T(t) & x^T(t-\tau_1) & \frac{1}{\tau_1}\left(\int_{t-\tau_1}^{t} x(s)\mathrm{d}s\right)^T \end{bmatrix}^{\mathrm{T}},
$$

$$
\Omega_1 = \begin{bmatrix} -4V_1 & -2V_1 & 6V_1 \\ * & -4V_1 & 6V_1 \\ * & * & -12V_1 \end{bmatrix}^{\mathrm{T}},
$$

$$
\Omega_2 = \begin{bmatrix} -4V_2 & -2V_2 & 6V_2 \\ * & -4V_2 & 6V_2 \\ * & * & -12V_2 \end{bmatrix}^{\mathrm{T}}.
$$

Furthermore, for any matrices $W_1 > 0$ and $W_2 > 0$ and utilizing Assumption 2, we have

$$
\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -L_1 W_1 & L_2 W_1 \\ * & -W_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \ge 0, \tag{15}
$$

$$
\begin{bmatrix} x(t-h(t)) \\ \bar{f}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -L_1 W_2 & L_2 W_2 \\ * & -W_2 \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ \bar{f}(t) \end{bmatrix} \ge 0. \tag{16}
$$

where $\bar{f}(t) = f(x(t-h(t)))$.

In the following, we consider two cases in calculating the derivative of Lyapunov–Krasovskii functional: $t \in [kT, kT+\delta)$ and $t \in [kT+\delta, (k+1)T)$.

Case 1 For $t \in [kT, kT+\delta)$. The first subsystem of (5) can be written as

$$
\begin{aligned}
\dot{x}(t) = {} & (C + \Delta C(t))\dot{x}(t-\tau(t)) + (B + \Delta B(t))f(x(t)) \\
& - (A + \Delta A(t) - EK)x(t) \\
& + (D + \Delta D(t))f(x(t-h(t))), \quad t \in [kT, kT+\delta).
\end{aligned}
$$

It is easy to see that

$$
\begin{aligned}
& 2\mu\dot{x}^T(t)P[(C + \Delta C(t))\dot{x}(t-\tau(t)) - (A + \Delta A(t) \\
& \quad - EK)x(t) + (B + \Delta B(t))f(x(t)) \\
& \quad + (D + \Delta D(t))f(x(t-h(t))) - \dot{x}(t)] = 0.
\end{aligned} \tag{17}
$$

Setting

$$
Z = PEK, \tag{18}
$$

and combining (10)–(17), we get that

$$
\dot{V}(t) + 2\alpha V(t) \le \xi^T(t)\tilde{\Pi}_1\xi(t), \tag{19}
$$

where

$$
\tilde{\Pi}_1 = \begin{bmatrix} \tilde{\Xi}_{11,1} & \tilde{\Xi}_{12,1} \\ * & \tilde{\Xi}_{22} \end{bmatrix},
$$

$$
\tilde{\Xi}_{11,1} = \begin{bmatrix}
\tilde{\Psi}_1 & O_{12} & 0 & O_{14} & 0 & 0 & 0 \\
* & \Pi_{22} & 0 & 0 & 0 & 0 & 0 \\
* & * & O_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 \\
* & * & * & * & O_{55} & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 \\
* & * & * & * & * & * & \Pi_{77}
\end{bmatrix},
$$

$$
\tilde{\Xi}_{12,1} = [\Theta_1 \quad \Theta_2],
$$

$$
\Theta_1 = \begin{bmatrix}
\tilde{\Pi}_{18} & P(D + \Delta D(t)) & \tilde{\Phi}_1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & L_2 W_2 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\Theta_2 = \begin{bmatrix}
P(C + \Delta C(t)) & 6e^{-2\alpha h_1}\frac{1}{h_1}V_1 & 6e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 6\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
$$

$$
\tilde{\Xi}_{22} = \begin{bmatrix} F_1 & F_2 \\ * & F_3 \end{bmatrix},
$$

$$
F_1 = \begin{bmatrix}
\tilde{\Pi}_{88} & 0 & \mu[P(B + \Delta B(t))]^T & 0 \\
* & \tilde{\Pi}_{99} & \mu[P(D + \Delta D(t))]^T & 0 \\
* & * & \Pi_{10,10} & 0 \\
* & * & * & \Pi_{11,11}
\end{bmatrix},
$$

$$
\begin{aligned}
F_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mu[P(C + \Delta C(t))] & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
F_3 &= \operatorname{diag}(\tilde{\Pi}_{12,12}, \Pi_{13,13}, \Pi_{14,14}), \\
\tilde{\Psi}_1 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + h_1 R_1 \\
&\quad - L_1 W_1 - P(A + \Delta A(t)) - (A + \Delta A(t))^T P \\
&\quad + Z + Z^T + \tau_1 R_2 - 4\mathrm{e}^{-2\alpha h_1} \frac{1}{h_1} V_1 - 4\mathrm{e}^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2, \\
\tilde{\Pi}_{18} &= P(B + \Delta B(t)) + L_2 W_1, \\
\tilde{\Phi}_1 &= [-\mu P(A + \Delta A(t))]^T + \mu Z^T, \\
\Pi_{22} &= -\mathrm{e}^{-2\alpha h_1} Q_1 - 4\mathrm{e}^{-2\alpha h_1} \frac{1}{h_1} V_1, \\
\Pi_{44} &= -\mathrm{e}^{-2\alpha\tau_1} Q_3 - 4\mathrm{e}^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2, \\
\Pi_{66} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2} Q_5 - L_1 W_2, \\
\Pi_{77} &= -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2} Q_6, \ \tilde{\Pi}_{88} = U_1 - W_1, \\
\tilde{\Pi}_{99} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2} U_1 - W_2, \\
\Pi_{10,10} &= S_1 + S_2 + h_1 V_1 + \tau_1 V_2 - 2\mu P, \\
\Pi_{11,11} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2} S_1, \\
\tilde{\Pi}_{12,12} &= -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2} S_2, \\
\Pi_{13,13} &= -h_1 \mathrm{e}^{-2\alpha h_1} R_1 - 12\mathrm{e}^{-2\alpha h_1} \frac{1}{h_1} V_1, \\
\Pi_{14,14} &= -\tau_1 \mathrm{e}^{-2\alpha\tau_1} R_2 - 12\mathrm{e}^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2. \\
\xi(t) &= \left[\xi_1^T(t), \xi_2^T(t), \xi_3^T(t)\right]^T, \\
\xi_1(t) &= [x^T(t), x^T(t - h_1), x^T(t - h_2), x^T(t - \tau_1), \\
&\quad x^T(t - \tau_2), x^T(t - h(t))]^{\mathrm{T}}, \\
\xi_2(t) &= [x^T(t - \tau(t)), f^T(x(t)), f^T(x(t - h(t))), \dot{x}(t), \\
&\quad \dot{x}(t - h(t)), \dot{x}(t - \tau(t))]^{\mathrm{T}}, \\
\xi_3(t) &= \left[\frac{1}{h_1}\left(\int_{t-h_1}^{t} x(s)\mathrm{d}s\right)^T, \frac{1}{\tau_1}\left(\int_{t-\tau_1}^{t} x(s)\mathrm{d}s\right)^T\right]^{\mathrm{T}}.
\end{aligned}
$$

