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ORIGINAL ARTICLE

A novel chaotic differential evolution hybridized with quadratic programming for short-term hydrothermal coordination

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Abstract In this paper, a viable global optimizer based on chaotic differential evolution is hybridized with sequential quadratic programming, an efficient local search technique to exploit short-term hydrothermal coordination (STHTC) involved for power generation and its efficient management. A multi-objective optimization framework is established for minimizing the total cost of thermal generators with valve point loading effects satisfying power balance constraint as well as generator operating and hydrodischarge limits, respectively. The proposed model is implemented on various systems comprising hydrogenerating units as well as different thermal units. The results are compared with state-of-the-art heuristic techniques recently employed on STHTC problems, while the reliability, stability and effectiveness of the proposed framework are validated through the comprehensive analysis of Monte Carlo simulations.

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Keywords Short-term hydrothermal coordination · Chaotic differential evolution · Sequential quadratic programming · Multi-objective optimization · Stability analysis

List of symbols

$x_{\mathrm{TH}i}$, $y_{\mathrm{TH}i}$, $z_{\mathrm{TH}i}$, $u_{\mathrm{TH}i}$, $e_{\mathrm{TH}i}$	Cost coefficients for thermal power generation
$P_{\mathrm{TH}it}$	Generated output power in time t of thermal unit i
$P_{\mathrm{TH}i}^{\min}, P_{\mathrm{TH}i}^{\max}$	Minimum and maximum thermal generation limits for unit i
$P_{\mathrm{d}t}$	Power demand at time t
$P_{\mathrm{l}t}$	Losses due to power transmission at time t
N_{TH}, N_{h}	Total number of thermal and hydroelectric units
DR_i, UR_i	Down and up ramp rate limits of thermal unit i
$Q_{\mathrm{h}jt}$, $V_{\mathrm{h}jt}$	Water discharge rate and storage volume of jth reservoir at time
$P_{\mathrm{h}jt}$	Generated power from jth hydroelectric unit at time t
C_{1j}, C_{2j}, C_{3j}, C_{4j}, C_{5j}, C_{6j}	Power generation coefficients of jth hydroelectric unit
$P_{\mathrm{h}j}^{\min}, P_{\mathrm{h}j}^{\max}$	Lower and upper limits of jth hydroelectric unit
$Q_{\mathrm{h}j}^{\min}, Q_{\mathrm{h}j}^{\max}$	Minimum and maximum water discharge rate of jth reservoir
t, T	Time index and scheduling period

Pop	Randomly generated population of candidate solutions
P_{size}	Population size
$\mathrm{STHTC}_{\mathrm{size}}$	Problem size
B	Weight factor
Crate	Crossover rate
ζ	Chaotic variable
f_{val}	Cost of the fitness evaluation function

1 Introduction

In power sector, optimum coordination of demand and generation impacts the budget substantially that demands the researchers and engineers minimization of cost and power losses [1, 2]. Modern gigantic power system comprises of many thermal as well as hydropower plant connected through transmission networks [3]. A very important objective in the operation of huge power system is meeting power demand economically by finding optimal mix of different power plants. Utilizing the available hydroresources completely without wasting a drop of water and using minimum thermal fuel will lead to huge saving in fuel cost [4]. The main objective of hydrothermal coordination is minimizing the cost of mixed hydrothermal system by trading off with transmission losses, available resources, fuel cost, dynamic loads and valve point loadings.

The research age of hydrothermal coordination problem is almost a century [5–7]; however, the development of the state-of-the-art optimization solvers and sophisticated hardware kept it a demanding area even in the last decade [8–10]. A number of conventional techniques have been employed to solve this nonlinear, non-deterministic and polynomial time hard problem like lambda gamma iteration method [11], gradient method [12], dynamic programming [13] and Newton–Raphson method [14] which have their own strengths and weaknesses [15, 16]. Although the conventional techniques are capable enough to provide a reasonably good solution in an appropriate time, however, they lack in handling more than a few constraints and are more probable to get stuck in the local minima and may lead toward an odd solution. The schemes like priority list method a class of weighted procedures and forward dynamic programming (DP) approach took lots of attention in the research community [17, 18] for handling the problems of unit commitment while constraints of getting stuck in the local minimum still exist. The similar problems have been observed in the Lagrange relaxation method and the methods of the class based on Karush–Kuhn–Tucker equations [16, 19].

The derivative-free methods have the strengths like converging to the optimal solution even on a very vague start points, less probable to get stuck in the local minima's, state of the art for getting global solution and their computational complexity also lie in an acceptable domain [20–22]. Since the last decade of twentieth century, many intelligent computational techniques have merged especially evolutionary computational methods remained more attractable to power system optimizers [23, 24]. The techniques that are mostly used are simulated annealing, artificial bee colony, genetic algorithm (GA), differential evolution (DE), particle swarm optimization (PSO) and ant colony optimization. These up-to-date techniques solved the shortcomings of conventional techniques of being caught to local optima and ability of handling a few constraints only [25–27].

In this paper, a nature-inspired evolutionary technique hybridized with an efficient local search optimizer sequential quadratic programming (SQP) for economic dispatch of nonlinear, dynamic and multi-constrained STHTC problem is developed. A multi-objective optimization framework is established for minimizing the total cost of thermal generators with valve point loading effects satisfying power balance constraint as well as generator operating and hydrodischarge limits, respectively. The exploitations of three computational techniques like chaotic DE on the four different cases of hydrothermal coordination are applied in mean square error sense as a fitness evaluation function. The comparison is made based on fitness evaluation, mean square error, computational complexity behavior in terms of time, fuel cost and power generation demand.

The organization of this paper is as follows: In Sect. 2, a detailed problem formulation has been provided containing STHTC problem and its constraints. The proposed scheme based on chaotic DE, SQP and hybrid approach is revealed in Sect. 3. The scenario-based results and discussion on the case studies [1, 28–30] and equivalent hydropower plant is provided in Sect. 4. The conclusions drawn from the results have been provided at the end.

2 Problem formulation

Hydrothermal coordination problem aims to minimize the fuel cost while meeting the different constraints fulfilling the power demand. The objective function for hydrothermal coordination and different constraints are formulated in the following way.

