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ORIGINAL ARTICLE

Extension of the VIKOR method for group decision making with extended hesitant fuzzy linguistic information

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Abstract Group decision-making approaches are very important due to the complexity and uncertainty of many real-world decision-making problems. Some of the decision-making problems are defined in qualitative frameworks. Extended hesitant fuzzy linguistic term set (EHFLTS) is proposed as a new and powerful tool for elicitation of hesitant qualitative information in group decision-making process. In this paper, we first introduced the comparison laws and a family of distance and similarity measures for extended hesitant fuzzy linguistic terms (EHFLTs) and EHFLTSs, respectively. Next, we developed the extended hesitant fuzzy linguistic (EHFL)-VIKOR method as a qualitative multi-attributes group decision-making approach based on the EHFLTS distance measures to deal with the qualitative hesitancy in group decision making. Finally, we presented an application example about selection of suitable telecommunications service provider of small- and medium-sized enterprises to verify applicability and validation of proposed method in the process of qualitative group decision making.

Keywords Extended hesitant fuzzy linguistic term set · Extended hesitant fuzzy linguistic distance measures · Extended hesitant fuzzy linguistic similarity measures · EHFL-VIKOR

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1 Introduction

Multi-attribute group decision making (MAGDM) refers to ranking alternatives and selecting the best choice in the presence of multiple, usually conflicting, attributes in which the information is provided by a group of decision makers [1, 2]. The main challenge in dealing with real cases is the inherent vagueness of human judgment as well as complexity and uncertainty of socio-economic environment [3, 4]. Fuzzy set theory is an effective tool to handle such vaguenesses and imprecisions [5, 6]. Some extensions of fuzzy sets such as type-2 fuzzy sets [7], intuitionistic fuzzy sets (IFSs) [8], fuzzy multisets [9] and hesitant fuzzy sets (HFSs) [10] are suitable for quantitative situations. Some other approaches such as fuzzy linguistic approach are used to model qualitative problems [5, 11].

Zadeh [12] first developed the fuzzy linguistic approach. Up to now, several extensions of fuzzy linguistic approach have been introduced, such as the linguistic model based on type-2 fuzzy sets [13, 14], 2-tuple fuzzy linguistic model [15–17] and the proportional 2-tuple model [18]. In all of these extensions, the value of a linguistic variable is expressed by only one linguistic term [5, 19]. But in many MADM cases, due to complexity and high degree of uncertainty, an expert cannot use a single linguistic term to evaluate an alternative. He/she may hesitate among different linguistic terms and prefer to use more than one linguistic term simultaneously [20, 21]. For example, when evaluating the speed of a car, an expert says, “its speed is a little high” and another expert says “its speed is at least medium.”

Hesitant fuzzy linguistic term set (HFLTS) is introduced by Rodriguez et al. [5] to solve abovementioned problem. The HFLTS approach evaluates a linguistic variable using several linguistic terms and consequently provides flexible elicitation of linguistic information by using context-free grammars [6, 11]. Considering the ability of HFLTS to improve the flexibility of the elicitation of linguistic information, more attention has been recently drawn to this approach. Liu and Rodriguez [22] introduced a fuzzy envelope for HFLTS and presented an illustrative example of its application through the use of fuzzy TOPSIS. Rodriguez et al. [6] proposed a linguistic group decision model to improve the elicitation of linguistic information by using of comparative linguistic expressions that is closed to human cognitive process. Wang et al. [23] suggested an outranking approach for multi-criteria decision making with HFLTSs. Liao and Xu [21] developed a family of cosine distant and similarity measures for the HFLTSs. Afterward, they extended the cosine-distance-based HFL-TOPSIS method and the cosine-distance-based HFL-VIKOR method. Liao et al. [20] developed a family of distance and similarity measures between two HFLTSs and a variety of distance and similarity measures between two collections of HFLTSs. Lee and Chen [24] proposed a fuzzy decision-making approach based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets. Lee and Chen [19] suggested a fuzzy group decision-making method on the basis of the likelihood-based comparison relations of HFLTSs and some HFLTS operators. Some typical operators are hesitant fuzzy linguistic weighted average (HFLWA), hesitant fuzzy linguistic weighted geometric (HFLWG), hesitant fuzzy linguistic ordered weighted average (HFLOWA) and hesitant fuzzy linguistic ordered weighted geometric (HFLOWG).

Wang et al. [25] introduced the concept of interval-valued hesitant fuzzy linguistic set (IVHFLS) as an extension of both a linguistic term set and an interval-valued hesitant fuzzy set. They proposed two kinds of prioritized aggregation operators and an MCDM method based on these operators for interval-valued hesitant fuzzy linguistic situation. Meng et al. [26] defined generalized linguistic hesitant fuzzy hybrid weighted averaging (GLHFHWA) operator, generalized linguistic hesitant fuzzy hybrid geometric mean (GLHFHGM) operator, generalized linguistic hesitant fuzzy hybrid Shapley weighted averaging (GLHFHSWA) operator and generalized linguistic hesitant fuzzy hybrid Shapley geometric mean (GLHFHSGM) operator. Then, they developed a linguistic hesitant fuzzy multi-attribute decision-making approach. Chen and Hong [27] presented an MCDM method based on the aggregation of linguistic terms and HFLTSs. Zhang and Wu [11] defined the multiplicative consistency of hesitant fuzzy linguistic preference relations (HFLPRs) and characterized the multiplicative consistency of HFLPRs. Liao et al. [28] focused on correlation coefficients of HFLTSs. They defined several types of correlation coefficients, the weighted correlation coefficients and ordered weighted correlation coefficients. Then they applied the suggested correlation coefficients of HFLTSs in qualitative decision-making process. Liu et al. [29] introduced some generalized aggregation operators to aggregate the interval-valued hesitant uncertain linguistic information. They developed a multiple attribute group decision-making approach with unknown weight information for interval-valued hesitant uncertain linguistic situation.

However, it can be seen from previous studies that applying HFLTSs to group decision making is not a facile process [6]. Wang [30] proposed the concept of extended hesitant fuzzy linguistic term sets (EHFLTSs) as an ordered subset of linguistic terms. This approach was applied to group decision making. Wang and Xu [31] developed the concept of extended hesitant fuzzy linguistic preference relations (EHFLPRs) based on EHFLTSs. They developed additive consistency and weak consistency measures, respectively. In addition, they proposed two algorithms to handle the consistency issued by the perspective of graphs.

A robust approach for MADM problem solving is the Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method. This approach was proposed by Opricovic and Tzeng [32]. It focuses on ranking alternatives and selecting the best choice in the presence of multiple conflicting attributes. VIKOR determines a compromise solution which is feasible solution as well as the nearest to the ideal and provides a maximum utility for the majority and a minimum regret for the opponent [33, 34]. In recent years, many scholars have used the VIKOR method as one of the most well-known MADM techniques in diverse field studies, which reflect the importance of this approach. Some of these studies are summarized as follows: supplier segmentation and evaluation in an automobile manufacturing company [35], evaluating the hospital service quality [36], service supplier evaluation and selection [37], measurement of customer satisfaction in mobile service [38], site selection in waste management [39, 40], facility location selection [41], conservation development in a coastal area [42], prioritization of climate change adaptation strategies [43], evaluation of product development partners [44], hesitant fuzzy VIKOR method [45, 46], human error identification and risk prioritization [47], risk assessment for project bidding selection [48], prospect theory-based VIKOR method [49], evaluation of green supply chain management practices [50], material selection of microelectromechanical systems electrostatic actuators [51], evaluation of alternative-fuel vehicles [52] and decision making in the housing market [53]. Group decision making is one of the most common approaches to solve real-world decision-making problems. The EHFLTS is a flexible tool to manage a group's evaluations under qualitative uncertainty of socio-economic environment. Due to the ability of the EHFLTS in modeling unquantifiable information and the importance of the VIKOR method, we notably modified the VIKOR method to solve the qualitative multi-attribute group decision-making (QMAGDM) problems with extended hesitant fuzzy linguistic information.