Note that $\tilde{\Pi}_1 < 0$ is not standard LMI due to the existence of parameter uncertainties, which will be further dealt with via the following approach. $\tilde{\Pi}_1$ can be written as

$$\tilde{\Pi}_1 = \hat{\Pi}_1 + \Delta\Pi(t),$$

where

$$
\begin{aligned}
\hat{\Pi}_1 &= \begin{bmatrix} \hat{\Xi}_{11,1} & \hat{\Xi}_{12,1} \\ * & \hat{\Xi}_{22} \end{bmatrix}, \\
\hat{\Xi}_{11,1} &= \begin{bmatrix}
\hat{\Psi}_1 & O_{12} & 0 & O_{14} & 0 & 0 & 0 \\
* & \Pi_{22} & 0 & 0 & 0 & 0 & 0 \\
* & * & O_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 \\
* & * & * & * & O_{55} & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 \\
* & * & * & * & * & * & \Pi_{77}
\end{bmatrix}, \\
\hat{\Xi}_{12,1} &= \begin{bmatrix}
\hat{\Pi}_{18} & PD & \hat{\Phi}_1 & 0 & PC & \bar{O}_{16} & 6\mathrm{e}^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2 \\
0 & 0 & 0 & 0 & 0 & \bar{O}_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6\mathrm{e}^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_2 W_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
\hat{\Xi}_{22} &= \begin{bmatrix}
\hat{\Pi}_{88} & 0 & \mu B^T P & 0 & 0 & 0 & 0 \\
* & \hat{\Pi}_{99} & \mu D^T P & 0 & 0 & 0 & 0 \\
* & * & \Pi_{10,10} & 0 & \mu PC & 0 & 0 \\
* & * & * & \Pi_{11,11} & 0 & 0 & 0 \\
* & * & * & * & \hat{\Pi}_{12,12} & 0 & 0 \\
* & * & * & * & * & \Pi_{13,13} & 0 \\
* & * & * & * & * & * & \Pi_{14,14}
\end{bmatrix}, \\
\hat{\Psi}_1 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + h_1 R_1 \\
&\quad + \tau_1 R_2 - PA - A^T P + Z + Z^T - 4\mathrm{e}^{-2\alpha h_1} \frac{1}{h_1} V_1 - 4e^{-2\alpha\tau_1} \frac{1}{\tau_1} V_2 - L_1 W_1, \\
\hat{\Pi}_{18} &= PB + L_2 W_1, \ \hat{\Phi}_1 = -\mu A^T P + \mu Z^T, \\
\hat{\Pi}_{88} &= U_1 - W_1, \ \hat{\Pi}_{99} = -(1 - h_d)\mathrm{e}^{-2\alpha h_2} U_1 - W_2, \hat{\Pi}_{12,12} = -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2} S_2, \\
\Delta\Pi(t) &= \begin{bmatrix} \tilde{U}_1 & \tilde{U}_2 \\ * & \tilde{U}_3 \end{bmatrix}, \\
\tilde{U}_1 &= \begin{bmatrix}
\Upsilon & 0 & 0 & 0 & 0 & 0 & 0 & P\Delta B(t) & P\Delta D(t) \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
\tilde{U}_2 &= \begin{bmatrix}
-\mu\Delta A^T(t)P & 0 & P\Delta C(t) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\mu\Delta B^T(t)P & 0 & 0 & 0 & 0 \\
\mu\Delta D^T(t)P & 0 & 0 & 0 & 0
\end{bmatrix}, \\
\tilde{U}_3 &= \begin{bmatrix}
0 & 0 & \mu P\Delta C(t) & 0 & 0 \\
* & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 \\
* & * & * & 0 & 0 \\
* & * & * & * & 0
\end{bmatrix}, \\
\Upsilon &= -P\Delta A(t) - \Delta A^T(t)P.
\end{aligned}
$$

According to Assumption 1, $\tilde{\Pi}_1$ could be rewritten as

$$\tilde{\Pi}_1 = \hat{\Pi}_1 + Y_1^T F(t) Y_2 + Y_2^T F^T(t) Y_1,$$

where

$$
\begin{aligned}
Y_1 &= \begin{bmatrix} H^T P^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu H^T P^T & 0 & 0 & 0 & 0 \end{bmatrix}, \\
Y_2 &= [\,-X_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad X_2 \quad X_4 \quad 0 \quad 0 \quad X_3 \quad 0 \quad 0\,].
\end{aligned}
$$

By Lemma 3 and $\|F(t)\| \leq I$, $\tilde{\Pi}_1 < 0$ holds if and only if there exists a positive scalar ϵ such that,

$$\hat{\Pi}_1 + \epsilon^{-1} Y_1^T Y_1 + \epsilon Y_2^T Y_2 < 0.$$

As a result, since (6) holds, it is deduced from (19) that

$$\dot{V}(t) + 2\alpha V(t) < 0. \tag{21}$$

Then, we have

$$V(t) \leq V(kT) e^{-2\alpha(t-kT)}, \tag{22}$$

$$V(kT+\delta) \leq V(kT)\mathrm{e}^{-2\alpha\delta}. \tag{23}$$

Case 2 For $t \in [kT+\delta, (k+1)T)$. The second subsystem of (5) can be written as

$$
\begin{aligned}
\dot{x}(t) = {} & (C + \Delta C(t))\dot{x}(t-\tau(t)) - (A + \Delta A(t))x(t) \\
& + (B + \Delta B(t)) f(x(t)) + (D + \Delta D(t)) f(x(t-h(t))), \\
& t \in [kT+\delta, (k+1)T).
\end{aligned}
$$

