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ORIGINAL ARTICLE

A multi-item transportation problem with mode of transportation preference by MCDM method in interval type-2 fuzzy environment

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Abstract In this paper, we employ all the parameters as trapezoidal interval type-2 fuzzy numbers to cope with ambiguity and vagueness problem. There are two issues being addressed in this paper. The first is the selection of the most convenient transportation mode. We present a method for solving multi-criteria decision-making problem to deal with evaluating and ranking alternatives from the best to the worst with respect to decision maker(s) preferences. This is applied to find the most preferred transportation mode among available modes concerning some evaluation criteria for a transportation problem. A possibility degree is used for comparisons between the overall values of alternatives to raise a possibility degree matrix. Based on that matrix, the alternatives are ranked according to the ranking vector derived from the matrix, and the best one is selected. The second is to construct a multi-item transportation problem using that preferred mode of transportation. To get the crisp model, a defuzzification approach is adopted. To convert multi-objective transportation problem into a single-objective problem, two different techniques (i) fuzzy goal programming method and (ii) convex combination method are used. Then the reduced single-objective problem is solved by generalized reduced gradient method (LINGO-14.0) and a set of optimal solutions are obtained and presented graphically.

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1 Introduction

Transportation problem is one of the most important and earliest network-structured linear programming problems that comes up in several perspective and has obtained ample of interest in the literature. A transportation problem (TP) mainly deals with the linear optimization problem, which aspires to locate the best way to accomplish the demand of some destinations using the capacities of some sources or origins. While trying to find the best way, usually a variable minimum cost of shipping the product from one supply point to a demand point or a similar constraint should be taken into consideration. This traditional TP considers only two types of constraints related to sources and destinations. The solid transportation problem (STP), first introduced by Haley [1] in 1962, is a generalization of the well-known TP in which three-dimensional constraints are taken into account in the objective. This extra constraint is mainly due to modes of transportation (conveyances). In reality, as the conveyance capacity is another critical parameter in transportation activities, numerous researchers have concentrated their studies on the STP. For example, Ojha et al. [3] considered a STP for an item with fixed charge, vehicle cost and price discounted varying charge. Many researchers developed multi-objective solid transportation problems [8–10].

The objective of a STP is to satisfy some needs, which essentially means to deliver items from some sources to destinations through different types of conveyances with minimization of cost and time. But time or cost may not be the only factor. There may be several other factors such as product volume or weight, safety factor, flexibility. Depending on these several factors, all modes may not be equally preferable for selection. Generally, the available modes of transportation are rail, road, water, air, etc. The selection of the best mode is such complex decision situation in which the decision maker(s), while evaluating, assessing and selecting alternatives must take into account several often contradictory points of view. Therefore, this is a multi-criteria decision-making problem, and its solution requires the use of MCDM methods. In the multi-criterion decision-making problem, the decision makers (DMs) may provide the information about attribute weights with value ranges or order relations, because of the fact that a decision may be made under time pressure or lack of knowledge of experts, or the DMs may have limited attention and information processing capabilities. However, there is little research work accessible to handle such cases. MCDM is a field which originates in operations research and which has been broadly studied by researchers and practitioners [22, 23]. It concerns with evaluating, assessing and selecting alternatives from the best to the worst under conflicting criteria with respect to decision maker(s) preferences. In some decision-making problems, there are multiple experts analysing the problem who add uncertainty and it makes more sense than exact numbers. The assessment ratings of the alternatives and criterion weights generally are not always crisp, to a certain extent it may be expressed in linguistic terms, which are usually represented by fuzzy numbers and then the problem is called fuzzy multi-criteria decision-making (FMCDM). By this means, fuzzy multi-criteria decision-making problem is appeared as an area that integrates fuzzy sets and MCDM problems [6, 24, 25]. There are some available methods for solving FMCDM problems such as fuzzy analytical network process (FANP) [29, 30], fuzzy analytical hierarchy process (FAHP) [27, 28], fuzzy preference relation-based decision-making [26, 32], fuzzy TOPSIS [23, 31]. FAHP and FANP methods consisting of large number of fuzzy pair-wise comparison which creates the methods difficult for computation. This is the major disadvantage of these methods. Chen's TOPSIS method [31] needs some complex operational rules such as square root. Again Lee's method [32] using extended fuzzy preference relation is computationally efficient, but in case of two alternatives, this method constantly gives total performance index of one alternative 1 and that of another 0. So it is not possible to compare the alternatives with each other in the sense that how much one is preferable than the other.

In decision-making problem like transportation, the available data, i.e. the possible values of the system parameters cannot be always exactly determined and known. There are numerous causes for this ambiguity like lack of input information, fluctuating nature of parameter values, multiple source of data, noise in data, uncertainty in judgement, bad statistical analysis. For example, transportation cost depends upon fuel price, labour charges, tax charges, etc., each of which fluctuates from time to time. So it is not so easy to calculate the exact transportation cost of a route for certain time phase. Such type of vagueness can be approximated by type-2 fuzzy set. The idea of type-2 fuzzy set (T2 FS), at first established by Zadeh [7], is an extension of the concept of type-1 fuzzy set (T1 FS) which was introduced by Zadeh [5]. The main difference between the two types of fuzzy sets is that the membership grade of a T1 FS [33–35] is a real number in [0, 1], whereas the membership grade of a T2 FS is a fuzzy number with a support bounded by the interval [0, 1], that is T2 FS was introduced to capture the fuzziness of the membership functions in fuzzy set theory. The T1 FSs cannot completely handle all of the uncertainty present in real world, that may be due to indiscretions like lack of input information, bad statistical analysis, noise in data. It is sometimes not so easy to establish exact membership grades and hence to create the problems in terms of T2 FSs. Consequently, T2 FS has emerged as a result of fuzziness in the membership function. But because of this, the computational complexity is very high to deal with T2 FSs. Yang et al. [12] considered a STP in type-2 fuzzy environment. Jana et al. [15] applied type-2 fuzzy set to a multi-level supply chain. [16–21] are to name a few who have made recent contributions in this field. For the high computational complexity of general T2 FS, it has not widely applied to real-world applications [11]. Interval type-2 fuzzy sets (IT2FS) are the most regularly used type-2 fuzzy sets due to their easiness and lessen computational effort in comparison with general T2 FSs. This paper tries to focus on possibility degree to solve MCDM problems within the decision environment of IT2FSs. Keeping all this in mind, we are motivated to consider some transportation mode selection problems to get the most convenient mode with respect to several criteria for a particular transportation system under interval type-2 fuzzy environment. And also we are motivated to consider some innovative transportation problems using that preferred mode of transportation under uncertain environments, i.e. interval type-2 fuzzy environment. The main contributions of this paper are summarized as follows:

- A method for solving multi-criteria decision-making problem has been presented to rank the transportation mode using possibility degree.
- Bi-objective multi-item interval type-2 fuzzy transportation problem with fixed charge costs has been developed.
- Multi-objective problems have been converted into single objective using fuzzy goal method and convex combination method.
- Reduced crisp problem has been solved by LINGO-14.0.
- The models are illustrated by some numerical examples, and optimal results are presented in tabular forms.

The organization of this paper is as follows:

1. To characterize the uncertainty in the practical decision environment, this paper treats all the parameters as trapezoidal interval type-2 fuzzy numbers. So, in Sect. 2 some preliminaries of interval type-2 fuzzy number are precised, and in Sect. 3 conversion techniques for multi-objective into single objective are discussed.
2. We present a method for solving multi-criteria decision-making (MCDM) problem for which a possibility degree is used for comparisons between the overall values of alternatives to raise a possibility degree matrix and accordingly the transportation mode is ranked. In Sect. 4, it is mentioned elaborately.
3. To identify the most preferred transportation mode among available modes for a solid transportation problem (STP), the MCDM method is applied which is shown in Sect. 5.
4. A multi-item TP is formulated using the most preferred transportation mode in Sect. 7, and a crisp model is obtained by applying defuzzification method in Sect. 8.
5. In Sect. 9, we provide some numerical experiment and a set of optimal solutions are obtained. Then conclusions are given.

