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ORIGINAL ARTICLE

New multi-criteria LNN WASPAS model for evaluating the work of advisors in the transport of hazardous goods

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Abstract
Successfully organizing the transport of hazardous materials and handling them correctly is a very important logistical task that affects both the overall flow of transport and the environment. Safety advisors for the transport of hazardous materials have a very important role to play in the proper and safe development of the transport flow for these materials; their task is primarily to use their knowledge and effort to prevent potential accidents from happening. In this research, a total of 21 safety advisors for the transport of hazardous materials in Serbia are assessed using a new model that integrates Linguistic Neutrosophic Numbers (LNN) and the WASPAS (Weighted Aggregated Sum Product Assessment) method. In this way, two important contributions are made, namely a completely new methodology for assessing the work of advisors and the new LNN WASPAS model, which enriches the field of multi-criteria decision making. The advisors are assessed by seven experts on the basis of nine criteria. After performing a sensitivity analysis on the results, validation of the model is carried out. The results obtained by the LNN WASPAS model are validated by comparing them with the results obtained by LNN extensions of the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution), LNN CODAS (COmbinative Distance-based ASsessment), LNN VIKOR (Multi-criteria Optimization and Compromise Solution) and LNN MABAC (Multi-Attributive Border Approximation area Comparison) models. The LNN CODAS, LNN VIKOR and LNN MABAC are also further developed in this study, which is an additional contribution made by the paper. After the sensitivity analysis, the SCC (Spearman Correlation Coefficient) is calculated which confirms the stability of the previously obtained results.

Keywords Linguistic neutrosophic numbers · WASPAS · Multi-criteria decision making · Hazardous goods

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1 Introduction

The total organization of the transportation of hazardous goods is a technologically demanding task. The standards and conditions that must be fulfilled by all participants, as well as the infrastructure and basic resources (packaging, mobile pressure equipment, tanks, vehicles, tank cars and/or ships) far exceed the standards and requirements placed on the transport of other types of goods. Generations of experts from various branches, primarily chemists and engineers of all necessary profiles, have been constantly developing technical and technological systems for the safer transport of hazardous goods. Today, it is one of the most organized logistical areas at an international level.

Based on recommendations by the United Nations (UN), the Economic Commission for Europe (UNECE) and the Land Transport Committee are responsible for the adoption of regulations in the field of transporting hazardous goods. Interested countries have accepted and confirmed these regulations in the form of agreements. In the Annex to these agreements, the Technical Regulations define the conditions, norms, standards and rules for transporting hazardous goods safely on international roads [55] railways [44] and inland waterways [12].

The European Union (EU) adopted [10] on the land transport of hazardous goods, the amendments of which are ADR, RID and ADN, with a given deadline stated and agreed by the regulations of all EU member countries. This makes it possible to have more uniform, safe and cheap transportation of hazardous goods in the territory of all EU member states.

In securing the safe international transport of hazardous goods, the agreements have established the rights and obligations of their signatories, the management and working bodies, the manner of decision making and the amendments to the Technical Regulations attached to them. The technical regulations stipulate the obligations of the participants in the transport of hazardous goods: those who pack (fill) and dispatch them; those who tranship and store them; and those who receive the hazardous goods. The regulations also determine the packaging, the means of transport and the obligations of the professionals who fill, pack and transport the hazardous goods.

The safety advisor for the transport of hazardous goods is a legal institute in the technical regulations attached to an agreement. Each company whose activity is related to the transport of hazardous goods, or connected with their packaging, loading or unloading, must appoint one or more safety advisors [55]. The main task of this individual is to find the appropriate means and methods for the activities of the company, and the appropriate measures for applying the existing regulations, under optimal safety conditions, in order to make the activities of the company run more smoothly. The safety officer's role is to help to prevent risk to persons, property and the environment. Evaluation of the work of advisors is of great importance for every employer, taking into account the risks and possible consequences that occur during the transport of hazardous goods.

The present research aims to achieve the objective, already stated as evaluating and prioritizing advisors in the transport of hazardous goods. To resolve the difficulty of implementing adequate tools to evaluate advisors from a management perspective, this research raises the following questions:

1. What are the factors (criteria) that need to be considered in order to successfully evaluate and prioritize advisors in the transport of hazardous goods?
2. What is the novelty and present contribution of this multi-criteria decision-making approach?
3. What methodology or technique needs to be presented in order to prioritize advisors in the transport of hazardous goods?

In order to define the justification for using linguistic neutrosophic numbers, the following section chronologically presents the occurrence of linguistic neutrosophic numbers and their advantages identified by the authors for applying the concept in this study.

In the traditional decision-making process, researchers most commonly use the fuzzy technique and its various modifications [9]. The intent of the fuzzy technique is the transformation of crisp numbers into fuzzy numbers that depict the uncertainties of the real environment by means of membership functions [11]. According to Zadeh [62] and Zimmermann [66], fuzzy linguistic variables presented as fuzzy sets can be used very successfully to quantify uncertainty in complex and uncertain situations. Nevertheless, Karnik and Mendel [24] consider that the representation of linguistic expressions using traditional fuzzy sets (fuzzy sets of type 1) is not sufficiently clear and precise. Karnik and Mendel [24] further consider it more natural and precise to present linguistic expressions using fuzzy sets. Fuzzy linguistic variables can provide greater flexibility when presenting inaccurate and unclear information, especially in the process of group decision making characterized by a high degree of uncertainty [38]. Therefore, the application of linguistic variables in multi-criteria decision making (MCDM) presents itself as a logical step to ensure a sufficiently clear presentation of linguistic expressions by decision makers [66]. After introducing the concept of linguistic variables and their use in fuzzy logic [62], Herrera et al. [18] and Herrera and Herrera-Viedma [19] presented the possibility of using linguistic information in mathematical decision-making models. Then, Xu [59] set up a linguistic hybrid arithmetic averaging operator for group multi-criteria decision making using linguistic information. Later, Xu [60] developed a goal programming model for use in multi-criteria decision making with linguistic information. By combining the intuitive fuzzy numbers (IFN) proposed by Atanassov [3] and fuzzy linguistic variables proposed by Zadeh [62], Chen et al. [9] proposed intuitionistic linguistic fuzzy numbers (LIFN) denoted as $s = (l_p, l_q)$, where l_p and l_q are linguistic variables which, respectively, describe the degree of membership or non-membership of the given set. Since LIFN cannot successfully deal with all types of uncertainties in various real problems (such as problems with indeterminate information), Ye [61] introduced a single-valued neutrosophic linguistic number (SVNLN) consisting of linguistic variables and a single-valued neutrosophic number. With SVNLN the linguistic variable represents the assessment of the decision maker with regard to the object of evaluation, and the single-valued neutrosophic number expresses the reliability of the given linguistic variable.

However, SVNLN cannot be successfully used to represent truth, indeterminacy or falsity on the basis of linguistic variables. In order to overcome the above-mentioned deficiencies of LIFN and SVNLN, one of the solutions is to independently represent the degree of truth, indeterminacy and falsity of the object being evaluated using three independent linguistic variables. Based on this idea, the concept of a linguistic neutrosophic number (LNN) is proposed, which is a combination of a single-valued neutrosophic number (SVNN) and a linguistic variable [14]. In LNN, independent linguistic variables are used to represent the degree of truth, indeterminacy and falsity, not a precise numerical value as in SVNN, or a linguistic variable and SVNN, as with SVNLN. With the introduction of the LNN concept the previously mentioned deficiencies of LIFN and SVNLN are eliminated. For this reason, LNN is a very interesting concept to study since it enables the presentation of the uncertain and inconsistent linguistic information present in human reasoning in complex systems. LNN are very suitable for representing linguistic information about the complex attributes of decisions, especially when it comes to qualitative attributes, since LNNs simultaneously use the advantages of both SVNN and linguistic variables.

Thus the main question arising in this paper is: How does this present methodology aid the evaluation and prioritizing of advisors in the sector for transporting hazardous goods, and what is the main advantage behind it?

Taking into account the above information, as well as the relevant literature and judgments of both academic and transport experts, the authors of the present paper propose an LNN-based framework for the WASPAS approach, with an original LNN model for determining the weight coefficients of the evaluation criteria. An LNN-based approach for determining the weights of the criteria and ranking the alternatives (advisors) provides deeper insight into the decision makers' perceptions from the perspective of management. For the purpose of accepting the imprecision and subjectivity in the collective decision-making process, this paper modifies the WASPAS method by applying LNN. Therefore, the main goal of this research paper is to explore an effective procedure for evaluating advisors in the transport sector for hazardous goods, using an LNN-based MCDM approach. Finally, the work of 21 advisors for the transport of hazardous goods was evaluated from 21 companies in Serbia in a case study, with the purpose of validating the proposed model.

The paper is organized so that after the introduction, Sect. 2 contains a literature review. Section 3 presents the linguistic neutrosophic concept and its basic arithmetic operations with LNN. The model for evaluating the work of advisors using LNN WASPAS is formed in Sect. 4. Sections 5 and 6 contain the discussion and validation of the results. Finally, this is followed by the conclusion and list of references.

2 Literature review

Since the problem of evaluating advisors in the transport sector for hazardous goods has not yet been considered in the literature, this section presents a brief review of the literature that includes the application of multi-criteria models in different areas of transport. Over the last two decades, various decision-making methods have been proposed to address various transportation problems. This study provides an empirical investigation in the field of transporting hazardous goods and evaluating personnel in the field of transport, and it conducts a literature survey to fill the gaps in the identification of crucial criteria for evaluating advisors. This paper aims to answer the following research questions (RQs):

RQ1 What multiple criteria decision-making trends have recently developed in the area of transporting hazardous goods and evaluating personnel in the field of transport in general?
RQ2 What are the relevant criteria generally considered in the area of evaluating personnel in the field of transport?
RQ3 What are the recent works in the transport sector that relate to the evaluation of advisors in the transport sector?

To answer these questions, we analyze academic peer-reviewed articles, published recently from 2015 to 2017.

According to Stević et al. [53], rationalizing the activities in a transport system plays a very important role in achieving the necessary effects, both as a transport system and as a traffic system considered from the aspect of a country's economy. By using methods of multi-criteria decision making [33, 36, 50] while taking into account the uncertainty and imprecision that exist in them it is possible to make correct and valid decisions in the field of transport. Lately, neutrosophic sets have often been used in these processes since they enable and support the above. Interval-valued neutrosophic sets (IVNS) were used in Kour and Basu [26] to evaluate the types of transport in four transport companies based on five criteria to select the best company. Neutrosophic sets were used by Thamaraiselvi and Santhi [54] to formulate and solve a transport problem with the goal of minimizing costs between the source and the destination, and in the following year in a study by Singh et al. [47] this approach was modified and perfected for the same purpose. A simplified neutrosophic linguistic MCDM approach based on MULTIMOORA [57] was applied in a company that produces transport construction machines. Rizk-Allah et al. [45] presented a novel neutrosophic compromise programming approach (NCPA) to deal with multi-objective transportation.

MCDM methods in combination with neutrosophic sets have been used to create various new approaches to solving problems in different areas. Ji et al. [22] proposed an extension of the TODIM method with multi-valued neutrosophic sets (MVNSs) in the field of selecting personnel. The new method makes it possible to eliminate the faults in the Fuzzy TODIM method. Bausys et al. [6] developed a new approach in their study by integrating the COPRAS method with single-valued neutrosophic sets (SVNS) to solve the location problem for selecting liquid gas terminals. Peng and Dai [41] used the single-valued neutrosophic MABAC and TOPSIS method, the application of which supports a reduction in the loss of information and to a great extent retains its originality. The integration of single-valued neutrosophic sets and the WASPAS method was used in the selection of a lead–zinc flotation circuit design in Zavadskas et al. [63], as well as in Baušys and Juodagalvienė [4] where it was used to determine the location problem of a garage for a residential house. WASPAS was extended in Nie et al. [32] with interval neutrosophic sets for the solar–wind power station location selection problem. An approach based on a single-valued neutrosophic set and the MULTIMOORA method in Zavadskas et al. [64] was successfully applied to the selection of materials in the construction industry. Huang et al. [20] evaluated five possible emerging technology enterprises on the basis of four criteria using a new approach that was the result of integrating interval neutrosophic numbers (INNs) and the VIKOR method. Otay and Kahraman [35] carried out the selection of Six Sigma projects using the interval neutrosophic TOPSIS method, while Bolturk and Kahraman [8] developed a new approach to the Analytic Hierarchy Process (AHP) method with interval-valued neutrosophic sets in the selection of energy alternatives. The Neutrosophic Analytic Hierarchy Process approach was also used in Radwan et al. [43] for the selection of a learning management system. Tian et al. [56] evaluated segments of the market using QFD, the single-valued neutrosophic DEMATEL and single-valued neutrosophic TODIM methods. In their research, Stanujkic et al. [51] used SWARA and a single-valued neutrosophic set to evaluate four restaurants, while Bausys and Zavadskas [5] applied interval-valued neutrosophic sets and the VIKOR method to select the location for a logistics terminal. A single-valued trapezoidal neutrosophic decision-making trial and evaluation laboratory was used by Liang et al. [28] to evaluate e-websites.

Karaşan and Kahraman [23] evaluated suppliers through the development of a new approach that integrates interval-valued neutrosophic sets and the EDAS method, while for the same purpose [46] applied the single-valued neutrosophic TOPSIS method. Abdel-Basset et al. [1] also evaluate and select suppliers using a combination of MCDM and neutrosophic sets. However, according to Ali et al. [2] a single-valued complex neutrosophic set has some difficulties in defining the degree of membership, and so these authors propose the application of an interval complex neutrosophic set (ICNS) approach, which is verified through the selection of suppliers in the green supply chain for a transport company.

In view of the fact that this field is developing very rapidly and is constantly striving to eliminate any drawbacks, Fang and Ye [14] proposed the concept of linguistic neutrosophic numbers (LNNs), which overcomes the deficiencies in linguistic intuitionistic fuzzy numbers (LIFNs). Only linguistic membership degrees and linguistic non-membership degrees are reflected in LIFNs. The concept of linguistic neutrosophic numbers (LNNs) is based on linguistic terms and simplified neutrosophic numbers [30, 40]. The difference between LNNs and neutrosophic linguistic numbers (NLNs) according to [58] is that NLNs have only a linguistic value, and the truth-membership, indeterminacy-membership, and false-membership are crisp numbers. Although only a short time has passed since the creation of this concept, the application of LNNs can already be seen in a few publications [14, 29, 30, 58].

By reviewing the available literature, there were no works found in which criteria for evaluating the work of advisors for transport of dangerous goods were defined. In the works [27, 39, 31, 16], the subject of research was the work of advisers in the fields of finance, industry, agriculture, etc. The authors of the papers included the classical procedures of the work of advisors or procedures specific to certain fields, while the subject of the research of any work is not narrowly specialized for the work of advisers in the transport of dangerous goods. That is why the contribution of this work is significant, since the authors in cooperation with the experts from this specific field first defined and selected relevant criteria for evaluating the work of the advisors for the transport of dangerous goods.

Here, we will discuss the research gaps (limitations) in our literature study, summarize the findings of the earlier sections and derive possible trends of MCDM applications in transport: (1) regarding the limitations of our study (literature survey), the review was restricted to academic peer-reviewed articles. Textbooks, master theses and doctoral dissertations were thus not selected; furthermore, only articles in English were considered; (2) moreover, our investigation is based on a keyword search in the ISI Web of Science database. Hence, it is possible that some relevant articles did not match our search terms or were not listed in the databases searched; (3) this review has applications rather than theoretical orientation and integrates many techniques in a simplified framework; (4) hence, there is a gap in the literature on the applications of MCDM in the process of evaluating advisors in the sector for transporting hazardous goods in recent years, specifically focusing on empirical challenges and the pros and cons of alternative MCDM techniques; (5) so far in the literature, the criteria for evaluating advisors in the sector for transporting hazardous goods have not yet been considered, neither has the application of the MCDM technique been considered for solving this problem. This study presents criteria for the evaluation of advisors from research that included a literature analysis and interviewing experts from the Department for Transport of Dangerous Goods at the Ministry of Construction and Infrastructure of the Republic of Serbia, traffic inspectors and persons who lead professional associations in this field in the Republic of Serbia.

