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ORIGINAL ARTICLE

# A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information

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**Abstract** Single-valued trapezoidal neutrosophic numbers (SVTNNs) have a strong capacity to depict uncertain, inconsistent, and incomplete information about decision-making problems. Preference relations represent a practical tool for presenting decision makers' preference information regarding various alternatives. The purpose of this paper is to propose single-valued trapezoidal neutrosophic preference relations (SVTNPRs) as a strategy for tackling multi-criteria decision-making problems. First, this paper briefly reviews basic concepts about neutrosophic sets and SVTNNs and defines a new comparison method and new operations for SVTNNs. Next, two aggregation operators, the single-valued trapezoidal neutrosophic weighted arithmetic average operator and the single-valued trapezoidal neutrosophic weighted geometric average operator, are proposed for applications in information fusion. Then, this paper discusses the definitions of completely consistent SVTNPRs and acceptably consistent SVTNPRs. Finally, we outline a decision-making method based on SVTNPRs to address green supplier selection problems, and we conduct a comparison study and discussion to illustrate the rationality and effectiveness of the decision-making method.

**Keywords** Multi-criteria decision-making · Single-valued trapezoidal neutrosophic preference relations · Aggregation operators · Completely consistent · Acceptably consistent

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## 1 Introduction

Recently, growing concerns about environmental issues have attracted worldwide attention to innovative business practices that alleviate or prevent negative environmental effects [1]. One potentially effective way of managing a company's environmental policy is by linking it closely with its purchasing function activities, i.e., through supplier selection [2]. Taking the suppliers' environmental performance into consideration, organizations and governments have attached great value to green supply chain management (GSCM). The processes of green supplier evaluation and selection are critical issues in GSCM [3], because incorporating environmental criteria into these processes can contribute to achieving GSCM goals [4]. Thus, it is critical and necessary to study green supplier evaluation and selection problems.

Green supplier selection problems involve strategic and complex decision making that demands consideration of different criteria, such as green products, green knowledge transfer, and environmental management systems [3]. Selecting the appropriate green supplier can have a direct impact on the reduction in enterprise costs, increase in enterprise flexibility, and the promotion of core competitiveness. Such complex problems can be solved using multi-criteria decision-making (MCDM) techniques, which can not only facilitate reaching clear decisions but also deal with various, often conflicting criteria [5]. MCDM techniques are very useful tools for addressing many real-life green supplier selection problems.

To the best of our knowledge, existing research into green supplier evaluation and selection can roughly be classified into the following MCDM approaches: the analytic hierarchy process (AHP) [6, 7], the analytic network process (ANP) [8–10], mathematical programming

[11, 12], and some other approaches [13–16]. Because decision makers (DMs) often have limited time and knowledge, the vagueness of their opinions must also be taken into consideration; for this reason, fuzzy logic and fuzzy sets (FSs), initially proposed by Zadeh [17], have been integrated in the methods listed above. For example, Chan et al. [18] incorporated fuzzy logic into the fuzzy-AHP model to measure the environmental and organizational performance of different designs for eco-friendly products. Kumar et al. [19] used an integrated fuzzy-AHP and fuzzy multi-objective linear programming approach for order allocation among green suppliers. AHP and fuzzy-AHP are recognized as good tools for addressing MCDM problems, as they can provide techniques for flexibly deciding among various options [20]. However, real-world complexity has generated the need to extend other fuzzy concepts to help organizations make more thoughtful and precise decisions. An extension of FSs, intuitionistic fuzzy sets (IFSs) [21] have demonstrated a strong ability to represent vagueness and uncertainty, such that they can describe MCDM problems in a more accurate way. Büyüközkan and Güleryüz [20] integrated the intuitionistic fuzzy analytic hierarchy process (IF-AHP) and intuitionistic fuzzy technique for order preference by similarity to ideal solution (IF-TOPSIS) methods to effectively evaluate product development partners. Furthermore, interval type-2 fuzzy sets (IT2FSs) [22], which are another extension of FSs, are remarkably flexible in modeling the uncertainty of MCDM problems. Within environments characterized by IT2FSs, researchers made some significant discoveries; for example, Yu et al. [23] proposed a new multi-attributive border approximation area comparison method to solve hotel selection problems in a tourism websites. Ghorabaee et al. [24] extended the Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for handling fuzzy multi-criteria group decision-making (MCGDM) problems using IT2FSs. In another study, Ghorabaee et al. [25] presented a new method for ranking interval type-2 fuzzy numbers (IT2FNs) and extended the complex proportional assessment method for supplier selection problems. Later, in order to consider environmental criteria, Ghorabaee et al. [4] proposed an integrated approach based on the weighted aggregated sum product assessment method to deal with green supplier selection problem using IT2FSs.

Although prior studies have contributed to advancing the study of green supplier evaluation and selection problems, these problems can feature incomplete and inconsistent information that remains beyond the scope of FSs, IFSs or other extensions [26–28]. In order to cope with indeterminate and inconsistent information to the greatest extent possible, Smarandache [29, 30] proposed neutrosophic sets (NSs) from a philosophical point of view [31]. Recently, Smarandache [32] extended NSs to propose refined NSs, introducing for the first time the degree of dependence or independence of (sub)components of NSs. However, NSs cannot be applied in real scientific and engineering areas since their description is not specified. To meet this critical challenge, multiple researchers have studied extensions of NSs, including simplified neutrosophic sets [33, 34], multi-valued neutrosophic sets (MVNSs) [35–37], and independent inputs simplified neutrosophic linguistic sets (SNLSs) [38]. Up to this point, these extensions have been applied to MCDM problems to remarkable effect [1]. However, the domains of these extensions are still discrete sets; for example, in independent inputs SNLSs, the three membership degrees are relative to a discrete fuzzy concept “excellent” or “good.” Naturally, this may lead to the loss of information, such that it is worthwhile to extend the discrete set to a continuous one.

To tackle issues analyzed above, Ye [39] proposed single-valued trapezoidal neutrosophic sets (SVTNSs) as another extension of NSs, while Deli and Şubaş [40] defined single-valued triangular neutrosophic numbers (SVTrN-numbers), which can be regarded as special cases of single-valued trapezoidal neutrosophic numbers (SVTNNs). As these studies demonstrate, information from DMs can be expressed in different dimensions and continuous sets. Other researchers have also performed significant work in this field. For example, both Deli and Şubaş [41] and Biswas et al. [42] proposed a new ranking method by defining the concept of cut sets for SVTNNs, which they applied to tackle MCDM problems. Thamaraiselvi and Santhi [43] introduced the mathematical representation of a transportation problem in the SVTNN environment. Based on the extent studies, Liang et al. [44] improved the existing operations and operations of SVTNNs and proposed a new MCGDM approach based on interdependent inputs of SVTNN information. In other fields, however, very little researches exist based on SVTNNs. Considering the positive characteristics of SVTNNs in representing incomplete and inconsistent information, this paper explores further applications of SVTNNs in green supplier selection problems.

In real-life MCDM problems, it seems more flexible for DMs to offer comparisons among alternatives rather than providing evaluation values for all alternatives with respect to each criterion. Moreover, it has been pointed out that building a preference relation by pairwise comparisons between alternatives is more accurate than non-pairwise comparisons [45]. Preference relations fall into two main categories: multiplicative preference relations [46, 47] and fuzzy preference relations (FPRs) [48–50]. Saaty [51] proposed the traditional AHP method based on multiplicative preference relations, using the 1 through $-9$

linguistic scale to represent judgments made by pairwise comparisons between alternatives, in which all judgments are crisp values. However, deterministic values cannot reflect the vagueness of real-life decision-making information. As a result, Orlovsky [50] introduced FPRs, in which each element has a membership degree assigned from 0 to 1. FSs cannot take the non-membership into consideration, while IFSs can cover this deficiency; therefore, Xu [52] constructed intuitionistic fuzzy preference relations (IFPRs). Using this type of representation technique, DMs can express their imprecise cognitions from positive, negative, and hesitant points of view when comparing alternatives $A_i$ and $A_j$ [53]. Although IFPRs can express DMs' preferences in a more comprehensive and flexible way than FPRs [53], some weaknesses still exist in IFPRs that go beyond the capacity of intuitionistic fuzzy numbers (IFNs) to handle inconsistent information, which might be common in complex decision-making situations.