2.1 Objective function

$$\text{Min Cost Function} = \sum_{t=1}^{T}\sum_{i=1}^{N_{\text{TH}}} \left[x_{\text{TH}i} + y_{\text{TH}i}P_{\text{TH}it} + z_{\text{TH}i}P_{\text{TH}it}^{2} + \left| u_{\text{TH}i} \times \sin\left\{ e_{\text{TH}i} \times \left(P_{\text{TH}i}^{\min} - P_{\text{TH}it}\right) \right\} \right| \right]. \quad (1)$$

2.2 System constraints

2.2.1 Power balancing constraints

In each interval of scheduling time, the combined output power of hydroelectric and thermal power plants must balance the expected power demand and transmission line losses

$$\sum_{i=1}^{N_{\text{TH}}} P_{\text{TH}it} + \sum_{j=1}^{N_{\text{h}}} P_{\text{h}jt} = P_{\text{d}t} + P_{\text{l}t}, \quad t \in T. \quad (2)$$

The hydroelectric power production depends on water discharge and water head that is directly related to storage volume of the reservoir

$$P_{\text{h}jt} = C_{1j}\,V_{\text{h}jt}^{2} + C_{2j}\,Q_{\text{h}jt}^{2} + C_{3j}\,V_{\text{h}jt}\,Q_{\text{h}jt} + C_{4j}\,V_{\text{h}jt} + C_{5j}\,V_{\text{h}jt} + C_{6j}, \quad j\varepsilon N_{\text{h}}, \quad t\,\varepsilon\,T. \quad (3)$$

Power losses due to transmission is given as

$$P_{\text{l}t} = \sum_{i=1}^{N_{\text{TH}}+N_{\text{h}}}\sum_{j=1}^{N_{\text{TH}}+N_{\text{h}}} P_{it}B_{ij}P_{jt} + \sum_{i=1}^{N_{\text{TH}}+N_{\text{h}}} B_{oi}P_{it} + B_{oo}. \quad (4)$$

2.2.2 Generation upper and lower constraints

Following equation indicates that each thermal generation unit has a certain upper and lower generation limit. The output power from every unit must be in the given generation range.

$$P_{\text{TH}i}^{\min} \le P_{\text{TH}it} \le P_{\text{TH}i}^{\max}, \quad i\,\varepsilon\,N_{\text{TH}}, \quad t\,\varepsilon\,T. \quad (5)$$

Similarly we have, hydroelectric power generation limits

$$P_{\text{h}j}^{\min} \le P_{\text{h}jt} \le P_{\text{h}j}^{\max}, \quad j\,\varepsilon\,N_{\text{h}}, \quad t\,\varepsilon\,T. \quad (6)$$

2.2.3 Up and down ramp limits for thermal generation units

The power from any thermal unit 'i' during an interval should not exceed from the power generated in previous interval by more than a certain amount. It should not be less than the power generated in the previous interval by more than a specified limit. It can be represented mathematically in the following constraints

$$P_{\text{TH}it} - P_{\text{TH}i(t-1)} \le UR_i, \quad i\varepsilon N_{\text{TH}}, \quad t\varepsilon T \quad (7)$$

$$P_{\text{TH}i(t-1)} - P_{\text{TH}it} \le DR_i, \quad i\varepsilon N_{\text{TH}}, \quad t\varepsilon T. \quad (8)$$

2.2.4 Hydroelectric system constraints

The operation of hydropower plants comprises constraints like balancing the input–output water of reservoir, power plant limitations, reservoir storage bounds and the multi-purpose storage. The mathematical model of these constraints is presented in following equations.

Limitations of storage volume and discharge rates in reservoir are;

$$V_{\text{h}j}^{\min} \le V_{\text{h}jt} \le V_{\text{h}j}^{\max}, \quad j\varepsilon N_{\text{h}}, \quad t\varepsilon T \quad (9)$$

$$Q_{\text{h}j}^{\min} \le Q_{\text{h}jt} \le Q_{\text{h}j}^{\max}, \quad j\varepsilon N_{\text{h}}, \quad t\varepsilon T. \quad (10)$$

3 Chaotic differential evolution and quadratic programming

A robust meta-heuristic method well known as differential evolution (DE) algorithm having strong capability of function minimization or maximization is proposed by Ken and Storn in 1997 [31]. DE is a population-based stochastic algorithm with very a few parameters but providing excellent solutions to the non-smooth, multimodal and non-convex problems [32] because it is not a gradient-based method [33]. As compared to other evolutionary algorithms, DE is less stochastic but more greedy and uses simple arithmetic operators for evolving starting population to final solution [34]. The main difference between genetic algorithm and DE is that DE uses the perturbing vectors that cause the diversity in the sample space and amplification factor searches the candidate solution from every knock and corner of the solution surface [35].

DE applications are widespread almost in every field of research, e.g., electrical power simulations, optical systems optimization, radio network designs, chemistry of carbon materials and water pumping systems optimization [36]. To overcome the basic drawbacks of DE, chaotic theory is exploited to make it chaotic DE that assures the diversification so that handle can move through the whole search space for improving the chances of not being caught in the local optima and it tunes the parameters control strategy as well.

More precisely, to create the diversity in the search space the behavior of a chaotic system is encapsulated in differential evolution algorithm as a chaotic parameter from the algebraic manipulation of initial random values passed from the random system that create the scatteredness and the algorithm will search every knock and corner of the candidate solutions. The vector representation and factor of DE algorithm are also coupled with the chaotic variable which guarantees the parallelism in n-dimensions and improves the computational searching of the proposed scheme as well.

SQP is an iterative method fall under the class of barrier methods efficiently used in nonlinear optimization of complex system having linear and nonlinear constraints. The capability of transforming a complex problem into the subproblems has been exploited by keeping in view the constraints like load demand, generating limits and valve point loading effects in STHTC problem. The SQP requires the objective function and its constraints as a lagrangian function to minimize the energy cost subject to the defined constraints. The applications of this local search schemes are found in combinatorial problems, image classification, numerical approximation of the control and power system stability analysis. The built-in subroutine of the sequential quadratic programming (SQP) is used as a local optimizer.