The reminder of this paper is organized as follows: In Sect. 2, the basic concepts of HFLTSs and EHFLTSs are reviewed. In Sect. 3, comparison laws and distance and similarity measures of EHFLTs are proposed. In Sect. 4, the comparison laws and distance and similarity measures between two EHFLTs are introduced. In Sect. 5, the principles and steps of the proposed EHFL-VIKOR method are introduced. A numerical example is given in Sect. 6 to demonstrate the applicability and validation of the proposed method. Finally, the conclusions are depicted in Sect. 7.

2 Preliminaries

The basic concepts of hesitant fuzzy linguistic term sets and extended hesitant fuzzy linguistic term sets are reviewed in this section.

2.1 Hesitant fuzzy linguistic term sets

The qualitative nature of many real-world problems leads to high degree of uncertainty and vagueness in information. The fuzzy linguistic approach is a common tool to address uncertainty in qualitative problems [5]. However, this approach has a main shortcoming, which expresses a linguistic variable by using a single linguistic term [5, 19]. Thus, Rodriguez et al. [5] proposed the concept of HFLTS. As HFSs are used to model hesitancy in quantitative problems, hesitant fuzzy linguistic term sets are used when experts hesitate between several linguistic terms in qualitative problems [52].

Definition 1 [5] Let $S = \{s_0, s_1, \ldots, s_\tau\}$ be a linguistic term set, and then an HFLTS H_S is an ordered finite subset of the consecutive linguistic terms of S.

Example 1 Let $S = \{s_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}\}$ be a linguistic term set. $H_s = \{s_1, s_2\}$, $H_s^1 = \{s_3, s_4\}$ and $H_s^2 = \{s_5\}$ are three HFLTSs on S.

By Definition 1, the subscripts asymmetry in linguistic term set S may cause some problems. For example, if $S = \{s_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}\}$ be a linguistic term set, then $s_2 \oplus s_3 = s_5$. It means the aggregated result of linguistic terms “low” and “medium” is “very high.” This result is incongruent with our intuition. To overcome this problem, Xu [53] proposed the subscript-symmetric linguistic term set $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ as a substitute of $S = \{s_0, s_1, \ldots, s_\tau\}$.

As mentioned above, the HFLTS is very effective in managing hesitation of qualitative information. But it is not similar to human manner of thought and cognition [54]. Rodriguez et al. [5] developed a context-free grammar G_H to generate comparative linguistic expressions similar to the human’s expressions and a transformation function to transform linguistic expressions into HFLTS.

Definition 2 [5] Let $G_H = (V_N, V_T, I, P)$ be a context-free grammar so that V_N is a set of nonterminal symbols, V_T is a set of terminals where $V_N \cap V_T = \varnothing$, I is a set of rules and P is the start symbol.

In order to construct a HFLTS from linguistic expressions, the following transformation functions E_{G_H} are used.

Definition 3 [5, 27] The linguistic expression obtained by the context-free grammar G_H transforms into an HFTS H_s as follows:

$$E_{G_H}(s_i) = \{s_i | s_i \in S\},$$
$$E_{G_H}(\text{greater than } s_i) = \{s_k | s_k \in S \text{ and } s_k \succ s_i\},$$
$$E_{G_H}(\text{lower than } s_i) = \{s_k | s_k \in S \text{ and } s_k \prec s_i\},$$
$$E_{G_H}(\text{at least } s_i) = \{s_k | s_k \in S \text{ and } s_k \geq s_i\}$$
$$E_{G_H}(\text{at most } s_i) = \{s_k | s_k \in S \text{ and } s_k \leq s_i\},$$
$$E_{G_H}(\text{between } s_i \text{ and } s_j) = \left\{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\right\}.$$

2.2 Extended hesitant fuzzy linguistic term sets

Despite the broad utility of the HFLTSs in qualitative decision-making problem by one expert, it cannot be very applicable in group decision making [30]. For example, two authorized group of experts intend to evaluate the performance of a company using the linguistic term set S as follows:

$$S = \{s_{-3} = \text{very weak}, s_{-2} = \text{weak}, s_{-1} = \text{slightly weak}, s_0 = \text{fair}, s_1 = \text{slightly good}, s_2 = \text{good}, s_3 = \text{very good}\}.$$

The experts of group 1 say “at least good.” Some experts of group 2 insist on “weak,” while others say between “fair” and “good.” The opinions of group 1 experts can be expressed by the HFLTS $\{s_2, s_3\}$, while the opinions of group 2 experts lead to a nonconsecutive subset of S. It can be represented by $\{s_{-2}, s_2, s_3\}$. To deal with such problems, Wang [30] defined the extended hesitant fuzzy linguistic term sets (EHFLTSs) as follows:

Definition 4 [30] Given linguistic term set S, the EHFLTS is an ordered subset of linguistic terms of S. It is symbolized by $\tilde{H}_s(x) = \{s_i | s_i \in S\}$.

An investigation of the relationship between the HFLTS and the EHFLTS indicated that the HFLTS is the special case of the EHFLTS [30]. It is noteworthy that the union of two HFLTSs result an EHFLTS [30]. For example, the union of two HFLTSs $H_s = \{s_0\}$ and $H_s^1 = \{s_2, s_3\}$ is an EHFLTS $\tilde{H}_s = \{s_0, s_2, s_3\}$. Thus, it can be very useful in group decision making, when every expert's opinion is in the form of HFLTS and group's opinion is in the form of EHFLTS constructed by the union of these HFLTSs. In comparison with aggregation operators which may lead to missing some important information, all possible linguistic terms can be considered by uniting the EHFLTSs [30, 31]. Note that in computation process of information, virtual linguistic terms extended by Xu [55] are applied to preserve all the available information. For more details about the syntax and semantics of virtual linguistic terms refer to [56]. Generally, given a fixed set X, $\tilde{H}_s(x)$ can be denoted as follows:

$$\tilde{H}_s(x) = \{\langle x, h_s(x)\rangle | x \in X\} \tag{1}$$

where $h_s(x)$ is a set of p linguistic terms in virtual linguistic terms set $\bar{S}$, i.e.,

$$h_s(x) = \{s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_p}\} \tag{2}$$

$h_s(x)$, which is abbreviated as h_s, displays the extended hesitant fuzzy linguistic term (EHFLT). For convenience, single linguistic term is regarded as an EHFLT [30]. Some operations for EHFLTs are defined as follows:

Definition 5 [30] Let h_s, h_s^1 and h_s^2 be three EHFLTs, then:

(1) $h_s^1 \oplus h_s^2 = \bigcup_{s_{\alpha_1} \in h_s^1, s_{\alpha_2} \in h_s^2} \{s_{\alpha_1} \oplus s_{\alpha_2}\} = \bigcup_{s_{\alpha_1} \in h_s^1, s_{\alpha_2} \in h_s^2} \{s_{\alpha_1 + \alpha_2}\}$;

(2) $h_s^1 \otimes h_s^2 = \bigcup_{s_{\alpha_1} \in h_s^1, s_{\alpha_2} \in h_s^2} \{s_{\alpha_1} \otimes s_{\alpha_2}\} = \bigcup_{s_{\alpha_1} \in h_s^1, s_{\alpha_2} \in h_s^2} \{s_{\alpha_1 . \alpha_2}\}$;

(3) $\lambda h_s = \bigcup_{s_\alpha \in h_s} \{\lambda s_\alpha\} = \bigcup_{s_\alpha \in h_s} \{s_{\lambda\alpha}\}, \quad \lambda \geq 0$;

(4) $(h_s)^\lambda = \bigcup_{s_\alpha \in h_s} \left\{(s_\alpha)^\lambda\right\} = \bigcup_{s_\alpha \in h_s} \left\{s_{(\alpha)^\lambda}\right\}, \quad \lambda \geq 0.$

3 Comparison laws and distance measures of EHFLTs

In this section, we propose a new comparison method for more accurate discriminant of EHFLTs. Also, some distance measures are introduced between two EHFLTs.