It is easy to see that

$$
\begin{aligned}
& 2\mu\dot{x}^T(t)P[(C + \Delta C(t))\dot{x}(t-\tau(t)) \\
& \quad - (A + \Delta A(t))x(t) + (B + \Delta B(t)) f(x(t)) \\
& \quad + (D + \Delta D(t)) f(x(t-h(t))) - \dot{x}(t)] = 0.
\end{aligned} \tag{24}
$$

From (10)–(16) and (24), we get that

$$
\begin{aligned}
\dot{V}(t) + 2\alpha V(t) & \leq \xi^T(t)\tilde{\Pi}_2\xi(t) + 2\gamma x^T(t)Px(t) \\
& \leq \xi^T(t)\tilde{\Pi}_2\xi(t) + 2\gamma V(t),
\end{aligned} \tag{25}
$$

that is,

$$\dot{V}(t) - 2(\gamma - \alpha)V(t) \leq \xi^T(t)\tilde{\Pi}_2\xi(t),$$

where $\tilde{\Pi}_2 = \hat{\Pi}_2 + \Delta\Pi(t)$, and

$$
\hat{\Pi}_2 = \begin{bmatrix} \hat{\Xi}_{11,2} & \hat{\Xi}_{12,2} \\ * & \hat{\Xi}_{22} \end{bmatrix},
$$

$$
\hat{\Xi}_{11,2} = \begin{bmatrix}
\hat{\Psi}_2 & O_{12} & 0 & O_{14} & 0 & 0 & 0 \\
* & \Pi_{22} & 0 & 0 & 0 & 0 & 0 \\
* & * & O_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 \\
* & * & * & * & O_{55} & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 \\
* & * & * & * & * & * & \Pi_{77}
\end{bmatrix},
$$

$$
\hat{\Xi}_{12,2} = \begin{bmatrix}
\hat{\Pi}_{18} & PD & -\mu A^T P & 0 & PC & \bar{O}_{16} & 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 0 & 0 & 0 & 0 & \bar{O}_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_2 W_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\hat{\Xi}_{22} = \begin{bmatrix}
\hat{\Pi}_{88} & 0 & \mu B^T P & 0 & 0 & 0 & 0 \\
* & \hat{\Pi}_{99} & \mu D^T P & 0 & 0 & 0 & 0 \\
* & * & \Pi_{10,10} & 0 & \mu PC & 0 & 0 \\
* & * & * & \Pi_{11,11} & 0 & 0 & 0 \\
* & * & * & * & \hat{\Pi}_{12,12} & 0 & 0 \\
* & * & * & * & * & \Pi_{13,13} & 0 \\
* & * & * & * & * & * & \Pi_{14,14}
\end{bmatrix},
$$

$$
\begin{aligned}
\hat{\Psi}_2 = {} & 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + h_1 R_1 \\
& + \tau_1 R_2 - PA - A^T P - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1 \\
& - 4\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 - L_1 W_1.
\end{aligned}
$$

According to Assumption 1, $\tilde{\Pi}_2$ could be rewritten as

$$\tilde{\Pi}_2 = \hat{\Pi}_2 + Y_1^T F(t) Y_2 + Y_2^T F^T(t) Y_1.$$

By Lemma 3 and $F^T(t)F(t) \leq I$, $\tilde{\Pi}_2 < 0$ holds if and only if there exists a positive scalar ϵ such that,

$$\hat{\Pi}_2 + \epsilon^{-1} Y_1^T Y_1 + \epsilon Y_2^T Y_2 < 0.$$

By using the condition (6), we have

$$\dot{V}(t) - 2(\gamma - \alpha)V(t) = \dot{V}(t) - 2\rho V(t) \leq 0, \tag{26}$$

where $\rho = \gamma - \alpha$.

Then, we have

$$V(t) \leq V(kT+\delta)\mathrm{e}^{2\rho(t-kT-\delta)}, \tag{27}$$

$$V((k+1)T) \leq V(kT+\delta)\mathrm{e}^{2\rho(T-\delta)}. \tag{28}$$

From (23) and (28), we have

$$V((k+1)T) \le V(0)\mathrm{e}^{-(k+1)[2\alpha\delta - 2\rho(T-\delta)]}, \tag{29}$$

$$V(kT+\delta) \le V(0)\mathrm{e}^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}. \tag{30}$$

Therefore, on the one hand, for $t \in [kT, kT+\delta)$, from (22) (29) and (7) we get

$$\begin{aligned} V(t) &\le V(kT)\mathrm{e}^{-2\alpha(t-kT)} \\ &\le V(0)\mathrm{e}^{-k[2\alpha\delta-2\rho(T-\delta)]}\mathrm{e}^{-2\alpha(t-kT)} \\ &\le V(0)\mathrm{e}^{-k[2\alpha\delta-2\rho(T-\delta)]} \\ &= V(0)\mathrm{e}^{2\alpha\delta-2\rho(T-\delta)}\mathrm{e}^{-(2\alpha\delta-2\rho(T-\delta))\frac{(kT+\delta)+(T-\delta)}{T}} \\ &\le \beta_1 V(0)\mathrm{e}^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \end{aligned} \tag{31}$$

where $\beta_1 = \mathrm{e}^{\frac{2\alpha\delta-2\rho(T-\delta)}{T}\delta}$. On the other hand, for $t \in [kT+\delta, (k+1)T)$, from (27), (30) and (7), we get

$$\begin{aligned} V(t) &\le V(kT+\delta)\mathrm{e}^{2\rho(t-kT-\delta)} \\ &\le V(0)\mathrm{e}^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}\mathrm{e}^{2\rho(t-kT-\delta)} \\ &\le V(0)\mathrm{e}^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}\mathrm{e}^{2|\rho|[(k+1)T-kT-\delta]} \\ &= V(0)\mathrm{e}^{\frac{-(2\alpha\delta-2\rho(T-\delta))(k+1)T}{T}}\mathrm{e}^{2(|\rho|-\rho)(T-\delta)} \\ &\le \beta_2 V(0)\mathrm{e}^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \end{aligned} \tag{32}$$

where $\beta_2 = \mathrm{e}^{2(|\rho|-\rho)(T-\delta)}$. Let $\beta = \max\{\beta_1, \beta_2\}$. From (31) and (32), we have

$$V(t) \le \beta V(0)\mathrm{e}^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \quad \forall t \ge 0. \tag{33}$$