The necessary explanation about the logical steps used for optimization of hybrid approach DE-SQP is presented here:

Step 1 *Parameter setup* An initial weight vector is generated randomly with the real bounded values of the length equal to the number of design parameters involved in STHTC. The user-defined population size, length of one vector in the population, the boundary constraints of the optimization, the mutation factor, stopping criteria and other necessary parameter settings and values are taken as given in Table 1.

Step 2 *Initialization of an individual population* Set generation $N = 0$ with a population of $i = 1,\ldots,M$ individuals (real-valued sufficiently large n-dimensional candidate solution) with random values generated according to a uniform probability distribution in the n-dimensional problem space in order to avoid premature convergence.

Step 3 *Fitness evaluation* Evaluate the energy function as defined for each objective function up to an acceptable range of the fitness value ε.

Step 4 *Differential operation* The mutation operation adds a vector differential to a population vector of individuals with a real mutation factor that controls the amplification of the difference between two individuals to avoid search stagnation and is usually taken from the range [0.1, 1].

Step 5 *Recombination operation* Recombination is employed to generate a trial vector by replacing certain parameters of the target vector with the corresponding parameters of a randomly generated donor vector; the recombination rate is taken in logarithmic manner to get a mature exploitation of the search space.

Step 6 *Selection operator* The procedure of producing better offsprings is obtained in this step with a criterion of comparing the fitness of the current individual as stability to stain in the next generation. Similarly, the fitness cost of each trial vector is compared with that of its parent target vector. If the cost of the target vector is lower than that of trial vector, the target is allowed to advance to the next generation; otherwise, the target vector is replaced by the trial vector in the next generation.

Table 1 Parameter values/settings for chaotic DE and SQP

Chaotic differential evolution		Sequential quadratic programming	
Parameters	Values/settings	Parameters	Values/settings
Generations	500	Start point	Random or best result of DE
Population size	360	No of variables	Generation unit dependent
Population range	[−1 1]	Iterations	1000
Function tolerance	10^{-30}	Max. function evaluations	50,000
Stall generation limit	200	Function tolerance	0
Nonlinear constraints tolerance	10^{-30}	Nonlinear constraints tolerance	0
Fitness limit	10^{-35}	Derivative approach	Finite forward difference
X-tolerance	10^{-18}	X-tolerance	10^{-18}
Bounds	As given in thermal units	Bounds	As given in thermal units
Others	Default	Others	Default

Step 7 *Stopping criteria for chaotic differential evolution* Set the generation number for $N = N + 1$ and proceed to step 3 until a stopping criteria is met that is problem dependent based on the following conditions:

1. Maximum number of generations achieved
2. Fitness value ε is less than 10^{-12} is achieved
3. Function tolerance is lower than a certain predefined criteria

Step 8 *Hybrid with SQP* The one of the best individual obtained with Chaotic DE is passed as a start point to the SQP algorithm for fine-tuning of the unknown adaptive parameters of the STHTC problem.

The detailed pseudo-code of the proposed scheme is given as follows:

```
Input: Psize, STHTCsize, β,C_rate,ζ
Output: R_best=[P_THit P_hj]
 Population ←initilize population(Psize, STHTCsize)
Evaluatepopulation(Pop)
R_best ← getbestcandiadte(Pop)
   While(stopingcriteria() not met)
newpop←Ø
forpop_i∈ Pop
R_i←newsample(pop_i,population, STHTCsize, β,C_rate,ζ)
        If (f_val (R_i)≤f_val (pop_i))
newpapulation← R_i
   Else
newpapulation← pop_i
End
       End
Pop ←newpopulation
Evaluatepopulation(Pop)
R_best←getbestcandidate(Pop)
    End
Return(R_best)
```

4 Simulation and results

Four scenario-based hydrothermal test systems have been investigated, and the simulation results based on various performance criterions like cost in dollars, fitness evaluation of the energy functions, absolute error of the load and thermal generations and computational complexity in terms of time are presented. The level and percentage of the convergence for chaotic DE, SQP and hybrid approach are also computed for sufficiently large number of independent runs. The algorithms used in this article are implemented by using MATLAB version 7.12.0 (R 2011a) on the Intel(R) Core(TM) i3-4010U CPU @ 1.70 GHz machine with 4 GB RAM.

4.1 Case study I

This test system considers a multi-chain cascade of four reservoirs hydropower plants along with the three steam power plants to fulfill the overall generation of 1050 MW. The entire scheduling period is one week and divided into 100 intervals. The system has been simulated using the optimization solvers based on chaotic DE, SQP and DE-SQP; the parameters values/settings of the solvers are provided in Table 1.

200 MW of the power is obtained from the hydropower plants, while the remaining power is fed to the system using three available steam power plants economically. The optimal values of the hydrothermal generation and steam generation are presented in Table 2 while the behavior of the steam power plant is shown in Fig. 1a as a fitness evaluation of the energy function while Fig. 1b describes the absolute difference of the remaining power load and thermal generation. It is quite evident from the figure that the value of the fitness achieved lie in the range from 10^{-11} to 10^{-13}, while difference of thermal generation is from 10^{-05} to 10^{-06} for the hybrid approach. It is worth to mention that the cost for DE-SQP is much lesser than that of DE and SQP alone.

4.2 Case study II

This test system considers six steam power plants and an equivalent hydrogeneration plant to meet the requirement of 400 MW. The entire scheduling period is one week and divided into 100 equal intervals. 116.6 MW of the power is fed by the hydropower plants, and the remaining power is provided by the six steam plants. The optimal values of the hydrothermal generation and steam generation are presented in Table 3 while the steam power plants behavior is shown in Fig. 2a as a fitness evaluation of the energy

Table 2 Optimal hydrothermal generation (MW) for case study I

Solver	DE	SQP	DE-SQP
Optimal hydrogeneration			
Ph (MW)	200	200	200
Optimal thermal generation			
Ps1 (MW)	368.0882	415.7895	389.2278
Ps2 (MW)	239.6326	289.4737	260.7722
Ps3 (MW)	242.2792	144.7368	200
Cost ($)	8678.374	8596.647	8421.025