3 The multi-criteria LNN WASPAS model based on linguistic neutrosophic numbers

In the following section (Sect. 3.1), the basic model of the neutrosophic concept is given, as well as the basic arithmetic operations for LNN. Then, in Sect. 3.2 the LNN WASPAS multi-criteria model is presented based on the LNN concept (Fig. 1).

3.1 Some concepts of LNN

According to the definition of a neutrosophic set, neutrosophic set A is universal set X which is characterized by truth-membership function $T_A(x)$, by indeterminacy-membership function $I_A(x)$ and by falsity-membership function $F_A(x)$ [48, 49] where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets from $[^-0, 1^+]$, so that all of these three neutrosophic components satisfy the condition that $T_A(x) \rightarrow [^-0, 1^+]$, $I_A(x) \rightarrow [^-0, 1^+]$ and $F_A(x) \rightarrow [^-0, 1^+]$.

Set $I_A(x)$ can be used to represent not only indeterminacy, but also lack of clarity, uncertainty, imprecision, errors, contradictions, and the undefined, unknown, incomplete, redundant, etc. [7, 15]. In order to include all unclear information, the indeterminacy-membership degree can be divided into subcomponents, such as "contradiction," "uncertainty" and "unknown" [49].

The sum of these three membership functions in a neutrosophic set $T_A(x)$, $I_A(x)$ and $F_A(x)$ should satisfy the following condition [7] $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. A component of neutrosophic set A for all values of $x \in X$

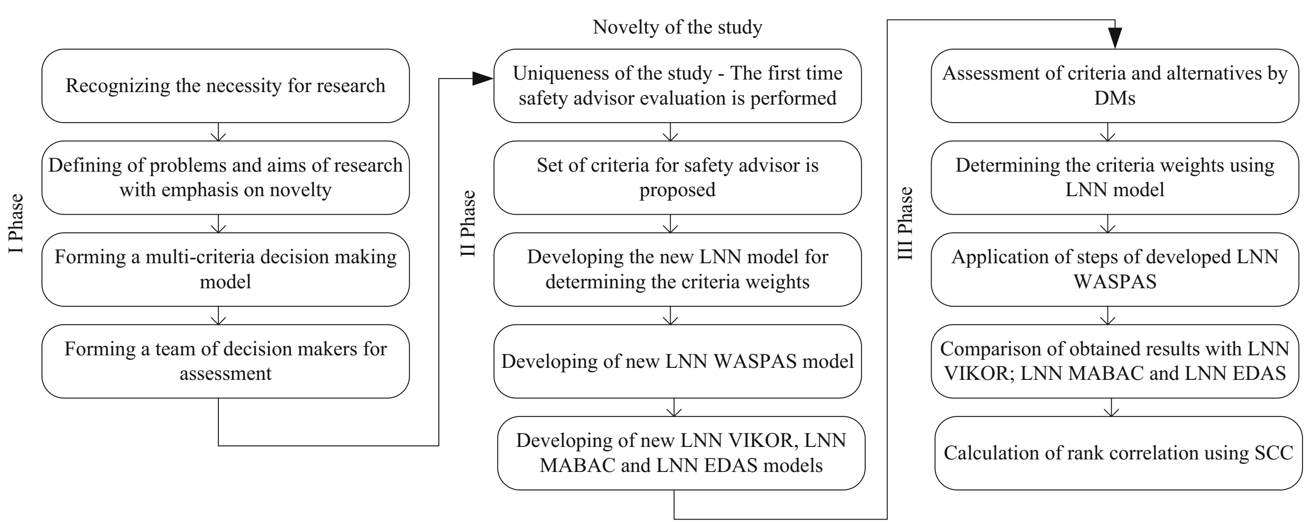

Fig. 1 LNN WASPAS model

is determined with A^C so that $T_A^c(x) = 1^+ - T_A(x)$, $I_A^c(x) = 1^+ - I_A(x)$ and $F_A^c(x) = 1^+ - F_A(x)$. Neutrosophic set A is contained in another set, neutrosophic set B (i.e., $A \subseteq B$) if and only if for each value of $x \in X$ the following conditions are satisfied: $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$ and $\sup F_A(x) \ge \sup F_B(x)$.

Because of the ambiguity of human thinking, the reasoning of experts and their preferences in complex decision-making conditions is difficult to present with numerical values. The use of linguistic terms makes it possible to have a much more convenient and more reliable presentation of the expert preferences, especially when it comes to qualitative attributes that describe certain phenomena. Based on the idea of neutrosophic sets, the concept of linguistic neutrosophic numbers (LNN) is proposed, which is a combination of single-valued neutrosophic numbers [65] and linguistic variables. In LNN, independent linguistic variables are used to represent the degree of truth, indeterminacy and falsity, but not like in single-valued neutrosophic numbers in which exact numerical values are used.

Definition 1 Suppose that $\Omega = \{\varphi_0, \varphi_1, \ldots, \varphi_t\}$ is a linguistic set with odd cardinality $t+1$. If $\varphi = \langle \xi_p, \xi_q, \xi_r \rangle$ is defined for $\xi_p, \xi_q, \xi_r \in \Omega$ and $p, q, r \in [0, t]$, where ξ_p, ξ_q and ξ_r represent linguistic expressions which independently express the degree of truth, indeterminacy and falsity, then e is called an LNN.

Definition 2 Let $\varphi = \langle \xi_p, \xi_q, \xi_r \rangle$, $\varphi_1 = \langle \xi_{p_1}, \xi_{q_1}, \xi_{r_1} \rangle$ and $\varphi_2 = \langle \xi_{p_2}, \xi_{q_2}, \xi_{r_2} \rangle$ be three LNNs in Ω and $k > 0$, then we can define the arithmetic operations on LNN [30]:

(1) Adding LNN "+"

$$\begin{aligned}\varphi_1 + \varphi_2 &= \langle \xi_{p_1}, \xi_{q_1}, \xi_{r_1} \rangle + \langle \xi_{p_2}, \xi_{q_2}, \xi_{r_2} \rangle \\ &= \left\langle \xi_{p_1 + p_2 - \frac{p_1 p_2}{t}}, \xi_{\frac{q_1 q_2}{t}}, \xi_{\frac{r_1 r_2}{t}} \right\rangle\end{aligned} \tag{1}$$

(2) Multiplying LNN "×"

$$\begin{aligned}\varphi_1 \times \varphi_2 &= \langle \xi_{p_1}, \xi_{q_1}, \xi_{r_1} \rangle \times \langle \xi_{p_2}, \xi_{q_2}, \xi_{r_2} \rangle \\ &= \left\langle \xi_{\frac{p_1 p_2}{t}}, \xi_{q_1 + q_2 - \frac{q_1 q_2}{t}}, \xi_{r_1 + r_2 - \frac{r_1 r_2}{t}} \right\rangle\end{aligned} \tag{2}$$

(3) Multiplying the LNN by a scalar, where $k > 0$

$$k \times \varphi = k\langle \xi_p, \xi_q, \xi_r \rangle = \left\langle \xi_{t - t\left(1 - \frac{p}{t}\right)^k}, \xi_{t\left(\frac{q}{t}\right)^k}, \xi_{t\left(\frac{r}{t}\right)^k} \right\rangle \tag{3}$$

(4) LNN exponent, where $k > 0$

$$\varphi^k = \langle \xi_p, \xi_q, \xi_r \rangle^k = \left\langle \xi_{t\left(\frac{p}{t}\right)^k}, \xi_{t - t\left(1 - \frac{q}{t}\right)^k}, \xi_{t - t\left(1 - \frac{r}{t}\right)^k} \right\rangle \tag{4}$$

Definition 3 Let $\varphi = \langle \xi_p, \xi_q, \xi_r \rangle$ be an LNN in Ω, then we can define the score function and the accuracy function according to the following [14]:

$$\chi(\xi) = (2t + p - q - r)/(3t), \quad \forall \chi(\xi) \in [0, 1] \tag{5}$$

$$T(\xi) = (p - r)/t, \quad \forall T(\xi) \in [-1, 1] \tag{6}$$

Definition 4 Let $\varphi_1 = \langle \xi_{p_1}, \xi_{q_1}, \xi_{r_1} \rangle$ and $\varphi_2 = \langle \xi_{p_2}, \xi_{q_2}, \xi_{r_2} \rangle$ be two random LNNs. Let $\Omega = \{\varphi_i | i \in [0, t]\}$ be a linguistic set and let $f(\varphi_i) = \frac{i}{t}$ be a linguistic function. Then, we can determine the distance between e_1 and e_2 using the following expressions:

$$\begin{aligned}d(\varphi_1, \varphi_2) = \Bigg\{ \frac{1}{3} \Big[&\left|f(\xi_{p_1}) - f(\xi_{p_2})\right|^k + \left|f(\xi_{t-q_1}) \right. \\ &\left. - f(\xi_{t-q_2})\right|^k + \left|f(\xi_{t-r_1}) - f(\xi_{t-r_2})\right|^k \Big] \Bigg\}^{\frac{1}{k}}, \quad k > 0\end{aligned} \tag{7}$$

For any three LNNs $\varphi = \langle \xi_p, \xi_q, \xi_r \rangle$, $\varphi_1 = \langle \xi_{p_1}, \xi_{q_1}, \xi_{r_1} \rangle$ and $\varphi_2 = \langle \xi_{p_2}, \xi_{q_2}, \xi_{r_2} \rangle$ from linguistic set $\Omega = \{\varphi_0, \varphi_1, \ldots, \varphi_t\}$ with odd cardinality $t+1$, where for $\xi_p, \xi_q, \xi_r \in \Omega$ and $p, q, r \in [0, t]$, the following properties apply:

(1) $0 \le d(\varphi_1, \varphi_2) \le 1$;
(2) $d(\varphi_1, \varphi_2) = d(\varphi_2, \varphi_1)$;
(3) $d(\varphi_1, \varphi_2) = 0$ if $\varphi_1 = \varphi_2$;
(4) $d(\varphi_1, \varphi) \le d(\varphi_1, \varphi_2) + d(\varphi_2, \varphi)$.

3.2 LNN WASPAS model

The LNN WASPAS method has eight steps that are described in the following part of this section.

Step 1 Forming the expert correspondence matrices ($N^{(l)}$). Starting from the assumption that m experts take part in the decision-making process who evaluate the sets of alternatives $A = \{a_1, a_2, \ldots, a_b\}$ (where b denotes the final number of alternatives) in relation to the defined set of evaluation criteria $C = \{c_1, c_2, \ldots, c_n\}$ (where n represents the total number of criteria). The experts $\{e_1, e_2, \ldots, e_m\}$ are given weight coefficients $\{\delta_1, \delta_2, \ldots, \delta_m\}$, $0 \le \delta_l \le 1, (l = 1, 2, \ldots, m)$ and $\sum_{l=1}^{m} \delta_l = 1$. Evaluation of the alternatives is based on a predefined set of linguistic variables $\Omega = \{\varphi_i | i \in [0, t]\}$.

In order to achieve the final ranking of the alternatives a_i $(i = 1, 2, \ldots, b)$ from the set of alternatives A, each expert

e_l $(l = 1, 2, \ldots, m)$ evaluates the alternatives according to a defined set of alternatives $C = \{c_1, c_2, \ldots, c_n\}$. So for each expert, we construct a correspondence initial decision matrix

$$N^{(l)} = \left[\varphi_{ij}^{(l)}\right]_{b\times n} = \begin{bmatrix} \varphi_{11}^{(l)} & \varphi_{12}^{(l)} & \cdots & \varphi_{1n}^{(l)} \\ \varphi_{21}^{(l)} & \varphi_{22}^{(l)} & \cdots & \varphi_{2n}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{b1}^{(l)} & \varphi_{b2}^{(l)} & \cdots & \varphi_{bn}^{(l)} \end{bmatrix}$$
$$= \begin{bmatrix} \left\langle \xi_{p_{11}}^{(l)}, \xi_{r_{11}}^{(l)}, \xi_{q_{11}}^{(l)} \right\rangle & \left\langle \xi_{p_{12}}^{(l)}, \xi_{r_{12}}^{(l)}, \xi_{q_{12}}^{(l)} \right\rangle & \cdots & \left\langle \xi_{p_{1n}}^{(l)}, \xi_{r_{1n}}^{(l)}, \xi_{q_{1n}}^{(l)} \right\rangle \\ \left\langle \xi_{p_{21}}^{(l)}, \xi_{r_{21}}^{(l)}, \xi_{q_{21}}^{(l)} \right\rangle & \left\langle \xi_{p_{22}}^{(l)}, \xi_{r_{22}}^{(l)}, \xi_{q_{22}}^{(l)} \right\rangle & \cdots & \left\langle \xi_{p_{2n}}^{(l)}, \xi_{r_{2n}}^{(l)}, \xi_{q_{2n}}^{(l)} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \xi_{p_{b1}}^{(l)}, \xi_{r_{b1}}^{(l)}, \xi_{q_{b1}}^{(l)} \right\rangle & \left\langle \xi_{p_{b2}}^{(l)}, \xi_{r_{b2}}^{(l)}, \xi_{q_{b2}}^{(l)} \right\rangle & \cdots & \left\langle \xi_{p_{bn}}^{(l)}, \xi_{r_{bn}}^{(l)}, \xi_{q_{bn}}^{(l)} \right\rangle \end{bmatrix} \tag{8}$$

where the basic elements of matrix $N^{(l)}$ $(\varphi_{ij}^{(l)})$ represent the linguistic variables from the sets $\Omega = \{\varphi_i | i \in [0, t]\}$, $\xi_p, \xi_q, \xi_r \in \Omega$ and $p, q, r \in [0, t]$. Linguistic expressions $\varphi_{ij}^{(l)} = \left\langle \xi_{p_{ij}}^{(l)}, \xi_{q_{ij}}^{(l)}, \xi_{r_{ij}}^{(l)} \right\rangle$, i.e., $\xi_{p_{ij}}^{(l)}$, $\xi_{q_{ij}}^{(l)}$ and $\xi_{r_{ij}}^{(l)}$ independently provide information on the degree of truth, indeterminacy and falsity when evaluating the alternatives a_i $(i = 1, 2, \ldots, b)$ according to the defined set of criteria $C = \{c_1, c_2, \ldots, c_n\}$.

Step 2 Calculating the elements of the normalized expert correspondence matrices $(\widehat{Y}^{(l)})$. The elements of normalized matrix $\widehat{Y}^{(l)} = \left[\widehat{y}_{ij}^{(l)}\right]_{b\times n}$ are calculated using expression (9)

$$\widehat{y}_{ij}^{(l)} = \left\langle \widehat{\xi}_{p_{ij}}^{(l)}, \widehat{\xi}_{q_{ij}}^{(l)}, \widehat{\xi}_{r_{ij}}^{(l)} \right\rangle = \begin{cases} \widehat{\xi}_{p_{ij}}^{(l)} = \xi_{t-p_{ij}}^{(l)};\ \widehat{\xi}_{q_{ij}}^{(l)} = \xi_{t-q_{ij}}^{(l)};\ \widehat{\xi}_{r_{ij}}^{(l)} = \xi_{t-r_{ij}}^{(l)} & \text{if } \widehat{y}_{ij}^{(l)} \in C; \\ \widehat{\xi}_{p_{ij}}^{(l)} = \xi_{p_{ij}}^{(l)};\ \widehat{\xi}_{q_{ij}}^{(l)} = \xi_{q_{ij}}^{(l)};\ \widehat{\xi}_{r_{ij}}^{(l)} = \xi_{r_{ij}}^{(l)} & \text{if } \widehat{y}_{ij}^{(l)} \in B. \end{cases} \tag{9}$$

where B and C, respectively, represent sets of criteria of the benefit and cost type, and $\widehat{y}_{ij}^{(l)} = \left\langle \widehat{\xi}_{p_{ij}}^{(l)}, \widehat{\xi}_{q_{ij}}^{(l)}, \widehat{\xi}_{r_{ij}}^{(l)} \right\rangle$ represents the elements of normalized matrix $\widehat{Y}^{(l)}$.