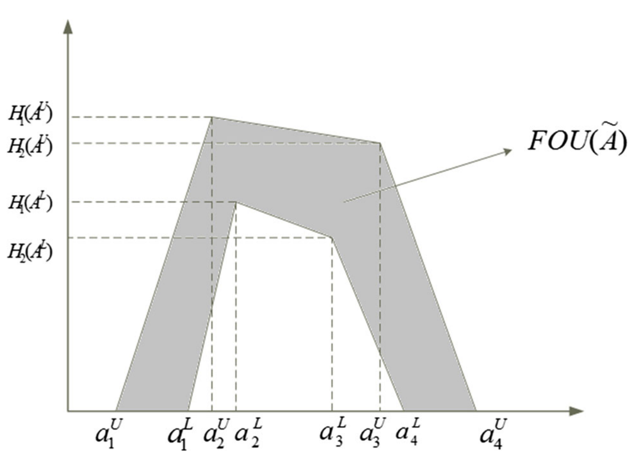

Fig. 1 Trapezoidal interval type-2 fuzzy number ($\widetilde{A}$)

2 Preliminaries

In this section, we recall some basic knowledge of interval type-2 fuzzy sets, which are characterized by primary and secondary membership functions and are the extensions of type-1 fuzzy sets.

Definition 1 A T2 fuzzy set $\widetilde{A}$ in X is defined as $\widetilde{A} = \{((x, u), \mu_{\widetilde{A}}(x,u)) : \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\widetilde{A}}(x,u) \leq 1\}$, where J_x is the primary membership of $x \in X$. Here X is the domain of $\widetilde{A}$ and $\mu_{\widetilde{A}}$, denotes the membership function of $\widetilde{A}$.

$\widetilde{A}$ can be expressed as:

$$\widetilde{A} = \int_{x\in X}\int_{\mu\in J_x} \mu_{\widetilde{A}}(x,u)/(x,u), J_x \subseteq [0,1]$$

Definition 2 A IT2 fuzzy set $\widetilde{A}$ is said to be an IT2 fuzzy set if all the secondary membership grades are 1 (i.e. $\mu_{\widetilde{A}}(x,u) = 1, \forall x, u$)

So IT2FS can be expressed as a special case of the general T2FS:

$$\widetilde{A} = \int_{x\in X}^{A}\int_{\mu\in J_x} 1/(x,u), J_x \subseteq [0,1]$$

Definition 3 The uncertainty in the primary membership of a IT2 fuzzy set $\widetilde{A}$ consists of a bounded region called the footprint of uncertainty (FOU). FOU is the union of all primary memberships J_x. That is, $FOU(\widetilde{A}) = \bigcup_{x\in X} J_x$. It is bounded by an upper membership function (UMF) $\bar{\mu}_{\widetilde{A}}(x)$ and a lower membership function (LMF) $\underline{\mu}_{\widetilde{A}}(x)$, which are type-1 membership functions so that $J_x = [\bar{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{A}}(x)]$.

Definition 4 A TrIT2 fuzzy variable $\widetilde{A}$ can be represented as $\widetilde{A} = (A^U, A^L) = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(A^U), H_2(A^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(A^L), H_2(A^L)))$ where, $a_1^U, a_2^U, a_3^U, a_4^U$ are four real numbers associated with upper membership function taking the membership values $0, H_1(A^U), H_2(A^U)$ and 0, respectively, whereas $a_1^L, a_2^L, a_3^L, a_4^L$ are associated with the lower membership function taking the membership values $0, H_1(A^L), H_2(A^L)$ and 0, respectively (in Fig. 1).

2.1 The arithmetic operations of IT2 fuzzy sets

Definition 5 $\widetilde{A}_1$ and $\widetilde{A}_2$ be two TrIT2 fuzzy numbers such as

$$\widetilde{A}_1 = (A_1^U, A_1^L) = \left((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)) \right)$$

$$\widetilde{A}_2 = (A_2^U, A_2^L) = \left((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_2(A_2^U), H_2(A_2^U)), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_2(A_2^L), H_2(A_2^L)) \right)$$

then,

(1) Addition:

$$\widetilde{A}_1 + \widetilde{A}_2 = (A_1^U, A_1^L) + (A_2^U, A_2^L) = \big((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; H_1(A_1^U) + H_1(A_2^U) - H_1(A_1^U).H_1(A_2^U)), \times (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; H_1(A_1^L) + H_1(A_2^L) - H_1(A_1^L).H_1(A_2^L)) \big)$$

(2) Subtraction:

$$\widetilde{A}_1 - \widetilde{A}_2 = (A_1^U, A_1^L) - (A_2^U, A_2^L) = \big((a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U; H_1(A_1^U) + H_1(A_2^U) - H_1(A_1^U).H_1(A_2^U)), \times (a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L; H_1(A_1^L) + H_1(A_2^L) - H_1(A_1^L).H_1(A_2^L)) \big)$$

$$Y = \frac{\sum_{T\in\{L,U\}} \left((a_{13}^T + a_{14}^T) - (a_{21}^T + a_{22}^T) \right) + \sum_{k=1}^{2} \left(\max\,(H_k(A_1^U) - H_k(A_2^U), 0) + \max\,(H_k(A_1^L) - H_k(A_2^L), 0) \right)}{\sum_{k=1}^{4} \operatorname{len}(v_k) + \sum_{k=1}^{2} |H_k(A_1^U) - H_k(A_2^U)| + \sum_{k=1}^{2} |H_k(A_1^L) - H_k(A_2^L)|} \quad (2)$$

(3) Multiplication

$$\widetilde{A}_1 \times \widetilde{A}_2 = (A_1^U, A_1^L) \times (A_2^U, A_2^L) = \big((\alpha_{11}^U, \alpha_{12}^U, \alpha_{13}^U, \alpha_{14}^U; H_1(A_1^U).H_1(A_2^U), H_2(A_1^U).H_2(A_2^U)), (\alpha_{11}^L, \alpha_{12}^L, \alpha_{13}^L, \alpha_{14}^L; H_1(A_1^L).H_1(A_2^L), H_2(A_1^L).H_2(A_2^L)) \big)$$

where $\alpha_{1i}^T = \min\left(a_{1i}^T, a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right)$, $T \in \{U, L\}$, $i \in \{1, 2\}$

(4) Multiplication with an ordinary number k:

$$k\widetilde{A}_1 = k(A_1^U, A_1^L) = \left(\left(ka_{11}^U, ka_{12}^U, ka_{13}^U, ka_{14}^U; 1 - (1 - H_1(A_1^U))^k, 1 - (1 - H_2(A_1^U))^k \right), \left(ka_{11}^L, ka_{12}^L, ka_{13}^L, ka_{14}^L; 1 - (1 - H_1(A_1^L))^k, 1 - (1 - H_2(A_1^L))^k \right) \right)$$

(5) Exponent:

$$\widetilde{A}_1^k = (A_1^U, A_1^L)^k = \left(\left((a_{11}^U)^k, (a_{12}^U)^k, (a_{13}^U)^k, (a_{14}^U)^k; (H_1(A_1^U))^k, (H_2(A_1^U))^k \right), \left((a_{11}^L)^k, (a_{12}^L)^k, (a_{13}^L)^k, (a_{14}^L)^k; (H_1(A_1^L))^k, (H_2(A_1^L))^k \right) \right)$$

Since these operations are more reasonable, here these are utilized [2].

2.2 Possibility degree of trapezoidal interval type-2 fuzzy number

Definition 6 $\widetilde{A}_1$ and $\widetilde{A}_2$ be two TrIT2 fuzzy numbers such that

$$\widetilde{A}_1 = (A_1^U, A_1^L) = \left((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)), \times (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)) \right)$$

$$\widetilde{A}_2 = (A_2^U, A_2^L) = \left((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_2(A_2^U), H_2(A_2^U)), \times (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_2(A_2^L), H_2(A_2^L)) \right)$$

Then, the possibility degree of $\widetilde{A}_1$ over $\widetilde{A}_2$ is defined as:

$$p\left(\widetilde{A}_1 \geq \widetilde{A}_2\right) = \min\,(\max\,(Y, 0), 1)) \quad (1)$$

where

where $\operatorname{len}(v_1) = a_{14}^L + a_{13}^L - a_{11}^L - a_{12}^L$, $\operatorname{len}(v_2) = a_{14}^U + a_{13}^U - a_{11}^U - a_{12}^U$, $\operatorname{len}(v_3) = a_{24}^L + a_{23}^L - a_{21}^L - a_{22}^L$ and $\operatorname{len}(v_4) = a_{24}^U + a_{23}^U - a_{21}^U - a_{22}^U$.