Obviously, we have

$$\begin{aligned} V(0) \ &= x^T(0)Px(0) + \int_{-h_1}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_1x(s)\mathrm{d}s \\ &+ \int_{-h_2}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_2x(s)\mathrm{d}s + \int_{-\tau_1}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_3x(s)\mathrm{d}s \\ &+ \int_{-\tau_2}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_4x(s)\mathrm{d}s + \int_{-h(0)}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_5x(s)\mathrm{d}s \\ &+ \int_{-\tau(0)}^{0} \mathrm{e}^{2\alpha s}x^T(s)Q_6x(s)\mathrm{d}s + \int_{-h_1}^{0}\int_{\theta}^{0} \mathrm{e}^{2\alpha s}x^T(s)R_1x(s)\mathrm{d}s\mathrm{d}\theta \\ &+ \int_{-\tau_1}^{0}\int_{\theta}^{0} \mathrm{e}^{2\alpha s}x^T(s)R_2x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h(0)}^{0} \mathrm{e}^{2\alpha s}\dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s \\ &+ \int_{-\tau(0)}^{0} \mathrm{e}^{2\alpha s}\dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s + \int_{-h_1}^{0}\int_{\theta}^{0} \mathrm{e}^{2\alpha s}\dot{x}^T(s)V_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &+ \int_{-\tau_1}^{0}\int_{\theta}^{0} \mathrm{e}^{2\alpha s}\dot{x}^T(s)V_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h(0)}^{0} \mathrm{e}^{2\alpha s}f^T(x(s))U_1f(x(s))\mathrm{d}s \\ &\le N\|\phi\|^2, \end{aligned} \tag{34}$$

where

$$\begin{aligned} N \ &= \lambda_{\max}(P) + h_1\lambda_{\max}(Q_1) + h_2\lambda_{\max}(Q_2) \\ &+ \tau_1\lambda_{\max}(Q_3) + \tau_2\lambda_{\max}(Q_4) + h_2\lambda_{\max}(Q_5) \\ &+ \tau_2\lambda_{\max}(Q_6) + \frac{h_1^2}{2}\lambda_{\max}(R_1) + \frac{\tau_1^2}{2}\lambda_{\max}(R_2) \\ &+ h_2\lambda_{\max}(S_1) + \tau_2\lambda_{\max}(S_2) + \frac{h_1^2}{2}\lambda_{\max}(V_1) \\ &+ \frac{\tau_1^2}{2}\lambda_{\max}(V_2) + h_2\bar{\delta}\lambda_{\max}(U_1), \\ \bar{\delta} &= \max\left\{(l_1^-)^2, (l_1^+)^2, (l_2^-)^2, (l_2^+)^2, \ldots, (l_n^-)^2, (l_n^+)^2\right\}. \end{aligned}$$

Hence from (9), (33) and (34), we get

$$\|x(t)\| \le \sqrt{\frac{\beta N}{\lambda_{\min}(P)}}\mathrm{e}^{\frac{-(\alpha\delta-\rho(T-\delta))t}{T}}\|\phi\|, \quad \forall t \ge 0. \tag{35}$$

As a result, according to Definition 1 and (35), neutral neural network (1) with multiple time-varying delays is robust α-exponentially stabilization under the intermittent controller (4). Furthermore, the state-feedback intermittent gain matrix is $K = (PE)^{-1}Z$. The completes the proof of Theorem 1.

We give the following assumption.

Assumption 3 The time delays $h(t)$ and $\tau(t)$ are time-varying function that satisfies

$$\begin{gathered} 0 \le h_1 \le h(t) \le h_2 < \infty, \\ 0 \le \tau_1 \le \tau(t) \le \tau_2 < \infty, \quad \bar{h} = \max\{h_2, \tau_2\}, \end{gathered} \tag{36}$$

where h_1 and τ_1 are the lower bound of $h(t)$ and $\tau(t)$, h_2 and τ_2 are the upper bound of $h(t)$ and $\tau(t)$ respectively.

Remark 1 Theorem 1 gives the robustly α-exponentially stabilization criterion for system (1) with

$$\begin{aligned} &0 \le h_1 \le h(t) \le h_2 < \infty, \quad \dot{h}(t) \le h_d, \\ &0 \le \tau_1 \le \tau(t) \le \tau_2 < \infty, \quad \dot{\tau}(t) \le \tau_d, \end{aligned}$$

where h_d and τ_d are given constants. In many cases, h_d and τ_d are unknown. Considering this case, the following criteria independent of derivatives of time delays are derived as follows.

Theorem 2 *Suppose that Assumption 2 and 3 are satisfied. For given constants $\alpha > 0$ and γ, system (1) is robustly α-exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0$ $(i = 1,2,3,4)$, $R_j > 0$, $V_j > 0$, $W_j > 0$ $(j = 1,2)$, Z, K and positive scalars $\mu > 0$ and $\epsilon > 0$ such that*

$$\bar{\Pi}_j = \begin{bmatrix} \bar{\Xi}_{11,j} & \bar{\Xi}_{12,j} \\ * & \bar{\Xi}_{22} \end{bmatrix} < 0, \quad j = 1,2, \tag{37}$$

$$\alpha\delta - \rho(T-\delta) > 0, \tag{38}$$

where

$$\bar{\Xi}_{11,j} = \begin{bmatrix} \bar{\Psi}_j & O_{12} & 0 & O_{14} & 0 & 0 \\ * & \bar{\Pi}_{22} & 0 & 0 & 0 & 0 \\ * & * & -e^{-2\alpha h_2}Q_2 & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{44} & 0 & 0 \\ * & * & * & * & -e^{-2\alpha\tau_2}Q_4 & 0 \\ * & * & * & * & * & \bar{\Pi}_{66} \end{bmatrix},$$

$$\bar{\Xi}_{12,j} = \begin{bmatrix} \bar{\Pi}_{17} & PD - \epsilon X_1^T X_4 & \bar{\Phi}_j & \bar{\Gamma}_j & \bar{O}_{16} & \Lambda & PH \\ 0 & 0 & 0 & 0 & \bar{O}_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2W_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_{22} = \begin{bmatrix} \tilde{G}_1 & \tilde{G}_2 \\ * & \tilde{G}_3 \end{bmatrix},$$