3.1 Comparison laws of EHFLTs

Comparing and sorting fuzzy information has key roles in decision-making procedure [57]. For example, it is necessary to arrange linguistic terms in ascending order to calculate distance measure between two HFLTs [20]. Thus, the comparison of HFLTs has essential function in distance-based MCDM methods, such as the HFL-TOPSIS method and the HFL-VIKOR method [21, 58]. To establish an order between HFLTs, Wang proposed a comparison law by defining an expected linguistic term and a hesitancy degree as follows:

Definition 6 [30] Let h_s be an HFLT, then $E(h_s) = \frac{1}{\#h}(\oplus_{s_\alpha \in h_s} s_\alpha)$ is the expected linguistic term of h_s, where $\#h$ is the number of linguistic terms s_α, in h_s.

Clearly, the expected linguistic term is equal to average linguistic term of an EHFLT. We can consider the expected linguistic term as $E(h_s) = s_\mu$, where $\mu = \frac{1}{\#h}\sum_{s_\alpha \in h_s} \alpha$.

Definition 7 [30] Given an EHFLT h_s, s_{α_L} and s_{α_U} are the smallest and the biggest linguistic terms of h_s, and then $D(h_s) = \frac{(\alpha_U - \alpha_L)}{(2\tau+1)}$ is the degree of hesitancy of h_s, where $2\tau + 1$ is the cardinality of linguistic term set S. Degree of hesitancy denoted the degree of uncertainty in evaluation.

Given two EHFLTs, the order relationship is as follows:

Definition 8 [30] Let h_s^1 and h_s^2 be two EHFLTs, then

If $E(h_s^1) \succ E(h_s^2)$, then h_s^1 is greater than $h_s^2 (h_s^1 \succ h_s^2)$.
If $E(h_s^1) = E(h_s^2)$ and $D(h_s^1) \prec D(h_s^2)$, then h_s^1 is greater than $h_s^2 (h_s^1 \succ h_s^2)$.
If $E(h_s^1) = E(h_s^2)$ and $D(h_s^1) = D(h_s^2)$, then h_s^1 and h_s^2 represented the same information $(h_s^1 \cong h_s^2)$.

According to Definition 7, the hesitancy degree of an EHFLT is constructed by the indices of the upper and lower bounds linguistic terms. Indeed, the impact of middle linguistic terms is disregarded which can lead to missing some information. Thereupon, we will point out with an example that the comparison law introduced by Wang [30] cannot excellently discriminate some EHFLTs due to abovementioned drawback.

Example 2 Let $S = \{s_t | t = -3, -2, -1, 0, 1, 2, 3\}$ be a linguistic term set $h_s^1 = \{s_{-3}, s_1, s_2\}$ and $h_s^2 = \{s_{-2}, s_{-1}, s_0, s_3\}$ be two EHLTs, then:

$$E\left(h_s^1\right) = s_0, \quad E\left(h_s^2\right) = s_0, \quad D\left(h_s^1\right) = 5, \quad D\left(h_s^2\right) = 5.$$

According to Wang [30] proposed method, these two EHFLTs represent the same information, which is at odds with our intuition. In this section, we define a new comparison method for better comparison of EHFLTs. To this aim, the expected linguistic term and the hesitancy degree of EHFLTs are considered simultaneously. As mentioned above, the expected linguistic term is equal to average linguistic term of an EHFLT. Also to express the degree of hesitancy, we use the concept of standard deviation in this study. Thus, the hesitancy degree of an EHFLT is obtained by the indices of all linguistic terms (the upper bound, the lower bound and middle linguistic terms). In order to measure the hesitancy degree of an EHFLT, the standard deviation of the subscripts of its constitutive linguistic terms is calculated as follows:

Definition 9 Let h_s be an EHFLT, then $\sigma(h_s) = \sqrt{\frac{1}{\#h}\sum_{s_\alpha \in h_s}(\mu - \alpha)^2}$ is the degree of hesitancy of h_s, where $\#h$ is the number of linguistic terms s_α, in h_s.

According to above discussion and for the sake of better discrimination of EHFLTs, a new operator namely coefficient of variation of EHFLT is proposed which simultaneously takes into account the average linguistic term and the hesitancy degree of EHFLTs.

Definition 10 For an EHFLT h_s with given $E(h_s)$ and $\sigma(h_s)$, the coefficient of variation (CV) is defined as follows:

$$\mathrm{CV}(h_s) = \frac{\mu}{\sigma(h_s) + c} - \frac{\sigma(h_s)}{\mu + c} \tag{3}$$

where $\mu = \frac{1}{\#h}\sum_{s_\alpha \in h_s} \alpha$ and c is an arbitrary positive number and large enough so that $\mu + c \succ 0$.

Subsequently, the proposed comparison method is defined below.

Definition 11 Given two EHFLTs h_s^1 and h_s^2, then

If $\mathrm{CV}(h_s^1) \succ \mathrm{CV}(h_s^2)$, then h_s^1 is greater than $h_s^2 (h_s^1 \succ h_s^2)$.
If $\mathrm{CV}(h_s^1) = \mathrm{CV}(h_s^2)$, then h_s^1 and h_s^2 represented the same information $(h_s^1 \cong h_s^2)$.

The total orders of EHFLTSs are usually necessary when comparing any EHFLTSs [59]. This paper proposed a partial order method to order EHFLTs. Wang and Xu [60] developed the total orders of extended hesitant fuzzy linguistic term sets. Also, they proposed total orders of hesitant fuzzy sets in another paper.

Example 3 Using Example 2, we compare the two EHFLTs by applying the proposed comparison method as follows:

$$E(h_s^1) = s_0, \quad E(h_s^2) = s_0 \Rightarrow \mu_1 = \mu_2 = 0$$
$$\sigma(h_s^1) = 2.16, \quad \sigma(h_s^2) = 1.87$$

for $c = 1$ we have

$$\mathrm{CV}(h_s^1) = \frac{0}{2.16 + 1} - \frac{2.16}{0 + 1} = -2.16$$

and

$$\mathrm{CV}(h_s^2) = \frac{0}{1.87 + 1} - \frac{1.87}{0 + 1} = -1.87.$$

So, $\mathrm{CV}(h_s^2) \succ \mathrm{CV}(h_s^1)$, then h_s^2 is greater than $h_s^1 (h_s^2 \succ h_s^1)$.

3.2 Distance measures between two EHFLTs

Distance and similarity measures are extensively used in many research fields including decision making, pattern recognition, machine learning and market prediction [61–63]. In recent years, many scholars directed their attention to extension and application of distance and similarity measures between HFLTSs [20, 21, 28, 58, 64, 65]. None of the previous studies have extended distance measures between two EHFLTs. Therefore, in this section, we develop the distance measures between two EHFLTs.

According to Xia and Xu [66], the Hamming distance, the Euclidean distance and the Hausdorff metric are some of the most common distance measures between two fuzzy sets. In this paper, we extend the aforesaid measures between two EHFLTs. For this purpose, we first propound the axioms of distance and similarity measures for EHFLTs below.

Definition 12 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set, h_s^1 and h_s^2 be two EHFLTs. The distance measure between h_s^1 and h_s^2 is denoted by $d(h_s^1, h_s^2)$, which satisfies:

(1) $0 \le d(h_s^1, h_s^2) \le 1$;
(2) $d(h_s^1, h_s^2) = 0$ if and only if $h_s^1 = h_s^2$;
(3) $d(h_s^1, h_s^2) = d(h_s^2, h_s^1)$.