Definition 7 [2] Let $\widetilde{A}_i, i = 1\ldots n$ be n trapezoidal interval type-2 fuzzy numbers. We can get all the possibility degree $p_{ij}\,(i.e.\, p(\widetilde{A}_i \geq \widetilde{A}_j))$ by comparing every two trapezoidal interval type-2 numbers using Eqs. 1 and 2 and can construct a matrix $P = (p_{ij})_{n\times n}$, and then we can find the rank as $\operatorname{Rank}(A_i) = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} p_{ij} + \frac{1}{2} - 1\right)$ and then $\widetilde{A}_i \succeq \widetilde{A}_j$ if and only if $\operatorname{Rank}(\widetilde{A}_i) \geq \operatorname{Rank}(\widetilde{A}_j)$.

2.3 Defuzzification method for type-2 fuzzy sets

Defuzzification of a type-2 fuzzy set consists of two steps. In the first step, a type-2 fuzzy set is established as a type-1 fuzzy set by using the type reduction process. Then one of the defuzzification methods for ordinary (type-1) fuzzy sets is employed to get the correspondence of the type-2 fuzzy set.

The defuzzification approach proposed by Kahraman et al. [4] is adopted here. The defuzzified value of $\widetilde{A}$ is computed as

$$df\left(\widetilde{A}\right) = \frac{1}{2}\left\{\frac{1}{4}\left(\left(a_4^U - a_1^U\right) + \left(H_2\left(A^U\right) * a_2^U - a_1^U\right) + \left(H_1\left(A^U\right) * a_3^U - a_1^U\right)\right) + a_1^U + \frac{1}{4}\left(\left(a_4^L - a_1^L\right) + \left(H_2\left(A^L\right) * a_2^L - a_1^L\right) + \left(H_1\left(A^L\right) * a_3^L - a_1^L\right)\right) + a_1^L\right\} \quad (3)$$

3 Conversion techniques of multiple objectives to a weighted function of single objective

In this segment, we will have a discussion about two different techniques: (i) convex combination method and (ii) fuzzy goal method. We will change the proposed multi-objective optimization problems into single-objective optimization problem with the help of above-mentioned methods.

3.1 Convex combination method (CCM)

We have considered the multi-objective model as follows:

$$\begin{cases} \min & [z_i(x),\ i = 1,2,\ldots,M] \\ \text{s.t} & f_j \geq 0;\ j = 1,2,\ldots,N \\ & x \in X \end{cases} \quad (4)$$

Then by the convex combination method (Tanino et al. [14]), we transfer the above problem into following form as

$$\begin{cases} \min & \sum_{i=1}^{M} w_i z_i(x),\ \text{where } \sum_{i=1}^{M} w_i = 1, 0 < w_i < 1 \\ \text{s.t} & f_j \geq 0;\ j = 1,2,\ldots,N \\ & x \in X \end{cases} \quad (5)$$

Corresponding x and $z_i(x)$ are the solutions of the problem in equation (14).

3.2 Fuzzy goal programming method (FGPM)

The fuzzy goal method (FGM) was proposed by Sakawa [13] to solve linear and nonlinear multi-objective programming problems (MOPPs). The MOPPs can be considered as:

$$\begin{cases} \max[z_1(x), z_2(x), \ldots, z_m(x)] \\ \{x \in X. \end{cases} \quad (6)$$

Let us consider that decision makers have fixed the membership function $\mu_i(z_i(x))$ and given the goal membership function value $(i = 1,2,\ldots,m)$. Let us assume the following programming problem as:

$$\begin{cases} \max \sum_{i=1}^{m} d_i^- \\ \begin{cases} \mu_i(z_i(x)) + d_i^+ - d_i^- = \bar{\mu}_i \\ x \in X \\ d_i^+ d_i^- = 0, d_i^+, d_i^- \geq 0, i = 0,1,2,\ldots,m \end{cases} \end{cases} \quad (7)$$

where d_i^+, d_i^- denotes the positive and negative deviations. Then, if x^* is the optimal solution of the problem (7) and $0 < \mu_i(z_i(x^*)) < 1$, $d_i^+ = 0 (i = 1,2,\ldots,m)$ holds, then x^* is an efficient solution of the problem in equation (6).

4 MCDM based on ranking alternatives using possibility degree

Suppose $A_1, A_2, \ldots, A_m$ are m alternatives and $C_1, C_2, \ldots, C_n$ are n criteria. Let $W_1, W_2, \ldots, W_n$ are the weights of the different criteria such that $W_j > 0$ and $\sum_{j=1}^{n} W_j = 1$, where W_j is the weight of the criterion C_j specifying its importance. The multi-criteria decision-making method to get the best alternative is as follows:

Step 1: Aggregate all the criteria value for the alternatives depending on the criteria weights to get the overall values.

Step 2: Compute the possibility degree of every two alternatives and construct a possibility matrix P.

Step 3: Rank all the alternatives according to the ranking vectors calculated based on matrix P and select the best one.

5 An application of the MCDM on transportation mode selection

Suppose in a solid transportation problem there are two modes of available transportation, those are rail and road. The mode of selection depends on different criteria which are very important for a transportation policy. The decision makers want to rank the two modes with respect to eight criteria, which are as follows:

(1) **Transportation cost** (C_1): Cost for transportation of goods from source to destination.

Table 1 Linguistic terms and their corresponding trapezoidal interval type-2 fuzzy number

Linguistic terms	Trapezoidal interval type-2 fuzzy number
Very high (VH)	((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))
High (H)	((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))
Medium high (MH)	((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9))
Medium (M)	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Medium low (ML)	((0.1, 0.3, 0.3, 0.5; 1, 1), (0.2, 0.3, 0.3, 0.4; 0.9, 0.9))
Low (L)	((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Low (L)	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))

Table 2 Linguistic ratings of the alternatives with respect to each criteria

	C_1	C_2	S	T	P	F	A	D
Road	MH	MH	H	VH	H	VH	MH	H
Rail	VH	H	H	H	VH	MH	H	H

Table 3 Linguistic importance weights with respect to each criteria

Criteria	C_1	C_2	S	T	P	F	A	D
Weight	0.15	0.15	0.1	0.1	0.1	0.1	0.15	0.15

(2) **Fixed cost** (C_2): Cost due to transport equipment, maintenance, terminal facilities etc.

(3) **Speed** (S): The average speed that the conveyance can provide.

(4) **Loading or unloading time** (T): The time taken for loading and unloading of goods.

(5) **Product characteristics** (P): The volume or the weight of the goods for shipment that the conveyance can allow.

(6) **Flexibility** (F): The ability to change the route of transportation or the predetermined time schedule due to unexpected cause at the time of transportation.

(7) **Accidental rate** (A): The accidental rate of the vehicle.

(8) **Damageability** (D): The rate of product being damaged during transportation.

Accordingly at this juncture, we have the two alternatives (road and rail) and eight criteria (C_1, C_2, S, T, P, F, A and D). Here the decision makers employ the linguistics terms and related trapezoidal interval type-2 fuzzy numbers (as shown in Table 1). Seven-point scale is referred to appraise the ratings which are divided into very high (VH), high (H), medium high (MH), medium (M), medium low (ML), low (L) and very low (VL). Each decision maker has their own evaluation for each alternative based on their knowledge and experience. But at last after the conversation, they come to a agreement on evaluation results for the alternatives which is shown in Table 2. The decision makers also provides the weight for each criteria as shown in Table 3. Then the overall values of the alternatives are calculated, as shown in Table 4.