$$\tilde{G}_1 = \begin{bmatrix} \bar{\Pi}_{77} & \epsilon X_2^T X_4 & \mu B^T P & \epsilon X_2^T X_3 - \mu B^T P \\ * & \bar{\Pi}_{88} & \mu D^T P & \epsilon X_4^T X_3 - \mu D^T P \\ * & * & \bar{\Pi}_{99} & \mu PC + \mu P \\ * & * & * & \bar{\Pi}_{10,10} \end{bmatrix},$$

$$\tilde{G}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu PH \\ 0 & 0 & -\mu PH \end{bmatrix},$$

$$\tilde{G}_3 = \mathrm{diag}(\bar{\Pi}_{11,11}, \bar{\Pi}_{12,12}, -\epsilon I),$$

$$\begin{aligned}
\bar{\Psi}_1 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + h_1 R_1 - L_1 W_1 \\
&\quad + \tau_1 R_2 - PA - A^T P + Z + Z^T \\
&\quad - 4e^{-2\alpha h_1}\frac{1}{h_1}V_1 - 4e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 + \epsilon X_1^T X_1, \\
\bar{\Psi}_2 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + h_1 R_1 \\
&\quad + \tau_1 R_2 - PA - A^T P - 4e^{-2\alpha h_1}\frac{1}{h_1}V_1 \\
&\quad - 4e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 - L_1 W_1 + \epsilon X_1^T X_1, \\
\bar{\Pi}_{17} &= PB + L_2 W_1 - \epsilon X_1^T X_2, \\
\bar{\Gamma}_1 &= PC + \mu PA - \mu Z - \epsilon X_1^T X_3, \\
\bar{\Gamma}_2 &= PC + \mu PA - \epsilon X_1^T X_3, \\
\bar{\Phi}_1 &= -\mu A^T P + \mu Z^T, \bar{\Phi}_2 = \mu A^T P, \\
\bar{\Pi}_{22} &= -e^{-2\alpha h_1}Q_1 - 4e^{-2\alpha h_1}\frac{1}{h_1}V_1, \\
\Pi_{44} &= -e^{-2\alpha\tau_1}Q_3 - 4e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2, \\
\bar{\Pi}_{66} &= -L_1 W_2, \bar{\Pi}_{77} = -W_1 + \epsilon X_2^T X_2, \\
\bar{\Pi}_{88} &= -W_2 + \epsilon X_4^T X_4, \rho = \gamma - \alpha, \\
\bar{\Pi}_{99} &= h_1 V_1 + \tau_1 V_2 - 2\mu P, \\
\bar{\Pi}_{10,10} &= -\mu PC - \mu C^T P + \epsilon X_3^T X_3, \\
\bar{\Pi}_{11,11} &= -h_1 e^{-2\alpha h_1} R_1 - 12e^{-2\alpha h_1}\frac{1}{h_1}V_1, \\
\bar{\Pi}_{12,12} &= -\tau_1 e^{-2\alpha\tau_1} R_2 - 2\Lambda - \mu PC - \mu C^T P, \\
O_{12} &= -2e^{-2\alpha h_1}\frac{1}{h_1}V_1,\ O_{14} = -2e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2, \\
\bar{O}_{16} &= 6e^{-2\alpha h_1}\frac{1}{h_1}V_1,\ \Lambda = 6e^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2.
\end{aligned}$$

Moreover, the gain matrix in the periodically intermittent controller (4) is $K = (PE)^{-1}Z$.

Proof In (8), let $Q_5 = Q_6 = 0$, $S_1 = S_2 = 0$, $U_1 = 0$, and replace (17) and (24) by the following equalities respectively

$$\begin{aligned}
&2\mu[\dot{x}^T(t) - \dot{x}^T(t-\tau(t))]P[(C+\Delta C(t))\dot{x}(t-\tau(t)) \\
&\quad - (A + \Delta A(t) - EK)x(t) + (B + \Delta B(t))f(x(t)) \\
&\quad + (D + \Delta D(t))f(x(t-h(t))) - \dot{x}(t)] = 0, \\
&2\mu[\dot{x}^T(t) - \dot{x}^T(t-\tau(t))]P[(C+\Delta C(t))\dot{x}(t-\tau(t)) \\
&\quad - (A + \Delta A(t))x(t) + (B + \Delta B(t))f(x(t)) \\
&\quad + (D + \Delta D(t))f(x(t-h(t))) - \dot{x}(t)] = 0.
\end{aligned}$$

The proof is similar to the proof of Theorem 1, which is omitted. □

Remark 2 Consider the neural network (1) with $\tau(t) = h(t)$, system (1) can be written as

$$\begin{aligned}
\dot{x}(t) - (C + \Delta C(t))\dot{x}(t-h(t)) &= -(A + \Delta A(t))x(t) \\
&\quad + (B + \Delta B(t))f(x(t)) + (D + \Delta D(t))f(x(t-h(t))) \\
&\quad + Eu(t), \quad t \geq 0, \\
x(t) &= \phi(t), \quad \forall t \in [-\bar{h}, 0],
\end{aligned} \tag{39}$$

and then, we have the following corollary.

Corollary 1 *Suppose that Assumption 1 and 2 are satisfied. For given constants $\alpha > 0$ and γ, system (39) is robustly α-exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0\,(i = 1,2,3,4,5,6)$, $R_j > 0$, $S_j > 0$, $V_j > 0$, $W_j > 0\,(j = 1,2)$, $U_1 > 0$, Z, K and positive scalars $\mu > 0$ and $\epsilon > 0$ such that*

$$\check{\Pi}_j = \begin{bmatrix} \check{\Xi}_{11,j} & \check{\Xi}_{12,j} \\ * & \check{\Xi}_{22} \end{bmatrix} < 0, \quad j = 1,2, \tag{40}$$

$$\alpha\delta - \rho(T - \delta) > 0, \tag{41}$$

where

$$\check{\Xi}_{11,j} = \begin{bmatrix} \check{\Psi}_j & -2T_1 & 0 & 0 \\ * & \check{\Pi}_{22} & 0 & 0 \\ * & * & -e^{-2\alpha h_2}(Q_2 + Q_4) & 0 \\ * & * & * & \check{\Pi}_{44} \end{bmatrix},$$