To overcome the defects described above, this paper proposes single-valued trapezoidal neutrosophic preference relations (SVTNPRs). In real-life decision-making situations, SVTNPRs are more widespread and can cover more DM preference information than FPRs and IFPRs. For example, suppose that teams are assembled from several constituencies to serve as DMs (reviewers) in order to solve a green supplier selection problem; then, data are collected over two sessions. First, experts are asked to offer their preference degree between each pair of alternatives $A_i$ and $A_j$ $(i, j = 1, 2, \ldots, m)$ with respect to each criterion. Due to the limited knowledge and indeterminacy inherent in the DMs' cognitions, they are more likely to use linguistic information rather numeric values to denote their preference values; for example, when asked about the performance of one car, the reviewers might tend to describe the performance as "good" or "poor." However, based on the preceding discussion about independent inputs SLNSs, it is worthwhile to translate the first part of a linguistic term into a trapezoidal fuzzy number (TFN) using the techniques by Wang and Hao [54], such that the preference degree can be obtained in a continuous way. In the second session, reviewers are asked to evaluate the obtained preference degree TFN by voting in favor, voting against, or abstaining on each evaluation index. In this way, the final preference relations can be obtained with respect to each criterion as assigned by SVTNNs. When represented in this way, the preference information is considerably more comprehensive and accurate. Additionally, the experts complete the survey anonymously, not communicating with each other so as not to influence each other. Using this strategy, the evaluation information expressed by SVTNNs is composed of independent components on $T$, $I$, and $F$. This paper only considers SVTNNs with independent inputs.

The rest of this paper is organized as follows. Section 2 reviews some preliminaries regarding SVTNNs and their operations. Section 2 also revisits existing comparison methods and analyzes their deficiencies. In order to overcome shortcomings in operations and comparison methods, Sect. 3 proposes new operations, an improved comparison method, and two aggregation operators to fuse decision information. Section 4 presents SVTNPR, exploring their complete consistency and acceptable consistency conditions. Based on these foundations, Sect. 5 presents a MCDM method using SVTNPRs. To verify the feasibility of the method, Sect. 6 provides an example of a green supplier selection problem and conducts a comparison analysis. Finally, Sect. 7 presents the main conclusions.

## 2 Preliminaries

This section introduces some basic concepts, operations, and comparison methods related to SVTNNs; in addition, this section briefly reviews the concepts of FPRs and IFPRs, which are utilized in the subsequent analyses.

### 2.1 NSs and SVTNNs

**Definition 1** [55] Suppose that $K = [a_1, a_2, a_3, a_4]$ is a TFN on the real number set $R$, and $a_1 \le a_2 \le a_3 \le a_4$. Then, its membership function $\mu_K : R \to [0, 1]$ is defined as follows:

$$\mu_K(x) = \begin{cases} (x - a_1)\mu_K/(a_2 - a_1), & a_1 \le x < a_2; \\ \mu_K, & a_2 \le x \le a_3; \\ (a_4 - x)\mu_K/(a_4 - a_3), & a_3 < x \le a_4; \\ 0, & \text{otherwise}. \end{cases}$$

When $a_2 = a_3$, the TFN $K = [a_1, a_2, a_3, a_4]$ is reduced to a triangular fuzzy number.

**Definition 2** [56] Let $X$ be a space of points or objects, with a generic element in $X$ denoted by $x$. A single-valued neutrosophic set (SVNS) $V$ in $X$ is characterized by three independent parts, namely truth-membership function $T_V$, indeterminacy-membership function $I_V$, and falsity-membership function $F_V$, such that $T_V : X \to [0, 1]$, $I_V : X \to [0, 1]$, and $F_V : X \to [0, 1]$.

For notational convenience, $V$ is often denoted as $V = \{ < x, (T_V(x), I_V(x), F_V(x)) > | x \in X\}$, satisfying $0 \le T_V(x) + I_V(x) + F_V(x) \le 3$.

A SVNN, which is an element in a SVNS, is denoted by crisp numbers; it is related to a discrete set and cannot represent very much fuzzy information. In order to overcome this challenge, Ye [39] extended the discrete set to a continuous one by combining the concept of TFNs with SVNSs and defined the SVTNNs.

**Definition 3** [39] Let $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \in [0,1]$; then, a SVTNN $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ is a special NS on the real number set $R$, whose truth-membership function $\mu_{\tilde{a}}$, indeterminacy-membership function $v_{\tilde{a}}$, and falsity-membership function $\lambda_{\tilde{a}}$ are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)T_{\tilde{a}}/(a_2-a_1) & a_1 \le x \le a_2, \\ T_{\tilde{a}} & a_2 \le x \le a_3, \\ (a_4-x)T_{\tilde{a}}/(a_4-a_3) & a_3 \le x \le a_4, \\ 0 & \text{otherwise.} \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} (a_2-x+I_{\tilde{a}}(x-a_1))/(a_2-a_1) & a_1 \le x \le a_2, \\ I_{\tilde{a}} & a_2 \le x \le a_3, \\ (x-a_3+I_{\tilde{a}}(a_4-x))/(a_4-a_3) & a_3 \le x \le a_4, \\ 1 & \text{otherwise.} \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (a_2-x+F_{\tilde{a}}(x-a_1))/(a_2-a_1) & a_1 \le x \le a_2, \\ F_{\tilde{a}} & a_2 \le x \le a_3, \\ (x-a_3+F_{\tilde{a}}(a_4-x))/(a_4-a_3) & a_3 \le x \le a_4, \\ 1 & \text{otherwise.} \end{cases}$$

When $a_1 > 0$, $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ is called a positive SVTNN, denoted by $\tilde{a} > 0$. Similarly, when $a_4 \le 0$, $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ becomes a negative SVTNN, denoted by $\tilde{a} < 0$. When $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \in [0,1]$, $\tilde{a}$ is called a normalized SVTNN.

When $I_{\tilde{a}} = 1 - T_{\tilde{a}} - F_{\tilde{a}}$, the SVTNN is reduced to a trapezoidal intuitionistic fuzzy number (TIFN). When $a_2 = a_3$, $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ turns out to be a single-valued triangular neutrosophic number (SVTrNN). When $I_{\tilde{a}} = 0$, $F_{\tilde{a}} = 0$, a SVTNN is reduced to a generalized TFN, $\tilde{a} = \langle [a_1, a_2, a_3, a_4], T_{\tilde{a}} \rangle$.

## 2.2 Operations and comparison methods for SVTNNs

**Definition 4** [39] Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ and $\tilde{b} = \langle [b_1, b_2, b_3, b_4], (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) \rangle$ be two arbitrary SVTNNs, and $\zeta \ge 0$; then, their operations are defined as follows:

1. $\tilde{a} \oplus \tilde{b} = \langle [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4], (T_{\tilde{a}} + T_{\tilde{b}} - T_{\tilde{a}}T_{\tilde{b}}, I_{\tilde{a}}I_{\tilde{b}}, F_{\tilde{a}}F_{\tilde{b}}) \rangle$;
2. $\tilde{a} \otimes \tilde{b} = \langle [a_1b_1, a_2b_2, a_3b_3, a_4b_4], (T_{\tilde{a}}T_{\tilde{b}}, I_{\tilde{a}} + I_{\tilde{b}} - I_{\tilde{a}}I_{\tilde{b}}, F_{\tilde{a}} + F_{\tilde{b}} - F_{\tilde{a}}F_{\tilde{b}}) \rangle$;
3. $\zeta\tilde{a} = \langle [\zeta a_1, \zeta a_2, \zeta a_3, \zeta a_4], \left(1 - (1 - T_{\tilde{a}})^{\zeta}, (I_{\tilde{a}})^{\zeta}, (F_{\tilde{a}})^{\zeta}\right) \rangle$; and
4. $\tilde{a}^{\zeta} = \left\langle \left[a_1^{\zeta}, a_2^{\zeta}, a_3^{\zeta}, a_4^{\zeta}\right], \left((T_{\tilde{a}})^{\zeta}, 1 - (1 - I_{\tilde{a}})^{\zeta}, 1 - (1 - F_{\tilde{a}})^{\zeta}\right) \right\rangle$.