Table 4 Overall values of the alternatives

Alternatives	Overall values
Road	((0.65,0.83,0.83,0.955;1,1),(0.74,0.83,0.83, 0.8925;0.9,0.9))
Rail	((0.73,0.905,0.905,0.99;1,1),(0.8175,0.905,0.905, 0.9475;0.9,0.9))

The possibility degree matrix P is as follows: $P = \begin{pmatrix} 0.5 & 0.2 \\ 0.8 & 0.5 \end{pmatrix}$.

Now ranking vectors are calculated as: Rank(Road) $= 0.1$, Rank(Rail) $= 0.4$. As a result, the alternatives are ranked as Rail $\succ$ Road . So, rail is preferred than road.

5.1 Comparative study

The above example is also solved with some other techniques proposed by Chen et al. [26] based on ranking value method. The results are as follows:

Using the same data, according to Chens ranking value method, let the two preference matrices are constructed. Then the ranking value of the alternatives is calculated as Rank(Road) $= 0.350957$ and Rank(Rail) $= 0.649043$. So the alternatives can be ranked as Rail $\succ$ Road . It can be observed that the overall ranking order is same for the two methods. But in Chens method two preference matrices are to be constructed, whereas here the possibility matrix is calculated just only once, ensuing compact computing time, which is the main advantage.

6 Assumptions and notations

6.1 Assumptions

In this solid profit transportation problem, the following assumptions are made: (a) No damageability of the units occurs, and (b) a single item of a homogeneous product should be transported from sources to destinations.

6.2 Notations

To formulate the transportation model with respect to the above suitable transportation mode, i.e. the rail, the following notations are used:

- (i) m = number of sources.
- (ii) n = number of destinations.
- (iii) L = number of items.
- (iv) z_k = the objective functions, where $k = 1, 2$.
- (v) $\tilde{p}_{il}$ = the purchasing price of the lth item at the ith source.
- (vi) $\tilde{s}_{jl}$ = the selling price of the lth item at the jth destination.
- (vii) $\tilde{c}_{ijl}$ = the unit transportation cost of lth item from ith source to jth destination.
- (viii) $\tilde{f}_{ij}$ = the fixed charge of the transportation problem from the ith source to jth destination.
- (ix) x_{ijl} = the decision variable which is the amount of lth item to be transported from ith source to jth destination.
- (x) $\tilde{t}_{il}$ = transportation time with respect to the transportation activity from ith source to jth destination (hrs).
- (xi) $\tilde{\alpha}_l$ = loading and unloading time with respect to the transportation activity of lth item (hrs).
- (xii) $\tilde{a}_{il}$ = the amount of the lth item available at the ith source.
- (xiii) $\tilde{b}_{jl}$ = the demand of the lth item at the jth destination.

Here, $\tilde{}$ denotes IT2FNs throughout this investigation.

7 Model formulation

We assume m origins (or sources) $O_i\,(i = 1, 2, \ldots, m)$, n destinations (i.e. demands) $D_j(j = 1, 2, \ldots, n)$ and l items.

Furthermore, in this model, the objectives are to maximize the profit incurred by the transportation activities and to minimize the total transportation time. The constraints, respectively, are the supply constraints and demand constraints. In reality, as it is not so easy to specifically estimate the amount of related parameters, it is suitable to consider the parameters as trapezoidal interval type-2 fuzzy variables due to the complexity, changeability and non-decidability of the decision environment. With this concern, we formulate the problem assuming that the selling prices, purchasing prices, transportation costs, fixed charge costs, supplies and demands are all trapezoidal interval type-2 fuzzy variables.

One objective of the problem is to maximize the total profit, which is as follows:

$$\max \quad \tilde{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\left(\tilde{s}_{jl} - \tilde{p}_{il} - \tilde{c}_{ijl}\right)\ x_{ijl} - \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{f}_{ij}y_{ij}$$

Here if the products are transported from ith source to jth destination, then only the fixed-charged cost will be taken. So, we introduce a binary relation as

$$y_{ij} = \begin{cases} 1, & \text{if } \sum_{l=1}^{L} x_{ijl} > 0; \\ 0, & \text{if } \sum_{l=1}^{L} x_{ijl} = 0 \end{cases} \tag{8}$$

The other objective is to minimize the total transportation time, which is as follows:

$$\min \quad \tilde{z}_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}y_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\tilde{\alpha}_l x_{ijl}$$

Now, as the quantity of a particular product from a source cannot exceed the supply capacity of that, we have

$$\sum_{j=1}^{n} x_{ijl} \le \tilde{a}_{il}, \qquad i = 1, 2, 3, \ldots, m; \quad l = 1, 2, \ldots, L$$

Again the quantity of a particular product transported to a destination should not be less than its demand, that is

$$\sum_{i=1}^{m} x_{ijl} \ge \tilde{b}_{jl}, \qquad j = 1, 2, 3, \ldots, n; \quad l = 1, 2, \ldots, L$$

It is usual to have the nonnegativity of decision variable x_{ijl}, that is

$$x_{ijl} \ge 0, \qquad i = 1, 2, 3; \quad j = 1, 2, 3, \ldots, n; \quad l = 1, 2, \ldots, L$$

So, the multi-objective transportation problem can be written as:

$$\max \quad \tilde{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}(\tilde{s}_{jl} - \tilde{p}_{il} - \tilde{c}_{ijl})\ x_{ijl} - \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{f}_{ij}y_{ij} \tag{9}$$

$$\min \quad \tilde{z}_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}y_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\tilde{\alpha}_l x_{ijl} \quad \text{and} \quad \text{eq. (8)} \tag{10}$$

subject to

$$\begin{cases} \sum_{j=1}^{n} x_{ijl} \le \tilde{a}_{il}, & i = 1, 2, 3, \ldots, m; \quad l = 1, 2, \ldots, L \\ \sum_{i=1}^{m} x_{ijl} \ge \tilde{b}_{jl}, & j = 1, 2, 3, \ldots, n; \quad l = 1, 2, \ldots, L \\ x_{ijl} \ge 0, & i = 1, 2, 3; \quad j = 1, 2, 3, \ldots, n; \quad l = 1, 2, \ldots, L \end{cases} \tag{11}$$

Here, the solid transportation problem is formulated with uncertain market demands and costs. So since the selling prices, purchasing prices, transportation costs, fixed charge costs, supplies and demands of the above model are all trapezoidal interval type-2 fuzzy variables, we apply defuzzification method presented in Sect. 2.3.

8 Equivalent crisp problem

Since $\tilde{s}_{jl}\,\tilde{p}_{il}$, $\tilde{c}_{ijl}$, $\tilde{f}_{ij}$, $\tilde{t}_{ij}$,$\tilde{\alpha}_l$,$\tilde{a}_{il}$ and $\tilde{b}_{jl}$ are trapezoidal interval type-2 fuzzy numbers, these can be denoted as:

$$\begin{aligned}
\tilde{s}_{jl} &= \left(s^U_{jl}, s^L_{jl}\right) = \left(\left(s^U_{jl1}, s^U_{jl2}, s^U_{jl3}, s^U_{jl4}; H_1\left(s^U_{jl}\right), H_2\left(s^U_{jl}\right)\right), \left(s^L_{jl1}, s^L_{jl2}, s^L_{jl3}, s^L_{jl4}; H_1\left(s^L_{jl}\right), H_2\left(s^L_{jl}\right)\right)\right)\\
\tilde{p}_{il} &= \left(p^U_{il}, p^L_{il}\right) = \left(\left(p^U_{il1}, p^U_{il2}, p^U_{il3}, p^U_{il4}; H_1\left(p^U_{il}\right), H_2\left(p^U_{il}\right)\right), \left(p^L_{il1}, p^L_{il2}, p^L_{il3}, p^L_{il4}; H_1\left(p^L_{il}\right), H_2\left(p^L_{il}\right)\right)\right)\\
\tilde{c}_{ijl} &= \left(c^U_{ijl}, c^L_{ijl}\right) = \left(\left(c^U_{ijl1}, c^U_{ijl2}, c^U_{ijl3}, c^U_{ijl4}; H_1\left(c^U_{ijl}\right), H_2\left(c^U_{ijl}\right)\right), \left(c^L_{ijl1}, c^L_{ijl2}, c^L_{ijl3}, c^L_{ijl4}; H_1\left(c^L_{ijl}\right), H_2\left(c^L_{ijl}\right)\right)\right)\\
\tilde{f}_{ij} &= \left(f^U_{ij}, f^L_{ij}\right) = \left(\left(f^U_{ij1}, f^U_{ij2}, f^U_{ij3}, f^U_{ij4}; H_1\left(f^U_{ij}\right), H_2\left(f^U_{ij}\right)\right), \left(f^L_{ij1}, f^L_{ij2}, f^L_{ij3}, f^L_{ij4}; H_1\left(f^L_{ij}\right), H_2\left(f^L_{ij}\right)\right)\right)\\
\tilde{t}_{ij} &= \left(t^U_{ij}, t^L_{ij}\right) = \left(\left(t^U_{ij1}, t^U_{ij2}, t^U_{ij3}, {}^U_{ij4}; H_1\left(t^U_{ij}\right), H_2\left(t^U_{ij}\right)\right), \left(t^L_{ij1}, t^L_{ij2}, t^L_{ij3}, t^L_{ij4}; H_1\left(t^L_{ij}\right), H_2\left(t^L_{ij}\right)\right)\right)\\
\tilde{\alpha}_{l} &= \left(\alpha^U_{l}, \alpha^L_{l}\right) = \left(\left(\alpha^U_{l1}, \alpha^U_{l2}, \alpha^U_{l3}, \alpha^U_{l4}; H_1\left(\alpha^U_{l}\right), H_2\left(\alpha^U_{l}\right)\right), \left(\alpha^L_{l1}, \alpha^L_{l2}, \alpha^L_{l3}, \alpha^L_{l4}; H_1\left(\alpha^L_{l}\right), H_2\left(\alpha^L_{l}\right)\right)\right)\\
\tilde{a}_{il} &= \left(a^U_{il}, a^L_{il}\right) = \left(\left(a^U_{il1}, a^U_{il2}, a^U_{il3}, a^U_{il4}; H_1\left(a^U_{il}\right), H_2\left(a^U_{il}\right)\right), \left(a^L_{il1}, a^L_{il2}, a^L_{il3}, a^L_{il4}; H_1\left(a^L_{il}\right), H_2\left(a^L_{il}\right)\right)\right)\\
\tilde{b}_{jl} &= \left(b^U_{jl}, b^L_{jl}\right) = \left(\left(b^U_{jl1}, b^U_{jl2}, b^U_{jl3}, b^U_{jl4}; H_1\left(b^U_{jl}\right), H_2\left(b^U_{jl}\right)\right), \left(b^L_{jl1}, b^L_{jl2}, b^L_{jl3}, b^L_{jl4}; H_1\left(b^L_{jl}\right), H_2\left(b^L_{jl}\right)\right)\right)
\end{aligned}$$

respectively. Then the earlier transportation model takes the following form

$$\max \quad df[\tilde{z}_1] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\left[\left(df[\tilde{s}_{jl}] - df[\tilde{p}_{il}] - df[\tilde{c}_{ijl}]\right)\ x_{ijl}\right] - \sum_{i=1}^{m}\sum_{j=1}^{n} df[\tilde{f}_{ij}]y_{ij} \tag{12}$$

$$\min \quad df[\tilde{z}_2] = \sum_{i=1}^{m}\sum_{j=1}^{n} df\left[\tilde{t}_{ij}\right]y_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} df[\tilde{\alpha}_l]x_{ijl} \quad \text{and} \quad \text{eq. (8)} \tag{13}$$

subject to

$$\begin{cases}
\sum_{j=1}^{n} x_{ijl} \le df[\tilde{a}_{il}] & i = 1,2,3,\ldots,m;\quad l = 1,2,\ldots,L\\
\sum_{i=1}^{m} x_{ijl} \ge df[\tilde{b}_{jl}] & j = 1,2,3,\ldots,n;\quad l = 1,2,\ldots,L\\
x_{ijl} \ge 0, \quad i = 1,2,3; & j = 1,2,3,\ldots,n;\quad l = 1,2,\ldots,L
\end{cases} \tag{14}$$

Using Eq. (3) the above Eqs. (12)–(14) reduce to the deterministic form as

$$\begin{aligned}
\max \quad df[\tilde{z}_1] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\Bigg[\Bigg\{&\frac{1}{2}\Bigg(\frac{1}{4}\Bigg(\left(s^U_{jl4} - s^U_{jl1}\right) + \left(H_2\left(s^U_{jl}\right) * s^U_{jl2} - s^U_{jl1}\right)\\
&+ \left(H_1(s^U_{jl}) * s^U_{jl3} - s^U_{jl1}\right)\Bigg) + s^U_{jl1}\\
&+ \frac{1}{4}\left(\left(s^L_{jl4} - s^L_{jl1}\right) + \left(H_2\left(s^L_{jl}\right) * s^L_{jl2} - s^L_{jl1}\right) + \left(H_1\left(s^L_{jl}\right) * s^L_{jl3} - s^L_{jl1}\right)\right) + s^L_{jl1}\Bigg)\Bigg\}\\
&+ \left\{\frac{1}{2}\left(\frac{1}{4}\left((\tilde{p}^U_{il4} - p^U_{il1}) + (H_2(p^U_{il}) * p^U_{il2} - p^U_{il1}) + (H_1(p^U_{il}) * p^U_{il3} - p^U_{il1})\right) + p^U_{il1}\right.\right.\\
&\left.\left.+ \frac{1}{4}\left((p^L_{il4} - p^L_{il1}) + (H_2(p^L_{il}) * p^L_{il2} - p^L_{il1}) + (H_1(p^L_{il}) * p^L_{il3} - p^L_{il1})\right) + p^L_{il1}\right)\right\}\\
&- \left\{\frac{1}{2}\left(\frac{1}{4}\left(\left(\tilde{c}^U_{ijl4} - c^U_{ijl1}\right) + \left(H_2(c^U_{ijl}) * c^U_{ijl2} - c^U_{ijl1}\right) + \left(H_1(c^U_{ijl}) * c^U_{ijl3} - c^U_{ijl1}\right)\right) + c^U_{ijl1}\right.\right.\\
&\left.\left.+ \frac{1}{4}\left(\left(c^L_{ijl4} - c^L_{ijl1}\right) + \left(H_2(c^L_{ijl}) * c^L_{ijl2} - c^L_{ijl1}\right) + \left(H_1(c^L_{ijl}) * c^L_{ijl3} - c^L_{ijl1}\right)\right) + c^L_{ijl1}\right)\right\}\Bigg]\ x_{ijl}\\
&- \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{1}{2}\left\{\frac{1}{4}\left(\left(\tilde{f}^U_{ij4} - f^U_{ij1}\right) + \left(H_2(f^U_{ij}) * f^U_{ij2} - f^U_{ij1}\right) + \left(H_1(f^U_{ij}) * f^U_{ij3} - f^U_{ij1}\right)\right) + f^U_{ij1}\right.\right.\\
&\left.\left.+ \frac{1}{4}\left(\left(f^L_{ij4} - f^L_{ij1}\right) + \left(H_2(f^L_{ij}) * f^L_{ij2} - f^L_{ij1}\right) + \left(H_1(f^L_{ij}) * f^L_{ij3} - f^L_{ij1}\right)\right) + f^L_{ij1}\right\}\right]y_{ij}
\end{aligned} \tag{15}$$

$$\min \quad df[\tilde{z}_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{1}{2}\left\{\frac{1}{4}\left(\left(\tilde{t}^U_{ij4} - t^U_{ij1}\right) + \left(H_2(t^U_{ij}) * t^U_{ij2} - t^U_{ij1}\right) + \left(H_1(t^U_{ij}) * t^U_{ij3} - t^U_{ij1}\right)\right) + t^U_{ij1} + \frac{1}{4}\left(\left(t^L_{ij4} - t^L_{ij1}\right) + \left(H_2(t^L_{ij}) * t^L_{ij2} - t^L_{ij1}\right) + \left(H_1(t^L_{ij}) * t^L_{ij3} - t^L_{ij1}\right)\right) + t^L_{ij1}\right\}\right] y_{ij} \tag{16}$$