Step 3 Calculating the elements of the aggregated normalized matrix. We obtain the final aggregated decision matrix N by averaging the elements $\widehat{y}_{ij}^{(l)} = \left\langle \widehat{\xi}_{p_{ij}}^{(l)}, \widehat{\xi}_{q_{ij}}^{(l)}, \widehat{\xi}_{r_{ij}}^{(l)} \right\rangle$ of matrix $\widehat{Y}^{(l)} = \left[\widehat{y}_{ij}^{(l)}\right]_{b\times n}$ using expression (11)

$$\widehat{Y} = \left[\widehat{y}_{ij}\right]_{b\times n} = \begin{bmatrix} \widehat{y}_{11} & \widehat{y}_{12} & \cdots & \widehat{y}_{1n} \\ \widehat{y}_{21} & \widehat{y}_{22} & \cdots & \widehat{y}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{y}_{b1} & \widehat{y}_{b2} & \cdots & \widehat{y}_{bn} \end{bmatrix}$$
$$= \begin{bmatrix} \left\langle \widehat{\xi}_{p_{11}}, \widehat{\xi}_{r_{11}}, \widehat{\xi}_{q_{11}} \right\rangle & \left\langle \widehat{\xi}_{p_{12}}, \widehat{\xi}_{r_{12}}, \widehat{\xi}_{q_{12}} \right\rangle & \cdots & \left\langle \widehat{\xi}_{p_{1n}}, \widehat{\xi}_{r_{1n}}, \widehat{\xi}_{q_{1n}} \right\rangle \\ \left\langle \widehat{\xi}_{p_{21}}, \widehat{\xi}_{r_{21}}, \widehat{\xi}_{q_{21}} \right\rangle & \left\langle \widehat{\xi}_{p_{22}}, \widehat{\xi}_{r_{22}}, \widehat{\xi}_{q_{22}} \right\rangle & \cdots & \left\langle \widehat{\xi}_{p_{2n}}, \widehat{\xi}_{r_{2n}}, \widehat{\xi}_{q_{2n}} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \widehat{\xi}_{p_{b1}}, \widehat{\xi}_{r_{b1}}, \widehat{\xi}_{q_{b1}} \right\rangle & \left\langle \widehat{\xi}_{p_{b2}}, \widehat{\xi}_{r_{b2}}, \widehat{\xi}_{q_{b2}} \right\rangle & \cdots & \left\langle \widehat{\xi}_{p_{bn}}, \widehat{\xi}_{r_{bn}}, \widehat{\xi}_{q_{bn}} \right\rangle \end{bmatrix} \tag{10}$$

whereby we obtain elements $\widehat{y}_{ij} = \left\langle \widehat{\xi}_{p_{ij}}, \widehat{\xi}_{q_{ij}}, \widehat{\xi}_{r_{ij}} \right\rangle$ using the LNN normalized weighted geometric Bonferroni mean (LNNNWGBM) operator [13]

$$\widehat{y}_{ij} = \text{LNNWGBM}^{p,q}\left(\widehat{y}_{ij}^{(1)}, \widehat{y}_{ij}^{(2)}, \ldots, \widehat{y}_{ij}^{(m)}\right) = \frac{1}{p+q}\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\left(p\widehat{y}_{ij}^{(i)} + q\widehat{y}_{ij}^{(j)}\right)^{\frac{\delta_i\delta_j}{1-\delta_i}}$$
$$= \left\langle \widehat{\xi}_{t-t}\left(1 - \prod_{i=1}^{m}\prod_{\substack{j=1\\ j\neq i}}^{m}\left(1 - \left(1 - \frac{p_i}{t}\right)^p\left(1 - \frac{p_j}{t}\right)^q\right)^{\frac{\delta_i\delta_j}{1-\delta_i}}\right)^{\frac{1}{p+q}}, \widehat{\xi}_t\left(1 - \prod_{i=1}^{m}\prod_{\substack{j=1\\ j\neq i}}^{m}\left(1 - \left(\frac{q_i}{t}\right)^p\left(\frac{q_j}{t}\right)^q\right)^{\frac{\delta_i\delta_j}{1-\delta_i}}\right)^{\frac{1}{p+q}}, \widehat{\xi}_t\left(1 - \prod_{i=1}^{m}\prod_{\substack{j=1\\ j\neq i}}^{m}\left(1 - \left(\frac{r_i}{t}\right)^p\left(\frac{r_j}{t}\right)^q\right)^{\frac{\delta_i\delta_j}{1-\delta_i}}\right)^{\frac{1}{p+q}} \right\rangle \tag{11}$$

where elements $\widehat{y}_{ij}^{(l)} = \left\langle \widehat{\xi}_{p_{ij}}^{(l)}, \widehat{\xi}_{q_{ij}}^{(l)}, \widehat{\xi}_{r_{ij}}^{(l)} \right\rangle$ represent the elements of the expert correspondence matrix (8), and $\delta_{i,j}$ represent the weight coefficients of the experts.

Step 4 Determining the values of the weight coefficients Determining the values of the weight coefficients is based on the model of maximum deviation (MMD). After normalizing the expert correspondence matrices, we obtain aggregated normalized decision matrix $\widehat{Y} = \left[\widehat{y}_{ij}\right]_{b\times n}$. The aggregated normalized decision matrix $\widehat{Y}$ is further transformed into weighted matrix $D = \left[d_{ij}\right]_{b\times n}$, $d_{ij} = \left\langle \widehat{\xi}_{p_{ij}}^{*}, \widehat{\xi}_{q_{ij}}^{*}, \widehat{\xi}_{r_{ij}}^{*} \right\rangle = w_j \cdot \left\langle \widehat{\xi}_{p_{ij}}, \widehat{\xi}_{q_{ij}}, \widehat{\xi}_{r_{ij}} \right\rangle$.

In matrix D, we can calculate the degree of deviation of an observed element in relation to other elements within the criteria c_j $(j = 1, 2, \ldots, n)$

$$D_{ij}(w_j) = \sum_{u=1}^{b} d(d_{ij}, d_{uj}) = \sum_{u=1}^{b} d(\hat{y}_{ij}, \hat{y}_{uj}) w_j \tag{12}$$

where $d(\hat{y}_{ij}, \hat{y}_{uj})$ represents the distance between $\hat{y}_{ij}$ and $\hat{y}_{uj}$.

From expression (12), we can clearly see that for a higher value of $D_{ij}(w_j)$ alternative a_i $(i = 1, 2, \ldots, b)$ is better. The MMD model is based on the following starting points: (1) If there are small deviations between the observed values of $\hat{y}_{ij}$ and all other values within the evaluation criteria c_j $(j = 1, 2, \ldots, n)$, then criterion c_j has little influence on the ranking of the alternatives (c_j has a low value of the weight coefficient w_j); (2) In contrast to this, if there is significant deviation between the observed values of $\hat{y}_{ij}$ and all other values within the evaluation criteria c_j $(j = 1, 2, \ldots, n)$, then criterion c_j has a great influence on the ranking of the alternatives (c_j has a high value of the weight coefficient w_j) and (3) If all values of $\hat{y}_{ij}$ are identical within the evaluation criteria c_j $(j = 1, 2, \ldots, n)$, then criterion c_j does not affect the ranking of the alternatives (c_j has the value of the weight coefficient $w_j = 0$).

Step 4.1 Calculating the degree of deviation between all elements within the observed criterion c_j $(j = 1, 2, \ldots, n)$

$$D_j(w_j) = \sum_{i=1}^{b} D_{ij}(w_j) = \sum_{i=1}^{b} \sum_{u=1}^{b} d(\hat{y}_{ij}, \hat{y}_{uj}) w_j \tag{13}$$

Namely, calculation of the total deviation of all alternatives according to the criteria

$$D(w) = \sum_{j=1}^{n} \sum_{i=1}^{b} D_{ij}(w_j) = \sum_{j=1}^{n} \sum_{i=1}^{b} \sum_{u=1}^{b} d(\hat{y}_{ij}, \hat{y}_{uj}) w_j \tag{14}$$

Step 4.2 The weight coefficients w_j are obtained by solving the optimization model that is based on the maximum deviation

$$\max D(w) = \sum_{j=1}^{n} \sum_{i=1}^{b} \sum_{u=1}^{b} d(\hat{y}_{ij}, \hat{y}_{uj}) w_j$$

$$\text{s.t.} \quad \begin{cases} \sum_{j=1}^{n} w_j^2 = 1; \\ 0 \le w_j \le 1; \quad j = 1, 2, \ldots, n \end{cases} \tag{15}$$

The Lagrangian function is introduced in order to obtain a solution for the model (15).

$$L(w, p) = \sum_{j=1}^{n} \sum_{i=1}^{b} \sum_{u=1}^{b} d(\hat{y}_{ij}, \hat{y}_{uj}) w_j + \frac{p}{2} \left(\sum_{j=1}^{n} w_j^2 - 1 \right) \tag{16}$$

After partial deviation according to w, and then according to p two equations are obtained $D(w) + p w_j = 0$ and $\sum_{j=1}^{n} w_j^2 = 1$, we get

$$w_j = \frac{\sum_{i=1}^{b} \sum_{u=1}^{b} \left\{ \frac{1}{3} \left[\left| f\left(\hat{\xi}_{pij}\right) - f\left(\hat{\xi}_{p_{uj}}\right) \right|^{\varphi} + \left| f\left(\hat{\xi}_{t-q_{ij}}\right) - f\left(\hat{\xi}_{t-q_{uj}}\right) \right|^{\varphi} + \left| f\left(\hat{\xi}_{t-r_{ij}}\right) - f\left(\hat{\xi}_{t-r_{uj}}\right) \right|^{\varphi} \right] \right\}^{\frac{1}{\varphi}}}{\sqrt{\sum_{j=1}^{n} \left(\sum_{i=1}^{b} \sum_{u=1}^{b} \left\{ \frac{1}{3} \left[\left| f\left(\hat{\xi}_{pij}\right) - f\left(\hat{\xi}_{p_{uj}}\right) \right|^{\varphi} + \left| f\left(\hat{\xi}_{t-q_{ij}}\right) - f\left(\hat{\xi}_{t-q_{uj}}\right) \right|^{\varphi} + \left| f\left(\hat{\xi}_{t-r_{ij}}\right) - f\left(\hat{\xi}_{t-r_{uj}}\right) \right|^{\varphi} \right] \right\}^{\frac{1}{\varphi}} \right)^2}} \tag{17}$$

Step 4.3 Calculating the final values of the weight coefficients. By normalizing the values (17), we obtain the final values of the weight coefficients.

$$\omega_j = \frac{w_j}{\sum_{j=1}^{n} w_j} \tag{18}$$

Step 5 Calculating the weighted sum and the weighted product of the optimality criteria. In the WASPAS method, we obtain the final values of the criterion functions of the alternatives using the weighted sum model (WSM) and the weighted product model (WPM), i.e., using expressions (19) and (20).

According to WSM, we obtain the weighted values using expression (19)

$$Q_i = \sum_{l=1}^{m} \hat{y}_{ij} \omega_j = \left\langle \hat{\xi}_{t - t\prod_{l=1}^{m} \left(1 - \frac{p_{ij_l}}{t}\right)^{\omega_l}}, \hat{\xi}_{t\prod_{l=1}^{m} \left(\frac{q_{ij_l}}{t}\right)^{\omega_l}}, \hat{\xi}_{t\prod_{l=1}^{m} \left(\frac{r_{ij_l}}{t}\right)^{\omega_l}} \right\rangle \tag{19}$$

According to WPM, we obtain the weighted values using expression (20)

$$P_i = \prod_{l=1}^{m} \left(\widehat{y}_{ij}\right)^{\omega_j} = \left\langle \widehat{\xi}_{t\prod_{l=1}^{m}\left(\frac{p_{ij_l}}{t}\right)^{\omega_l}}, \widehat{\xi}_{t-t\prod_{l=1}^{m}\left(1-\frac{q_{ij_l}}{t}\right)^{\omega_l}}, \widehat{\xi}_{t-t\prod_{l=1}^{m}\left(1-\frac{r_{ij_l}}{t}\right)^{\omega_l}} \right\rangle \quad (20)$$

where $\widehat{y}_{ij} = \left\langle \widehat{\xi}_{p_{ij}}, \widehat{\xi}_{q_{ij}}, \widehat{\xi}_{r_{ij}} \right\rangle$ represents the aggregated elements of matrix (10), while ω_j represents the weight coefficient of criterion j.

Step 6 Calculating the resulting functions of the WSM and WPM models

$$\chi(Q_i) = (2t + p - q - r)/(3t), \quad \forall \chi(Q_i) \in [0, 1] \quad (21)$$

$$\chi(P_i) = (2t + p - q - r)/(3t), \quad \forall \chi(P_i) \in [0, 1] \quad (22)$$

Step 7 Calculating the integrated value of the criterion function. Since in the previous step of the WASPAS method we defined the WSM and WPM resulting functions, in this step we obtain the integrated value of the function

$$K_i = \lambda \sum_{j=1}^{n} \chi(Q_i) + (1 - \lambda) \sum_{j=1}^{n} \chi(P_i) \quad (23)$$

where coefficient λ takes its value from interval $[0, 1]$ and is obtained from the assumption that the sum of all WSM values according to the alternatives should be equal to the sum of all WPM values according to the alternatives.

$$\lambda = \frac{\sum_{i=1}^{m} \chi(P_i)}{\sum_{i=1}^{m} \chi(Q_i) + \sum_{i=1}^{m} \chi(P_i)} \quad (24)$$

Step 8 Selection of the optimal alternative. The alternatives are ranked based on the values of K_i, whereby the best alternative is the one with the highest value of K_i.

4 Application of the LNN WASPAS model to evaluate the work of the advisors

An advisor is an individual who in a company, other legal entity or for a contractor carries out activities that ensure the implementation of regulations in the transport of hazardous goods, organizes the transport of hazardous goods and increases the level of safety. Table 1 presents the criteria for evaluating the work of an advisor and gives a brief description of them. Authors and experts from the field of transporting hazardous goods selected nine criteria which they believe to be most significant in the evaluation of the work of advisors. As experts from this field of research, professionals from the Department for the transport of hazardous goods at the Ministry of construction and traffic infrastructure, traffic inspectors and persons who lead professional associations from this field were surveyed.

Seven experts took part in the research (e_i, $i = 1, 2, \ldots, 7$) with weight coefficients of $\delta_1 = 0.1183$, $\delta_2 = 0.1491$, $\delta_3 = 0.1096$, $\delta_4 = 0.1554$, $\delta_5 = 0.1656$, $\delta_6 = 0.1144$ and $\delta_7 = 0.1877$. A total of 21 advisors were evaluated who were denoted as A1 to A21. The names of these advisors are not given in order to protect their personal data. The advisors (alternatives) were

Table 1 Criteria for evaluating the work of the advisors

Number	Criteria	Description of the criteria
1.	Knowledge of regulations and professional development	Knowledge of regulations governing the transport of hazardous goods and related regulations, in line with the specifics of the employer
		The advisor constantly renews and expands their knowledge, monitors changes to regulations, attends scientific/professional conferences, etc.
2.	Analytical processing of the established requirements	Familiarization with the specifics of the employer's mode of work
		Establishing and processing the prescribed requirements in the transport of hazardous goods for the employer
3.	The quality of the proposed measures	By proposing appropriate measures, the advisor prevents the occurrence of accidents and contributes to reducing risk in the transport of hazardous goods
4.	Level of implementation of the proposed measures	The advisor points out the importance of applying the proposed measures, determines priorities and influences the implementation
5.	Quality of professional training of employees	Training employees and raising awareness of the possible risks that occur during the transportation, storage and manipulation of hazardous goods
6.	Response to emergency situations	Implementing appropriate measures in emergencies
7.	Preparation of documents	Preparation of a safety plan and annual and periodic reports according to the employer's degree of compliance with regulations, as well as other documents
8.	Method of solving professional questions	Communication of the advisor with competent institutions
		The level of cooperation with other advisors, especially in the same transport process
9.	Activity in professional bodies	Engagement in the group for the transport of hazardous goods, the cluster of hazardous goods, the association of industrial gas and other bodies

evaluated according to the criteria using a set of linguistic variables $\Omega = \{\varphi_i | i \in [0, 6]\}$, whereby $\varphi = \{\varphi_0$-exceedingly low, φ_1-low, φ_2-slightly low, φ_3-medium, φ_4-slightly high, φ_5-high, φ_6-exceedingly high$\}$.