However, some drawbacks exist in operations (1) and (3) in Definition 4, and they will be discussed in Example 1 and Example 2, respectively.

*Example 1* Let $\tilde{a} = \langle [0.1, 0.1, 0.2, 0.3], (0, 0, 1) \rangle$ and $\tilde{b} = \langle [0.1, 0.1, 0.2, 0.3], (1, 0, 0) \rangle$ be two SVTNNs. According to Definition 4, the following result can be calculated: $\tilde{a} + \tilde{b} = \langle [0.2, 0.2, 0.4, 0.6], (1, 0, 0) \rangle$. However, this result is inaccurate because it does not consider the falsity-membership of $\tilde{a}$, the correlations among TFNs and the membership degrees of $\tilde{a}$ and $\tilde{b}$. Therefore, these operations are imprecise.

*Example 2* Let $\tilde{a}_1 = \langle [0.03, 0.05, 0.07, 0.09], (0.3, 0.5, 0.5) \rangle$ be a SVTNN and $\zeta = 10$. Then, the result $\zeta\tilde{a}_1$ obtained using Definition 4 is

$$10\tilde{a}_1 = \langle [0.3, 0.5, 0.7, 0.9], (0.9718, 0.001, 0.001) \rangle.$$

According to Example 2, the former TFN and the latter SVNN in $\tilde{a}_1$ are calculated simultaneously; therefore, repeated calculations occur between the three membership degrees and the TFN of the SVTNN, which significantly distort the result and conflict with common sense.

Therefore, some new operations for SVTNNs must be defined in order to overcome these anomalies. The new operations are discussed in Sect. 3.

To compare any two SVTNNs, Ye [39] and Deli and Şubaş [57] proposed comparison methods based on the score function and accuracy function.

**Definition 5** [39] Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ be a SVTNN. The score function of $\tilde{a}$ is defined as

$$S(\tilde{a}) = \frac{1}{12}[a_1 + a_2 + a_3 + a_4] \times (2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}). \quad (1)$$

**Definition 6** [39] Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ and $\tilde{b} = \langle [b_1, b_2, b_3, b_4], (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) \rangle$ be two SVTNNs. Then,

1. when $S(\tilde{a}) > S(\tilde{b})$, $\tilde{a} > \tilde{b}$; and
2. when $S(\tilde{a}) = S(\tilde{b})$, $\tilde{a} = \tilde{b}$.

However, some flaws exist in Definition 5, which are discussed in Example 3.

*Example 3* Let $\tilde{a} = \langle [0.3, 0.4, 0.5, 0.8], (0.5, 0.3, 0.7) \rangle$ and $\tilde{b} = \langle [0.5, 0.7, 0.8, 1], (0.2, 0.8, 0.4) \rangle$ be two SVTNNs; then, it is clear that $\tilde{a} \ne \tilde{b}$. However, according to Definitions 5 and 6, $S(\tilde{a}) = S(\tilde{b}) = 0.25$, and $\tilde{a} = \tilde{b}$, which does not conform to our intuition.

In order to overcome the deficiency in this comparison method, Deli and Şubaş [57] defined a new comparison method for SVTNNs.

**Definition 7** [57] Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ be a SVTNN. The score function and accuracy function of $\tilde{a}$ are defined, respectively, as follows:

$$S'(\tilde{a}) = \frac{1}{16}[a_1 + a_2 + a_3 + a_4] \times (2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}), \quad (2)$$

$$H'(\tilde{a}) = \frac{1}{16}[a_1 + a_2 + a_3 + a_4] \times (2 + T_{\tilde{a}} - I_{\tilde{a}} + F_{\tilde{a}}). \quad (3)$$

**Definition 8** [57] Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ and $\tilde{b} = \langle [b_1, b_2, b_3, b_4], (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) \rangle$ be two arbitrary SVTNNs.

1. When $S'(\tilde{a}) < S'(\tilde{b})$, $\tilde{a} < \tilde{b}$;
2. when $S'(\tilde{a}) = S'(\tilde{b})$ and $H'(\tilde{a}) < H'(\tilde{b})$, $\tilde{a} < \tilde{b}$; and
3. when $S'(\tilde{a}) = S'(\tilde{b})$ and $H'(\tilde{a}) = H'(\tilde{b})$, $\tilde{a} = \tilde{b}$.

Although the comparison method proposed in Deli and Şubaş [57] addressed the problem in Ye [39], some drawbacks still exist in the operations, as described in Example 4.

*Example 4* Let $\tilde{a} = \langle [0.2, 0.3, 0.5, 0.8], (0.1, 0.8, 0) \rangle$ and $\tilde{b} = \langle [0.1, 0.4, 0.5, 0.8], (0.2, 0.9, 0) \rangle$ be two SVTNNs; then, it is clear that $\tilde{a} \neq \tilde{b}$. According to Definition 7, we can determine that $S'(\tilde{a}) = S'(\tilde{b}) = 0.146$, $H'(\tilde{a}) = H'(\tilde{b}) = 0.146$, and according to Definition 8, $\tilde{a} = \tilde{b}$, which is counterintuitive.

As a result, it is worthwhile to define a new comparison method for SVTNNs that overcomes the shortcomings of the extant research.

### 2.3 Fuzzy preference relation and intuitionistic fuzzy preference relation

**Definition 9** [53] A FPR $B$ on the set $A = \{A_1, A_2, \ldots, A_m\}$ is represented by a matrix $B = (b_{ik})_{m \times m}$, where $b_{ik}$ is the intensity of preference of $A_i$ over $A_k$, and satisfies

$$b_{ik} + b_{ki} = 1, \quad b_{ij} \in [0, 1], \quad \forall A_i, A_k \in A. \quad (4)$$

In the FPRs, the preference degree is represented by a crisp number. However, this seems to be counterintuitive for several reasons: (1) The experiences and knowledge of experts are limited and they may be not familiar with the content of the decision-making problems; (2) even if the experts are familiar with the decision-making problems, scarcity of information and time pressure may make it difficult for DMs to determine the exact values of the preference values; and (3) the evaluation information for alternatives usually contain some incomplete, inconsistent, or indeterminate types. All of the situations described above can create challenges when experts attempt to construct a FPR when comparing alternatives.

In order to express both the vagueness and uncertainty existing in DMs' pairwise judgments, Xu [52] proposed a standard definition of an IFPR, in which experts can express their opinions of the alternatives from three perspectives: preferred, non-preferred, and indeterminate.

**Definition 10** [52] An IFPR $R$ on the set $X = \{x_1, x_2, \ldots, x_m\}$ is represented by a matrix $R = (r_{ij})_{m \times m}$, where $r_{ij} = \langle (x_i, x_j), (u(x_i, x_j), v(x_i, x_j), \pi(x_i, x_j)) \rangle$ for all $i, j = 1, 2, \ldots, m$. For simplicity, $r_{ij}$ is denoted as $r_{ij} = (u_{ij}, v_{ij}, \pi_{ij})$, and $u_{ij} + v_{ij} + \pi_{ij} = 1$, $u_{ij}$, $v_{ij} \in [0, 1]$, for all $i, j = 1, 2, \ldots, m$, satisfying

1. $u_{ij} + v_{ij} \leq 1$;
2. $u_{ij} = v_{ji}$ and $v_{ij} = u_{ji}$; and
3. $u_{ii} = v_{ii} = 0.5$;

where $u_{ij}$ indicates the certainty degree to which alternative $X_i$ is preferred to $X_j$, $v_{ij}$ denotes the certainty degree to which the alternative $X_i$ is non-preferred to $X_j$, and $\pi_{ij}$ is interpreted as the indeterminate degree to which the alternative $X_i$ is superior to $X_j$.