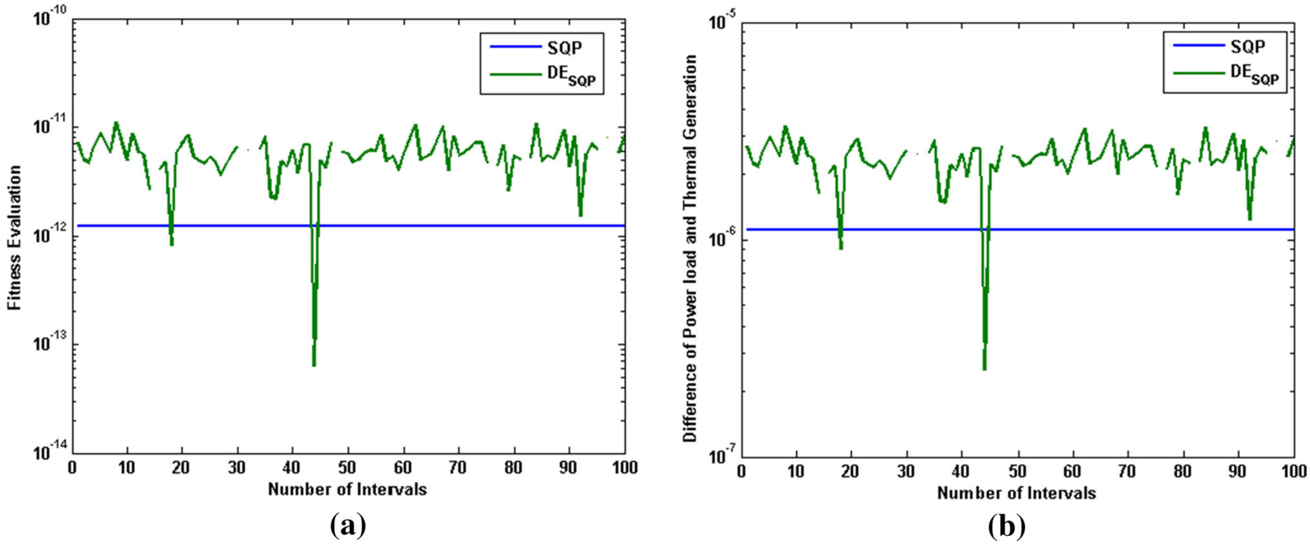

Fig. 1 Behavior of the fitness function evaluation (**a**) and absolute error of thermal generation for 100 intervals in (**b**)

Table 3 Optimal hydrothermal generation (MW) for case study II

Solver	DE	SQP	DE-SQP
Optimal hydrogeneration			
Ph (MW)	116.6	116.6	116.6
Optimal thermal generation			
Ps1 (MW)	81.85596539	128.4905694	87.05768382
Ps2 (MW)	38.06031749	51.39622726	43.26203736
Ps3 (MW)	47.48569106	33.31446592	48.08028332
Ps4 (MW)	37.23094576	23.08176139	34.99999835
Ps5 (MW)	33.40607996	20.46540911	29.9999984
Ps6 (MW)	45.36100034	26.65156693	39.99999832
Cost ($)	123,020.8913	4394.600398	1648.481294

function while Fig. 2b shows the absolute difference between power required and the power generated.

It is very clear from the figure that the value of the fitness achieved lie in the range from 10^{-12} to 10^{-15}, while difference of thermal generation is from 10^{-06} to 10^{-07} for the hybrid approach. We observed that the cost for DE-SQP is much lesser than that of DE and SQP alone.

4.3 Case study III

In this test system, we consider a system comprising 13 steam power plants and equivalent hydropower generation. The total power demand is 2200 MW of which 400 MW is provided by hydropower plants free of cost, and the left

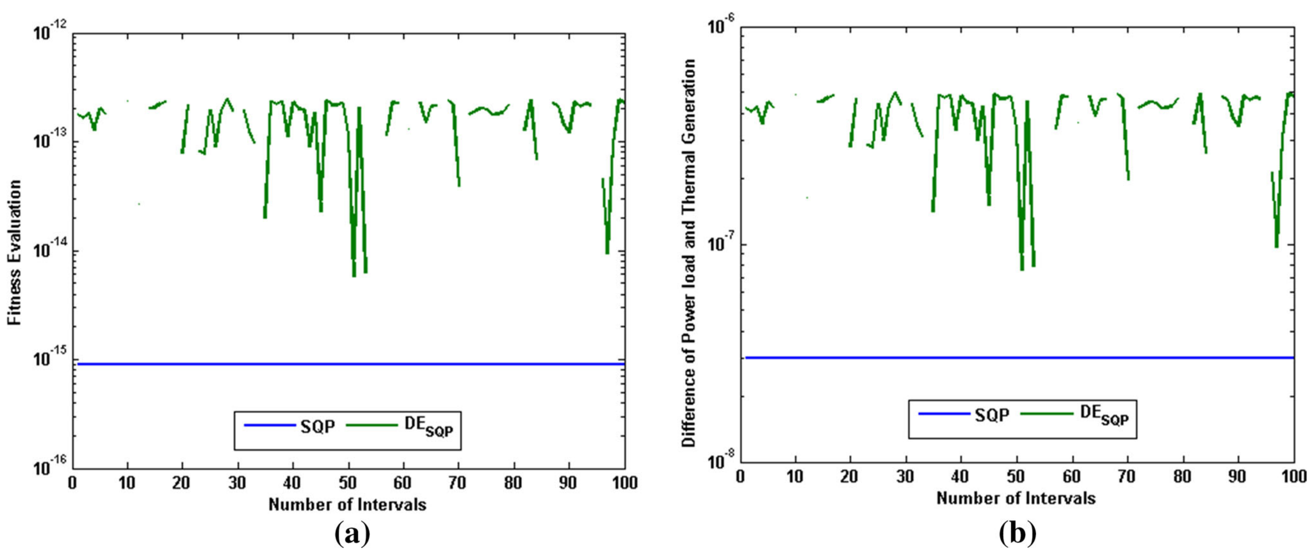

Fig. 2 Behavior of the fitness function value (**a**) and absolute error of thermal generation for 100 intervals in (**b**)