Definition 13 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set, h_s^1 and h_s^2 be two EHFLTs. The similarity measure between h_s^1 and h_s^2 is denoted by $s(h_s^1, h_s^2)$, which satisfies:

(1) $0 \le s(h_s^1, h_s^2) \le 1$;
(2) $s(h_s^1, h_s^2) = 1$ if and only if $h_s^1 = h_s^2$;
(3) $s(h_s^1, h_s^2) = s(h_s^2, h_s^1)$.

Referring to Definitions 12 and 13, it is obvious that $s(h_s^1, h_s^2) = 1 - d(h_s^1, h_s^2)$. We predominantly focus on the distance measures for EHFLTs in this paper, and the corresponding similarity measures can be obtained easily. Liao et al. [20] developed the Hamming distance, the Euclidean distance and the Hausdorff metric between two HFLTSs. Motivated by Liao et al. [20], we extended the concept of distance measures between two EHFLTs below.

Assumption 1 Note that in most cases two EHFLTs have different number of linguistic terms. To make a correct comparison, we should optimistically extend the shorter one by repeating its biggest linguistic term as far as both of them have the same length.

Assumption 2 All the linguistic terms in both of EHFLTs are arranged in ascending order.

Definition 14 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set, $h_s^1 = \{s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_p}\}$ and $h_s^2 = \{s_{\beta_1}, s_{\beta_2}, \ldots, s_{\beta_q}\}$ be two EHFLTs, where p and q are the numbers of linguistic terms in h_s^1 and h_s^2, respectively. The designated distance measures between two EHFLTs are defined as follows.

The Hamming distance of h_s^1 and h_s^2:

$$d_{\mathrm{hd}}\left(h_s^1, h_s^2\right) = \frac{1}{L}\sum_{l=1}^{L} \frac{|\alpha_l - \beta_l|}{2\tau + 1}. \tag{4}$$

The Euclidean distance of h_s^1 and h_s^2:

$$d_{\mathrm{ed}}\left(h_s^1, h_s^2\right) = \left(\frac{1}{L}\sum_{l=1}^{L} \left(\frac{|\alpha_l - \beta_l|}{2\tau + 1}\right)^2\right)^{1/2}. \tag{5}$$

Generalized distance measure as a generalization of the Hamming distance and the Euclidean distance is represented below:

$$d_{\mathrm{gd}}\left(h_s^1, h_s^2\right) = \left(\frac{1}{L}\sum_{l=1}^{L} \left(\frac{|\alpha_l - \beta_l|}{2\tau + 1}\right)^\lambda\right)^{1/\lambda}, \quad \lambda \succ 0. \tag{6}$$

For $\lambda = 1$ the above generalized distance becomes the Hamming distance, and for $\lambda = 2$ it becomes the Euclidean distance.

The generalized Hausdorff metric of h_s^1 and h_s^2:

$$d_{\mathrm{ghaud}}\left(h_s^1, h_s^2\right) = \left(\max_{l=1,2,\ldots,L} \left(\frac{|\alpha_l - \beta_l|}{2\tau + 1}\right)^\lambda\right)^{1/\lambda}, \quad \lambda \succ 0. \tag{7}$$

Especially, for $\lambda = 1$, the generalized Hausdorff distance becomes the Hamming–Hausdorff distance:

$$d_{\mathrm{hhaud}}\left(h_s^1, h_s^2\right) = \max_{l=1,2,\ldots,L} \frac{|\alpha_l - \beta_l|}{2\tau + 1}. \tag{8}$$

And for $\lambda = 2$, the generalized Hausdorff distance becomes the Euclidean–Hausdorff distance:

$$d_{\mathrm{ehaud}}\left(h_s^1, h_s^2\right) = \left(\max_{l=1,2,\ldots,L} \left(\frac{|\alpha_l - \beta_l|}{2\tau + 1}\right)^2\right)^{1/2}. \tag{9}$$

Note that in all of the above equations, $L = \max(p, q)$.

We can easily check whether the distance measures can satisfy the axioms of distance measures given in Definition 12. As an example, for the Hamming distance, it is obvious that $-\tau \le \alpha_l \le \tau$ and $-\tau \le \beta_l \le \tau$, then we have $0 \le |\alpha_l - \beta_l| \le 2\tau$ and finally $0 \le d_{\mathrm{hd}}(h_s^1, h_s^2) \le 1$. Additionally, if $h_s^1 = h_s^2$, i.e., for $l = 1, \ldots, L$, then $d_{\mathrm{hd}}(h_s^1, h_s^2) = 0$. It is clear that $d_{\mathrm{hd}}(h_s^1, h_s^2) = d_{\mathrm{hd}}(h_s^2, h_s^1)$. Therefore, the hamming distance can satisfy the mentioned axioms. Similarly, the ability of the other distance measures to satisfy these axioms is verifiable and thus we ignore the details here.

Example 4 For two EHFLTs $h_s^1 = \{s_{-3}, s_1, s_2\}$ and $h_s^2 = \{s_{-2}, s_{-1}, s_0, s_3\}$ given in Example 2, the Hamming distance, the Euclidean distance and the Hausdorff metric are calculated as follows.

We can firstly extend h_s^1 to $h_s^1 = \{s_{-3}, s_1, s_2, s_2\}$ by repeating the linguistic term s_2. Then, the Hamming distance between h_s^1 and h_s^2 is

$$d_{\mathrm{hd}}\left(h_s^1, h_s^2\right) = \frac{1}{4}\left(\frac{|-3-(-2)|}{7} + \frac{|1-(-1)|}{7} + \frac{|2-0|}{7} + \frac{|2-3|}{7}\right) = 0.2143.$$

The Euclidean distance between h_s^1 and h_s^2 is

$$d_{\mathrm{ed}}\left(h_s^1, h_s^2\right) = \left(\frac{1}{4}\left(\left(\frac{-3-(-2)}{7}\right)^2 + \left(\frac{1-(-1)}{7}\right)^2 + \left(\frac{2-0}{7}\right)^2 + \left(\frac{2-3}{7}\right)^2\right)\right)^{1/2} = 0.2259.$$

The Hamming–Hausdorff distance between h_s^1 and h_s^2 is

$$d_{\mathrm{hhaud}}\left(h_s^1, h_s^2\right) = \max\left\{\frac{|-3-(-2)|}{7}, \frac{|1-(-1)|}{7}, \frac{|2-0|}{7}, \frac{|2-3|}{7}\right\} = 0.2857.$$

The Euclidean–Hausdorff distance between h_s^1 and h_s^2 is

$$d_{\mathrm{ehaud}}\left(h_s^1, h_s^2\right) = \left(\max\left\{\left(\frac{-3-(-2)}{7}\right)^2, \left(\frac{1-(-1)}{7}\right)^2, \left(\frac{-3-(-2)}{7}\right)^2, \left(\frac{2-0}{7}\right)^2, \left(\frac{2-3}{7}\right)^2\right\}\right)^{1/2} = 0.2857.$$

4 Comparison laws and distance measures of EHFLTSs

As discussed in previous sections, group decision making is very common in complicated decision making environment. Also, the EHFLTS is introduced as an effective tool to handle hesitancy in group decision-making problem [30, 31]. Thus, we focus on comparison laws of EHFLTSs and distance and similarity measures between two EHFLTSs in this section.

4.1 Comparison laws of EHFLTSs

In order to propose the comparison method, it is necessary to define some concepts. According to the previous section, we simultaneously take into account the expected linguistic term and the degree of hesitancy of EHFLTSs for better comparison of EHFLTSs. To this aim, we define these concepts as follows:

Definition 15 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set. For a given EHFLTS $\tilde{H}_s = \{h_s^1, h_s^2, \ldots, h_s^m\}$, the expected linguistic term is defined as:

$$E(\tilde{H}_s) = \frac{1}{m} \oplus_{i=1,\ldots,m} \left(\frac{1}{\# h_i} \left(\oplus_{s_{\alpha^i} \in h_s^i} s_{\alpha^i} \right) \right) \tag{10}$$

where $\# h_i$ is the number of linguistic terms s_{α^i}, in h_s^i.