$$+ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\left[\frac{1}{2}\left\{\frac{1}{4}\left((\tilde{\alpha}^U_{l4} - \alpha^U_{l1}) + (H_2(\alpha^U_l) * \alpha^U_{l2} - \alpha^U_{l1}) + (H_1(\alpha^U_l) * \alpha^U_{l3} - \alpha^U_{l1})\right) + \alpha^U_{l1} + \frac{1}{4}\left((\alpha^L_{l4} - \alpha^L_{l1}) + (H_2(\alpha^L_l) * \alpha^L_{l2} - \alpha^L_{l1}) + (H_1(\alpha^L_l) * \alpha^L_{l3} - \alpha^L_{l1})\right) + \alpha^L_{l1}\right\}\right] x_{ijl}$$

$$\text{and} \quad \text{eq. (8)} \tag{17}$$

subject to

9.1 Input data

Here all the relevant costs, i.e. $\tilde{s}_{jl}$, $\tilde{p}_{il}$, $\tilde{c}_{jl}$, $\tilde{f}_{ij}$ and times, i.e. $\tilde{t}_{ij}$, $\tilde{\alpha}_l$, which are represented by trapezoidal interval type-2 fuzzy variables, are, respectively, given in Tables 5, 6, 7, 8, 9, 10. Again the supplies and demands which are also trapezoidal interval type-2 fuzzy variables are, respectively, given in Tables 11 and 12.

9.2 Optimum results

The deterministic optimization problems given by Eqs. (15) and (18) of the model in Eqs. (11) and (8) are solved for the above data. The solution techniques used here are the generalized reduced gradient (GRG) technique (using LINGO-14.0 solver). We have calculated the values of $z^0_k\ z^1_k, (k = 1, 2)$. These values are given as follows:

$z^0_1 = 2522.947,\quad z^1_1 = 4518.138,\quad z^0_2 = 821.4156,$
$z^1_2 = 586.5237,$

$$\begin{cases} \sum_{j=1}^{n} x_{ijl} \le \frac{1}{2}\left\{\frac{1}{4}\left((\tilde{a}^U_{il4} - a^U_{il1}) + (H_2(a^U_{il}) * a^U_{il2} - a^U_{il1}) + (H_1(a^U_{il}) * a^U_{il3} - a^U_{il1})\right) + a^U_{il1} + \frac{1}{4}\left((a^L_{il4} - a^L_{il1}) + (H_2(a^L_{il}) * a^L_{il2} - a^L_{il1}) + (H_1(a^L_{il}) * a^L_{il3} - a^L_{il1})\right) + a^L_{il1}\right\} \\ \sum_{i=1}^{m} x_{ijl} \ge \frac{1}{2}\left\{\frac{1}{4}\left(\left(\tilde{b}^U_{jl4} - b^U_{jl1}\right) + \left(H_2\left(b^U_{jl}\right) * b^U_{jl2} - b^U_{jl1}\right) + \left(H_1\left(b^U_{jl}\right) * b^U_{jl3} - b^U_{jl1}\right)\right) + b^U_{jl1} + \frac{1}{4}\left(\left(b^L_{jl4} - b^L_{jl1}\right) + \left(H_2\left(b^L_{jl}\right) * b^L_{jl2} - b^L_{jl1}\right) + \left(H_1\left(b^L_{jl}\right) * b^L_{jl3} - b^L_{jl1}\right)\right) + b^L_{jl1}\right\} \\ x_{ijl} \ge 0 \\ \quad \text{where } i = 1,2,3,\ldots,m; \quad j = 1,2,3,\ldots,n; \quad l = 1,2,\ldots,L \end{cases} \tag{18}$$

9 Numerical experiment

In this set of experiments, we consider a transportation problem with two sources, three destinations and two types of items. So $m = 2$, $n = 3$, and $l = 3$. The selling prices, purchasing costs of products, availabilities of the corresponding origins (i.e. resources), demands of the destinations, unit transportation costs, fixed charge costs are assumed as interval type-2 fuzzy numbers which are tabulated.

So we now formulate the membership functions of z_1 and z_2 as follows:

$$\mu_1(z_1(x)) = \begin{cases} 1 & \text{for} \quad z_1(x) > 4518.138 \\ \dfrac{z_1(x) - 2522.947}{4518.138 - 2522.947} & \text{for} \quad 2522.947 < z_1(x) < 4518.138 \\ 0 & \text{for } z_1(x) < 2522.947 \end{cases}$$

$$\mu_2(z_2(x)) = \begin{cases} 1 & \text{for} \quad z_2(x) > 821.4156 \\ \dfrac{821.4156 - z_2(x)}{821.4156 - 586.5237} & \text{for} \quad 586.5237 < z_2(x) < 821.4156 \\ 0 & \text{for } z_2(x) < 586.5237 \end{cases}$$

Table 5 Input data for TrIT2F unit selling price $\tilde{s}_{jl}$

$\tilde{s}_{11} = ((68, 75, 85, 87; 0.9, 0.8), (74, 77, 81, 86; 0.7, 0.5))$	$\tilde{s}_{12} = ((55, 60, 68, 79; 0.9, 0.8), (58, 61, 65, 75; 0.6, 0.6))$
$\tilde{s}_{13} = ((45, 53, 60, 68; 1, 0.9), (50, 55, 58, 63; 0.7, 0.5))$	$\tilde{s}_{21} = ((50, 58, 63, 70; 1, 0.8), (54, 61, 62, 64; 0.7, 0.6))$
$\tilde{s}_{22} = ((52, 58, 70, 75; 0.9, 0.8), (55, 60, 65, 72; 0.7, 0.5))$	$\tilde{s}_{23} = ((60, 65, 73.5, 75; 1, 0.9), (62, 72, 73, 74; 0.6, 0.5))$
$\tilde{s}_{31} = ((60, 70, 83, 87; 0.9, 0.8), (65, 75, 80, 84; 0.7, 0.5))$	$\tilde{s}_{32} = ((70, 75, 85, 89; 1, 0.8), (74, 77, 82, 87; 0.7, 0.6))$
$\tilde{s}_{33} = ((65, 76, 83, 85; 0.9, 0.8), (72, 78, 80, 84; 0.6, 0.5))$	–

Table 6 Input data for TrIT2F unit purchasing price $\tilde{p}_{il}$

$\tilde{p}_{11} = ((30, 34, 38, 42; 1, 0.9), (33, 36, 37, 40; 0.7, 0.5))$	$\tilde{p}_{12} = ((30, 41, 44, 48; 0.9, 0.8), (38, 42, 43, 46; 0.7, 0.6))$
$\tilde{p}_{13} = ((29, 37, 45, 48; 0.9, 0.8), (36, 38, 40, 46; 0.7, 0.6))$	$\tilde{p}_{21} = ((23, 26, 35, 40; 0.9, 0.8), (25, 27, 30, 38; 0.7, 0.5))$
$\tilde{p}_{22} = ((25, 29, 38, 42; 0.9, 0.8), (26, 30, 35, 40; 0.7, 0.6))$	$\tilde{p}_{23} = ((24, 28, 41, 47; 1, 0.8), (26, 30, 36, 45; 0.7, 0.6))$