Step 1 In the first step, the experts evaluated the 21 alternatives (advisors) in relation to nine evaluation criteria denoted as C1 to C9. Thus, for each expert, one correspondence matrix was formed (Table 2). Evaluation of the alternatives was carried out using a predefined set of linguistic variables $\Omega = \{\varphi_i | i \in [0, 6]\}$.

Step 2 In the second step, using expression (9) the expert correspondence matrices shown in Table 2 were normalized. Since all of the criteria C1–C9 fall into the group of benefit (max) criteria (a higher value is desirable) for normalization of the values from Table 2, only the second part of expression (9) was used, since the first part refers to the normalization of the cost (min) group of criteria. The normalization of elements A1–C1 ($\varphi_{11}^{(1)}$) in matrix $N^{(1)} = \left[\varphi_{ij}^{(1)}\right]_{21\times 9}$ was carried out using the following expression

$$\widehat{y}_{11}^{(1)} = \left\langle \widehat{\xi}_{p_{11}}^{(1)}, \widehat{\xi}_{q_{11}}^{(1)}, \widehat{\xi}_{r_{11}}^{(1)} \right\rangle = \left\langle \widehat{\xi}_5^{(1)}, \widehat{\xi}_5^{(1)}, \widehat{\xi}_5^{(1)} \right\rangle = \begin{cases} \widehat{\xi}_5^{(1)} = \xi_5^{(1)}; \\ \widehat{\xi}_5^{(1)} = \xi_5^{(1)}; \\ \widehat{\xi}_3^{(1)} = \xi_3^{(1)}. \end{cases}$$

Normalization of the remaining elements from the expert correspondence matrices $\widehat{Y}^{(l)} = \left[\widehat{y}_{ij}^{(l)}\right]_{21\times 9}, l = 1, 2, \ldots, 7$; (Table 2) was carried out in a similar way.

Step 3 In order to evaluate the alternatives, the normalized expert correspondence matrices were aggregated into a single normalized initial decision matrix (Table 3). Aggregation of the expert matrices $N^{(l)}$ $(l = 1, 2, \ldots, 7)$ was carried out using LNNNWGBM, expression (11).

The element in position C11–A1 ($\widehat{y}_{11}$) was aggregated using expression (11) in the following way

$$\widehat{y}_{11} = \text{LNNWGBM}^{p,q}\left(\widehat{y}_{11}^{(1)}, \widehat{y}_{11}^{(2)}, \ldots, \widehat{y}_{11}^{(7)}\right) = \frac{1}{p+q} \sum_{i=1}^{7} \sum_{\substack{j=1 \\ j \neq i}}^{7} \left(p\widehat{y}_{11}^{(i)} + q\widehat{y}_{11}^{(j)}\right)^{\frac{\delta_i \delta_j}{1-\delta_i}}; \quad p = q = 1;$$

$$= \left\langle \begin{array}{l} \widehat{\xi}_{6-6\left(1-\left(\left(1-\left(1-\frac{5}{6}\right)\left(1-\frac{4}{6}\right)\right)^{\frac{0.118 \cdot 0.149}{1-0.118}} \cdot \left(1-\left(1-\frac{5}{6}\right)\left(1-\frac{4}{6}\right)\right)^{\frac{0.118 \cdot 0.1096}{1-0.118}} \cdot \left(1-\left(1-\frac{5}{6}\right)\left(1-\frac{4}{6}\right)\right)^{\frac{0.118 \cdot 0.1554}{1-0.118}} \cdot, \ldots, \cdot \left(1-\left(1-\frac{6}{6}\right)\left(1-\frac{4}{6}\right)\right)^{\frac{0.1877 \cdot 0.1144}{1-0.1877}}\right)\right)^{\frac{1}{2}}}, \\ \widehat{\xi}_{6\left(1-\left(\left(1-\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\right)^{\frac{0.118 \cdot 0.149}{1-0.118}} \cdot \left(1-\left(\frac{5}{6}\right)\left(\frac{2}{6}\right)\right)^{\frac{0.118 \cdot 0.1096}{1-0.118}} \left(1-\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\right)^{\frac{0.118 \cdot 0.1554}{1-0.118}} \cdot, \ldots, \cdot \left(1-\left(\frac{4}{6}\right)\left(\frac{3}{6}\right)\right)^{\frac{0.1877 \cdot 0.1144}{1-0.1877}}\right)\right)^{\frac{1}{2}}}, \\ \widehat{\xi}_{6\left(1-\left(\left(1-\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\right)^{\frac{0.118 \cdot 0.149}{1-0.118}} \cdot \left(1-\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\right)^{\frac{0.118 \cdot 0.1096}{1-0.118}} \cdot \left(1-\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\right)^{\frac{0.118 \cdot 0.1554}{1-0.118}} \cdot, \ldots, \cdot \left(1-\left(\frac{1}{6}\right)\left(\frac{3}{6}\right)\right)^{\frac{0.1877 \cdot 0.1144}{1-0.1877}}\right)\right)^{\frac{1}{2}}} \end{array} \right\rangle$$

$$= \left\langle \widehat{\xi}_{4.36}, \widehat{\xi}_{2.39}, \widehat{\xi}_{2.21} \right\rangle$$

where $\delta_{i,j}$ (with coefficients $\delta_1 = 0.1183$, $\delta_2 = 0.1491$, $\delta_3 = 0.1096$, $\delta_4 = 0.1554$, $\delta_5 = 0.1656$, $\delta_6 = 0.1144$ and $\delta_7 = 0.1877$) represents the weight coefficients of the experts. Aggregation of the remaining elements of the aggregated normalized matrix in Table 3 was carried out in the same way.

Step 4 After determining the initial decision matrix (Table 3), the deviations between the elements of the aggregated matrix were calculated using expressions (12)–(16). Thus, for criteria (C1–C9) the deviations shown in Table 4 were obtained.

The weight coefficients of criteria C1–C9 were obtained using expression (17)

Table 2 The expert correspondent matrices

Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9
Expert 1									
A1	$\langle\xi_5,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_4\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$
A2	$\langle\xi_6,\xi_3,\xi_5\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$
A3	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_3\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$
A4	$\langle\xi_5,\xi_3,\xi_5\rangle$	$\langle\xi_5,\xi_4,\xi_2\rangle$	$\langle\xi_6,\xi_5,\xi_6\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$	$\langle\xi_6,\xi_4,\xi_6\rangle$	$\langle\xi_5,\xi_5,\xi_4\rangle$	$\langle\xi_6,\xi_5,\xi_5\rangle$	$\langle\xi_6,\xi_5,\xi_4\rangle$	$\langle\xi_5,\xi_5,\xi_3\rangle$
A5	$\langle\xi_4,\xi_2,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_6,\xi_6\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_5,\xi_4,\xi_5\rangle$
A6	$\langle\xi_6,\xi_4,\xi_3\rangle$	$\langle\xi_5,\xi_2,\xi_2\rangle$	$\langle\xi_5,\xi_4,\xi_4\rangle$	$\langle\xi_5,\xi_3,\xi_4\rangle$	$\langle\xi_6,\xi_1,\xi_2\rangle$	$\langle\xi_6,\xi_2,\xi_2\rangle$	$\langle\xi_5,\xi_3,\xi_2\rangle$	$\langle\xi_5,\xi_2,\xi_1\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$
A7	$\langle\xi_3,\xi_1,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_1\rangle$	$\langle\xi_6,\xi_5,\xi_5\rangle$	$\langle\xi_5,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_3,\xi_5\rangle$	$\langle\xi_5,\xi_4,\xi_3\rangle$	$\langle\xi_6,\xi_5,\xi_5\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$
A8	$\langle\xi_5,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$
A9	$\langle\xi_2,\xi_2,\xi_0\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$
A10	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_4\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_6,\xi_2,\xi_3\rangle$	$\langle\xi_6,\xi_2,\xi_4\rangle$
A11	$\langle\xi_6,\xi_5,\xi_5\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_5,\xi_4,\xi_4\rangle$	$\langle\xi_6,\xi_4,\xi_6\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$
A12	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_2,\xi_5,\xi_3\rangle$	$\langle\xi_3,\xi_4,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_4\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$
A13	$\langle\xi_6,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$
A14	$\langle\xi_3,\xi_1,\xi_0\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$
A15	$\langle\xi_2,\xi_1,\xi_0\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_0\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$
A16	$\langle\xi_6,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$	$\langle\xi_3,\xi_3,\xi_5\rangle$	$\langle\xi_2,\xi_2,\xi_0\rangle$	$\langle\xi_1,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_4,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_5,\xi_5,\xi_2\rangle$	$\langle\xi_6,\xi_3,\xi_5\rangle$
A17	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_6,\xi_4,\xi_5\rangle$	$\langle\xi_3,\xi_2,\xi_5\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_4\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_5,\xi_4,\xi_4\rangle$
A18	$\langle\xi_5,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_2,\xi_5\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_6,\xi_5,\xi_4\rangle$	$\langle\xi_6,\xi_6,\xi_5\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_5,\xi_4,\xi_2\rangle$	$\langle\xi_1,\xi_2,\xi_1\rangle$
A19	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_4\rangle$	$\langle\xi_5,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_5,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_4\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$	$\langle\xi_6,\xi_5,\xi_4\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$
A20	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_6,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_5,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_4\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_2,\xi_3,\xi_1\rangle$
A21	$\langle\xi_4,\xi_1,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_6\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$
…									
Expert 7									
A1	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_2,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$	$\langle\xi_5,\xi_4,\xi_3\rangle$
A2	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_5,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_5,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_5,\xi_4,\xi_4\rangle$	$\langle\xi_6,\xi_4,\xi_5\rangle$	$\langle\xi_6,\xi_4,\xi_5\rangle$
A3	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_6,\xi_3,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_3,\xi_5\rangle$	$\langle\xi_5,\xi_4,\xi_2\rangle$	$\langle\xi_6,\xi_5,\xi_6\rangle$
A4	$\langle\xi_6,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_3,\xi_0\rangle$	$\langle\xi_4,\xi_1,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$	$\langle\xi_5,\xi_3,\xi_2\rangle$
A5	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_2,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_6,\xi_4,\xi_3\rangle$
A6	$\langle\xi_5,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_5,\xi_3,\xi_4\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_6,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_5,\xi_3,\xi_5\rangle$	$\langle\xi_5,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$
A7	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_0\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_5,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_1\rangle$
A8	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_4,\xi_1\rangle$	$\langle\xi_2,\xi_3,\xi_1\rangle$	$\langle\xi_1,\xi_2,\xi_0\rangle$	$\langle\xi_4,\xi_3,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_1,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_3\rangle$
A9	$\langle\xi_3,\xi_1,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_1,\xi_1,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$
A10	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_5,\xi_2,\xi_3\rangle$	$\langle\xi_5,\xi_5,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$
A11	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_5,\xi_4,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_6,\xi_5,\xi_4\rangle$	$\langle\xi_4,\xi_4,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$
A12	$\langle\xi_3,\xi_3,\xi_0\rangle$	$\langle\xi_4,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_5,\xi_4,\xi_3\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$
A13	$\langle\xi_3,\xi_1,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_4\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_3,\xi_1,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$
A14	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_6,\xi_4,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$
A15	$\langle\xi_2,\xi_4,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_1,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_2,\xi_3,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_2\rangle$	$\langle\xi_2,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$
A16	$\langle\xi_3,\xi_2,\xi_1\rangle$	$\langle\xi_4,\xi_4,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_6,\xi_5,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$
A17	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_2,\xi_3\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_4,\xi_3\rangle$	$\langle\xi_4,\xi_3,\xi_1\rangle$	$\langle\xi_5,\xi_4,\xi_3\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$
A18	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_4,\xi_2,\xi_2\rangle$	$\langle\xi_2,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_1,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_3,\xi_1,\xi_1\rangle$	$\langle\xi_1,\xi_2,\xi_1\rangle$
A19	$\langle\xi_2,\xi_1,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_1,\xi_1,\xi_3\rangle$	$\langle\xi_1,\xi_3,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_1\rangle$	$\langle\xi_1,\xi_1,\xi_1\rangle$	$\langle\xi_1,\xi_2,\xi_2\rangle$
A20	$\langle\xi_3,\xi_4,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_1\rangle$	$\langle\xi_3,\xi_3,\xi_0\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_2,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_2,\xi_2,\xi_2\rangle$	$\langle\xi_1,\xi_1,\xi_2\rangle$
A21	$\langle\xi_4,\xi_3,\xi_3\rangle$	$\langle\xi_6,\xi_4,\xi_1\rangle$	$\langle\xi_4,\xi_1,\xi_3\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$	$\langle\xi_4,\xi_3,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_1\rangle$	$\langle\xi_4,\xi_2,\xi_3\rangle$	$\langle\xi_3,\xi_2,\xi_2\rangle$	$\langle\xi_3,\xi_3,\xi_2\rangle$