Example 5 For EHFLTS $\tilde{H}_s = \{\{s_1, s_3\}, \{s_{-2}, s_{-1}, s_2, s_3\}, \{s_{-3}, s_0, s_2, s_3\}\}$, the expected linguistic term set is as follows:

$$E(\tilde{H}_s) = \frac{1}{3} \left(\frac{1}{2}(s_1 \oplus s_3) \oplus \frac{1}{4}(s_{-2} \oplus s_{-1} \oplus s_2 \oplus s_3) \oplus \frac{1}{4}(s_{-3} \oplus s_0 \oplus s_2 \oplus s_3) \right) = s_1.$$

Note that for the convenience of operations, we can consider the expected linguistic term as $E(\tilde{H}_s) = s_{\bar{\mu}}$, where $\bar{\mu} = \frac{1}{m} \sum_{i=1}^m \mu_i$ and $\mu_i = \frac{1}{\# h_i} \sum_{s_{\alpha^i} \in h_s^i} \alpha^i$.

In order to measure the hesitancy degree of an EHFLTS, we firstly measure the hesitancy degree of each EHFLT, obtained by computation of the standard deviation of the subscripts of its constitutive linguistic terms, and then calculate the average and standard deviation of these measured standard deviations as follows:

Definition 16 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set. For a given EHFLTS $\tilde{H}_s = \{h_s^1, h_s^2, \ldots, h_s^m\}$, the average and standard deviation of the standard deviations are defined as:

$$\bar{\sigma}(\tilde{H}_s) = \frac{1}{m} \sum_{i=1}^m \sigma(h_s^i) \tag{11}$$

$$\sigma_\sigma(\tilde{H}_s) = \sqrt{\frac{1}{m} \sum_{i=1}^m \left(\sigma(h_s^i) - \bar{\sigma} \right)^2} \tag{12}$$

where

$$\sigma(h_s^i) = \sqrt{\frac{1}{\# h_i} \sum_{s_{\alpha^i} \in h_s^i} (\mu_i - \alpha^i)^2}. \tag{13}$$

Definition 17 For an EHFLTS $\tilde{H}_s$ with given $\bar{\sigma}(\tilde{H}_s)$ and $\sigma_\sigma(\tilde{H}_s)$, the degree of hesitancy (DH) is defined as follows:

$$DH(\tilde{H}_s) = \frac{\bar{\sigma}}{\sigma_\sigma + \varepsilon} + \frac{\sigma_\sigma}{\bar{\sigma} + \varepsilon} \tag{14}$$

where ε is a very small positive number.

Definition 18 For an EHFLTS $\tilde{H}_s$ with given $E(\tilde{H}_s)$ and $DH(\tilde{H}_s)$, the coefficient of variation (CV) is defined as follows:

$$CV(\tilde{H}_s) = \frac{\bar{\mu}}{DH(\tilde{H}_s)}. \tag{15}$$

Finally, the proposed comparison method is defined below.

Definition 19 Given two EHFLTSs $\tilde{H}_s^1$ and $\tilde{H}_s^2$, then

If $\mathrm{CV}(\tilde{H}_s^1) \succ \mathrm{CV}(\tilde{H}_s^2)$, then $\tilde{H}_s^1$ is greater than $\tilde{H}_s^2$ ($\tilde{H}_s^1 \succ \tilde{H}_s^2$).

If $\mathrm{CV}(\tilde{H}_s^1) = \mathrm{CV}(\tilde{H}_s^2)$, then $\tilde{H}_s^1$ and $\tilde{H}_s^2$ represented the same information ($\tilde{H}_s^1 \cong \tilde{H}_s^2$).

Example 6 Suppose that $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ is a linguistic term set and

$\tilde{H}_s^1 = \{\{s_0, s_2\}, \{s_{-2}, s_{-1}, s_2\}, \{s_{-3}, s_{-1}, s_2, s_3\}\}$ and
$\tilde{H}_s^2 = \{\{s_{-2}, s_{-1}, s_1\}, \{s_{-1}, s_{-2}, s_3\}\{s_0, s_1, s_3\}\}$

are two EHFLTSs. We compare the given EHFLTSs by applying the proposed comparison method as follows:

$$\begin{aligned}
&E(\tilde{H}_s^1) = s_{0.194}, \quad \bar{\mu}(\tilde{H}_s^1) = 0.194 \\
&E(\tilde{H}_s^2) = s_{0.222}, \quad \bar{\mu}(\tilde{H}_s^2) = 0.222 \\
&\sigma(h_s^{11}) = 1, \quad \sigma(h_s^{12}) = 1.7, \quad \sigma(h_s^{13}) = 2.38 \\
&\quad \Rightarrow \bar{\sigma}(\tilde{H}_s^1) = 1.69, \quad \sigma_\sigma(\tilde{H}_s^1) = 0.56 \\
&\sigma(h_s^{21}) = 1.25, \quad \sigma(h_s^{22}) = 2.16, \quad \sigma(h_s^{23}) = 1.25 \\
&\quad \Rightarrow \bar{\sigma}(\tilde{H}_s^2) = 1.53, \quad \sigma_\sigma(\tilde{H}_s^1) = 0.43.
\end{aligned}$$

Then, for $\varepsilon = 0.001$, we have

$$\begin{aligned}
&\mathrm{DH}(\tilde{H}_s^1) = \frac{1.69}{0.56 + 0.001} + \frac{0.56}{1.69 + 0.001} = 3.343 \\
&\mathrm{DH}(\tilde{H}_s^2) = \frac{1.53}{0.43 + 0.001} + \frac{0.43}{1.53 + 0.001} = 3.831.
\end{aligned}$$

And finally

$$\begin{aligned}
&\mathrm{CV}(\tilde{H}_s^1) = \frac{0.194}{3.343} = 0.0580, \\
&\mathrm{CV}(\tilde{H}_s^2) = \frac{0.222}{3.831} = 0.0579.
\end{aligned}$$

So $\mathrm{CV}(\tilde{H}_s^1) \succ \mathrm{CV}(\tilde{H}_s^2)$ and referring to Definition 19 $\tilde{H}_s^1$ is greater than $\tilde{H}_s^2$ ($\tilde{H}_s^1 \succ \tilde{H}_s^2$).

4.2 Distance measures between two EHFLTSs

In this paper, we focus only on the distance measures for EHFLTSs and the corresponding similarity measures can be obtained using the following equation:

$$s\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = 1 - d\left(\tilde{H}_s^1, \tilde{H}_s^2\right). \tag{16}$$

In order to propose distance and similarity measures for EHFLTSs, we first express some assumptions.

Assumption 1 In most cases, the EHFLTs constituted two given EHFLTSs $\tilde{H}_s^1 = \left\{h_s^{11}, h_s^{12}, \ldots, h_s^{1m}\right\}$ and $\tilde{H}_s^2 = \left\{h_s^{21}, h_s^{22}, \ldots, h_s^{2n}\right\}$ have unequal numbers of linguistic terms. In order to operate correctly, we first determine $\#h_{sL}^1 = \max_{i=1,\ldots,m}\left(h_s^{11}, \ldots, \#h_s^{1i}, \ldots, \#h_s^{1m}\right)$ and $\#h_{sL}^2 = \max_{j=1,\ldots,n}\left(\#h_s^{21}, \ldots, \#h_s^{2j}, \ldots, \#h_s^{2n}\right)$, where $\#h_s^{1i}$ and $\#h_s^{2j}$ indicate the number of linguistic terms of h_s^{1i} and h_s^{2j}. Then, we have $L = \max(h_{sL}^1, h_{sL}^2)$. We should optimistically extent the shorter EHFLTs by repeating the biggest linguistic term of any of them as far as the length of all of them is equal to L.

Assumption 2 All the linguistic terms in each EHFLT and all the EHFLTs in both of EHFLTSs are arranged in ascending order.