$$\check{\Xi}_{12,j} = \begin{bmatrix} \check{\Pi}_{15} & T_2 & \check{\Phi}_j & PC - \epsilon X_1^T X_3 & 6T_1 & PH \\ 0 & 0 & 0 & 0 & 6T_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2W_2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\check{\Xi}_{22} = \begin{bmatrix} \check{\Pi}_{55} & \epsilon X_2^T X_4 & \mu B^T P & \epsilon X_2^T X_3 & 0 & 0 \\ * & \check{\Pi}_{66} & \mu D^T P & \epsilon X_4^T X_3 & 0 & 0 \\ * & * & \check{\Pi}_{77} & \mu PC & 0 & \mu PH \\ * & * & * & \check{\Pi}_{88} & 0 & 0 \\ * & * & * & * & \check{\Pi}_{99} & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix},$$

$$
\begin{aligned}
\breve{\Psi}_1 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 \\
&\quad + h_1(R_1 + R_2) - PA - A^T P + Z + Z^T \\
&\quad - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}(V_1 + V_2) - L_1 W_1 + \epsilon X_1^T X_1, \\
\breve{\Psi}_2 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 \\
&\quad + h_1(R_1 + R_2) - PA - A^T P \\
&\quad - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}(V_1 + V_2) - L_1 W_1 + \epsilon X_1^T X_1, \\
\breve{\Pi}_{15} &= PB + L_2 W_1 - \epsilon X_1^T X_2, \\
\breve{\Phi}_1 &= -\mu A^T P + \mu Z^T, \ \ \breve{\Phi}_2 = -\mu A^T P, \\
\breve{\Pi}_{22} &= -\mathrm{e}^{-2\alpha h_1}(Q_1 + Q_3) - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}(V_1 + V_2), \\
\breve{\Pi}_{44} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2}(Q_5 + Q_6) - L_1 W_2, \\
\breve{\Pi}_{55} &= U_1 - W_1 + \epsilon X_2^T X_2, \ \ T_2 = PD - \epsilon X_1^T X_4, \\
\breve{\Pi}_{66} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2} U_1 - W_2 + \epsilon X_4^T X_4, \\
\breve{\Pi}_{77} &= S_1 + S_2 + h_1(V_1 + V_2) - 2\mu P, \\
\breve{\Pi}_{88} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2}(S_1 + S_2) + \epsilon X_3^T X_3, \\
\breve{\Pi}_{99} &= -h_1 \mathrm{e}^{-2\alpha h_1}(R_1 + R_2) - 12\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}(V_1 + V_2), \\
T_1 &= \mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}(V_1 + V_2), \quad \rho = \gamma - \alpha.
\end{aligned}
$$

Moreover, the gain matrix in the periodically intermittent controller (4) is $K = (PE)^{-1}Z$.

Remark 3 Consider the neural network (1) without parametric uncertainty, system (1) can be written as

$$
\begin{aligned}
&\dot{x}(t) - C\dot{x}(t - \tau(t)) = -Ax(t) + Bf(x(t)) \\
&\quad + Df(x(t - h(t))) + Eu(t), \quad t \geq 0, \\
&\qquad x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0],
\end{aligned} \tag{42}
$$

and then, we have the following corollary.

Corollary 2 *Suppose that Assumption 1 and 2 are satisfied. For given constants $\alpha > 0$ and γ, system (42) is α-exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0\,(i = 1, 2, 3, 4, 5, 6)$, $R_j > 0$, $S_j > 0$, $V_j > 0$, $W_j > 0\,(j = 1, 2)$, $U_1 > 0$, Z, K and positive scalars $\mu > 0$ and $\epsilon > 0$ such that*

$$
\acute{\Pi}_j = \begin{bmatrix} \acute{\Xi}_{11,j} & \acute{\Xi}_{12,j} \\ * & \acute{\Xi}_{22} \end{bmatrix} < 0, \quad j = 1, 2, \tag{43}
$$

$$
\alpha\delta - \rho(T - \delta) > 0, \tag{44}
$$

where

$$
\acute{\Xi}_{11,j} = \begin{bmatrix}
\acute{\Psi}_j & O_{12} & 0 & O_{14} & 0 & 0 & 0 \\
* & \acute{\Pi}_{22} & 0 & 0 & 0 & 0 & 0 \\
* & * & O_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \acute{\Pi}_{44} & 0 & 0 & 0 \\
* & * & * & * & O_{55} & 0 & 0 \\
* & * & * & * & * & \acute{\Pi}_{66} & 0 \\
* & * & * & * & * & * & \acute{\Pi}_{77}
\end{bmatrix},
$$

$$
\acute{\Xi}_{12,j} = \begin{bmatrix}
\acute{\Pi}_{18} & PD & \acute{\Phi}_j & 0 & PC & \bar{O}_{16} & 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 0 & 0 & 0 & 0 & \bar{O}_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_2 W_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\acute{\Xi}_{22} = \begin{bmatrix}
\acute{\Pi}_{88} & 0 & \mu B^T P & 0 & 0 & 0 & 0 \\
* & \acute{\Pi}_{99} & \mu D^T P & 0 & 0 & 0 & 0 \\
* & * & \acute{\Pi}_{10,10} & 0 & \mu PC & 0 & 0 \\
* & * & * & \acute{\Pi}_{11,11} & 0 & 0 & 0 \\
* & * & * & * & \acute{\Pi}_{12,12} & 0 & 0 \\
* & * & * & * & * & \acute{\Pi}_{13,13} & 0 \\
* & * & * & * & * & * & \acute{\Pi}_{14,14}
\end{bmatrix},
$$

$$
\begin{aligned}
\acute{\Psi}_1 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + h_1 R_1 \\
&\quad + \tau_1 R_2 - PA - A^T P + Z + Z^T \\
&\quad - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1 - 4\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 - L_1 W_1, \\
\acute{\Psi}_2 &= 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 \\
&\quad + h_1 R_1 + \tau_1 R_2 - PA - A^T P \\
&\quad - 4\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1 - 4\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2 - L_1 W_1, \\
\acute{\Pi}_{18} &= PB + L_2 W_1, \quad \acute{\Phi}_1 = -\mu A^T P + \mu Z^T, \\
\acute{\Phi}_2 &= -\mu A^T P, \quad \acute{\Pi}_{22} = -\mathrm{e}^{-2\alpha h_1}Q_1 - 4e^{-2\alpha h_1}\frac{1}{h_1}V_1, \\
\acute{\Pi}_{44} &= -\mathrm{e}^{-2\alpha\tau_1}Q_3 - 4\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2,
\end{aligned}
$$