Table 3 Normalized initial decision matrix

Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	$\langle \xi_{4.36}, \xi_{2.39}, \xi_{2.21} \rangle$	$\langle \xi_{4.11}, \xi_{2.35}, \xi_{1.91} \rangle$	$\langle \xi_{3.73}, \xi_{2.18}, \xi_{2.61} \rangle$	$\langle \xi_{3.56}, \xi_{2.12}, \xi_{2.02} \rangle$	$\langle \xi_{3.28}, \xi_{2.23}, \xi_{1.85} \rangle$	$\langle \xi_{4.08}, \xi_{2.61}, \xi_{2.48} \rangle$	$\langle \xi_{4.48}, \xi_{2.37}, \xi_{3.52} \rangle$	$\langle \xi_{4.69}, \xi_{3.21}, \xi_{3.42} \rangle$	$\langle \xi_{4.87}, \xi_{3.33}, \xi_{2.94} \rangle$
A2	$\langle \xi_{4.27}, \xi_{2.73}, \xi_{2.76} \rangle$	$\langle \xi_{4.28}, \xi_{2.45}, \xi_{1.96} \rangle$	$\langle \xi_{4.57}, \xi_{3.24}, \xi_{2.73} \rangle$	$\langle \xi_{3.78}, \xi_{2.72}, \xi_{2.84} \rangle$	$\langle \xi_{4.16}, \xi_{2.47}, \xi_{2.34} \rangle$	$\langle \xi_{3.86}, \xi_{1.88}, \xi_{2.24} \rangle$	$\langle \xi_{4.21}, \xi_{3.15}, \xi_{2.5} \rangle$	$\langle \xi_{4.8}, \xi_{3.89}, \xi_{3.28} \rangle$	$\langle \xi_{5.25}, \xi_{3.78}, \xi_{3.47} \rangle$
A2	$\langle \xi_{4.19}, \xi_{2.96}, \xi_{2.27} \rangle$	$\langle \xi_{3.96}, \xi_{2.68}, \xi_{2.07} \rangle$	$\langle \xi_{3.7}, \xi_{2.58}, \xi_{2.5} \rangle$	$\langle \xi_{2.99}, \xi_{2.03}, \xi_{1.22} \rangle$	$\langle \xi_{3.55}, \xi_{1.52}, \xi_{2.75} \rangle$	$\langle \xi_{4.07}, \xi_{2.88}, \xi_{2.91} \rangle$	$\langle \xi_{4.43}, \xi_{2.14}, \xi_{2.84} \rangle$	$\langle \xi_{4.39}, \xi_{3.45}, \xi_{2.53} \rangle$	$\langle \xi_{4.66}, \xi_{2.79}, \xi_{3.81} \rangle$
A4	$\langle \xi_{3.97}, \xi_{2.63}, \xi_{2.34} \rangle$	$\langle \xi_{4.49}, \xi_{2.44}, \xi_{3.1} \rangle$	$\langle \xi_{4.01}, \xi_{2.19}, \xi_{2.1} \rangle$	$\langle \xi_{3.31}, \xi_{2.71}, \xi_{1.24} \rangle$	$\langle \xi_{4.13}, \xi_{2.63}, \xi_{2.9} \rangle$	$\langle \xi_{4.17}, \xi_{3.27}, \xi_{3.18} \rangle$	$\langle \xi_{4.33}, \xi_{2.57}, \xi_{2.92} \rangle$	$\langle \xi_{4.48}, \xi_{3.14}, \xi_{2.86} \rangle$	$\langle \xi_{4.92}, \xi_{3.39}, \xi_{3.2} \rangle$
A3	$\langle \xi_{3.28}, \xi_{2.62}, \xi_{2.35} \rangle$	$\langle \xi_{3.76}, \xi_{2.65}, \xi_{3.21} \rangle$	$\langle \xi_{3.89}, \xi_{2.71}, \xi_{2.33} \rangle$	$\langle \xi_{5.05}, \xi_{3.38}, \xi_{3.83} \rangle$	$\langle \xi_{4.51}, \xi_{6}, \xi_{3.75} \rangle$	$\langle \xi_{3.62}, \xi_{2.52}, \xi_{2.28} \rangle$	$\langle \xi_{4.23}, \xi_{3.15}, \xi_{3.26} \rangle$	$\langle \xi_{3.73}, \xi_{2.38}, \xi_{2.09} \rangle$	$\langle \xi_{5.17}, \xi_{3.96}, \xi_{3.53} \rangle$
A6	$\langle \xi_{5.04}, \xi_{2.99}, \xi_{3.03} \rangle$	$\langle \xi_{4.4}, \xi_{3.65}, \xi_{2.4} \rangle$	$\langle \xi_{4.72}, \xi_{3.01}, \xi_{3.37} \rangle$	$\langle \xi_{4.05}, \xi_{2.53}, \xi_{2.82} \rangle$	$\langle \xi_{5}, \xi_{2.63}, \xi_{3.49} \rangle$	$\langle \xi_{3.98}, \xi_{2.52}, \xi_{2.47} \rangle$	$\langle \xi_{4.66}, \xi_{2.93}, \xi_{2.93} \rangle$	$\langle \xi_{4.53}, \xi_{3.35}, \xi_{1.96} \rangle$	$\langle \xi_{3.61}, \xi_{3.06}, \xi_{2.79} \rangle$
A4	$\langle \xi_{4.19}, \xi_{2.35}, \xi_{1.81} \rangle$	$\langle \xi_{3.92}, \xi_{3.02}, \xi_{2.83} \rangle$	$\langle \xi_{3.25}, \xi_{2.41}, \xi_{1.51} \rangle$	$\langle \xi_{3.16}, \xi_{3.21}, \xi_{1.53} \rangle$	$\langle \xi_{4.25}, \xi_{2.81}, \xi_{2.53} \rangle$	$\langle \xi_{4.17}, \xi_{2.46}, \xi_{2.41} \rangle$	$\langle \xi_{4.04}, \xi_{2.58}, \xi_{3} \rangle$	$\langle \xi_{3.88}, \xi_{3.36}, \xi_{3.1} \rangle$	$\langle \xi_{3.72}, \xi_{2.22}, \xi_{2.47} \rangle$
A8	$\langle \xi_{4.71}, \xi_{2.73}, \xi_{3.76} \rangle$	$\langle \xi_{4.05}, \xi_{2.86}, \xi_{2.43} \rangle$	$\langle \xi_{3.57}, \xi_{2.26}, \xi_{1.89} \rangle$	$\langle \xi_{2.65}, \xi_{1.95}, \xi_{1.9} \rangle$	$\langle \xi_{4.63}, \xi_{3.35}, \xi_{3.25} \rangle$	$\langle \xi_{3.48}, \xi_{2}, \xi_{2} \rangle$	$\langle \xi_{3.89}, \xi_{2.35}, \xi_{2.59} \rangle$	$\langle \xi_{4.92}, \xi_{3.32}, \xi_{2.63} \rangle$	$\langle \xi_{3.85}, \xi_{2.35}, \xi_{3.04} \rangle$
A5	$\langle \xi_{2.8}, \xi_{1.61}, \xi_{1.45} \rangle$	$\langle \xi_{3.89}, \xi_{2.15}, \xi_{2.51} \rangle$	$\langle \xi_{2.76}, \xi_{2.32}, \xi_{1.92} \rangle$	$\langle \xi_{2.82}, \xi_{1.81}, \xi_{1.64} \rangle$	$\langle \xi_{2.97}, \xi_{2.27}, \xi_{2.75} \rangle$	$\langle \xi_{3.12}, \xi_{1.5}, \xi_{2.37} \rangle$	$\langle \xi_{3.29}, \xi_{2.33}, \xi_{2.7} \rangle$	$\langle \xi_{3.7}, \xi_{1.91}, \xi_{2.37} \rangle$	$\langle \xi_{4.16}, \xi_{3.01}, \xi_{2.92} \rangle$
A10	$\langle \xi_{3.91}, \xi_{2.98}, \xi_{2.17} \rangle$	$\langle \xi_{3.35}, \xi_{2.12}, \xi_{1.68} \rangle$	$\langle \xi_{3.72}, \xi_{2.39}, \xi_{2.34} \rangle$	$\langle \xi_{3.59}, \xi_{2.46}, \xi_{2.31} \rangle$	$\langle \xi_{3.26}, \xi_{2.39}, \xi_{1.91} \rangle$	$\langle \xi_{3.85}, \xi_{2.56}, \xi_{2.2} \rangle$	$\langle \xi_{3.22}, \xi_{2.08}, \xi_{2.13} \rangle$	$\langle \xi_{4.4}, \xi_{3.29}, \xi_{3.19} \rangle$	$\langle \xi_{3.78}, \xi_{1.79}, \xi_{2.52} \rangle$
A11	$\langle \xi_{4.36}, \xi_{3.58}, \xi_{1.89} \rangle$	$\langle \xi_{3.38}, \xi_{2.74}, \xi_{1.82} \rangle$	$\langle \xi_{4.71}, \xi_{3.22}, \xi_{3.02} \rangle$	$\langle \xi_{3.29}, \xi_{1.76}, \xi_{1.79} \rangle$	$\langle \xi_{3.9}, \xi_{3.17}, \xi_{2.3} \rangle$	$\langle \xi_{4.02}, \xi_{2.46}, \xi_{2.14} \rangle$	$\langle \xi_{4.51}, \xi_{2.61}, \xi_{3.72} \rangle$	$\langle \xi_{3.95}, \xi_{2.91}, \xi_{3.36} \rangle$	$\langle \xi_{3.4}, \xi_{2.63}, \xi_{2.38} \rangle$
A12	$\langle \xi_{3.32}, \xi_{2.15}, \xi_{0.98} \rangle$	$\langle \xi_{3.15}, \xi_{2.02}, \xi_{1.93} \rangle$	$\langle \xi_{2.96}, \xi_{2.17}, \xi_{1.54} \rangle$	$\langle \xi_{2.93}, \xi_{2.58}, \xi_{2.14} \rangle$	$\langle \xi_{2.84}, \xi_{2.57}, \xi_{2.19} \rangle$	$\langle \xi_{3.24}, \xi_{2.27}, \xi_{1.41} \rangle$	$\langle \xi_{3.75}, \xi_{2.7}, \xi_{2.56} \rangle$	$\langle \xi_{3.34}, \xi_{2.04}, \xi_{1.67} \rangle$	$\langle \xi_{2.43}, \xi_{1.76}, \xi_{1.49} \rangle$
A12	$\langle \xi_{3.18}, \xi_{2.16}, \xi_{1.93} \rangle$	$\langle \xi_{3.39}, \xi_{1.9}, \xi_{2.61} \rangle$	$\langle \xi_{3.79}, \xi_{2.75}, \xi_{2.33} \rangle$	$\langle \xi_{2.68}, \xi_{1.92}, \xi_{1.58} \rangle$	$\langle \xi_{2.77}, \xi_{1.82}, \xi_{1.89} \rangle$	$\langle \xi_{2.45}, \xi_{2.14}, \xi_{1.72} \rangle$	$\langle \xi_{3.93}, \xi_{2.56}, \xi_{2.24} \rangle$	$\langle \xi_{3.4}, \xi_{2.11}, \xi_{2.79} \rangle$	$\langle \xi_{2.44}, \xi_{1.49}, \xi_{1.62} \rangle$
A14	$\langle \xi_{3.03}, \xi_{2.04}, \xi_{1.76} \rangle$	$\langle \xi_{3.14}, \xi_{2.31}, \xi_{2} \rangle$	$\langle \xi_{3.67}, \xi_{2.7}, \xi_{2.4} \rangle$	$\langle \xi_{3.36}, \xi_{1.72}, \xi_{2.48} \rangle$	$\langle \xi_{3.34}, \xi_{2.24}, \xi_{2.91} \rangle$	$\langle \xi_{3.56}, \xi_{2.6}, \xi_{2.07} \rangle$	$\langle \xi_{4.26}, \xi_{2.5}, \xi_{2.8} \rangle$	$\langle \xi_{3.52}, \xi_{2.41}, \xi_{1.52} \rangle$	$\langle \xi_{3.38}, \xi_{2.41}, \xi_{2.25} \rangle$
A13	$\langle \xi_{2.68}, \xi_{2.18}, \xi_{1.6} \rangle$	$\langle \xi_{3.22}, \xi_{2.26}, \xi_{2.27} \rangle$	$\langle \xi_{2.61}, \xi_{2.49}, \xi_{1.44} \rangle$	$\langle \xi_{2.76}, \xi_{1.85}, \xi_{1.28} \rangle$	$\langle \xi_{3.04}, \xi_{2.69}, \xi_{2.06} \rangle$	$\langle \xi_{3.24}, \xi_{2.62}, \xi_{2.06} \rangle$	$\langle \xi_{3.13}, \xi_{2.18}, \xi_{1.99} \rangle$	$\langle \xi_{3.1}, \xi_{2.31}, \xi_{2.17} \rangle$	$\langle \xi_{3.67}, \xi_{2.23}, \xi_{2.38} \rangle$
A16	$\langle \xi_{4.06}, \xi_{1.97}, \xi_{2.29} \rangle$	$\langle \xi_{3.64}, \xi_{2.32}, \xi_{2.17} \rangle$	$\langle \xi_{3.39}, \xi_{2.16}, \xi_{2.39} \rangle$	$\langle \xi_{3.28}, \xi_{1.81}, \xi_{1.58} \rangle$	$\langle \xi_{3.39}, \xi_{2.26}, \xi_{2.24} \rangle$	$\langle \xi_{2.95}, \xi_{2.65}, \xi_{1.81} \rangle$	$\langle \xi_{3.46}, \xi_{2.07}, \xi_{1.8} \rangle$	$\langle \xi_{4.3}, \xi_{3.54}, \xi_{2.08} \rangle$	$\langle \xi_{4.06}, \xi_{2.25}, \xi_{2.52} \rangle$
A14	$\langle \xi_{3.54}, \xi_{2.03}, \xi_{1.71} \rangle$	$\langle \xi_{3.81}, \xi_{2.21}, \xi_{2.8} \rangle$	$\langle \xi_{3.15}, \xi_{1.41}, \xi_{2.44} \rangle$	$\langle \xi_{3.12}, \xi_{2.03}, \xi_{1.39} \rangle$	$\langle \xi_{3.54}, \xi_{2.52}, \xi_{2.55} \rangle$	$\langle \xi_{3.27}, \xi_{2.83}, \xi_{1.65} \rangle$	$\langle \xi_{3.71}, \xi_{2.66}, \xi_{2.27} \rangle$	$\langle \xi_{4.01}, \xi_{2.36}, \xi_{1.97} \rangle$	$\langle \xi_{2.95}, \xi_{1.8}, \xi_{2.06} \rangle$
A18	$\langle \xi_{3.21}, \xi_{1.51}, \xi_{2.25} \rangle$	$\langle \xi_{3.36}, \xi_{1.63}, \xi_{1.99} \rangle$	$\langle \xi_{3.6}, \xi_{1.53}, \xi_{2.31} \rangle$	$\langle \xi_{3.57}, \xi_{2.53}, \xi_{2.7} \rangle$	$\langle \xi_{4.14}, \xi_{2.25}, \xi_{2.41} \rangle$	$\langle \xi_{4.14}, \xi_{2.44}, \xi_{3.36} \rangle$	$\langle \xi_{3.87}, \xi_{2.55}, \xi_{2.72} \rangle$	$\langle \xi_{3.68}, \xi_{2.49}, \xi_{1.86} \rangle$	$\langle \xi_{1.77}, \xi_{1.9}, \xi_{1.48} \rangle$
A15	$\langle \xi_{2.99}, \xi_{2.16}, \xi_{1.71} \rangle$	$\langle \xi_{3.39}, \xi_{2.04}, \xi_{2.36} \rangle$	$\langle \xi_{3.83}, \xi_{2.39}, \xi_{1.93} \rangle$	$\langle \xi_{3.11}, \xi_{2.18}, \xi_{2.52} \rangle$	$\langle \xi_{3.87}, \xi_{3.75}, \xi_{1.73} \rangle$	$\langle \xi_{3.54}, \xi_{2.51}, \xi_{2.47} \rangle$	$\langle \xi_{4.21}, \xi_{2.85}, \xi_{2.73} \rangle$	$\langle \xi_{4.33}, \xi_{3.33}, \xi_{2.9} \rangle$	$\langle \xi_{2.06}, \xi_{1.76}, \xi_{1.64} \rangle$
A20	$\langle \xi_{2.86}, \xi_{2.11}, \xi_{1.86} \rangle$	$\langle \xi_{3.42}, \xi_{2.28}, \xi_{1.92} \rangle$	$\langle \xi_{3.36}, \xi_{2.38}, \xi_{1.64} \rangle$	$\langle \xi_{3.32}, \xi_{1.97}, \xi_{1.23} \rangle$	$\langle \xi_{3.21}, \xi_{2.5}, \xi_{2.08} \rangle$	$\langle \xi_{3.23}, \xi_{2.44}, \xi_{1.95} \rangle$	$\langle \xi_{4.08}, \xi_{2.88}, \xi_{2.63} \rangle$	$\langle \xi_{3.77}, \xi_{2.65}, \xi_{2.46} \rangle$	$\langle \xi_{1.77}, \xi_{1.85}, \xi_{1.64} \rangle$
A21	$\langle \xi_{3.58}, \xi_{2.74}, \xi_{2.3} \rangle$	$\langle \xi_{3.64}, \xi_{2.43}, \xi_{1.72} \rangle$	$\langle \xi_{3.11}, \xi_{1.64}, \xi_{2.54} \rangle$	$\langle \xi_{2.83}, \xi_{1.95}, \xi_{1.68} \rangle$	$\langle \xi_{3.34}, \xi_{2.69}, \xi_{1.93} \rangle$	$\langle \xi_{3.51}, \xi_{1.86}, \xi_{1.69} \rangle$	$\langle \xi_{3.72}, \xi_{2.47}, \xi_{3.19} \rangle$	$\langle \xi_{4.15}, \xi_{2.61}, \xi_{3.7} \rangle$	$\langle \xi_{3.14}, \xi_{2.39}, \xi_{1.61} \rangle$