Assumption 3 If two given EHFLTSs have different number of EHFLTs, we should optimistically extent the shorter one by repeating its greatest EHFLT as far as both of them have the same number of EHFLTs.

Definition 20 Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set, $\tilde{H}_s^1 = \left\{h_s^{11}, h_s^{12}, \ldots, h_s^{1m}\right\}$ and $\tilde{H}_s^2 = \left\{h_s^{21}, h_s^{22}, \ldots, h_s^{2n}\right\}$ be two EHFLTSs, where $m \succ n$. The designated distance measures between two EHFLTs are defined as follows:

The generalized distance of $\tilde{H}_s^1$ and $\tilde{H}_s^2$:

$$d_{\text{gd}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\frac{1}{m \cdot L}\sum_{i=1}^{m}\sum_{l=1}^{L}\left(\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}\right)^{\lambda}\right)^{1/\lambda}, \quad \lambda \succ 0. \tag{17}$$

The Hamming distance of $\tilde{H}_s^1$ and $\tilde{H}_s^2$:

$$d_{\text{hd}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \frac{1}{m \cdot L}\sum_{i=1}^{m}\sum_{l=1}^{L}\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}. \tag{18}$$

The Euclidean distance of $\tilde{H}_s^1$ and $\tilde{H}_s^2$:

$$d_{\text{ed}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\frac{1}{m \cdot L}\sum_{i=1}^{m}\sum_{l=1}^{L}\left(\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}\right)^{2}\right)^{1/2}. \tag{19}$$

It is obvious that the Hamming distance and the Euclidean distance are special cases of the generalized distance for $\lambda = 1$ and $\lambda = 2$, respectively.

The generalized Hausdorff metric of $\tilde{H}_s^1$ and $\tilde{H}_s^2$:

$$d_{\text{ghaud}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\max_{i=1,2,\ldots,m}\left(\max_{l=1,2,\ldots,L}\left(\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}\right)^{\lambda}\right)\right)^{1/\lambda}, \quad \lambda \succ 0. \tag{20}$$

Especially, for $\lambda = 1$ the generalized Hausdorff distance becomes the Hamming–Hausdorff distance:

$$d_{\text{hhaud}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \max_{i=1,2,\ldots,m}\left(\max_{l=1,2,\ldots,L}\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}\right). \tag{21}$$

And for $\lambda = 2$ the generalized Hausdorff distance becomes the Euclidean–Hausdorff distance:

$$d_{\text{ehaud}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\max_{i=1,2,\ldots,m}\left(\max_{l=1,2,\ldots,L}\left(\frac{\left|\alpha_l^i - \beta_l^i\right|}{2\tau + 1}\right)^{2}\right)\right)^{1/2}. \tag{22}$$

Therefore, the axioms of distance measures given in Definition 12 can be defined and verified for distance and similarity measures between two EHFLTSs. Here, we refrain from expressing details.

Example 7 Using Example 6, we calculate the Hamming distance, the Euclidean distance and the Hausdorff metric between two EHFLTs. We can firstly modify $\tilde{H}_s^1$ and $\tilde{H}_s^2$ by extension of the shorter EHFLTs and then arrange them in ascending order by comparison method of EHFLTs as follows:

$$\tilde{H}_s^1 = \{\{s_{-3}, s_{-1}, s_2, s_3\}, \{s_{-2}, s_{-1}, s_2, s_2\}, \{s_0, s_2, s_2, s_2\}\}$$
$$\tilde{H}_s^2 = \{\{s_{-2}, s_{-1}, s_1, s_1\}, \{s_{-1}, s_{-2}, s_3, s_3\}, \{s_0, s_1, s_3, s_3\}\}.$$

Then, the Hamming distance between h_s^1 and h_s^2 is

$$\begin{aligned} d_{\text{hd}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = {} & \frac{1}{3}\left(\frac{1}{4}\left(\frac{|-3-(-2)|}{7} + \frac{|-1-(-1)|}{7} + \frac{|2-1|}{7} + \frac{|3-1|}{7}\right)\right) \\ & + \frac{1}{3}\left(\frac{1}{4}\left(\frac{|-2-(-1)|}{7} + \frac{|-1-(-2)|}{7} + \frac{|2-3|}{7} + \frac{|2-3|}{7}\right)\right) \\ & + \frac{1}{3}\left(\frac{1}{4}\left(\frac{|0-0|}{7} + \frac{|2-1|}{7} + \frac{|2-3|}{7} + \frac{|2-3|}{7}\right)\right) = 0.139. \end{aligned}$$

The Euclidean distance between $\tilde{H}_s^1$ and $\tilde{H}_s^2$ is

$$d_{\mathrm{ed}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\frac{1}{3} \times \frac{1}{4}\left(\left(\frac{-3-(-2)}{7}\right)^2 + \left(\frac{-1-(-1)}{7}\right)^2 + \left(\frac{2-1}{7}\right)^2 + \left(\frac{3-1}{7}\right)^2\right)\right)^{1/2} + \left(\frac{1}{3} \times \frac{1}{4}\left(\left(\frac{-2-(-1)}{7}\right)^2 + \left(\frac{-1-(-2)}{7}\right)^2 + \left(\frac{2-3}{7}\right)^2 + \left(\frac{2-3}{7}\right)^2\right)\right)^{1/2} + \left(\frac{1}{3} \times \frac{1}{4}\left(\left(\frac{0-0}{7}\right)^2 + \left(\frac{2-1}{7}\right)^2 + \left(\frac{2-3}{7}\right)^2 + \left(\frac{2-3}{7}\right)^2\right)\right)^{1/2} = 0.255.$$

The Hamming–Hausdorff distance between $\tilde{H}_s^1$ and $\tilde{H}_s^2$ is

$$d_{\mathrm{hhaud}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \max\left(\frac{2}{7}, \frac{1}{7}, \frac{1}{7}\right) = 0.286.$$

The Euclidean–Hausdorff distance between $\tilde{H}_s^1$ and $\tilde{H}_s^2$ is

$$d_{\mathrm{hhaud}}\left(\tilde{H}_s^1, \tilde{H}_s^2\right) = \left(\max\left(\frac{4}{7}, \frac{1}{7}, \frac{1}{7}\right)\right)^{1/2} = 0.286.$$

5 EHFL-VIKOR method

Many real-world problems have qualitative nature and most of these problems should be solved with several groups of experts. As clarified above, some hesitant fuzzy linguistic MADM methods are on the basis of distance measures. Thus, we propose a distance-based VIKOR method to solve MAGDM problem with the extended hesitant fuzzy linguistic term sets under qualitative circumstances. To do this, let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of m possible alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a finite set of n attributes and $W = \{w_1, w_2, \ldots, w_n\}$ be the weight vector of attributes, such that $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. And let $G = \{G_1, G_2, \ldots, G_p\}$ be a set of p group of experts. Given linguistic term set $S = \{s_t | t = -\tau, \ldots, \tau\}$, a decision matrix which represented by EHFLTSs has the following form:

$$\tilde{D} = \begin{pmatrix} \tilde{H}_s^{11} & \tilde{H}_s^{12} & \cdots & \tilde{H}_s^{1n} \\ \tilde{H}_s^{21} & \tilde{H}_s^{22} & \cdots & \tilde{H}_s^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{H}_s^{m1} & \tilde{H}_s^{m2} & \cdots & \tilde{H}_s^{mn} \end{pmatrix} \tag{23}$$

where $\tilde{H}_s^{ij} = \{h_s^{ij1}, h_s^{ij2}, \ldots, h_s^{ijp}\}$ is an EHFLTS and h_s^{ijk} is an EHFLT provided by kth group of experts G_k for the ith alternative A_i with respect to jth attribute C_j. The proposed method involves the following steps:

Step 1 Determine the extended hesitant fuzzy linguistic positive ideal solution (EHFL-PIS) A^+ and extended hesitant fuzzy linguistic negative ideal solution (EHFL-NIS) A^-:

$$A^+ = \left\{\tilde{H}_s^{1+}, \tilde{H}_s^{2+}, \ldots, \tilde{H}_s^{n+}\right\} \tag{24}$$

$$A^- = \left\{\tilde{H}_s^{1-}, \tilde{H}_s^{2-}, \ldots, \tilde{H}_s^{n-}\right\} \tag{25}$$

where

$$\tilde{H}^{j+} = \begin{cases} \max\limits_{i=1,2,\ldots,m} \tilde{H}_s^{ij} & \text{for benefit criteria} \\ \min\limits_{i=1,2,\ldots,m} \tilde{H}_s^{ij} & \text{for cost criteria} \end{cases} \tag{26}$$

$$\tilde{H}^{j-} = \begin{cases} \min\limits_{i=1,2,\ldots,m} \tilde{H}_s^{ij} & \text{for benefit criteria} \\ \max\limits_{i=1,2,\ldots,m} \tilde{H}_s^{ij} & \text{for cost criteria} \end{cases} \tag{27}$$

and $j = 1, 2, \ldots, n$.