Table 4 Optimal hydrothermal generation (MW) for case study III

Solver	DE	SQP	DE-SQP
Optimal hydrogeneration			
Ph (MW)	400	400	400
Optimal thermal generation			
Ps1 (MW)	26.9588	352.8772	98.7181922
Ps2 (MW)	41.85882	186.676	99.2111282
Ps3 (MW)	42.8607	186.9355	100.032396
Ps4 (MW)	199.4378	122.1997	179.999958
Ps5 (MW)	200.0579	122.1997	179.999958
Ps6 (MW)	211.0128	122.1997	179.999958
Ps7 (MW)	152.4702	122.1997	157.433046
Ps8 (MW)	137.5302	122.1997	145.186288
Ps9 (MW)	203.039	122.1997	179.999958
Ps10 (MW)	119.2916	81.46589	119.419249
Ps11 (MW)	174.2788	81.46589	119.999956
Ps12 (MW)	143.426	88.69074	119.999956
Ps13 (MW)	147.7774	88.69074	119.999956
Cost ($)	123,020.8913	4394.600398	1648.481294

over power is met by the steam power plants. The system has been simulated using the optimization solvers based on chaotic DE, SQP and DE-SQP; the parameters values/settings of the solvers are provided in Table 1.

The optimal solution of the hydrothermal values is presented in Table 4. The behavior of steam power plants is shown in Fig. 3a as a fitness evaluation of the energy function, and Fig. 3b shows the absolute difference between power demand and the power generated. It is very clear from the figure that the value of the fitness achieved lie in the range from 10^{-12} to 10^{-16}, while difference of thermal generation is from 10^{-06} to 10^{-08} for the hybrid approach. We have observed in this case also the cost for DE-SQP is much lesser than that of DE and SQP alone.

4.4 Case study IV

In this case, we consider a system comprising of 40 steam power plants and equivalent hydropower generation. The total power demand is 12,000 MW of which 1500 MW is generated by hydropower plants and the remaining power is fed by the steam power plants; here, we are neglecting transmission losses. The optimal solution we found by the algorithms is given in Table 5. The behavior of steam power plants is shown in Fig. 4a as a fitness evaluation of the energy function, and Fig. 4b shows the absolute difference between power demand and the power generated. The results of the figure are drawn at semi-log scale in order to describe the clear difference in each independent run. It is quite evident from the table that the hybrid scheme shows supremacy in terms of cost per MW than that of chaotic DE and SQP, although the computational time complexity of hybrid scheme is slightly more but this effect can be ignored with the effect of the cost.

It is quite evident from the figure that the value of the fitness achieved lie in the range of 10^{-20} to 10^{-25}, while difference of thermal generation is from 10^{-10} to 10^{-12} for the hybrid approach. The cost of hybrid approach is much lesser than that of DE and SQP alone in this case also.

4.5 Comparative analysis of the results

The comparative study for case 1–4 is also thoroughly investigated based on behavior of their computation time complexity, analysis on fitness achieved, load error and statistical analysis for the computational derive. The

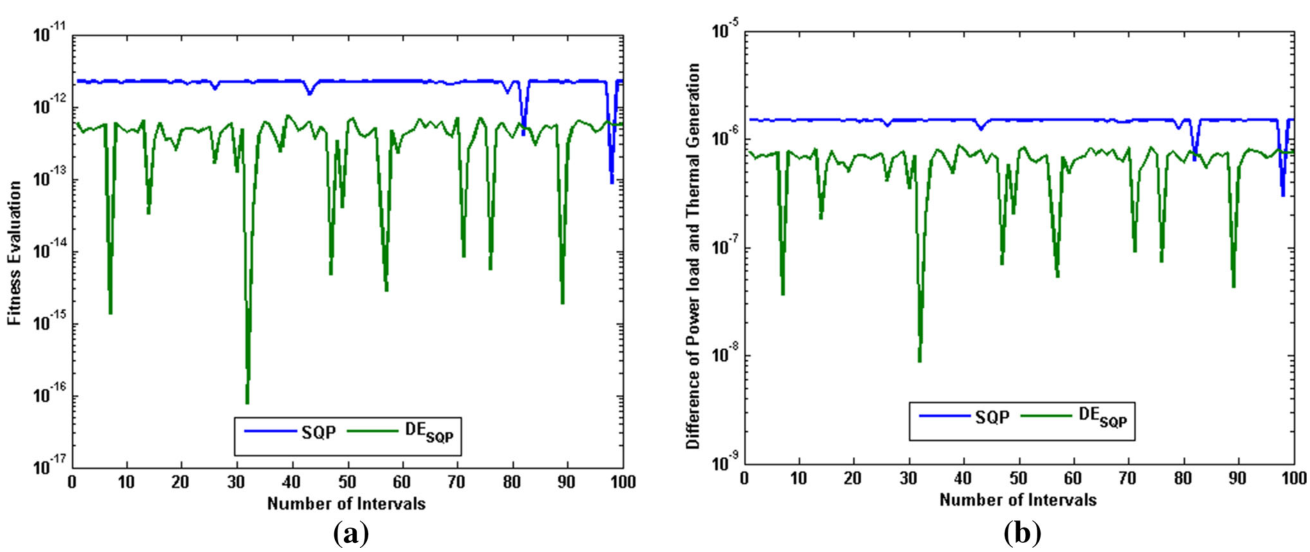

Fig. 3 Behavior of the fitness function evaluation (**a**) and absolute error of thermal generation for 100 intervals in (**b**)