Table 4 Deviations between the criteria in the initial decision matrix

Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	2.210	1.695	1.763	1.811	2.301	1.673	2.185	2.644	3.779
A2	2.485	1.842	2.987	2.696	2.212	1.816	1.797	3.173	4.852
A3	2.258	1.581	1.651	2.072	2.954	2.209	1.769	2.130	4.068
A4	1.958	2.583	1.794	2.361	2.376	2.969	1.432	2.018	3.978
A5	2.078	2.313	1.722	5.703	7.481	1.367	2.011	2.181	5.000
A6	3.715	3.079	3.551	2.826	3.880	1.550	1.939	2.492	2.871
A7	2.158	2.134	2.116	2.721	2.328	1.722	1.363	2.261	2.520
A8	4.162	1.778	1.659	2.002	3.473	1.655	1.376	2.507	2.807
A9	2.974	1.555	2.271	1.912	2.460	2.490	1.885	2.583	2.992
A10	2.130	1.821	1.540	1.990	2.170	1.430	2.340	2.238	2.869
A11	3.200	1.792	3.315	1.825	2.345	1.559	2.468	2.325	2.568
A12	2.862	1.820	2.328	2.000	2.398	2.152	1.387	3.058	3.735
A13	1.988	1.830	1.708	2.001	3.075	2.788	1.557	2.623	3.834
A14	2.179	1.674	1.662	2.215	2.343	1.420	1.359	2.852	2.546
A15	2.647	1.568	2.846	2.204	2.244	1.583	2.505	2.746	2.512
A16	2.191	1.340	1.673	1.842	2.015	2.002	2.496	2.428	2.692
A17	2.044	1.735	2.556	1.853	1.950	1.988	1.595	2.259	3.023
A18	2.622	2.067	2.238	2.327	2.294	2.746	1.282	2.329	4.444
A19	2.188	1.551	1.696	2.036	3.344	1.477	1.407	2.045	3.995
A20	2.294	1.487	1.912	2.089	2.055	1.581	1.397	1.948	4.362
A21	1.971	1.639	2.348	1.809	2.142	2.002	1.669	2.836	3.075
Sum	52.314	38.885	45.336	48.295	57.842	40.180	37.219	51.675	72.523

$$w_1 = 52.314 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.3456,$$

$$w_2 = 38.885 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.2568,$$

$$w_3 = 45.336 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.2995,$$

$$w_4 = 48.295 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.3190,$$

$$w_5 = 57.842 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.3821,$$

$$w_6 = 40.180 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.2654,$$

$$w_7 = 37.219 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.2459,$$

$$w_8 = 51.675 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.3414,$$

$$w_9 = 72.523 \Big/ \sqrt{(52.314)^2 + (38.885)^2 + \cdots + (72.523)^2} = 0.4791.$$

The normalized values of the weight coefficients of the criteria were obtained using expression (17). By normalizing the value $w_1 = 3456$, we obtained the final value of the weight coefficient for the first criterion

$$\omega_1 = \frac{w_1}{\sum_{j=1}^{9} w_j} = \frac{0.3456}{0.3456+0.2569+0.2995+\cdots+0.4791} = 0.1178$$

The weight coefficients of the remaining criteria were determined in a similar way:

$$\omega_j = (0.1178, 0.0875, 0.1020, 0.1087, 0.1302, 0.0904, 0.0838, 0.1163, 0.1632)^T$$

Step 5 The weighted sum and the weighted product of the optimality criteria (Table 5) were calculated using expressions (19) and (20).

According to WSM, we obtained the weighted values for alternative A1 using expression (19)

$$\sum_{j=1}^{9} Q_{1j} = \left\langle \begin{array}{l} \widehat{\xi}_{6-6\left(\left(1-\frac{4.36}{6}\right)^{0.1177}\cdot\left(1-\frac{4.311}{6}\right)^{0.0875}\cdot\left(1-\frac{3.73}{6}\right)^{0.1020}\cdot\left(1-\frac{3.56}{6}\right)^{0.1087}\cdot\ldots\left(1-\frac{4.87}{6}\right)^{0.1632}\right)}, \\ \widehat{\xi}_{6\left(\left(\frac{2.39}{6}\right)^{0.1177}\cdot\left(\frac{2.35}{6}\right)^{0.0875}\cdot\left(\frac{2.18}{6}\right)^{0.1020}\cdot\left(\frac{2.12}{6}\right)^{0.1087}\cdot\ldots,\cdot\left(\frac{3.33}{6}\right)^{0.1632}\right)}, \\ \widehat{\xi}_{6\left(\left(\frac{2.21}{6}\right)^{0.1177}\cdot\left(\frac{1.91}{6}\right)^{0.0875}\cdot\left(\frac{2.61}{6}\right)^{0.1020}\cdot\left(\frac{2.02}{6}\right)^{0.1087}\cdot\ldots,\cdot\left(\frac{2.94}{6}\right)^{0.1632}\right)}, \end{array} \right\rangle = \langle \xi_{4.23}, \xi_{2.55}, \xi_{2.49} \rangle$$

According to WPM, we obtained the weighted values for alternative A1 using expression (20)

$$\sum_{j=1}^{9} P_{1j} = \left\langle \begin{matrix} \hat{\xi}_{6\left(\left(\frac{4.36}{6}\right)^{0.1177}\cdot\left(\frac{4.11}{6}\right)^{0.0875}\cdot\left(\frac{3.73}{6}\right)^{0.1020}\cdot\left(\frac{3.56}{6}\right)^{0.1087}\cdot\ldots,\cdot\left(\frac{4.87}{6}\right)^{0.1632}\right)}, \\ \hat{\xi}_{6-6\left(\left(1-\frac{2.39}{6}\right)^{0.1177}\cdot\left(1-\frac{2.35}{6}\right)^{0.0875}\cdot\left(1-\frac{2.18}{6}\right)^{0.1020}\cdot\left(1-\frac{2.12}{6}\right)^{0.1087}\cdot\ldots\left(1-\frac{3.33}{6}\right)^{0.1632}\right)}, \\ \hat{\xi}_{6-6\left(\left(1-\frac{2.21}{6}\right)^{0.1177}\cdot\left(1-\frac{1.91}{6}\right)^{0.0875}\cdot\left(1-\frac{2.61}{6}\right)^{0.1020}\cdot\left(1-\frac{2.02}{6}\right)^{0.1087}\cdot\ldots\cdot\left(1-\frac{2.94}{6}\right)^{0.1632}\right)} \end{matrix} \right\rangle = \langle \xi_{1.77}, \xi_{3.45}, \xi_{3.51} \rangle$$

The WSM and WPM of the weighted values for the remaining alternatives were obtained in a similar way.

Step 6 The resulting functions of the WSM and WPM models were calculated using expressions (21) and (23) (Table 6).

Using expression (21), we obtained the resulting function of WSM for alternative A1

$$\chi(Q_1) = \chi(\langle \xi_{4.23}, \xi_{2.55}, \xi_{2.49} \rangle) \\ = (2 \cdot 6 + 4.23 - 2.55 - 2.49)/(3 \cdot 6) = 0.6221$$

According to WPM, we obtained the weighted values for alternative A1 using expression (20)

$$\chi(P_1) = \chi(\langle \xi_{1.77}, \xi_{3.45}, \xi_{3.51} \rangle) \\ = (2 \cdot 6 + 1.77 - 3.45 - 3.51)/(3 \cdot 6) = 0.3779$$

The resulting functions for the WSM and WPM models for the remaining alternatives were determined in a similar way (Table 6).

Step 7 The integrated values of the criterion functions of the alternatives were obtained using expressions (23) and (24), and they can be seen in Table 6. The integrated value of the criterion function for the first alternative (A1) was obtained using expression (23)

$$K_1 = \lambda \sum_{j=1}^{9} \chi(Q_{1j}) + (1-\lambda) \sum_{j=1}^{9} \chi(P_{1j}) \\ = 0.390 \cdot 0.6221 + (1 - 0.390) \cdot 0.3779 = 0.4730$$

where coefficient λ was obtained using expression (24)

Table 5 WSM and WPM values

Alternatives	$\sum_{j=1}^{n} Q_i$	$\sum_{j=1}^{n} Q_i$
A1	$\langle \xi_{4.23}, \xi_{2.55}, \xi_{2.49} \rangle$	$\langle \xi_{1.77}, \xi_{3.45}, \xi_{3.51} \rangle$
A2	$\langle \xi_{4.50}, \xi_{2.92}, \xi_{2.71} \rangle$	$\langle \xi_{1.50}, \xi_{3.08}, \xi_{3.29} \rangle$
A2	$\langle \xi_{4.08}, \xi_{2.49}, \xi_{2.50} \rangle$	$\langle \xi_{1.92}, \xi_{3.51}, \xi_{3.50} \rangle$
A4	$\langle \xi_{4.29}, \xi_{2.79}, \xi_{2.57} \rangle$	$\langle \xi_{1.71}, \xi_{3.21}, \xi_{3.43} \rangle$
A3	$\langle \xi_{4.35}, \xi_{3.23}, \xi_{2.92} \rangle$	$\langle \xi_{1.65}, \xi_{2.77}, \xi_{3.08} \rangle$
A6	$\langle \xi_{4.49}, \xi_{2.94}, \xi_{2.78} \rangle$	$\langle \xi_{1.51}, \xi_{3.06}, \xi_{3.22} \rangle$
A4	$\langle \xi_{3.87}, \xi_{2.67}, \xi_{2.27} \rangle$	$\langle \xi_{2.13}, \xi_{3.33}, \xi_{3.73} \rangle$
A8	$\langle \xi_{4.12}, \xi_{2.55}, \xi_{2.61} \rangle$	$\langle \xi_{1.88}, \xi_{3.45}, \xi_{3.39} \rangle$
A5	$\langle \xi_{3.37}, \xi_{2.10}, \xi_{2.25} \rangle$	$\langle \xi_{2.63}, \xi_{3.90}, \xi_{3.75} \rangle$
A10	$\langle \xi_{3.72}, \xi_{2.40}, \xi_{2.27} \rangle$	$\langle \xi_{2.28}, \xi_{3.60}, \xi_{3.73} \rangle$
A11	$\langle \xi_{3.97}, \xi_{2.75}, \xi_{2.40} \rangle$	$\langle \xi_{2.03}, \xi_{3.25}, \xi_{3.60} \rangle$
A12	$\langle \xi_{3.07}, \xi_{2.20}, \xi_{1.68} \rangle$	$\langle \xi_{2.93}, \xi_{3.80}, \xi_{4.32} \rangle$
A12	$\langle \xi_{3.10}, \xi_{2.01}, \xi_{2.00} \rangle$	$\langle \xi_{2.90}, \xi_{3.99}, \xi_{4.00} \rangle$
A14	$\langle \xi_{3.47}, \xi_{2.30}, \xi_{2.20} \rangle$	$\langle \xi_{2.53}, \xi_{3.70}, \xi_{3.80} \rangle$
A13	$\langle \xi_{3.09}, \xi_{2.30}, \xi_{1.90} \rangle$	$\langle \xi_{2.91}, \xi_{3.70}, \xi_{4.10} \rangle$
A16	$\langle \xi_{3.70}, \xi_{2.30}, \xi_{2.11} \rangle$	$\langle \xi_{2.30}, \xi_{3.70}, \xi_{3.89} \rangle$
A14	$\langle \xi_{3.45}, \xi_{2.13}, \xi_{2.04} \rangle$	$\langle \xi_{2.55}, \xi_{3.87}, \xi_{3.96} \rangle$
A18	$\langle \xi_{3.48}, \xi_{2.04}, \xi_{2.21} \rangle$	$\langle \xi_{2.52}, \xi_{3.96}, \xi_{3.79} \rangle$
A15	$\langle \xi_{3.49}, \xi_{2.47}, \xi_{2.11} \rangle$	$\langle \xi_{2.51}, \xi_{3.53}, \xi_{3.89} \rangle$
A20	$\langle \xi_{3.19}, \xi_{2.28}, \xi_{1.87} \rangle$	$\langle \xi_{2.81}, \xi_{3.72}, \xi_{4.13} \rangle$
A21	$\langle \xi_{3.45}, \xi_{2.30}, \xi_{2.13} \rangle$	$\langle \xi_{2.55}, \xi_{3.70}, \xi_{3.87} \rangle$

Table 6 The resulting functions of the WSM and WPM models

Alternatives	$\sum_{j=1}^{n} \chi(Q_i)$	$\sum_{j=1}^{n} \chi(P_i)$	K_i	Rank
A1	0.6221	0.3779	0.4730	18
A2	0.6039	0.3961	0.4771	4
A3	0.6156	0.3844	0.4745	15
A4	0.6074	0.3926	0.4763	8
A5	0.5667	0.4333	0.4853	1
A6	0.5983	0.4017	0.4783	2
A7	0.6072	0.3928	0.4763	7
A8	0.6087	0.3913	0.4760	9
A9	0.6118	0.3882	0.4753	11
A10	0.6143	0.3857	0.4748	14
A11	0.6011	0.3989	0.4777	3
A12	0.6217	0.3783	0.4731	17
A13	0.6160	0.3840	0.4744	16
A14	0.6097	0.3903	0.4758	10
A15	0.6054	0.3946	0.4767	5
A16	0.6271	0.3729	0.4719	21
A17	0.6266	0.3734	0.4720	20
A18	0.6240	0.3760	0.4726	19
A19	0.6062	0.3938	0.4766	6
A20	0.6132	0.3868	0.4750	13
A21	0.6120	0.3880	0.4753	12

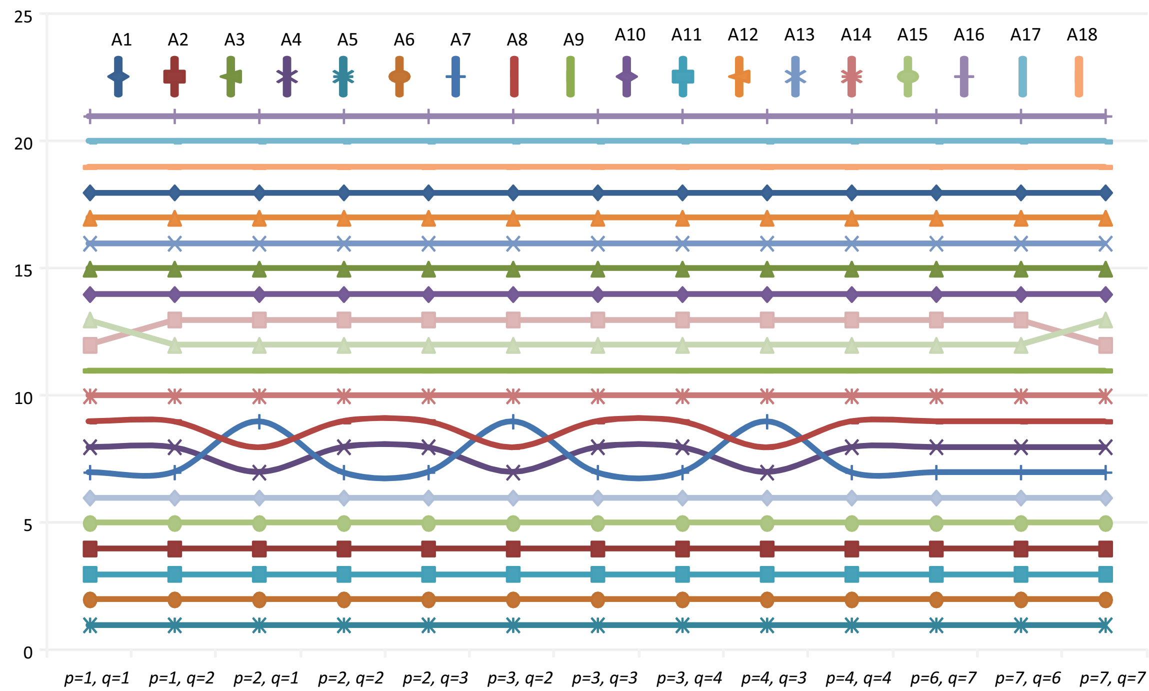

Fig. 2 Ranking order for different values of parameters p and q

$$\lambda = \frac{\sum_{i=1}^{21} \chi(Q_i)}{\sum_{i=1}^{21} \chi(Q_i) + \sum_{i=1}^{21} \chi(P_i)} = \frac{12.818}{12.818+8.181} = 0.390$$

and where we obtained the values $\sum_{i=1}^{21} \chi(Q_i) = 12.818$ and $\sum_{i=1}^{21} \chi(P_i) = 8.181$ by summing the elements from Table 6.