Step 2 Compute the distance-based group utility and individual regret measures. In this paper, we use the Hamming distance to calculate abovementioned measures as follows:

$$\tilde{S}_i = \sum_{j=1}^{n} w_j \frac{d_{\mathrm{hd}}\left(\tilde{H}_s^{ij}, \tilde{H}_s^{j+}\right)}{d_{\mathrm{hd}}\left(\tilde{H}_s^{j+}, \tilde{H}_s^{j-}\right)} \tag{28}$$

$$\tilde{R}_i = \max\left(w_j \frac{d_{\mathrm{hd}}\left(\tilde{H}_s^{ij}, \tilde{H}_s^{j+}\right)}{d_{\mathrm{hd}}\left(\tilde{H}_s^{j+}, \tilde{H}_s^{j-}\right)}\right) \tag{29}$$

where $\tilde{S}_i$ and $\tilde{R}_i$ express the hesitant fuzzy linguistic group utility measure and the hesitant fuzzy linguistic individual regret measure, respectively. Also, $d_{\mathrm{hd}}(\tilde{H}_s^{ij}, \tilde{H}_s^{j+})$ and $d_{\mathrm{hd}}(\tilde{H}_s^{j+}, \tilde{H}_s^{j-})$ are the Hamming distance calculated by Eq. (18).

Step 3 Calculate the extended hesitant fuzzy linguistic compromise measure for each alternative as follows:

$$\tilde{Q}_i = v\frac{S_i - S^+}{S^- - S^+} + (1 - v)\frac{R_i - R^+}{R^- - R^+} \tag{30}$$

where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$, $R^- = \max_i R_i$ and v is weight of the strategy of the majority of criteria, here suppose $v = 0.5$.

Step 4 Rank the alternatives, arranged by the values $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$ in descending order to obtain three ranking lists.

Step 5 Propose the alternative A' as a compromise solution. A' is the best ranked by Q (where $Q(A') =$

$\min_{1 \le i \le n} Q(A_i)$) and must satisfy the following two conditions:

1. Acceptable advantage: $Q(A'') - Q(A') \ge \frac{1}{m-1}$, where A'' is the alternative with the second position in the ranking list by Q and m is the number of alternatives.
2. Acceptable stability in decision making: the alternative A' must also be the best ranked by S or/and R.

If these two conditions are not satisfied simultaneously, then two or more alternatives are considered as compromise solution, which consists of:

- Alternative A' and A'' if the condition 2 is not satisfied.
- Alternatives $A', A'', \ldots, A^{(N)}$ If the condition 1 is not satisfied, where $A^{(N)}$ is obtained by $Q(A^{(i)}) - Q(A') \prec \frac{1}{m-1}$ for maximum N.

6 Numerical example

In this section, we consider a MAGDM problem that pertains to finding suitable telecommunications service provider of SMEs [65] to verify applicability of EHFL-VIKOR method. Suppose that a SME intends to find the best telecommunications service provider. There are four candidates $A = \{A_1, A_2, A_3, A_4\}$ to be assessed with respect to four attributes: C_1 (the satisfaction of price), C_2 (quality), C_3 (service) and C_4 (safeguard). The weight vector of attributes is supposed to be $W = (0.2, 0.15, 0.15, 0.5)^{\mathrm{T}}$. A detailed explanation of mentioned attributes is given in Table 1.

Table 1 Attributes to evaluate a telecommunications service

Attribute	Explanation of attribute
C_1 (price)	How the company is satisfied with the price, which will be paid for the telecommunications service
C_2 (quality)	What level the telecommunications service can reach
C_3 (service)	The maintenance and repair
C_4 (safeguard)	The reliability of information protection

There are three groups of experts $G = \{G_1, G_2, G_3\}$ and each group consists of two experts. The linguistic term set S used in providing evaluation values by experts is as follows:

$$S = \{s_{-3} = \text{none},\ s_{-2} = \text{very low},\ s_{-1} = \text{low},\ s_0 = \text{medium},\ s_1 = \text{high},\ s_2 = \text{very high},\ s_3 = \text{perfect}\}.$$

The evaluation information given by three groups of decision makers in linguistic expressions are shown in Tables 2, 3 and 4.

The symbol "bt" in Tables 2, 3 and 4 is an abbreviation for "between."

Afterward, the linguistic expressions are transformed into EHFLTSs and the associated decision matrix is established:

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array}$$
$$\begin{bmatrix}
\{\{s_{-1},s_0,s_2\},\{s_{-2},s_{-1},s_1\},\{s_{-1},s_0,s_1\}\} & \{\{s_0,s_2\},\{s_{-1},s_0,s_2\},\{s_{-1},s_1,s_2\}\} & \{\{s_{-2},s_{-1},s_1\},\{s_{-2},s_0\},\{s_{-3},s_{-2},s_1,s_2\}\} & \{\{s_{-2},s_0\},\{s_{-1},s_2\},\{s_0,s_2\}\} \\
\{\{s_{-3},s_{-2},s_0\},\{s_{-2},s_0,s_1\},\{s_{-2},s_1\}\} & \{\{s_{-1},s_0,s_2\},\{s_{-1},s_1,s_2\},\{s_{-2},s_1,s_2\}\} & \{\{s_{-1},s_1,s_2\},\{s_{-1},s_0\},\{s_{-1},s_0,s_1\}\} & \{\{s_0,s_2\},\{s_{-1},s_1,s_2\},\{s_{-1},s_2\}\} \\
\{\{s_{-3},s_{-1},s_0\},\{s_{-2},s_1\},\{s_{-1},s_1,s_2\}\} & \{\{s_{-2},s_1,s_2,s_3\},\{s_{-2},s_1,s_2\},\{s_{-1},s_0,s_2\}\} & \{\{s_0,s_2\},\{s_{-1},s_0,s_2\},\{s_{-1},s_1\}\} & \{\{s_1,s_3\},\{s_0,s_2\},\{s_0,s_1,s_3\}\} \\
\{\{s_{-1},s_0,s_2\},\{s_{-2},s_1\},\{s_{-2},s_{-1},s_1\}\} & \{\{s_{-2},s_{-1},s_2\},\{s_{-3},s_1,s_2\},\{s_{-2},s_0\}\} & \{\{s_{-2},s_1\},\{s_{-1},s_1,s_2\},\{s_{-1},s_0,s_1\}\} & \{\{s_0,s_2,s_3\},\{s_{-1},s_2,s_3\},\{s_{-1},s_0,s_3\}\}
\end{bmatrix}$$

In the following, we use the EHFL-VIKOR method to solve this MCDM problem.