$$
\begin{aligned}
\acute{\Pi}_{66} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2}Q_5 - L_1 W_2, \\
\acute{\Pi}_{77} &= -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2}Q_6, \acute{\Pi}_{88} = U_1 - W_1, \\
\acute{\Pi}_{99} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2}U_1 - W_2, \\
\acute{\Pi}_{10,10} &= S_1 + S_2 + h_1 V_1 + \tau_1 V_2 - 2\mu P, \\
\acute{\Pi}_{11,11} &= -(1 - h_d)\mathrm{e}^{-2\alpha h_2}S_1, \\
\acute{\Pi}_{12,12} &= -(1 - \tau_d)\mathrm{e}^{-2\alpha\tau_2}S_2, \\
\acute{\Pi}_{13,13} &= -h_1\mathrm{e}^{-2\alpha h_1}R_1 - 12\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}M, \\
\acute{\Pi}_{14,14} &= -\tau_1\mathrm{e}^{-2\alpha\tau_1}R_2 - 12\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}N, \ \rho = \gamma - \alpha. \\
O_{12} &= -2\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1, O_{14} = -2\mathrm{e}^{-2\alpha\tau_1}\frac{1}{\tau_1}V_2, \\
O_{33} &= -\mathrm{e}^{-2\alpha h_2}Q_2, \quad O_{55} = -\mathrm{e}^{-2\alpha\tau_2}Q_4, \\
\bar{O}_{16} &= 6\mathrm{e}^{-2\alpha h_1}\frac{1}{h_1}V_1.
\end{aligned}
$$

Moreover, the gain matrix in the periodically intermittent controller (4) is $K = (PE)^{-1}Z$.

Remark 4 For given $\alpha > 0$ and γ, we note that (6), (37), (40), and (43) are linear matrix inequalities which can be solved efficiently by MATLAB LMI Toolbox.

Remark 5 The proposed intermittent state-feedback controller can ensure robustly α-exponential stabilization of system (1) in Theorem 1. If $\alpha > 0$ and γ are given, the feasibility problem of LMI can be solved to get a suitable stabilization controller gain.

4 Numerical examples

In this section, we will provide numerical examples to illustrate the effectiveness and the merits of the obtained results.

Example 1 Consider the following uncertain neutral neural networks with mixed time-varying delays

$$\begin{aligned} &\dot{x}(t) - (C + \Delta C(t))\dot{x}(t - \tau(t)) = -(A + \Delta A(t))x(t) \\ &+ (B + \Delta B(t))f(x(t)) + (D + \Delta D(t))f(x(t - h(t))) \\ &+ Eu(t), \quad t \geq 0, \end{aligned} \tag{45}$$

where

$$A = \begin{bmatrix} 2.8 & 0 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.9 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.04 & 0 \\ 0.5 & 0.08 \end{bmatrix}, \quad E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$
$$f(x(t)) = (0.12\tanh(x_1(t)), \quad 0.12\tanh(x_2(t)))^T,$$
$$h(t) = 0.05\sin(t) + 0.2, \quad \tau(t) = 0.05\cos(t) + 0.2.$$

The parametric uncertainties are given by

$$[\Delta A(t)\ \Delta B(t)\ \Delta C(t)\ \Delta D(t)] = HF(t)[X_1\ X_2\ X_3\ X_4],$$

where

$$H = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 0.1 & 0 \\ 0.7 & 0.1 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix},$$
$$X_3 = \begin{bmatrix} 0.1 & 0.4 \\ 0 & 0.1 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 0.1 & 0.6 \\ 0.8 & 0.1 \end{bmatrix},$$

and $F(t)$ is unknown real time-varying matrix with Lebesgue measurable elements bounded by

$$F^T(t)F(t) \leq I.$$

We have that $l_1^- = l_2^- = 0$, $l_1^+ = l_2^+ = 0.12, h_1 = 0.15, h_2 = 0.25, h_d = 0.05, \tau_1 = 0.15, \tau_2 = 0.25, \tau_d = 0.05$. Take $T = 2, \delta = 1.95, \mu = 0.7, \alpha = 0.001, \gamma = 0.027$.

By utilizing the MATLAB LMI Toolbox solving (6) and (7), feasible solutions can be obtained as follows:

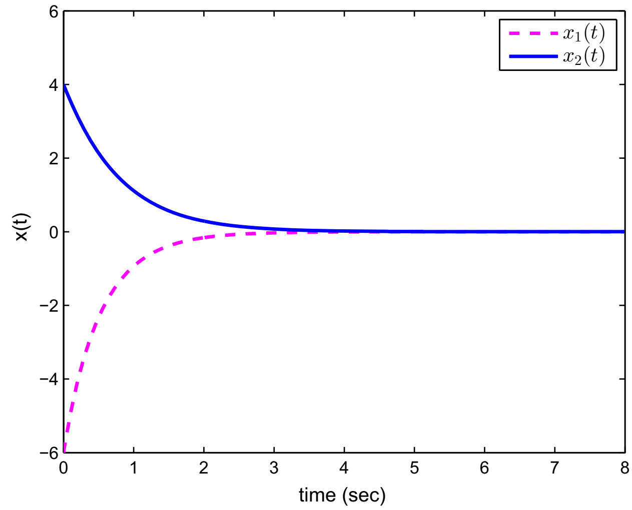

Fig. 1 The trajectories of $x_1(t)$ and $x_2(t)$ of system (45) via intermittent control

$$P = \begin{bmatrix} 4.9163 & 0.0484 \\ 0.0484 & 7.3599 \end{bmatrix},$$
$$Q_i = \begin{bmatrix} 0.9855 & 0.0751 \\ 0.0751 & 0.7639 \end{bmatrix}, \quad i = 1, 2, 3, 4, 5, 6,$$
$$R_1 = R_2 = \begin{bmatrix} 23.1202 & -7.6185 \\ -7.6185 & 14.7255 \end{bmatrix},$$
$$S_1 = S_2 = \begin{bmatrix} 3.8705 & 0.7055 \\ 0.7055 & 5.7581 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 16.2639 & -2.7538 \\ -2.7538 & 16.6249 \end{bmatrix}, V_2 = \begin{bmatrix} 16.2639 & -2.7538 \\ -2.7538 & 16.6249 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 543.3057 & -83.0160 \\ -83.0160 & 573.2388 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 242.7700 & -67.0821 \\ -67.0821 & 91.9672 \end{bmatrix},$$
$$U_1 = \begin{bmatrix} 110.7938 & -20.7812 \\ -20.7812 & 74.2520 \end{bmatrix}, Z = \begin{bmatrix} 4.1438 & -1.0334 \\ -1.0334 & 3.4752 \end{bmatrix}.$$