Step 8 The alternatives are ranked in step 8 based on the values obtained for the criterion functions K_i (Table 6). The best alternative is selected as the one with the highest value of K_i, which in this case is alternative A5. The final ranking of the alternatives is shown in Table 6.

5 Discussion and validation of the results

The discussion and validation of the results are in three parts: (1) the effect of a change in the p and q parameters in a Bonferroni aggregator on the results presented in Sect. 4; (2) a sensitivity analysis of the LNN WASPAS model to changes in the weight coefficients of the evaluation criteria and (3) validation of the results obtained by the LNN WASPAS model carried out by comparing them with LNN extensions of TOPSIS [28, 30], LNN CODAS (proposed), LNN VIKOR (proposed) and LNN MABAC (proposed) models. The details of these parts are presented in the following section.

5.1 The effect of a change in parameters *p* and *q* on the results of the evaluation

In this section, different values of parameters p and q are considered and the effects of changing their values on the results of the evaluation are analyzed. The analyzed values of parameters p and q in LNNNWGBM and their effect on the results of the ranking are shown in Fig. 2.

From Fig. 2, we see that parameters p and q in the LNNNWGBM aggregator have a certain effect on the final results of the ranking. In four scenarios, changes in parameters p and q led to minor changes in the ranking of the alternatives. For the parameter values $p = 1$, $q = 2$ and $p = 7$, $q = 7$, there was a change in the ranking of the alternatives that were 12th (A20) and 13th (A21), i.e., alternatives A20 and A21 changed places. The remaining alternatives retained their existing ranks. This change is the result of the small differences in the values of the criterion functions for alternatives A20 and A21. The values of the criterion functions for alternatives A20 and A21 are almost the same, which can be seen in Table 6 (K_{20} = 0.4750 and K_{21} = 0.4753). Since alternative A21 had a very small advantage over alternative A20 (only 0.063%), a change in parameters p and q caused a change in their ranking. Similar to the previous case, for the parameter values $p = 2$, $q = 1$; $p = 3$, $q = 2$ and $p = 4$, $q = 3$ there is a change in the ranking of alternatives A7, A4 and A8, which in the initial ranking were seventh, eighth and ninth,

Table 7 Sensitivity analysis scenarios

Scenario	Weights of criteria	Scenario	Weights of criteria
S1	$w_{c1} = 1.25 \times w_{c1(\text{old})}$	S28	$w_{c1} = 1.85 \times w_{c1(\text{old})}$
S2	$w_{c2} = 1.25 \times w_{c2(\text{old})}$	S29	$w_{c2} = 1.85 \times w_{c2(\text{old})}$
…	…	…	…
S9	$w_{c9} = 1.25 \times w_{c9(\text{old})}$	S36	$w_{c9} = 1.85 \times w_{c9(\text{old})}$
S10	$w_{c1} = 1.45 \times w_{c1(\text{old})}$	S37	$w_{c1} = 2.05 \times w_{c1(\text{old})}$
S11	$w_{c2} = 1.45 \times w_{c2(\text{old})}$	S38	$w_{c2} = 2.05 \times w_{c2(\text{old})}$
…	…	…	…
S18	$w_{c9} = 1.45 \times w_{c9(\text{old})}$	S45	$w_{c9} = 2.05 \times w_{c9(\text{old})}$
S19	$w_{c1} = 1.65 \times w_{c1(\text{old})}$	S46	$w_{c1} = 2.25 \times w_{c1(\text{old})}$
S20	$w_{c2} = 1.65 \times w_{c2(\text{old})}$	S47	$w_{c2} = 2.25 \times w_{c2(\text{old})}$
…	…	…	…
S27	$w_{c9} = 1.65 \times w_{c9(\text{old})}$	S54	$w_{c9} = 2.25 \times w_{c9(\text{old})}$

respectively. As can be seen in Fig. 2, the change in the ranking here is also due to their having approximately the same values of the criterion functions K_7= 0.47632, K_4= 0.47629 and K_8= 0.4760.

Based on the results shown (Fig. 2), we can conclude that for different values of parameters p and q the results of the ranking remain almost the same. Small and almost expected changes occur only in alternatives that have roughly the same value of their criterion functions. We can therefore conclude that the effect of changing parameters p and q is very small and does not influence the results of this decision-making problem.

5.2 Sensitivity analysis of the LNN WASPAS model based on the influence of variations in the value of the weight coefficients on the ranks of the alternatives

In this paper, a sensitivity analysis was performed to changes in the weight coefficients of the criteria on the ranking of the alternatives through 54 scenarios (Table 7).

The scenarios were grouped into six groups, so in the first group of nine criteria (S1–S9) in each scenario the value of one criterion was increased by 1.25. At the same time, the values of the remaining criteria were not changed. In the following groups, the process was repeated, but using different values. So, in the second group (S10–S18) the values of individual criteria were increased by 1.45, in the third (S19–S27) by 1.65, the fourth (S28–S36) by 1.85, and the fifth (S37–S45) by 2.05, while in the sixth group (S46–S54) the value of individual criteria was increased by 2.25. Both in the first group of scenarios and in the remaining scenarios, the values of the unfavored criteria remained unchanged. The changes in the ranks of the alternatives through 54 scenarios are shown in Fig. 3.

Changes in the weight coefficients through 54 scenarios show that assigning different weights to the criteria leads to a change in the ranks of individual alternatives, which confirms that the model is sensitive to changes in the weight coefficients. In Fig. 3, the horizontal axis (apical axis) represents the rank for the analyzed alternative, while the vertical axis (ordinate axis) represents the number of scenarios in which the observed alternative has the corresponding rank. So, for example, with Fig. 3, we see that alternative A5 remains first-ranked in all scenarios, while alternative A6 is ranked second in 52 scenarios third in one and fourth in another.

By comparing the initial ranks of the alternatives (Table 6) with the ranks obtained through the scenarios (Fig. 3), we can see that all of the alternatives retain their existing rank through the majority of scenarios. If we take the initial ranks from Table 6 as the reference values and compare them with those obtained through the scenarios, we can see a normal distribution of the ranks around the reference values. There is very little dispersion of the ranks around the reference values, and so all deviations are covered by ± two standard deviations (S_{dev}). The average value of the standard deviation through the scenarios is $S_{\text{dev}} = 0.428$, Fig. 4.

From Fig. 4, we can see that through the scenarios there is no deviation in the alternative ranked first, A5, while the alternative ranked second (A6) has very little deviation. For the remaining alternatives, the value of S_{dev} does not go above 0.96, and for 16 alternatives (in all 54 scenarios) it does not go above 0.40. The average value of S_{dev} for all alternatives is 0.428, which indicates that the correlation of the ranks is very high through the scenarios. Since the average value of S_{dev} is significantly lower than 0.5, we can conclude that there is a very high correlation (closeness) of the ranks and that the proposed ranking is confirmed and credible [42, 52].

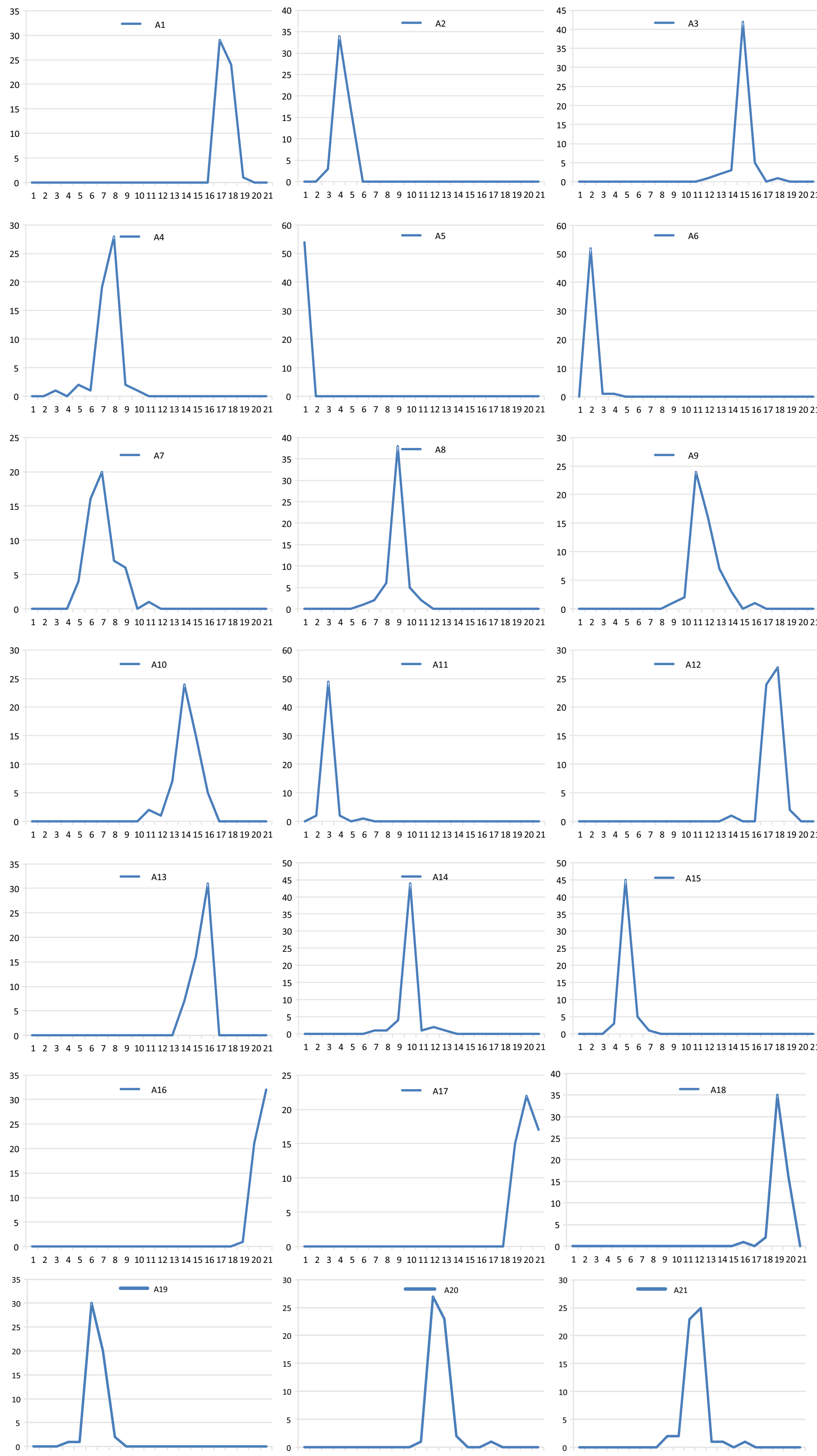

Fig. 3 Analysis of changes in the ranking of the alternatives through 54 scenarios

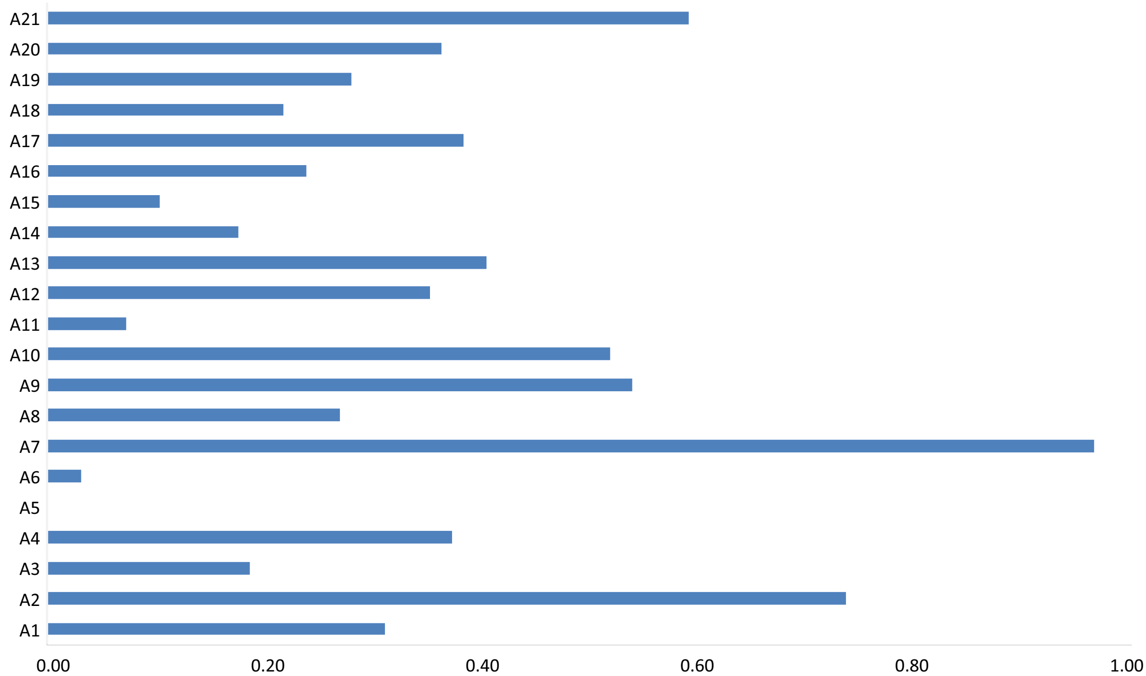

Fig. 4 Deviation of the ranks of the alternatives from the reference values through 54 scenarios

5.3 Verifying the stability of the solution based on different ranking methodologies

The stability of the solution was verified by comparing the results of the LNN WASPAS model with the results obtained using LNN extensions of traditional MCDM models: TOPSIS [21], MABAC [37], VIKOR [34] and EDAS [25]. These methods were chosen because their application so far has shown that they give stable and reliable results [52, 53]. For the purposes of this research, the authors made original extensions of the MABAC, VIKOR and EDAS multi-criteria techniques, based on the LNN concept.

In the following section, the LNN WASPAS model was validated by comparing the results with LNN TOPSIS [28, 30], LNN MABAC, LNN VIKOR and LNN EDAS models. The comparative ranking of the alternatives is shown in Fig. 5.

Figure 5 shows that the proposed model is stable, because most of the alternatives stay in the same position using different LNN methods. Alternatives A5, A6, A8 and A14 remain in the same position for all approaches, while A1, A12 and A16 are in the same position for all methods except for LNN EDAS. The biggest changes in position are for alternative A20. The results indicate the set {A5, A6} to be good alternatives.

The alternative ranked highest by LNN VIKOR, LNN MABAC, LNN TOPIS and LNN EDAS is the closest to the ideal solution. However, the alternative ranked highest by LNN TOPSIS is the best in terms of the ranking index, but this does not mean that it is always the closest to the ideal solution. Alternatives A5 and A6 are top ranked by LNN TOPSIS, and they are very close to each other. Some results by LNN TOPSIS are different from the results by LNN VIKOR, LNN MABAC and LNN EDAS, and the solution by LNN TOPSIS is not always the closest to the ideal. For certain weights, the alternative ranked highest by LNN TOPSIS is A6, whereas the closest to the ideal is A5. According to the LNN TOPSIS method Q_j, the best solution is A5 since Q_5 = 0.872. Alternative A6 is the best according to D^* = 0.1310. However, A5 is not the closest to the ideal since D_6^-= 0.402 and D_5^-= 0.392. From these values, we can see that A5 is ranked best by LNN TOPSIS, although it is not the closest to the ideal, because D_6^-= 0.402 and $D_5^- < D_6^-$. We can conclude that the LNN TOPSIS method determines a solution according to the distance from the ideal/negative-ideal solution, but the main issue in TOPSIS is the negligence of the relative importance of these distances.