Table 5 Optimal hydrothermal generation (MW) for case study IV

Solver	DE	SQP	DE-SQP
Optimal hydrogeneration			
Ph (MW)	1500	1500	1500
Optimal thermal generation			
Ps1 (MW)	132.2539582	92.06804958	114
Ps2 (MW)	139.2677773	92.06804958	114
Ps3 (MW)	163.4171438	103.1292689	120
Ps4 (MW)	177.1651195	159.0703263	183.7098418
Ps5 (MW)	140.9268812	82.94105742	97
Ps6 (MW)	167.7413554	119.7551227	140
Ps7 (MW)	176.1009267	246.5760182	239.2873291
Ps8 (MW)	237.2224228	253.6054895	269.2385135
Ps9 (MW)	185.7263532	253.6054895	244.0047197
Ps10 (MW)	232.3491264	252.1995952	266.850625
Ps11 (MW)	173.6844415	295.9887427	276.3542302
Ps12 (MW)	194.0661898	296.7075639	286.8509457
Ps13 (MW)	196.0262765	394.5579307	351.0504
Ps14 (MW)	186.0605753	394.5579307	346.1671317
Ps15 (MW)	229.7251196	394.5579307	367.5632823
Ps16 (MW)	268.7190231	394.5579307	386.6704321
Ps17 (MW)	285.1172317	421.2699216	394.7055833
Ps18 (MW)	327.2881577	421.2699216	415.3695108
Ps19 (MW)	339.1840995	463.3969137	446.6986891
Ps20 (MW)	340.9184334	463.3969137	447.5487986
Ps21 (MW)	400.4456321	466.77106	476.7176762
Ps22 (MW)	331.1034731	466.77106	442.7395591
Ps23 (MW)	315.5877954	466.77106	435.1361993
Ps24 (MW)	306.4064802	466.77106	430.6376298
Ps25 (MW)	362.7246052	466.77106	458.2338338
Ps26 (MW)	331.7683512	466.77106	443.0647432
Ps27 (MW)	119.3841058	110.6349608	134.9967919
Ps28 (MW)	107.1727912	110.6349608	129.013211
Ps29 (MW)	53.50256196	110.6349608	102.7158613
Ps30 (MW)	135.8759898	82.94105742	97
Ps31 (MW)	157.4549128	153.4467493	174.0534244
Ps32 (MW)	155.1604129	153.4467493	172.928325
Ps33 (MW)	153.1323907	153.4467493	171.934534
Ps34 (MW)	221.233994	169.0703263	200
Ps35 (MW)	174.2553217	169.0703263	187.384766
Ps36 (MW)	215.1853029	169.0703263	200
Ps37 (MW)	60.20898716	86.09979762	85.60153412
Ps38 (MW)	131.7594186	86.09979762	110
Ps39 (MW)	117.3141014	86.09979762	110
Ps40 (MW)	306.6807263	463.3969137	430.7718759
Cost ($)	993,176,931.4	3661678.233	2,777,718.62

behavior of the time analysis for 100 independent runs is shown in Fig. 5a–d for DE, SQP and DE-SQP approaches in case study 1 to case study 4, respectively. It is quite evident from the figure that the computational derive of the local search is very small; however, the probability of getting struck in the local minima is high while DE and DE-SQP time is approximately the same but the fitness values achieved in hybrid approach is better than DE.

The statistical mode mean and standard deviation always explain more realistically the effectiveness of an algorithm. The statistical parameters min, max, mean, standard deviation, variance and mode have been computed for a large number of independent runs to see the convergence of the optimizer, and the results are tabulated in Table 6. The mean fitness value achieved using 100 independent runs is computed using the relation given in Eq. (11).

$$\mathrm{MF} = \frac{1}{\Re}\sum_{j=1}^{\Re}\left(\frac{1}{N}\sum_{k=1}^{N}\left|f_{\mathrm{val}} - \hat{f}_{\mathrm{val}}^{k}\right|^{2}\right)_{j} \qquad (11)$$

This can be exploited from the table that the mean value of the fitness achieved for hybrid approach is consistently better than that of DE and SQP individually; the similar behavior has been observed in standard deviation as well. The comparative analysis of the load error is also determined for 100 independent runs of the DE, SQP and DE-SQP algorithms, and the results are shown in Table 7.

Similarly, the comparative study for DE, SQP and DE-SQP in terms of computational complexity for case study 1–4 is provided in Table 8 using the relation given below:

$$\mathrm{MET} = \frac{1}{\Re}\sum_{j=1}^{\Re}\mathrm{ET}^{j}. \qquad (12)$$

It is quite clear from the table that computational budget in terms of time complexity increases with the increase in number of generators. One of the important parameter in this research is cost in dollars for the production of power in MW; in this regard, a comparative study is also carried out for DE, SQP and DE-SQP algorithms whose results are given in Table 9. The dominancy of DE-SQP is observed in the table as the mean value of the cost for DE-SQP is lower than that of global search scheme and local search algorithm individually.

The convergence behavior is observed for 100 independent runs to validate the stability of the DE-SQP, and the results are drawn in Fig. 6 on the semi-log scale to make clarity among the various case studies. It is quite evident from the figure that overall convergence lies from 10^{-11} to 10^{-16}. Moreover, approximately 88% of the independent runs are found to be stable for case 1, case 3 and case 4 in the range from 10^{-11} to 10^{-13}, while it is 70% for case study 2 with a precision lies in 10^{-13}.

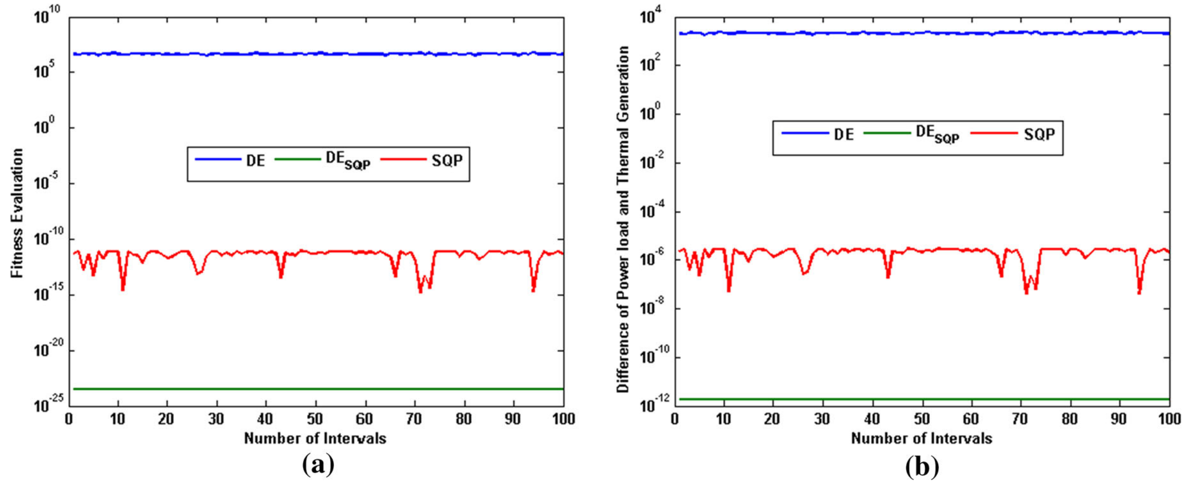

Fig. 4 Behavior of the fitness function evaluation (**a**) and absolute error of thermal generation for 100 intervals in (**b**)