Table 7 Input data for TrIT2F unit transportation cost $\tilde{c}_{ijl}$

$\tilde{c}_{111} = ((5, 7, 12, 15; 0.9, 0.8), (6, 9, 10, 13; 0.7, 0.5))$	$\tilde{c}_{112} = ((8, 11, 15, 18; 1, 0.8), (10, 12, 14, 16; 0.6, 0.5))$
$\tilde{c}_{113} = ((14, 17, 22, 25; 1, 0.8), (16, 18, 19, 23; 0.7, 0.5))$	$\tilde{c}_{121} = ((5, 8, 11, 14; 0.9, 0.8), (7, 9, 10, 12; 0.7, 0.6))$
$\tilde{c}_{122} = ((7, 10, 14, 16; 0.9, 0.8), (8, 11, 12, 15; 0.6, 0.5))$	$\tilde{c}_{123} = ((8, 10, 15, 17; 0.9, 0.7), (9, 12, 13, 16; 0.6, 0.4))$
$\tilde{c}_{131} = ((5, 9, 13, 15; 0.9, 0.8), (7, 10, 11, 14; 0.7, 0.5))$	$\tilde{c}_{132} = ((5, 6, 10, 13; 0.9, 0.7), (6, 7, 8, 11; 0.6, 0.5))$
$\tilde{c}_{133} = ((6, 9, 12, 15; 0.9, 0.8), (7, 10, 11, 13; 0.7, 0.6))$	$\tilde{c}_{211} = ((9, 12, 17, 20; 0.9, 0.7), (11, 14, 15, 19; 0.6, 0.4))$
$\tilde{c}_{212} = ((5, 8, 13, 15; 0.9, 0.8), (7, 10, 11, 14; 0.7, 0.6))$	$\tilde{c}_{213} = ((7, 12, 20, 25; 0.9, 0.7), (11, 14, 17, 22; 0.6, 0.3))$
$\tilde{c}_{221} = ((3, 5, 9, 11; 1, 0.8), (4, 6, 8, 10; 0.7, 0.6))$	$\tilde{c}_{222} = ((7, 12, 15, 19; 0.9, 0.7), (10, 13, 14, 17; 0.6, 0.4))$
$\tilde{c}_{223} = ((12, 17, 22, 27; 1, 0.8), (15, 19, 20, 25; 0.6, 0.5))$	$\tilde{c}_{231} = ((4, 8, 14, 16; 0.9, 0.8), (5, 10, 12, 15; 0.7, 0.5))$
$\tilde{c}_{232} = ((3, 5, 12, 16; 1, 0.8), (4, 8, 10, 14; 0.7, 0.5))$	$\tilde{c}_{233} = ((3, 7, 16, 20; 0.9, 0.8), (6, 10, 11, 18; 0.7, 0.6))$

Table 8 Input data for TrIT2F fixed charges $\tilde{f}_{ij}$

$\tilde{f}_{11} = ((1, 2.5, 3.5, 4.8; 0.8, 0.6), (2, 2.6, 3, 4; 0.5, 0.4))$	$\tilde{f}_{12} = ((2, 3, 5, 5.2; 0.9, 0.6), (2.5, 4, 4.5, 5.1; 0.5, 0.3))$
$\tilde{f}_{13} = ((0.5, 1, 2.2, 2.5; 0.8, 0.7), (0.8, 1.5, 1.8, 2.4; 0.6, 0.3))$	$\tilde{f}_{21} = ((1, 2.2, 3, 3.2; 1, 0.7), (2, 2.4, 2.5, 3.1; 0.6, 0.3))$
$\tilde{f}_{22} = ((0.3, 0.7, 1.2, 2.2; 1, 0.9), (0.5, 0.8, 1, 1.8; 0.8, 0.4))$	$\tilde{f}_{23} = ((1.5, 2, 3, 3.5; 0.9, 0.7), (1.8, 2.4, 2.8, 3.2; 0.6, 0.3))$

Table 9 Input data for TrIT2F transportation time $\tilde{t}_{ij}$

$\tilde{t}_{11} = ((18, 20, 21.5, 22.5; 1, 0.9), (19, 20.4, 21, 22.1; 0.8, 0.7))$	$\tilde{t}_{12} = ((15, 17, 19.5, 21; 1, 0.8), (16, 17.5, 18, 20; 0.7, 0.6))$
$\tilde{t}_{13} = ((20.6, 21.6, 22.5, 23.1; 1, 0.9), (21, 21.8, 22.2, 23; 0.8, 0.6))$	$\tilde{t}_{21} = ((17.5, 18.5, 20, 22; 1, 0.9), (18, 19, 19.8, 21; 0.8, 0.7))$
$\tilde{t}_{22} = ((7, 8, 10, 11; 1, 0.9), (7.5, 9, 9.6, 10.2; 0.7, 0.5))$	$\tilde{t}_{23} = ((11.5, 12.5, 13.7, 14.1; 1, 0.95), (12, 12.8, 13, 14; 0.8, 0.7))$

Table 10 Input data for TrIT2F loading and unloading time $\tilde{\alpha}_l$

$\tilde{\alpha}_1 = ((5, 5.3, 5.7, 5.82; 1, 0.9), (5.1, 5.4, 5.5, 5.8; 0.8, 0.65))$	$\tilde{\alpha}_2 = ((2.9, 3.2, 3.5, 3.8; 1, 0.85), (3, 3.3, 3.4, 3.7; 0.8, 0.7))$
$\tilde{\alpha}_3 = ((3.9, 4.3, 4.54, 4.8; 1, 0.9), (4.2, 4.4, 4.5, 4.6; 0.8, 0.6))$	–

Table 11 Input data for TrIT2F availabilities $\tilde{a}_{il}$

$\tilde{a}_{11} = ((18, 30, 45, 51; 0.9, 0.7), (25, 36, 43, 49; 0.6, 0.3))$	$\tilde{a}_{12} = ((22, 29, 49, 53; 1, 0.9), (24, 34, 43, 52; 0.6, 0.4))$
$\tilde{a}_{13} = ((20, 30, 43, 48; 1, 0.9), (25, 33, 40, 46; 0.7, 0.6))$	$\tilde{a}_{21} = ((15, 25, 34, 38; 1, 0.8), (20, 30, 32, 36; 0.7, 0.5))$
$\tilde{a}_{22} = ((20, 25, 35, 38; 1, 0.9), (22, 28, 33, 36; 0.8, 0.6))$	$\tilde{a}_{23} = ((27, 35, 45, 48; 1, 0.8), (30, 38, 42, 46; 0.7, 0.5))$

Table 12 Input data for TrIT2F unit demands $\tilde{b}_{jl}$

$\tilde{b}_{11} = ((9, 15, 19, 22; 1, 0.8), (12, 17, 18, 21; 0.7, 0.4))$	$\tilde{b}_{12} = ((12, 17, 23.5, 28; 0.9, 0.8), (14, 18, 20, 26; 0.6, 0.4))$
$\tilde{b}_{13} = ((7, 16, 19, 23; 1, 0.7), (12, 17, 17.5, 22; 0.6, 0.4))$	$\tilde{b}_{21} = ((10, 14, 18, 22; 1, 0.8), (12, 15, 16, 20; 0.7, 0.5))$
$\tilde{b}_{22} = ((6, 10, 18, 22; 0.9, 0.8), (8, 12, 15, 20; 0.7, 0.5))$	$\tilde{b}_{23} = ((11, 14, 20, 25; 1, 0.8), (13, 15, 16, 22; 0.7, 0.5))$
$\tilde{b}_{21} = ((10, 14, 22, 26; 1, 0.9), (13, 15, 17, 24; 0.7, 0.6))$	$\tilde{b}_{22} = ((5, 9, 18, 21; 1, 0.8), (8, 14, 16, 19; 0.7, 0.5))$
$\tilde{b}_{23} = ((4, 8, 15, 19; 1, 0.8), (6, 10, 12, 16; 0.7, 0.6))$	–

Table 13 Optimum results via fuzzy goal programming method

$\bar{\mu}_1$	$\bar{\mu}_2$	(z_1)	(z_2)	$\bar{\mu}_1$	$\bar{\mu}_2$	(z_1)	(z_2)
0.9	0.1	3811.835	797.9269	0.8	0.1	3826.228	797.9275
	0.2	3741.182	774.4376		0.2	3755.575	774.4381
	0.3	3662.751	750.9484		0.3	3684.579	750.9489
	0.4	3570.102	727.4591		0.4	3614.934	727.4595
	0.5	3477.115	703.9698		0.5	3477.116	703.9700
	0.6	3354.039	680.4815		0.6	3354.033	680.4806
	0.7	3257.035	656.9922		0.7	3257.037	656.9925
	0.8	3014.112	633.5033		0.8	3042.984	639.5935
	0.9	2902.764	610.0154		0.9	3042.984	639.5935
0.1	0.1	4389.626	797.9264	0.1	0.9	3452.122	658.5864
	0.2	4259.711	774.4372	0.2		2794.247	586.5237
	0.3	4129.415	750.9480	0.3		2791.398	586.5237
	0.4	3979.136	727.4588	0.4		2794.247	586.5237
	0.5	3809.342	703.9696	0.5		2794.247	586.5237
	0.6	3626.253	680.4805	0.6		2794.247	586.5237
	0.7	3452.122	658.5864	0.7		2791.398	586.5237
	0.8	3452.122	658.5864	0.8		2791.398	586.5237
	0.9	3452.122	658.5864	0.9		2791.398	586.5237