However, part of the information about the alternatives in complex decision-making problems, including inconsistent or unknown decision information, still cannot be depicted in depth. For this reason, it is worthwhile to extend IFPRs to SVTNPRs; this extension is discussed in Sect. 4.

## 3 New operations and comparison method for SVTNNs

This section defines new operations and proves their properties. Moreover, a new comparison method is proposed on the basis of score and accuracy functions in order to cover the limitations presented in Sect. 2.2. Finally, two aggregation operators are proposed.

### 3.1 New operations based on area of SVTNNs

**Definition 11** Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ be an arbitrary SVTNN; then, the areas under the three membership functions, denoted, respectively, by $\mathrm{ar}(T_{\tilde{a}})$, $\mathrm{ar}(I_{\tilde{a}})$ and $\mathrm{ar}(F_{\tilde{a}})$, can be defined as follows:

$$\mathrm{ar}(T_{\tilde{a}}) = \frac{a_3 - a_2 + a_4 - a_1}{2} \times T_{\tilde{a}}, \quad (5)$$

$$\mathrm{ar}(I_{\tilde{a}}) = \frac{a_3 - a_2 + a_4 - a_1}{2} \times (1 - I_{\tilde{a}}), \quad (6)$$

$$\mathrm{ar}(F_{\tilde{a}}) = \frac{a_3 - a_2 + a_4 - a_1}{2} \times (1 - F_{\tilde{a}}). \quad (7)$$

**Definition 12** Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ and $\tilde{b} = \langle [b_1, b_2, b_3, b_4], (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) \rangle$ be two arbitrary SVTNNs, and $\zeta \geq 0$; then, new operations for SVTNNs are defined as follows:

1. $\tilde{a} \oplus \tilde{b} = \left\langle [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4], \left( \frac{\varphi(\tilde{a})T_{\tilde{a}} + \varphi(\tilde{b})T_{\tilde{b}}}{\varphi(\tilde{a}) + \varphi(\tilde{b})}, \frac{\varphi(\tilde{a})(1 - I_{\tilde{a}}) + \varphi(\tilde{b})(1 - I_{\tilde{b}})}{\varphi(\tilde{a}) + \varphi(\tilde{b})}, \frac{\varphi(\tilde{a})(1 - F_{\tilde{a}}) + \varphi(\tilde{b})(1 - F_{\tilde{b}})}{\varphi(\tilde{a}) + \varphi(\tilde{b})} \right) \right\rangle$, where $\varphi(\tilde{a}) = \frac{a_3 - a_2 + a_4 - a_1}{2}$ and $\varphi(\tilde{b}) = \frac{b_3 - b_2 + b_4 - b_1}{2}$;

2. $\tilde{a} - \tilde{b} = \left\langle [a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 + b_1], \left( \frac{\varphi(\tilde{a})T_{\tilde{a}} + \varphi(\tilde{b})T_{\tilde{b}}}{\varphi(\tilde{a}) + \varphi(\tilde{b})}, \frac{\varphi(\tilde{a})(1 - I_{\tilde{a}}) + \varphi(\tilde{b})(1 - I_{\tilde{b}})}{\varphi(\tilde{a}) + \varphi(\tilde{b})}, \frac{\varphi(\tilde{a})(1 - F_{\tilde{a}}) + \varphi(\tilde{b})(1 - F_{\tilde{b}})}{\varphi(\tilde{a}) + \varphi(\tilde{b})} \right) \right\rangle$, where $\varphi(\tilde{a}) = \frac{a_3 - a_2 + a_4 - a_1}{2}$ and $\varphi(\tilde{b}) = \frac{b_3 - b_2 + b_4 - b_1}{2}$;

*Note* When $\varphi(\tilde{a}) = 0$ and $\varphi(\tilde{b}) = 0$, then $T_{\tilde{a} \oplus \tilde{b}} = T_{\tilde{a} - \tilde{b}} = \frac{T_{\tilde{a}} + T_{\tilde{b}}}{2}$, $I_{\tilde{a} \oplus \tilde{b}} = I_{\tilde{a} - \tilde{b}} = \frac{I_{\tilde{a}} + I_{\tilde{b}}}{2}$, and $F_{\tilde{a} \oplus \tilde{b}} = F_{\tilde{a} - \tilde{b}} = \frac{F_{\tilde{a}} + F_{\tilde{b}}}{2}$.

3. $\tilde{a} \otimes \tilde{b} = \langle [a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4], (T_{\tilde{a}} T_{\tilde{b}}, I_{\tilde{a}} + I_{\tilde{b}} - I_{\tilde{a}} I_{\tilde{b}}, F_{\tilde{a}} + F_{\tilde{b}} - F_{\tilde{a}} F_{\tilde{b}}) \rangle$;
4. $\zeta \tilde{a} = \langle [\zeta a_1, \zeta a_2, \zeta a_3, \zeta a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$, $\zeta \geq 0$;
5. $\tilde{a}^{\zeta} = \left\langle \left[ a_1^{\zeta}, a_2^{\zeta}, a_3^{\zeta}, a_4^{\zeta} \right], \left( (T_{\tilde{a}})^{\zeta}, 1 - (1 - I_{\tilde{a}})^{\zeta}, 1 - (1 - F_{\tilde{a}})^{\zeta} \right) \right\rangle$, $\zeta \geq 0$;

$$\mathrm{COG}(K) = \begin{cases} a & \text{if } \ a_1 = a_2 = a_3 = a_4 \\ \frac{1}{3}\left[ a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_2 a_1}{a_4 + a_3 - a_2 - a_1} \right] & \text{otherwise} \end{cases}. \quad (8)$$

6. $\mathrm{neg}(\tilde{a}) = \langle \mathrm{neg}(K), (F_{\tilde{a}}, 1 - I_{\tilde{a}}, T_{\tilde{a}}) \rangle$, where $K$ denotes the TFN in $\tilde{a}$.

*Example 5* Using the data in Example 1, let $\zeta = 2$; based on Definition 12, we can identify that

1. $\tilde{a} \oplus \tilde{b} = \langle [0.2, 0.2, 0.35, 0.7], (0.538, 0, 0.538) \rangle$;
2. $\tilde{a} \otimes \tilde{b} = \langle [0.01, 0.01, 0.03, 0.12], (0, 0, 0) \rangle$;
3. $2\tilde{a} = \langle [0.2, 0.2, 0.4, 0.6], (0, 0, 1) \rangle$; and
4. $\tilde{a}^2 = \langle [0.01, 0.01, 0.04, 0.09], (0, 0, 1) \rangle$.

Compared with the operations proposed by Ye [39], our new proposed operations for SVTNNs not only capture the correlations of TFNs and the three membership degrees of SVTNNs, but also effectively avoid the loss and distortion of information. Therefore, the newly defined operations are more reasonable than the existing ones.

In terms of the corresponding operations for SVTNNs, the following theorem can be easily proved.

**Theorem 1** *Let $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ be three SVTNNs and $\zeta \geq 0$; then, the following equations are true*:

1. $\tilde{a} \oplus \tilde{b} = \tilde{b} \oplus \tilde{a}$;
2. $(\tilde{a} \oplus \tilde{b}) \oplus \tilde{c} = \tilde{a} \oplus (\tilde{b} \oplus \tilde{c})$;
3. $\tilde{a} \otimes \tilde{b} = \tilde{b} \otimes \tilde{a}$;
4. $(\tilde{a} \otimes \tilde{b}) \otimes \tilde{c} = \tilde{a} \otimes (\tilde{b} \otimes \tilde{c})$;
5. $\zeta \tilde{a} \oplus \zeta \tilde{b} = \zeta (\tilde{b} \oplus \tilde{a})$; and
6. $(\tilde{a} \otimes \tilde{b})^{\zeta} = \tilde{a}^{\zeta} \otimes \tilde{b}^{\zeta}$.

The proof of Theorem 1 according to Definition 12 is self-explanatory, so it is omitted here.