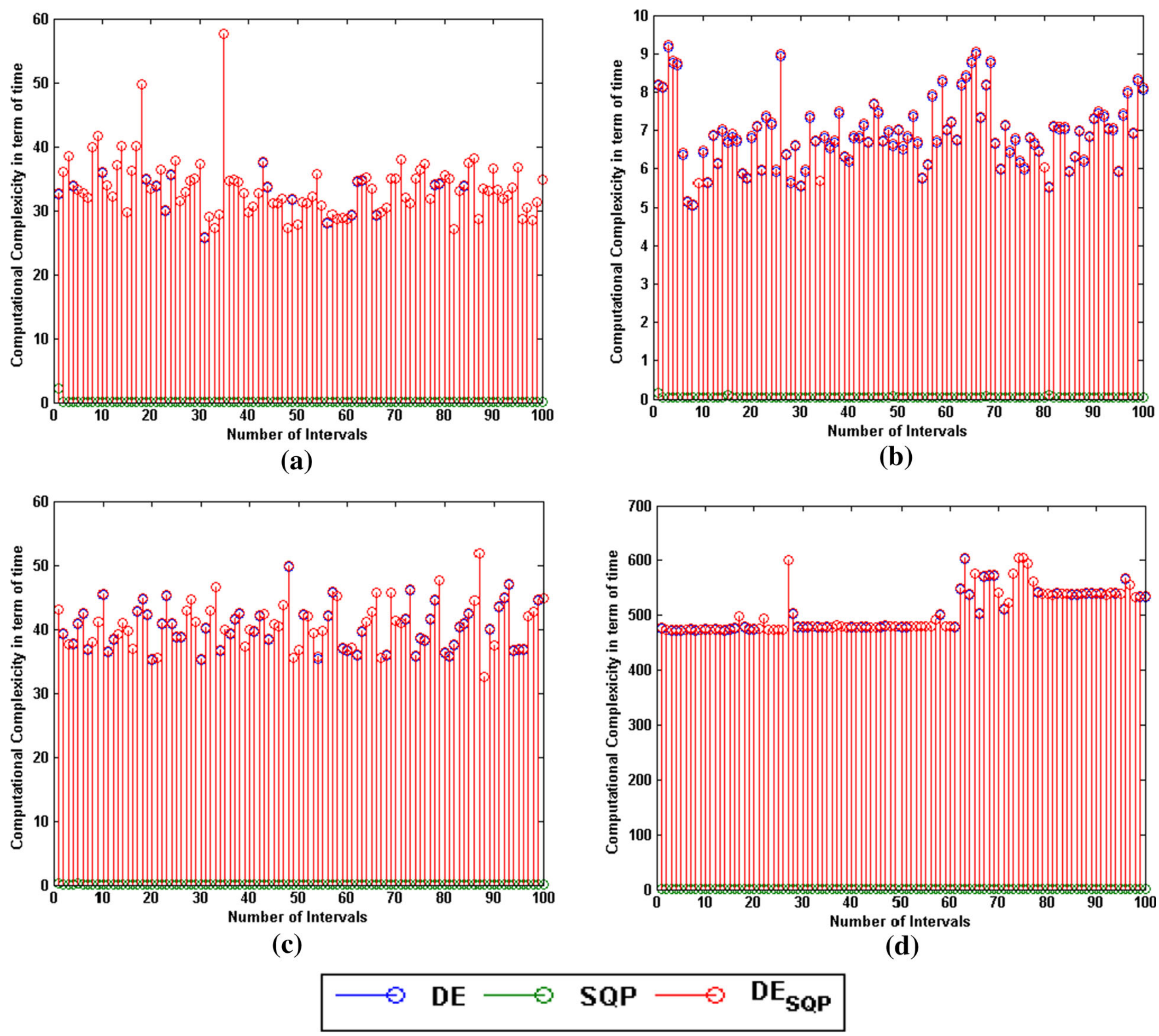

Fig. 5 Comparative analysis of the computational budget for case study I to case study IV

Table 6 Comparative study in DE, SQP and DE-SQP in terms of fitness achieved

Case study	Solver	Min	Max	Mean	STD	Var	Mode
1	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	1.2374E−12	1.2374E−12	5.3856E−12	6.0890E−28	3.7076E−55	1.2374E−12
	DE-SQP	0.0000E+00	1.1058E−11	1.2374E−12	2.4940E−12	6.2200E−24	0.0000E+00
2	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	8.8948E−16	8.8948E−16	1.3097E−13	0.0000E+00	0.0000E+00	8.8948E−16
	DE-SQP	0.0000E+00	2.4514E−13	8.8948E−16	9.2651E−14	8.5842E−27	0.0000E+00
3	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	8.7560E−14	2.2660E−12	2.1721E−12	3.0447E−13	9.2704E−26	2.2656E−12
	DE-SQP	7.7525E−17	7.6447E−13	4.3402E−13	1.8722E−13	3.5052E−26	7.7525E−17
4	DE	3.3162E+06	6.0206E+06	4.4814E+06	5.3298E+05	2.8407E+11	3.3162E+06
	SQP	3.3087E−24	3.3087E−24	5.1161E−12	0.0000E+00	0.0000E+00	3.3087E−24
	DE-SQP	1.6466E−15	8.3930E−12	3.3087E−24	2.6583E−12	7.0667E−24	1.6466E−15