Table 14 Optimum results via convex combination method

w_1	w_2	z_1	z_2
0	1	2522.947	586.5237
0.2	0.8	2652.122	625.8461
0.5	0.5	3452.122	658.5864
.8	0.2	4518.138	821.4156
0.9	0.1	4518.138	821.4156

Then we compute the following model to get the solution,

$$\begin{cases} \max d_1^- - d_2^- \\ \begin{cases} \dfrac{\bar{z}_1(x) - 2522.947}{4518.138 - 2522.947} + d_1^+ - d_1^- = \bar{\mu}_1 \\ \dfrac{821.4156 - \bar{z}_2(x)}{821.4156 - 586.5237} + d_2^+ - d_2^- = \bar{\mu}_2 \\ x \in X \\ d_i^+ d_i^- = 0, d_i^+, d_i^- \geq 0, i = 0, 1, 2, \ldots, m. \\ \text{and } (14) \end{cases} \end{cases} \tag{19}$$

After solving the model in Eq. (19), we get the solutions for total profit (z_1) and total transportation time (z_2) of proposed Model, which are listed in Table 13.

Here, the above results are the optimal solutions of the problem (19) for different values of $\bar{\mu}_1$ and $\bar{\mu}_2$. Moreover, $d_i^+ = 0 (i = 1, 2, \ldots, m)$ holds. So these are efficient solutions of the problem in Eqs. (15), (18).

Applying convex combination method stated in Sect. 3.1, for different weights on z_1 and z_2, we get the following Table 14.

The comparison between the optimum results calculating by different methods for the model is given in Table 15.

10 Discussion

Since mode of transportation plays vital role in transportation problem, the available modes are ranked to find the most preferred transportation mode. Here, we obtain Rank(Road) = 0.1, Rank(Rail) = 0.4, i.e. rail is preferred than road. Subsequently, the proposed multi-objective multi-item TP bears all the parameters with respect to rail. The multi-objective optimization problems are converted into single objective using two different techniques: (i) fuzzy goal programming method and (ii) convex combination method. The optimum results are presented in optimistic labels of different parameters. So decision maker only can choose the optimum values of total profit and time by the choice of different optimistic labels, i.e. different $\bar{\mu}_i (i = 1, 2)$, which is shown in Table 13. From Table 13, it is observed that if $\bar{\mu}_1$ is constant and $\bar{\mu}_2$ increases, both total profit and total transportation time z_1 and z_2 decrease. This phenomenon is depicted in Fig. 2, where the dotted line represents transportation time and the bold line represents profit. Similarly with constant μ_2 as μ_1 increases, z_1 and z_2 decrease. But at some of the different optimistic labels the values are same. Again in case of convex combination method, the optimum results are different for the choice of weight function $w_i (i = 1, 2)$ which is shown in Table 14. At this juncture, as $w_1 + w_2 = 1$, while we increase the value of w_1, we decrease the value of w_2. Now increase in

Table 15 Comparison of optimum results

Method	z_1	z_2	Values of decision variables
FGPM	3809.342	703.9696	14.3,0,0,0,12.09,15.11,5,21.1,1.28,0 12.75,25.23,13.99,0,0,11.06,14.34,8.82
CCM	3452.122	658.5864	14.3,0,0,0,12.09,15.11,5,21.1,0 0,12.75,13.94,13.99,0,0,11.06,14.34,10.10

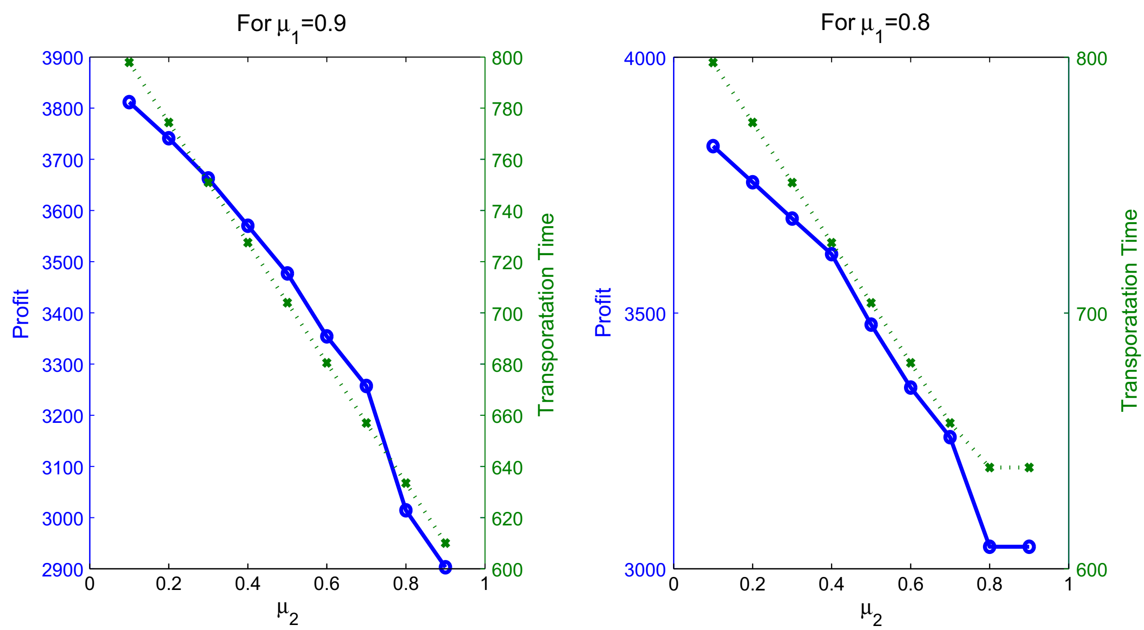

Fig. 2 Change of total profit and transportation time w.r.t different optimistic labels

the value of w_1 means we give more weightage in the first objective, i.e. total profit, and simultaneously decrease in the value of w_2 means we give less weightage in the second objective, i.e. total transportation time. Here we see in Table 14 that as we increase the value of w_1 and simultaneously decrease the value of w_2, the total profit increases and total transportation time also increases which is quite expected because we want to maximize the first objective and minimize the second objective.

11 Conclusions and future research

The selection of suitable transportation mode is one of the most important decision issues in transportation system. In this paper, we proposed a multi-criteria decision-making method to find the most ideal transportation mode among available modes concerning some evaluation criteria for a transportation problem. Moreover, there exists uncertainty in realistic TPs. In fact, uncertainty exists everywhere in the practical life. So here we made an effort to utilize trapezoidal interval type-2 fuzzy numbers to explain the uncertain information, and so all the parameters are considered as trapezoidal interval type-2 fuzzy numbers. On the other hand, a multi-item TP was addressed using the desired transportation mode. A crisp model was also established which was solved by one of the available software LINGO-14.0. There are numerous possible extensions of the current study that could constitute future research activities in this field. The methodologies used in this paper are quite general and computationally efficient. We expect that the methodologies which we used in the first part of this paper, i.e. in the selection of suitable transportation mode, have potential applications and it can be applied in many industry-based MCDM problems in the future. These can be applied to the decision-making problems in different areas such as inventory control system, supply chain, portfolio selection with type-2 fuzzy parameters. Again the presented transportation problem can be extended to different types of realistic transportation problems including space constraints, price discounts on the basis of amount of transported units, breakable/deteriorating items. Further research activities are dealing with the inclusion of different types of uncertainty level about other available vague information and test the formulation to transportation problem for supply chain network.

Compliance with ethical standards

Conflict of interest The authors have no conflict of interest for the publication of this paper.

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