Furthermore, a desired intermittent feedback gain matrix is derived by utilizing Theorem 1:

$$K = (PE)^{-1}Z = \begin{bmatrix} 0.4222 & -0.1074 \\ -0.0730 & 0.2368 \end{bmatrix}.$$

The intermittent feedback controller is

$$u(t) = \begin{cases} \begin{bmatrix} 0.4222 & -0.1074 \\ -0.0730 & 0.2368 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \\ \qquad t \in [2k, 2k + 1.95), \\ 0, \quad t \in [2k + 1.95, 2(k + 1)). \end{cases}$$

In the simulation, we take $F(t) = 0.5I$. The trajectories of $x_1(t)$ and $x_2(t)$ of the closed-loop system (45) are shown in Fig. 1. The results show that the exponential stability of the closed-loop system can be achieved by using the proposed method of this paper.

Remark 6 In [14], the system is not neutral type and has no regard for the uncertain factors. So, the conditions in [14] cannot deal with this problem in Example 1. However, in practical neural networks, it is necessary to take parameter uncertainties into account. Compared with [14], the results obtained in this paper have a greater range of applications.

Example 2 Consider the following neutral neural networks with mixed time-varying delays

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-\tau(t)) &= -Ax(t) + Bf(x(t)) \\ &\quad + Df(x(t-h(t))) + Eu(t), \quad t \geq 0,\end{aligned} \tag{46}$$

where

$$A = \begin{bmatrix} 2.8 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 1 \\ 1 & 0.9 \end{bmatrix}, C = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.04 & 0 \\ 0.6 & 0.08 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.5 \end{bmatrix},$$
$$f(x(t)) = (0.12\tanh(x_1(t)) \quad 0.12\tanh(x_2(t)))^T,$$
$$h(t) = 0.05\sin(t) + 0.2. \quad \tau(t) = 0.05\cos(t) + 0.2.$$

We have that $l_1^- = l_2^- = 0$, $l_1^+ = l_2^+ = 0.12$, $h_1 = 0.15, h_2 = 0.25, h_d = 0.05, \tau_1 = 0.15, \tau_2 = 0.25, \tau_d = 0.05$. We take $T = 2, \delta = 1.95, \mu = 0.7, \alpha = 0.01, \gamma = 0.027$.

By utilizing the MATLAB LMI Toolbox solving (43) and (44), feasible solutions can be obtained as follows:

$$P = \begin{bmatrix} 1011.7 & -115.4 \\ -115.4 & 1028.8 \end{bmatrix}, Q_i = \begin{bmatrix} 307.8868 & -29.1719 \\ -29.1719 & 277.9632 \end{bmatrix},$$
$$i = 1,2,3,4,5,6,$$
$$R_1 = R_2 = \begin{bmatrix} 543.1801 & -45.7232 \\ -45.7232 & 506.7260 \end{bmatrix},$$
$$S_1 = S_2 = \begin{bmatrix} 157.5384 & -23.6084 \\ -23.6084 & 210.0452 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 6.0984 & 0.1734 \\ 0.1734 & 6.2745 \end{bmatrix}, U_1 = \begin{bmatrix} 450.6355 & -31.1552 \\ -31.1552 & 356.8329 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 1673.8 & 484.6 \\ 484.6 & 1799.0 \end{bmatrix}, W_2 = \begin{bmatrix} 681.3510 & 38.4059 \\ 38.4059 & 530.1901 \end{bmatrix},$$
$$V_2 = \begin{bmatrix} 6.0984 & 0.1734 \\ 0.1734 & 6.2745 \end{bmatrix}, Z = \begin{bmatrix} 1000.4 & -236.9 \\ -236.9 & 323.9 \end{bmatrix}.$$

Furthermore, a desired intermittent feedback gain matrix is derived by utilizing Corollary 2 :

$$K = (PE)^{-1}Z = \begin{bmatrix} 1.0234 & -0.3177 \\ -0.2418 & 0.5847 \end{bmatrix}.$$

The intermittent feedback controller is

$$u(t) = \begin{cases} \begin{bmatrix} 1.0234 & -0.3177 \\ -0.2418 & 0.5847 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \\ \qquad t \in [2k, 2k+1.95), \\ 0, \quad t \in [2k+1.95, 2(k+1)). \end{cases}$$

The trajectories of $x_1(t)$ and $x_2(t)$ of the closed-loop system (46) are shown in Fig. 2. From Fig. 2, it is easy to see that the closed-loop system is exponentially stable.

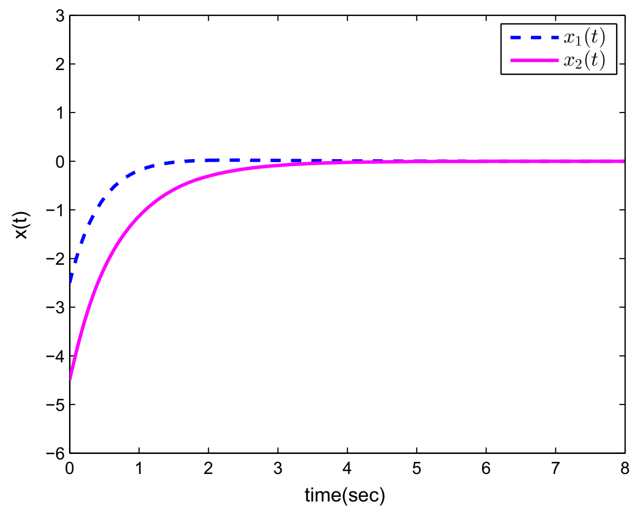

Fig. 2 The trajectories of $x_1(t)$ and $x_2(t)$ of system (46) via intermittent control

5 Conclusion

This paper has addressed the problem of robust exponential stabilization for a class of uncertain neutral neural networks with multiple time-varying delays via periodically intermittent control. Based on Lyapunov–Krasovskii functions method and some useful lemmas, the sufficient conditions are derived to guarantee the exponential stabilization of considered uncertain neutral neural networks in terms of linear matrix inequalities technique. The obtained sufficient conditions use more information of the upper and lower delay bounds and can be easily solved using any of the available standard software. Finally, numerical examples are given to show the effectiveness of the proposed method.

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Compliance with ethical standards

Conflict of interest The authors declared that they have no conflict of interest.

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