Step 1 Given that price (C_1) is a cost attribute and quality (C_2), service (C_3) and safeguard (C_4) are benefit attributes, the positive ideal solution A^+ and the negative ideal solution A^- are as follows:

$$A^{+} = \left\{ \begin{array}{l} \{\{s_{-3}, s_{-2}, s_0\}, \{s_{-2}, s_0, s_1\}, \{s_{-2}, s_1\}\}, \{\{s_{-2}, s_1, s_2, s_3\}, \{s_{-2}, s_1, s_2\}, \{s_{-1}, s_0, s_2\}\}, \\ \{\{s_0, s_2\}, \{s_{-1}, s_0, s_2\}, \{s_{-1}, s_1\}\}, \{\{s_1, s_3\}, \{s_0, s_2\}, \{s_0, s_1, s_3\}\} \end{array} \right\}$$

$$A^{-} = \left\{ \begin{array}{l} \{\{s_{-2}, s_{-1}, s_1\}, \{s_{-2}, s_1\}, \{s_{-1}, s_0, s_2\}\}, \{\{s_{-2}, s_{-1}, s_2\}, \{s_{-3}, s_1, s_2\}, \{s_{-2}, s_0\}\}, \\ \{\{s_{-2}, s_{-1}, s_1\}, \{s_{-2}, s_0\}, \{s_{-3}, s_{-2}, s_1, s_2\}\}, \{\{s_{-2}, s_0\}, \{s_{-1}, s_2\}, \{s_0, s_2\}\} \end{array} \right\}$$

Step 2 We compute $\tilde{S}_i$ and $\tilde{R}_i$ for different alternatives according to Eqs. (28) and (29). The Hamming distance measure for each alternative with respect to attributes is shown in Table 5, and the obtained values of $\tilde{S}_i$ and $\tilde{R}_i$ are shown in Table 6.

Step 3 The value of $\tilde{Q}$ for each alternative is calculated by using Eq. (30) as follows:

$\tilde{Q}_1 = 1, \quad \tilde{Q}_2 = 0.06, \quad \tilde{Q}_1 = 0, \quad \tilde{Q}_1 = 0.64.$

Step 4 In this step, we rank the alternatives by $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$ in decreasing order. The result is shown in Table 7.

Step 5 In this step we derive the compromise solution as follows.

Note that $\tilde{S}_3 \prec \tilde{S}_2 \prec \tilde{S}_4 \prec \tilde{S}_1$, $\tilde{R}_3 \prec \tilde{R}_2 \prec \tilde{R}_4 \prec \tilde{R}_1$ and $\tilde{Q}_3 \prec \tilde{Q}_2 \prec \tilde{Q}_4 \prec \tilde{Q}_1$. According to this result, the condition 2 is valid. But since $Q(A'') - Q(A') = 0.06 - 0 = 0.6 \not\succeq \frac{1}{4-1} = 0.33$ the condition 1 is not valid. Thus, the compromise solution consists of alternatives $A', A'', \ldots, A^{(N)}$ where $A^{(N)}$ is determined by the relation $Q(A^{(i)}) - Q(A') \prec \frac{1}{m-1}$ for maximum N as follows:

$$Q(A') - Q(A') = 0 \prec \frac{1}{4-1}$$

$$Q(A'') - Q(A') = 0.06 - 0 \prec \frac{1}{4-1}$$

$$Q(A^3) - Q(A') = 0.64 - 0 \not\prec \frac{1}{4-1}$$

$$Q(A^4) - Q(A') = 1 - 0 \not\prec \frac{1}{4-1}.$$

Table 5 The calculated Hamming distance measure

	C_1	C_2	C_3	C_4
$d_{hd}(\tilde{H}_s^{1j}, \tilde{H}_s^{j+})$	0.1111	0.0833	0.1905	0.1905
$d_{hd}(\tilde{H}_s^{2j}, \tilde{H}_s^{j+})$	0	0.0238	0.0595	0.0476
$d_{hd}(\tilde{H}_s^{3j}, \tilde{H}_s^{j+})$	0.0635	0	0	0
$d_{hd}(\tilde{H}_s^{4j}, \tilde{H}_s^{j+})$	0.1111	0.1071	0.0714	0.1270
$d_{hd}(\tilde{H}_s^{j+}, \tilde{H}_s^{j-})$	0.1111	0.1587	0.1905	0.1905
w_j	0.2	0.15	0.15	0.5

Table 6 The obtained values of $\tilde{S}_i$ and $\tilde{R}_i$ for each alternative

	$\tilde{S}_i$	$\tilde{R}_i$
A_1	0.93	0.50
A_2	0.19	0.12
A_3	0.11	0.11
A_4	0.69	0.33

Table 7 Ranking of alternatives via $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$

	A_1	A_2	A_3	A_4
By $\tilde{S}$	4	2	1	3
By $\tilde{R}$	4	2	1	3
By $\tilde{Q}$	4	2	1	3

Table 2 The evaluation information given by group 1

	C_1	C_2	C_3	C_4
A_1	bt l and m, vh	m, vh	bt vl and l, h	vl, m
A_2	bt n and vl, m	bt l and m, vh	l, bt h and vh	m, vh
A_3	n, bt l and m	vl, bt h and p	m, vh	h, p
A_4	bt l and m, vh	bt vl and l, vh	vl, h	m, bt vh and p

Table 3 The evaluation information given by group 2

	C_1	C_2	C_3	C_4
A_1	bt vl and l, h	bt l and m, vh	vl, m	l, vh
A_2	vl, bt m and h	l, bt h and vh	l, m	l, bt h and vh
A_3	vl, h	vl, bt h and vh	bt l and m, vh	m, vh
A_4	l, vh	n, bt h and vh	l, bt h and vh	l, bt vh and p

Table 4 The evaluation information given by group 3

	C_1	C_2	C_3	C_4
A_1	l, bt m and h	l, bt h and vh	bt n and vl, bt h and vh	m, vh
A_2	vl, bt m and h	vl, bt h and vh	l, bt m and h	l, vh
A_3	vl, h	l, bt m and vh	l, h	bt m and h, p
A_4	vl, vh	v, m	l, bt m and h	m, bt vh and p

Therefore, the compromise solution consists of A_3 and A_2.

7 Conclusion

Group decision making is very common in complicated decision-making environment. Most of the group decision-making problems have qualitative nature. Introduction of EHFLTS provides an effective tool to elicit the hesitancy and uncertainty of information in group decision making. As the novelty of EHFLTS, we first focused on the distance and similarity measures for EHFLTSs. We developed some distance and similarity measures for EHFLTs and EHFLTSs, respectively, such as the Hamming distance, the Euclidean distance and the Hausdorff metric. Afterward, we proposed the EHFL-VIKOR method based on these distance measures to deal with hesitancy in qualitative MAGDM problems. According to the literature review, Zhang and Wei developed the VIKOR method based on hesitant fuzzy set. In comparison with our proposed method, they used the hesitant fuzzy sets to deal with uncertainty. Since the HFSs are suitable for quantitative situations, their proposed method can be considered as a quantitative decision-making approach. Finally, an illustrative example concerning the selection of suitable telecommunications service provider of SMEs has been presented.

This study also has limitations which can provide some ideas for future research. These limitations include:

1. In this paper, we only focused on traditional distance measures such as the Hamming distance, the Euclidean distance and the Hausdorff metric. The other type of algebraic distance measures such as weighted distance operators and also geometric distance measures such as cosine-based distance measure can be investigated in future research works.
2. We applied the distance and similarity measures to develop EHFL-VIKOR method. It would be interesting to extent other distance-based decision-making methods, such as the EHFL-ELECTRE, the EHFL-TOPSIS and the EHFL-TODIM in future works.
3. In this paper, we supposed that the linguistic term sets for all criteria have the same cardinality and developed distance and similarity measures based on this assumption. Obviously, different experts may have different recognition from the same problem. Thus, they may use several linguistic term sets with different granularity of uncertainty. Considering multi-granularity in development of distance and similarity measures for EHFLTSs and consequently introduction of some EHFL-MAGDM approach based on these measures can be an interesting topic for future research works.

Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

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