### 3.2 A new comparison method for SVTNNs

Motivated by the centroid defuzzification method [54, 58] and related research on neutrosophic theory [35, 59], we redefine the score function, accuracy function, and certainty function of SVTNNs, based on which a new comparison method for SVTNNs is presented.

**Definition 13** [54] Let $K = [a_1, a_2, a_3, a_4]$ be a TFN on the real number set $R$, and $a_1 \leq a_2 \leq a_3 \leq a_4$; then, the center of gravity (COG) of $K$ can be defined as follows:

**Definition 14** Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ be a SVTNN; then, the score function, accuracy function, and certainty function of SVTNN $\tilde{a}$ are defined, respectively, as:

$$E(\tilde{a}) = \mathrm{COG}(K) \times \frac{(2 + T(\tilde{a}) - I(\tilde{a}) - F(\tilde{a}))}{3}, \quad (9)$$

$$A(\tilde{a}) = \mathrm{COG}(K) \times (T_{\tilde{a}} - F_{\tilde{a}}), \quad \text{and} \quad (10)$$

$$C(\tilde{a}) = \mathrm{COG}(K) \times T_{\tilde{a}}. \quad (11)$$

Based on the above three functions, we can define a novel comparison method for SVTNNs as follows.

**Definition 15** Let $\tilde{a} = \langle [a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ and $\tilde{b} = \langle [b_1, b_2, b_3, b_4], (T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}) \rangle$ be two SVTNNs. The comparison method for $\tilde{a}$ and $\tilde{b}$ can be defined as follows:

1. When $E(\tilde{a}) > E(\tilde{b})$, $\tilde{a} > \tilde{b}$, meaning that $\tilde{a}$ is superior to $\tilde{b}$;
2. when $E(\tilde{a}) = E(\tilde{b})$, and $A(\tilde{a}) > A(\tilde{b})$, $\tilde{a} > \tilde{b}$, meaning that $\tilde{a}$ is superior to $\tilde{b}$;
3. when $E(\tilde{a}) = E(\tilde{b})$, and $A(\tilde{a}) < A(\tilde{b})$, $\tilde{a} < \tilde{b}$, meaning that $\tilde{a}$ is inferior to $\tilde{b}$; and
4. when $E(\tilde{a}) = E(\tilde{b})$, $A(\tilde{a}) = A(\tilde{b})$, and $C(\tilde{a}) > C(\tilde{b})$, $\tilde{a} > \tilde{b}$, meaning that $\tilde{a}$ is superior to $\tilde{b}$; and $\tilde{a} < \tilde{b}$ when $C(\tilde{a}) < C(\tilde{b})$, meaning that $\tilde{a}$ is inferior to $\tilde{b}$; and $\tilde{a} = \tilde{b}$ when $C(\tilde{a}) = C(\tilde{b})$, meaning that $\tilde{a}$ is indifferent to $\tilde{b}$.

*Example 6* Utilizing the data in Example 1, we can calculate that $E(\tilde{a}) = 0.1986$ and $E(\tilde{b}) = 0.195$. Then, $\tilde{a} > \tilde{b}$; that is, $\tilde{b}$ is superior to $\tilde{a}$, which is consistent with our intuition.

### 3.3 Aggregation operators for SVTNNs

**Definition 16** Let $\tilde{a}_j = \left\langle [a_{j1}, a_{j2}, a_{j3}, a_{j4}], (T_{\tilde{a}_j}, I_{\tilde{a}_j}, F_{\tilde{a}_j}) \right\rangle$ $(j = 1, 2, \ldots, n)$ be a set of SVTNNs, and let $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ be the weight vector of $\tilde{a}_j$ $(j = 1, 2, \ldots, n)$ with $\varpi_j \in [0, 1]$ and $\sum_{j=1}^{n} \varpi_j = 1$; then, the SVTNWAA operator can be defined as follows:

$$\mathrm{SVTNWAA}_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \mathop{\oplus}_{j=1}^{n} \varpi_j \tilde{a}_j. \tag{12}$$

**Theorem 2** *Let* $\tilde{a}_j = \left\langle [a_{j1}, a_{j2}, a_{j3}, a_{j4}], (T_{\tilde{a}_j}, I_{\tilde{a}_j}, F_{\tilde{a}_j}) \right\rangle$ $(j = 1, 2, \ldots, n)$ *be a set of SVTNNs. Then, the aggregated value utilizing the SVTNWAA operator is still a SVTNN, which is shown as follows*:

$$\mathrm{SVTNWAA}_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \mathop{\oplus}_{j=1}^{n} \varpi_j \tilde{a}_j = \left\langle \begin{array}{l} \left[\sum_{j=1}^{n} \varpi_j a_{j1}, \sum_{j=1}^{n} \varpi_j a_{j2}, \sum_{j=1}^{n} \varpi_j a_{j3}, \sum_{j=1}^{n} \varpi_j a_{j4}\right], \\ \left(\frac{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j) T_{\tilde{a}}}{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j)}, \frac{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j)(1 - I_{\tilde{a}})}{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j)}, \frac{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j)(1 - F_{\tilde{a}})}{\sum_{j=1}^{n} \varpi_j \varphi(\tilde{a}_j)}\right) \end{array} \right\rangle, \tag{13}$$

*where* $\varphi(\tilde{a}_j) = \frac{a_{j3} - a_{j2} + a_{j4} - a_{j1}}{2}$.

In the following, we proof Theorem 2 using mathematical induction on $n$.

*Proof* 1. When $n = 2$, the following equation can be obtained:

$$\begin{aligned} \mathrm{SVTNWAA}_w(\alpha_1, \alpha_2) &= \varpi_1 \alpha_1 \oplus \varpi_2 \alpha_2 \\ &= \langle [\varpi_1 a_{11}, \varpi_1 a_{12}, \varpi_1 a_{13}, \varpi_1 a_{14}], (T_{\tilde{a}_1}, I_{\tilde{a}_1}, F_{\tilde{a}_1}) \rangle \oplus \langle [\varpi_2 a_{21}, \varpi_2 a_{22}, \varpi_2 a_{23}, \varpi_2 a_{24}], (T_{\tilde{a}_2}, I_{\tilde{a}_2}, F_{\tilde{a}_2}) \rangle \\ &= \left\langle \begin{array}{l} [\varpi_1 a_{11} \oplus \varpi_2 a_{21}, \varpi_1 a_{12} \oplus \varpi_2 a_{22}, \varpi_1 a_{13} \oplus \varpi_2 a_{23}, \varpi_1 a_{14} \oplus \varpi_2 a_{24}], \\ \left( \begin{array}{l} \frac{\varpi_1 \varphi(\tilde{a}_1) T_{\tilde{a}_1} + \varpi_2 \varphi(\tilde{a}_2) T_{\tilde{a}_2}}{\varpi_1 \varphi(\tilde{a}_1) + \varpi_2 \varphi(\tilde{a}_2)}, \frac{\varpi_1 \varphi(\tilde{a}_1)(1 - I_{\tilde{a}_1}) + \varpi_2 \varphi(\tilde{a}_2)(1 - I_{\tilde{a}_2})}{\varpi_1 \varphi(\tilde{a}_1) + \varpi_2 \varphi(\tilde{a}_2)}, \\ \frac{\varpi_1 \varphi(\tilde{a}_1)(1 - F_{\tilde{a}_1}) + \varpi_2 \varphi(\tilde{a}_2)(1 - F_{\tilde{a}_2})}{\varpi_1 \varphi(\tilde{a}_1) + \varpi_2 \varphi(\tilde{a}_2)} \end{array} \right) \end{array} \right\rangle \\ &= \left\langle \begin{array}{l} \left[\sum_{j=1}^{2} \varpi_j a_{j1}, \sum_{j=1}^{2} \varpi_j a_{j2}, \sum_{j=1}^{2} \varpi_j a_{j3}, \sum_{j=1}^{2} \varpi_j a_{j4}\right], \\ \left(\frac{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j) T_{\tilde{a}_j}}{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j)}, \frac{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j)(1 - I_{\tilde{a}_j})}{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j)}, \frac{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j)(1 - F_{\tilde{a}_j})}{\sum_{j=1}^{2} \varpi_j \varphi(\tilde{a}_j)}\right) \end{array} \right\rangle, \end{aligned}$$

where $\varphi(\tilde{a}_1) = \frac{a_{13} - a_{12} + a_{14} - a_{11}}{2}$ and $\varphi(\tilde{a}_2) = \frac{a_{23} - a_{22} + a_{24} - a_{21}}{2}$. Clearly, when $n = 2$, Theorem 2 is true.