Ranking by LNN VIKOR, LNN MABAC and LNN EDAS is almost the same and gives similar results to the ranking in LNN WASPAS. The initially best-ranked alternative according to LNN MABAC is A5 since the dominance index of alternative A5 in relation to alternative A6 (initially the second-ranked alternative) is higher than I_D= 0.045, so we conclude that A5 has enough advantage in relation to A6, and thus alternative A5 has the first rank. The other values of the dominance index are higher than

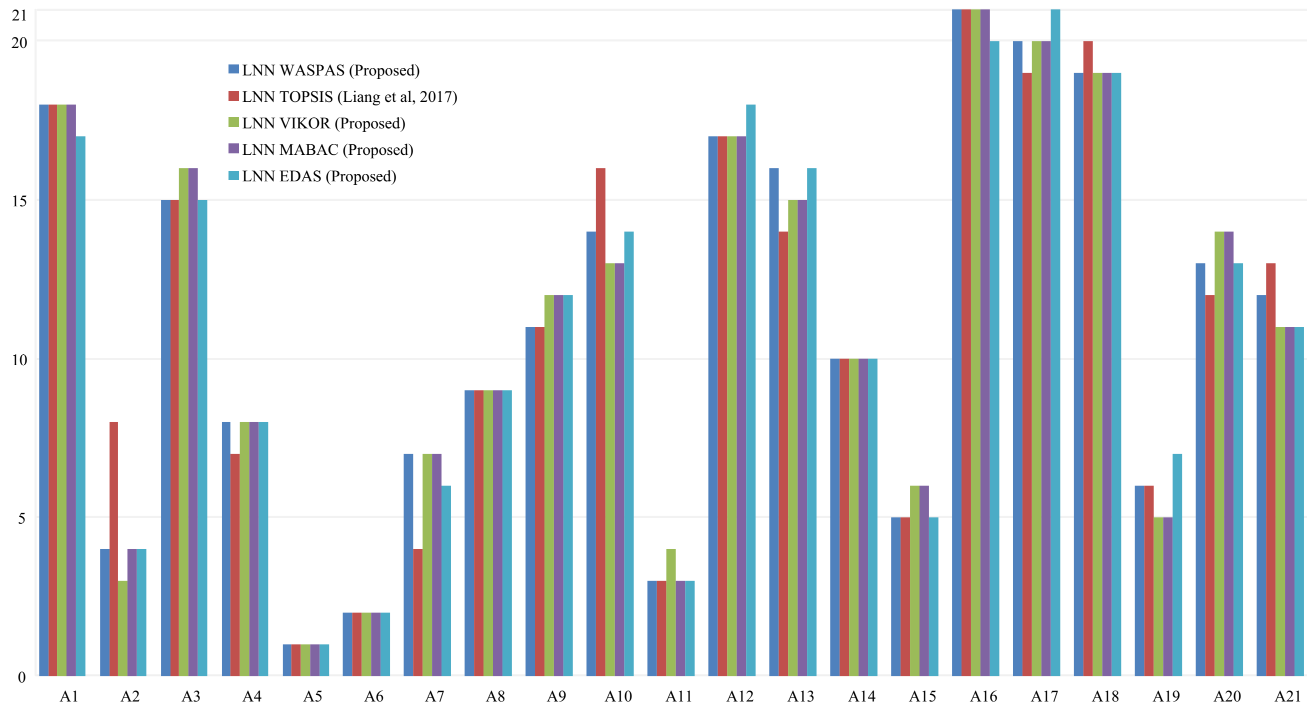

Fig. 5 Comparison of the ranks of the alternatives according to different methods

0.045, so the initial rank is retained for the other alternatives.

The LNN VIKOR, LNN MABAC, LNN TOPSIS and LNN EDAS results stand only for the given set of alternatives. Inclusion (or exclusion) of an alternative could affect the LNN VIKOR, LNN MABAC, LNN TOPSIS and LNN EDAS ranking of a new set of alternatives. By fixing the best and the worst values, this effect could be avoided, but that would mean that the decision maker could define a fixed ideal solution. The paper does not consider the trade-offs involved by normalization when obtaining the aggregating function in the LNN WASPAS method and this remains a topic for further research.

Spearman's rank correlation coefficient was used to determine the relationship between the results obtained by different approaches (Fig. 6). The results of the ranking comparison show an extremely high correlation between these models. Correlation with the LNN MABAC and LNN EDAS models is 0.995, while correlation with the LNN VIKOR and LNN TOPSIS models is 0.994 and 0.975, respectively. The mean SCC value for all of the models is 0.990. Since all of the SCC values are significantly higher than 0.9, we can conclude that there is a very high correlation (closeness) of the ranks and that the proposed ranking is confirmed and credible [42]. Based on this analysis, in addition to confirming the credibility of the ranking, we can also draw the conclusion that the LNN-based approach successfully exploits the uncertainties that arise in the group decision-making process.

6 Managerial implications and contributions

The practical application of this model makes it possible to obtain credible results when deciding under uncertainty conditions and when the data on which a decision is based are partially known and imprecise. The methodology's flexibility in the selection and weighting of performance measures to be used is also valuable. This flexibility will allow management to perform sensitivity analyses at many levels and thus obtain more robust and relevant solutions. Therefore, the application of this model is significant. This model practically helps managers to deal with their own subjectivity in prioritizing criteria and advisors in the transport of hazardous goods. The results of the research indicate the justification of the selected MCDM model.

The objective of this case study is to evaluate advisors in the transport of hazardous goods. The first benefit of this study is criteria selection based on a comprehensive literature review and practical research. The second benefit is not only the evaluation of advisors in the transport of hazardous goods, but also the analysis of advisors who do not satisfy the defined criteria. This tool will be acceptable to managers who have to deal with large magnitudes of uncertainties and imprecision in human resource management.

Thus, the main contribution and novelty of this study are as follows:

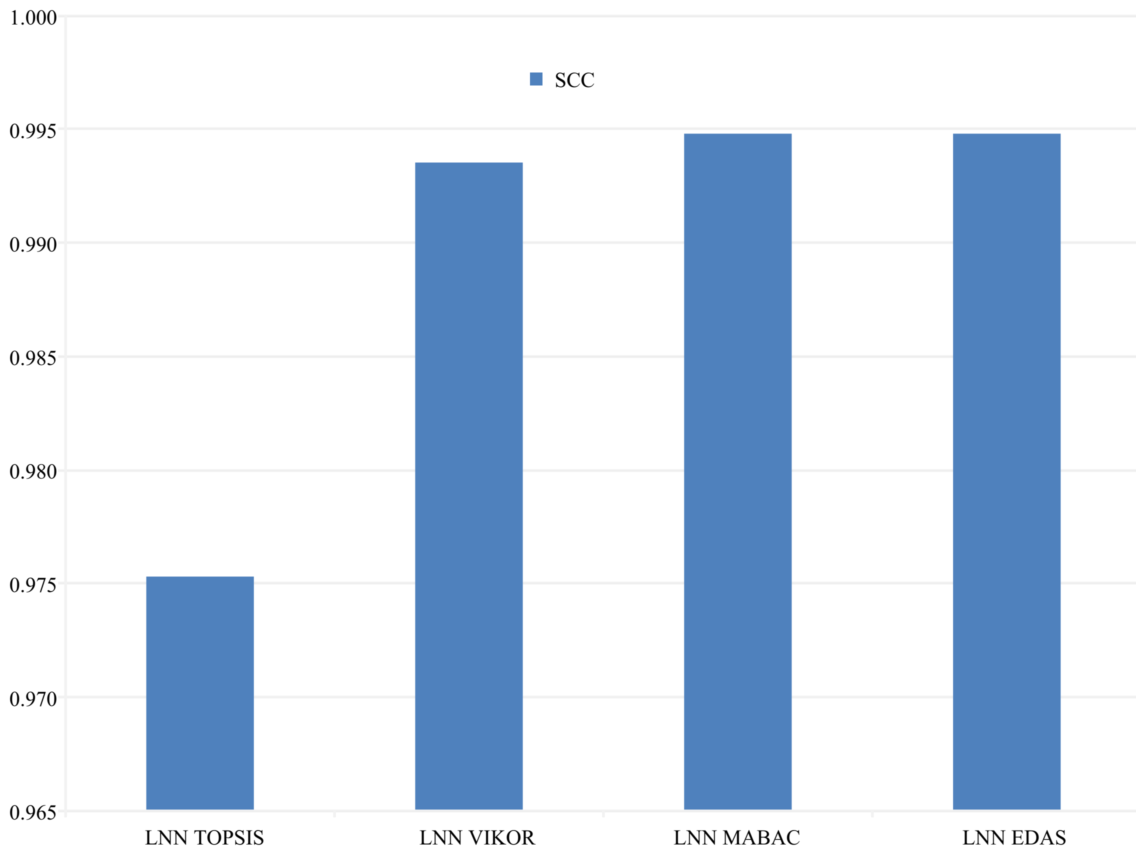

Fig. 6 Spearman's coefficient values for the MCDM models

- This paper introduces a unique hybrid LNN WASPAS MCDM model that provides more objective expert evaluation of criteria in a subjective environment. The present methodology enables the evaluation of alternative solutions (advisors) despite dilemmas in the decision-making process and lack of quantitative information. The multi-criteria model presented represents new methodology that gives employers the possibility of evaluating the work of advisors by means of predefined criteria, in order to reduce risk in the transport of hazardous goods, and to reduce the level of damage and extent of any consequences in the event of an accident. In order to validate the LNN WASPAS model in this study, original LNN VIKOR, LNN MABAC and LNN EDAS models were also developed, the results of which were compared with those of the original LNN WASPAS model.
- In this study, criteria are proposed for the evaluation of advisors, and they are evaluated using the new LNN model for determining the weight coefficients of the criteria. The model for determining the weight coefficients of the criteria was implemented within the LNN WASPAS model, which significantly improved the structure of the WASPAS model. The evaluation criteria provide a critical analysis of the content of empirical research in the sector for the transport of hazardous goods, which serves as a useful reference for researchers in the transport of hazardous goods or other operations fields. Defining criteria for evaluating advisors in the field of transporting hazardous goods improves the theoretical and practical framework for managing their work and effectiveness.
- Assessing the potentialities of advisors in the sector for the transport of hazardous goods, besides providing numerous objective indicators, helps managers to deal with their own subjectivity. The use of the LNN WASPAS approach significantly reduces imprecision when assessing the potentialities of advisors in the sector for the transport of hazardous goods that arises as a result of the lack of complete information on the criteria characteristics of the alternatives offered.
- The application of the LNN WASPAS approach significantly reduces uncertainty. The proposed hybrid model uses only internal linguistic knowledge (degree of truth, indeterminacy and falsity) to present the values of the attributes of a decision. In this way, subjectivity and assumptions when defining the indeterminacy and falsity degree in expert preferences are eliminated that

could affect the values of the attribute and the final choice of alternatives. The present methodology enables the evaluation of alternative solutions regardless of dilemmas in the decision-making process and lack of quantitative information.

7 Conclusion

The transport of hazardous materials plays a very important role with regard to environmental protection, i.e., the risk of accidents that can endanger the environment, whereby it is necessary to keep any possible risk to a minimum. In addition, the transport chain of hazardous materials must satisfy all participants in it without compromising the safety of any of them. Advisors for the safety of hazardous materials, whose role it is to prevent the occurrence of potential accidents, have an influence on the adequate organization and efficiency of the transport of hazardous materials. It is therefore very important to monitor and assess the work of these advisors, and to determine the level of professionalism with which they carry out their work. In this paper, a completely new methodology was proposed for assessing the work of safety advisors in the transport of hazardous goods, which is one of the contributions of the paper. The original LNN WASPAS model was developed for the purpose of evaluating the work of advisors, which is the second contribution of the paper. The results from the proposed model were verified by means of a sensitivity analysis that is made up of three parts. In the first part, different values of the parameters p and q were presented and the effect of a change in their values was analyzed on the results of the evaluation of the alternatives. In the second part, a sensitivity analysis is performed of the ranks of the alternatives to changes in the weight coefficients of the criteria through the formation of 54 scenarios, and in the third part the model was compared with the new models developed in this paper: LNN CODAS, LNN VIKOR and LNN MABAC. The development of these models in the sensitivity analysis is the third contribution of the paper. Lastly, Spearman's correlation coefficient was calculated which confirms the ranking of the alternatives.

In LNN, independent linguistic variables are used to represent the degree of truth, indeterminacy and falsity, but not like in single-valued neutrosophic numbers in which exact numerical values are used. In comparison with other concepts, a novel LNN WASPAS approach has some advantages that can be described as follows. The first reason is its advantage in comparison with gray theory. Gray relation analysis provides a well-structured analytical framework for the multi-criteria decision-making process, but it lacks the capability to characterize the subjective perceptions of designers in the evaluation process. LNNs may help here, because they can facilitate effective representation of vague information or imprecise data. In addition, the integration of linguistic neutrosophic numbers in MCDM methods makes it possible to explore the subjective and unclear evaluation of experts and to avoid assumptions, which is not the case when applying, for example, fuzzy theory. According to Hashemkhani Zolfani et al. [17], the main advantage of the WASPAS method is its high degree of reliability. The integration of linguistic neutrosophic numbers and the WASPAS method, with advantages of both concepts, presents a very important support in decision-making processes.

The research has shown that the LNN WASPAS model takes into consideration all parameters that affect the final decision, regardless of the degree and nature of their uncertainty. The model allows the processing of qualitative, subjective expert preferences, even when decisions are made on the basis of data that is partially known or even very little known. In this way, it is easier for decision makers to express their own preferences, while respecting subjectivity and the lack of information on specific criteria. In addition, the LNN model for determining the weight coefficients of the criteria, which improves the traditional WASPAS model, makes it possible to determine the weight coefficients for evaluating advisors. In addition to the above contributions, the benefit of this study is the development of selection criteria based on a comprehensive literature review and research involving experts from the Ministry of Construction and Infrastructure of the Republic of Serbia, as well as traffic inspectors and persons who lead professional association in this field in the Republic of Serbia. The second benefit is not only the evaluation of advisors in the transport of hazardous goods, but also the analysis of advisors who do not satisfy the defined criteria. The sustainable criteria provide a critical analysis of the content of empirical research that serves as a useful reference for researchers in operations fields.

Also, the result of this study helps managers to establish a systematic approach to the selection of the best advisors within the set of criteria. This tool will be acceptable to managers who have to deal with greater magnitudes of uncertainties and imprecision in the development and planning of transport. The results suggest the need for future workshops and discussion groups to coordinate the methodical procedures for evaluating advisors, and for improving the perception of transport resources. The MCDM methodology suggested here provides managers with a unique tool suitable for evaluating the work of advisors in relation to predefined sustainable criteria. The LNN WASPAS model is a tool for managing human resources in the process of transporting hazardous goods which gives managers insight into the quality of the work of their advisors. Good quality management of human resources contributes to reduced risks in the transport of hazardous goods, as well as a reduction in the level of damage and extent of the consequences in the event of accidents. From the aspect of the quality of the work of the safety advisors in the transport of hazardous goods, it is necessary to apply the proposed methodology at least twice a year, or to conduct constant monitoring and evaluation of the work of the advisors.

Bearing in mind the stated advantages, one of the improvements in this model would be the development and implementation of software for real-world applications. This would make the model much more within the reach of users and enable full exploitation of all the benefits stated in the paper.

As shown in Sect. 2, there are only a few papers that discuss the application of the LNN concept in MCDM. In view of this, the LNN WASPAS model represents an original MCDM approach that has not been considered in the literature so far and which provides promising results. The authors suggest that one of the pathways for future research is the application of LNN in other traditional MCDM models for determining the weight coefficients of criteria (e.g., Best–Worst method, DEMATEL method, FUCOM). Further integration of the LNN approach in traditional MCDM models, such as the Best–Worst method and the AHP method, would make it possible to determine the degree of consistency for expert comparisons. This would indirectly make it possible to determine the degree of reliability of the results, thus significantly contributing to the validation of the model. In addition, future research should include extension of the LNN WASPAS method with a stability analysis determining the weight stability intervals. That research should include determination of the weight stability interval $[w_{Li}, w_{Ui}]$ for each (ith) criterion, separately, with the initial values of the weights.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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