Table 7 Comparative study in DE, SQP and DE-SQP in terms of load error

Case study	Solver	Min	Max	Mean	STD	Var	Mode
1	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	1.1124E−06	1.1124E−06	1.1124E−06	0.0000E+00	0.0000E+00	1.1124E−06
	DE-SQP	0.0000E+00	3.3253E−06	2.1843E−06	7.8786E−07	6.2072E−13	0.0000E+00
2	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	2.9824E−08	2.9824E−08	2.9824E−08	0.0000E+00	0.0000E+00	2.9824E−08
	DE-SQP	0.0000E+00	4.9512E−07	3.0860E−07	1.8998E−07	3.6094E−14	0.0000E+00
3	DE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	SQP	2.9591E−07	1.5053E−06	1.4660E−06	1.5232E−07	2.3201E−14	1.5052E−06
	DE-SQP	8.8048E−09	8.7434E−07	6.2598E−07	2.0640E−07	4.2599E−14	8.8048E−09
4	DE	1.8210E+03	2.4537E+03	2.1133E+03	1.2490E+02	1.5601E+04	1.8210E+03
	SQP	1.8190E−12	1.8190E−12	1.8190E−12	0.0000E+00	0.0000E+00	1.8190E−12
	DE-SQP	4.0578E−08	2.8971E−06	2.1071E−06	8.2651E−07	6.8311E−13	4.0578E−08

Table 8 Comparative study in DE, SQP and DE-SQP in terms of computational complexity

Case study	Solver	Min	Max	Mean	STD	Var	Mode
1	DE	25.7830	57.6757	33.5441	4.4063	19.4158	25.7830
	SQP	0.0337	2.1092	0.0672	0.2065	0.0426	0.0337
	DE-SQP	25.7974	57.8232	33.5714	4.4207	19.416	25.7974
2	DE	5.0349	9.1769	6.8964	0.8990	0.8082	5.0349
	SQP	0.0409	0.1612	0.0514	0.0137	0.0002	0.0409
	DE-SQP	5.0569	9.2737	6.9402	0.912	0.8084	5.0772
3	DE	32.4815	51.7848	40.5578	3.6145	13.0643	32.4815
	SQP	0.0610	0.2841	0.0938	0.0343	0.0012	0.0610
	DE-SQP	32.54	52.0225	40.6578	3.6476	13.0654	32.54
4	DE	472.1110	604.7093	507.3392	39.0906	1528.0744	472.1110
	SQP	0.4338	0.7292	0.4999	0.0569	0.0032	0.4338
	DE-SQP	472.6688	606.6227	508.2392	39.3652	1528.15	472.6688

Table 9 Comparative study in DE, SQP and DE-SQP in terms of cost in dollars

Case study	Solver	Min	Max	Mean	STD	Var	Mode
1	DE	8.4411E+03	8.9003E+03	8.6525E+03	1.2192E+02	1.4865E+04	8.4411E+03
	SQP	8.5966E+03	8.5966E+03	8.5966E+03	5.4845E−12	3.0079E−23	8.5966E+03
	DE-SQP	8.4210E+03	8.7175E+03	8.5354E+03	8.5599E+01	7.3272E+03	8.4210E+03
2	DE	7.8557E+04	1.3717E+05	1.0585E+05	1.2439E+04	1.5472E+08	7.8557E+04
	SQP	1.7587E+03	4.3946E+03	1.9878E+03	3.0023E+02	9.0140E+04	1.7587E+03
	DE-SQP	1.2254E+03	2.3933E+03	1.5779E+03	1.9676E+02	3.8714E+04	1.2254E+03
3	DE	1.0103E+07	1.5460E+07	1.2160E+07	1.1092E+06	1.2303E+12	1.0103E+07
	SQP	3.7678E+04	1.0748E+05	4.7934E+04	1.0724E+04	1.1500E+08	3.7678E+04
	DE-SQP	3.4304E+04	8.1544E+04	4.5256E+04	8.7267E+03	7.6155E+07	3.4304E+04
4	DE	9.0242E+08	1.3276E+09	1.0549E+09	8.9049E+07	7.9297E+15	9.0242E+08
	SQP	1.6980E+06	2.7777E+06	3.2418E+06	9.6010E+05	4.3193E+10	1.6980E+06
	DE-SQP	2.0505E+06	6.7488E+06	1.9396E+06	2.0783E+05	9.2179E+11	2.0505E+06

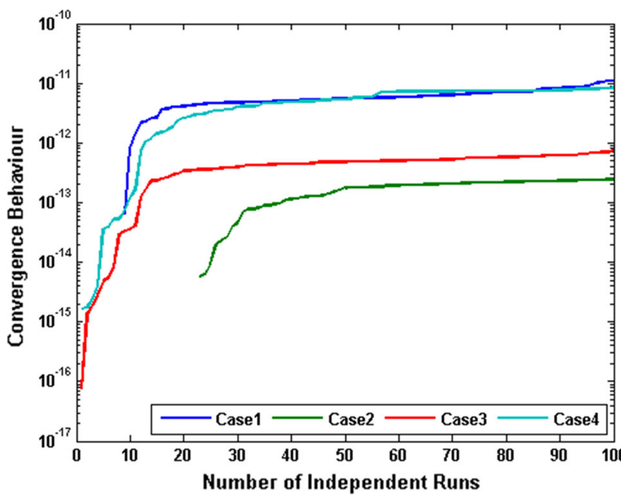

Fig. 6 Convergence behavior of the fitness for case study I–IV using hybrid approach

5 Conclusions

Based on the simulation, the following findings can be extracted:

- The stochastic optimizer based on chaotic differential evolution, sequential quadratic programming a viable local search and their hybrid scheme provides an alternate platform to optimize hydrothermal coordination problem.
- The fitness value obtained by the DE-SQP outperforms than that of DE and SQP schemes independently; similarly, the cost per MW for DE-SQP is lower as compared with other optimizers.
- The fitness value achieved lies in the range from 10^{-11} to 10^{-13}, 10^{-12} to 10^{-15}, 10^{-12} to 10^{-16} and 10^{-20} to 10^{-25} for case study 1–4, respectively.
- The convergence of the proposed scheme is validated through Monte Carlo simulations; it has been observed from the graphs that the convergence percentage for DE-DQP, SQP and DE is 100, 90 and 95, respectively.
- The computational complexity of the DE-SQP is slightly higher than DE and SQP, while this effect can be overshadow at the cost of supremacy in mean absolute error of the DE-SQP.
- Another advantage of the proposed scheme is simplicity in concept, ease in implementation, low computational complexity, good convergence and an acceptable range of accuracy in the solution.

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Compliance with ethical standards

Conflict of interest None.

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