2. Suppose that when $n = k$, Theorem 2 is true. That is,

$$\mathrm{SVTNWAA}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_k)=\mathop{\oplus}_{j=1}^{k}\varpi_j\tilde{a}_j = \left\langle \begin{array}{l}\left[\sum_{j=1}^{k}\varpi_j a_{j1},\sum_{j=1}^{k}\varpi_j a_{j2},\sum_{j=1}^{k}\varpi_j a_{j3},\sum_{j=1}^{k}\varpi_j a_{j4}\right],\\ \left(\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)T_{\tilde{a}_j}}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)\left(1-I_{\tilde{a}_j}\right)}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)\left(1-F_{\tilde{a}_j}\right)}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)}\right)\end{array}\right\rangle. \tag{14}$$

Then, when $n=k+1$, the following equation can be calculated:

$$\begin{aligned}&\mathrm{SVTNWAA}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_{k+1})=\mathop{\oplus}_{j=1}^{k}\varpi_j\tilde{a}_j\oplus\varpi_{k+1}\tilde{a}_{k+1}\\ &=\left\langle \begin{array}{l}\left[\sum_{j=1}^{k}\varpi_j a_{j1},\sum_{j=1}^{k}\varpi_j a_{j2},\sum_{j=1}^{k}\varpi_j a_{j3},\sum_{j=1}^{k}\varpi_j a_{j4}\right],\\ \left(\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)T_{\tilde{a}_j}}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)\left(1-I_{\tilde{a}_j}\right)}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)\left(1-F_{\tilde{a}_j}\right)}{\sum_{j=1}^{k}\varpi_j\varphi(\tilde{a}_j)}\right)\end{array}\right\rangle\\ &\oplus\left\langle\left[\varpi_{k+1}a_{k+1,1},\varpi_{k+1}a_{k+1,2},\varpi_{k+1}a_{k+1,3},\varpi_{k+1}a_{k+1,4}\right],\left(T_{\tilde{a}_{k+1}},I_{\tilde{a}_{k+1}},F_{\tilde{a}_{k+1}}\right)\right\rangle\\ &=\left\langle \begin{array}{l}\left[\sum_{j=1}^{k+1}\varpi_j a_{j1},\sum_{j=1}^{k+1}\varpi_j a_{j2},\sum_{j=1}^{k+1}\varpi_j a_{j3},\sum_{j=1}^{k+1}\varpi_j a_{j4}\right],\\ \left(\frac{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)T_{\tilde{a}_j}}{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)\left(1-I_{\tilde{a}_j}\right)}{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)},\frac{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)\left(1-F_{\tilde{a}_j}\right)}{\sum_{j=1}^{k+1}\varpi_j\varphi(\tilde{a}_j)}\right)\end{array}\right\rangle.\end{aligned}$$

That is, Theorem 2 is true for $n=k+1$.

Therefore, Theorem 2 holds for all $n$.

**Definition 17** Let $\tilde{a}_j=\left\langle\left[a_{j1},a_{j2},a_{j3},a_{j4}\right],\left(T_{\tilde{a}_j},I_{\tilde{a}_j},F_{\tilde{a}_j}\right)\right\rangle$ $(j=1,2,\ldots,n)$ be a set of SVTNNs, and $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ be the weight vector of $\tilde{a}_j$ $(j=1,2,\ldots,n)$ with $\varpi_j\in[0,1]$ and $\sum_{j=1}^{n}\varpi_j=1$; then, the SVTNWGA operator is defined as follows:

$$\mathrm{SVTNWGA}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\prod_{j=1}^{n}\tilde{a}_j^{\varpi_j}. \tag{15}$$

**Theorem 3** *Let* $\tilde{a}_j=\left\langle\left[a_{j1},a_{j2},a_{j3},a_{j4}\right],\left(T_{\tilde{a}_j},I_{\tilde{a}_j},F_{\tilde{a}_j}\right)\right\rangle$ $(j=1,2,\ldots,n)$ *be a set of SVTNNs. Then, the aggregated value utilizing the SVTNWGA operator is still a SVTNN. The aggregated result satisfies*

$$\mathrm{SVTNWGA}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\prod_{j=1}^{n}\tilde{a}_j^{\varpi_j} = \left\langle\begin{array}{l}\left[\prod_{j=1}^{n}a_{j1}^{\varpi_j},\prod_{j=1}^{n}a_{j2}^{\varpi_j},\prod_{j=1}^{n}a_{j3}^{\varpi_j},\prod_{j=1}^{n}a_{j4}^{\varpi_j}\right],\\ \left(\prod_{j=1}^{n}\left(T_{\tilde{a}_j}\right)^{\varpi_j},1-\prod_{j=1}^{n}\left(1-I_{\tilde{a}_j}\right)^{\varpi_j},1-\prod_{j=1}^{n}\left(1-F_{\tilde{a}_j}\right)^{\varpi_j}\right)\end{array}\right\rangle. \tag{16}$$

The proof is similar to Theorem 2, so it is omitted here.

## 4 Single-valued trapezoidal neutrosophic preference relations

This section extends the traditional FPRs and IFPRs to SVTNPRs and explores some of their prominent characteristics.

**Definition 18** Let a matrix $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$ on the set $A = \{A_1, A_2, \ldots, A_m\}$ be composed of SVTNNs, where $\tilde{a}_{ij} = \left\langle \left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right], \left(T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}}\right) \right\rangle$ for all $i,j = 1,2,\ldots,m$, and $0 \le a_{ij}^1 \le a_{ij}^2 \le a_{ij}^3 \le a_{ij}^4 \le 1$, $0 \le T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}} \le 1$. When comparing alternative $A_i$ over $A_j$, $T_{\tilde{a}_{ij}}$ indicates the certainty degree to which alternative $A_i$ is preferred to $A_j$ with a degree of $\left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right]$ which is represented as a TFN; $I_{\tilde{a}_{ij}}$ is the indeterminate degree to which alternative $A_i$ is preferred to $A_j$ with a degree of $\left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right]$; and $F_{\tilde{a}_{ij}}$ denotes the certainty degree to which alternative $A_i$ is non-preferred to $A_j$ with a degree of $\left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right]$.

The matrix $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$ is denoted as a SVTNPR on $A$, if the following rules can be satisfied:

1. $a_{ij}^1 + a_{ji}^4 = 1$, $a_{ij}^2 + a_{ji}^3 = 1$, $a_{ij}^3 + a_{ji}^2 = 1$, $a_{ij}^4 + a_{ji}^1 = 1$;
2. $T_{\tilde{a}_{ij}} = F_{\tilde{a}_{ji}}$, $I_{\tilde{a}_{ij}} = I_{\tilde{a}_{ji}}$, $F_{\tilde{a}_{ij}} = T_{\tilde{a}_{ji}}$;
3. $\tilde{a}_{ii} = \langle [1,1,1,1], (1,0,0) \rangle$; and
4. $0 \le T_{\tilde{a}_{ij}} + I_{\tilde{a}_{ij}} + F_{\tilde{a}_{ij}} \le 3$.

**Theorem 4** *A SVTNPR $\widetilde{R}$ on the set of $A = \{A_1, A_2, \ldots, A_m\}$ is represented by a matrix $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$, where $\tilde{a}_{ij} = \left\langle \left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right], \left(T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}}\right) \right\rangle$ for all $i,j = 1,2,\ldots,m$, when the elements in the ith row and ith column are removed from $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$, the preference relation composed by the remainder elements of $\widetilde{R}$ is still a SVTNPR.*

*Proof* The proof of Theorem 4 can be completed easily according to Definition 18, so it is omitted here.

**Definition 19** A SVTNPR $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$ with $\tilde{a}_{ij} = \left\langle \left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right], \left(T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}}\right) \right\rangle (i,j,k = 1,2,\ldots,m)$ is called a completely consistent SVTNPR when the following statements are equivalent for any $i,j,k$:

1. $a_{ij}^1 + a_{ij}^4 + a_{jk}^1 + a_{jk}^4 + a_{ki}^1 + a_{ki}^4 = a_{ij}^2 + a_{ij}^3 + a_{jk}^2 + a_{jk}^3 + a_{ki}^2 + a_{ki}^3 = 3$;
2. $T_{\tilde{a}_{ij}} T_{\tilde{a}_{jk}} T_{\tilde{a}_{ki}} = F_{\tilde{a}_{ji}} F_{\tilde{a}_{ik}} F_{\tilde{a}_{kj}}$;
3. $I_{\tilde{a}_{ij}} I_{\tilde{a}_{jk}} I_{\tilde{a}_{ki}} = I_{\tilde{a}_{ji}} I_{\tilde{a}_{ik}} I_{\tilde{a}_{kj}}$; and
4. $F_{\tilde{a}_{ij}} F_{\tilde{a}_{jk}} F_{\tilde{a}_{ik}} = T_{\tilde{a}_{ji}} T_{\tilde{a}_{ki}} T_{\tilde{a}_{kj}}$.

Let the symbols $\succ$ and $\sim$ be two binary relations on SVTNRs, interpreted as preferred and indifferent relations, respectively.

**Definition 20** ASVTNPR $\widetilde{R} = [\tilde{a}_{ij}]_{m\times m}$ with $\tilde{a}_{ij} = \left\langle \left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right], \left(T_{\tilde{a}_{ij}}, I_{\tilde{a}_{ij}}, F_{\tilde{a}_{ij}}\right) \right\rangle (i,j,k = 1,2,\ldots,m)$ is called an acceptably consistent SVTNPR when the following statements are equivalent for any $i,j,k$:

1. when $\tilde{a}_{ij} \succ \tilde{a}_{ii}$ and $\tilde{a}_{jk} \succ \tilde{a}_{ii}$, then $\tilde{a}_{ik} \succ \tilde{a}_{ii}$;
2. when $\tilde{a}_{ij} \sim \tilde{a}_{ii}$ and $\tilde{a}_{jk} \sim \tilde{a}_{ii}$, then $\tilde{a}_{ik} \sim \tilde{a}_{ii}$; and
3. when $\tilde{a}_{ij} \prec \tilde{a}_{ii}$ and $\tilde{a}_{jk} \prec \tilde{a}_{ii}$, then $\tilde{a}_{ik} \prec \tilde{a}_{ii}$.

It can be interpreted as follows: when $\tilde{a}_{ij}$ is preferred (inferior) to $\tilde{a}_{ii}$, and $\tilde{a}_{jk}$ is preferred (inferior) to $\tilde{a}_{ii}$, $\tilde{a}_{ij}$ should be preferred (inferior) to $\tilde{a}_{jk}$; similarly, when $\tilde{a}_{ij}$ is indifferent to $\tilde{a}_{ii}$, and $\tilde{a}_{jk}$ is indifferent to $\tilde{a}_{ii}$, $\tilde{a}_{ij}$ should be indifferent to $\tilde{a}_{jk}$, too.

Clearly, when $a_2 = a_3$, $T_{\tilde{a}_{ij}} = 1$, and $I_{\tilde{a}_{ij}} = F_{\tilde{a}_{ij}} = 0$, the SVTNPR is reduced to a triangular fuzzy preference relation (TrFPR); meanwhile, when $a_2 = a_3$ and $F_{\tilde{a}_{ij}} = 1 - T_{\tilde{a}_{ij}} - I_{\tilde{a}_{ij}}$, the SVTNPR is reduced to a triangular intuitionistic fuzzy preference relation (TrIPR). In essence, a SVTNPR is a generalized form of FPR.

## 5 Multi-criteria decision-making method based on SVTNPRs and complete weight information

This section proposes an MCDM approach based on SVTNPRs and provides the main procedures of the proposed method.

For an MCDM problem with a finite set of $m$ alternatives, let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of $m$ feasible alternatives, and let $C = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria. Assume that the criteria weight vector is $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^{\mathrm{T}}$ with $\varpi_j \in [0,1]$ and $\sum_{j=1}^{n} \varpi_j = 1$. The pairwise comparison analyses are conducted with respect to every criterion $C_j (j = 1,2,\ldots,n)$; then, the evaluation values represented by SVTNPRs $R_j = [\tilde{a}_{ik}^j]_{m\times m}$ with $\tilde{a}_{ik}^j = \left\langle \left[a_{ik}^{1j}, a_{ik}^{2j}, a_{ik}^{3j}, a_{ik}^{4j}\right], \left(T_{\tilde{a}_{ik}^j}, I_{\tilde{a}_{ik}^j}, F_{\tilde{a}_{ik}^j}\right) \right\rangle (i = 1,2,\ldots,m; k = 1,2,\ldots,m; j = 1,2,\ldots,n)$ can be obtained by transforming the preference values provided by DMs.

Then, we can derive the decision matrix $R_j = [\tilde{a}_{ik}^j]_{m\times m}$ as follows:

$$R_j = \begin{bmatrix} \tilde{a}_{11}^j & \tilde{a}_{12}^j & \cdots & \tilde{a}_{1m}^j \\ \tilde{a}_{21}^j & \tilde{a}_{22}^j & \cdots & \tilde{a}_{2m}^j \\ \vdots & \vdots & & \vdots \\ \tilde{a}_{m1}^j & \tilde{a}_{m2}^j & \cdots & \tilde{a}_{mm}^j \end{bmatrix}_{m\times m}.$$

Our proposed approach involves the following steps:

*Step 1* Normalize the SVTNPRs.

Normalize the decision-making information $\tilde{a}_{ik}^{j}$ in the matrices $R_j = [\tilde{a}_{ik}^{j}]_{m\times m}$. The criteria in the decision matrices are classified as benefit and cost types. In order to make the criterion type uniform, the cost criteria must be transformed into benefits using the negation operation defined in Definition 12. Because the elements in the TFN of SVTNNs are assigned values between 0 and 1, we conduct the negation operation as follows:

$$\mathrm{neg}(\tilde{a}) = \langle [1-a_4, 1-a_3, 1-a_2, 1-a_1], (F_{\tilde{a}}, 1-I_{\tilde{a}}, T_{\tilde{a}})\rangle. \tag{17}$$

The normalized evaluation information matrices are denoted as $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$.

*Step 2* Check the complete consistency or acceptable consistency of SVTNPR $\bar{R}_j$.

If $\bar{R}_j$ meets complete consistency or acceptable consistency, then proceed to Step 6. Otherwise, $\bar{R}_j$ should be modified based on Definitions 19 and 20, until it has complete consistency.

*Step 3* Improve the consistency degree of SVTNPR $\bar{R}_j$.

Determine the element with highest degree of confidence level when comparing with other alternatives in SVTNPR $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$. Assume that the experts are sure about the evaluation information of the first row in $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$; then, according to Definition 18, the experts are also sure about elements of the first column in $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$.

Let the elements of the TFNs in the $i$th row be divided by elements in the first row. If the obtained difference is a fixed value, and the subsequent three membership degrees meet the conditions in Definition 19, then there is no need to modify; otherwise, the elements that do not meet the conditions must be modified according to Definition 19.

*Step 4* Obtain the completely consistent SVTNPR.

Check every element in SVTNPR $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$ and modify the elements that do not meet the conditions until $\bar{R}_j = \left(\bar{a}_{ik}^{j}\right)_{m\times m}$ is completely consistent.

*Step 5* Obtain the overall preference information.

Utilizing Eqs. (13) and (16), the overall value of alternative $A_i$ can be aggregated. We can obtain the overall preference degrees of $A_i$ $(i = 1, 2, \ldots, m)$ when comparing it with other alternatives, and they are denoted by $t_{ik}$ and $t'_{ik}$ $(i = 1, 2, \ldots, m; k = 1, 2, \ldots, m)$, respectively; then, the matrices are denoted as $T = (t_{ik})_{m\times m}$ and $T' = \left(t'_{ik}\right)_{m\times m}$, respectively.

*Step 6* Calculate the ordering vector of each alternative.

Aggregate each row of the SVTNPRs $T = (t_{ik})_{m\times m}$ and $T' = \left(t'_{ik}\right)_{m\times m}$, and gain the matrices $U = (u_i)_{m\times 1}$ and $U' = \left(u'_i\right)_{m\times 1}$, which are composed of the ordering vector of each alternative.

$$U = \sum_{k=1}^{m} t_{ik}, \tag{18}$$

$$U' = \sum_{k=1}^{m} t'_{ik}. \tag{19}$$

Then, the elements $u_i = \left\langle \left[u_i^1, u_i^2, u_i^3, u_i^4\right], (T_{u_i}, I_{u_i}, F_{u_i})\right\rangle$ in $U$ should be normalized to $\tilde{u}_i = \left\langle \left[\tilde{u}_i^1, \tilde{u}_i^2, \tilde{u}_i^3, \tilde{u}_i^4\right], (T_{\tilde{u}_i}, I_{\tilde{u}_i}, .F_{\tilde{u}_i})\right\rangle$ in $V = (\tilde{u}_i)_{m\times 1}^{\mathrm{T}} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_m)$, where

$$\left[\tilde{u}_i^1, \tilde{u}_i^2, \tilde{u}_i^3, \tilde{u}_i^4\right] = \left[\frac{u_i^1}{\max u_i^4}, \frac{u_i^2}{\max u_i^4}, \frac{u_i^3}{\max u_i^4}, \frac{u_i^4}{\max u_i^4}\right], \tag{20}$$

$$T_{\tilde{u}_i} = T_{u_i}, \quad I_{\tilde{u}_i} = I_{u_i} \quad F_{\tilde{u}_i} = F_{u}. \tag{21}$$

Similarly, the elements $u'_i = \left\langle \left[u_i'^1, u_i'^2, u_i'^3, u_i'^4\right], (T_{u'}, I_{u'}, F_{u'})\right\rangle$ in $U'$ should be normalized to $u''_i = \left\langle \left[u_i''^1, u_i''^2, u_i''^3, u_i''^4\right], \left(T_{u''_i}, I_{u''_i}, .F_{u''_i}\right)\right\rangle$ in $V' = \left(u''_i\right)_{m\times 1}^{\mathrm{T}} = \left(u''_1, u''_2, \ldots, u''_m\right)$, where

$$\left[u_i''^1, u_i''^2, u_i''^3, u_i''^4\right] = \left[\frac{u_i''^1}{\max u_i''^4}, \frac{u_i''^2}{\max u_i''^4}, \frac{u_i''^3}{\max u_i''^4}, \frac{u_i''^4}{\max u_i''^4}\right], \tag{22}$$

$$T_{u''_i} = T_{u_i}, \quad I_{u''_i} = I_{u_i}, \quad F_{u''_i} = F_{u_i}. \tag{23}$$

*Step 7* Derive the score values of each alternative $A_i$.

Utilizing Definition 14, the score values for each alternative can be calculated.

*Step 8* Gain the final ranking order and select the optimal alternative(s).

By comparing the values obtained in Step 8, the final ranking results can be obtained, and the optimum option can be selected.

## 6 An numerical example

This section uses a green supplier selection problem adapted from Wan and Dong [60] to demonstrate the applicability of the proposed method.

Shanghai General Motors Company Limited (SGM) is planning to incorporate environmentally friendly features into the product design stage to protect the environment and achieve sustainable development of the social economy. For this reason, SGM wishes to select the most appropriate green supplier for one of the key elements in its manufacturing process. After pre-evaluation, four suppliers remain as candidates for further evaluation. They are Howden Hua Engineering Company ($A_1$), Sino Trunk ($A_2$), Taikai Electric Group Company Limited ($A_3$), and Shantui Construction Machinery Company Limited ($A_4$). Utilizing principal component analysis, the experts choose the following three independent criteria as evaluation principles: product quality ($C_1$), pollution control ($C_2$), and environment management ($C_3$). According to historical data, the weight vector of the three criteria is $\varpi = (0.4, 0.35, 0.25)^T$. The results of pairwise comparisons among these three alternatives with respect to the three criteria $C_j (j = 1, 2, 3)$ are listed as follows in the form of SVTNNs:

$$R_1 = \begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.3,0.5,0.5,0.7],(0.9,0.5,0.1)\rangle & \langle[0.4,0.5,0.5,0.6],(0.3,0.3,0.4)\rangle \\ A_2 & \langle[0.3,0.5,0.5,0.7],(0.1,0.5,0.9)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.4,0.5,0.6,0.7],(0.7,0,0.3)\rangle \\ A_3 & \langle[0.4,0.5,0.5,0.6],(0.4,0.3,0.3)\rangle & \langle[0.3,0.4,0.5,0.6],(0.3,0,0.7)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle \end{matrix},$$

$$R_2 = \begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.25,0.4,0.6,0.75],(0.5,0.2,0.4)\rangle & \langle[0.35,0.45,0.55,0.65],(0.4,0.2,0.4)\rangle \\ A_2 & \langle[0.25,0.4,0.6,0.75],(0.4,0.2,0.5)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.25,0.45,0.55,0.75],(0.6,0.3,0.3)\rangle \\ A_3 & \langle[0.35,0.45,0.55,0.65],(0.4,0.2,0.4)\rangle & \langle[0.25,0.45,0.55,0.75],(0.3,0.3,0.6)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle \end{matrix},$$

$$R_3 = \begin{matrix} & A_1 & A_2 & A_3 \\ A_1 & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.35,0.45,0.55,0.65],(0.5,0.1,0.4)\rangle & \langle[0.4,0.5,0.5,0.6],(0.4,0.2,0.5)\rangle \\ A_2 & \langle[0.35,0.45,0.55,0.65],(0.4,0.1,0.5)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle & \langle[0.25,0.4,0.6,0.75],(0.6,0.3,0.3)\rangle \\ A_3 & \langle[0.4,0.5,0.5,0.6],(0.5,0.2,0.4)\rangle & \langle[0.25,0.4,0.6,0.75],(0.3,0.3,0.6)\rangle & \langle[0.5,0.5,0.5,0.5],(1,0,0)\rangle \end{matrix},$$

### 6.1 Evaluation steps for green supplier selection problem

The evaluation steps of the proposed method proceed as follows.

*Step 1* Normalize the SVTNPRs.

Because all the criteria are of the benefit type, the decision information does not need to be normalized. In other words, $\bar{R}_j = \left(\bar{a}_{ik}^j\right)_{m\times m} = R_j = \left(\tilde{a}_{ik}^j\right)_{m\times m} (k = 1, 2, \ldots, m)$.

*Step 2* Check the complete consistency or acceptable consistency of SVTNPR $\widetilde{R}$.

Based on Definition 19, the results obtained by calculating the elements in SVTNPR $R_1$ are as follows: $a_{31}^{11} + a_{31}^{14} + a_{12}^{11} + a_{12}^{14} + a_{23}^{11} + a_{23}^{14} = 0.4 + 0.6 + 0.3 + 0.7 + 0.4 + 0.7 = 3.1 \neq 3$, and $a_{31}^{12} + a_{31}^{13} + a_{12}^{12} + a_{12}^{13} + a_{23}^{12} + a_{23}^{13} = 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.6 = 3.1 \neq 3$; therefore, $R_1$ is not completely consistent. However, $R_2$ and $R_3$ are both completely consistent according to Definition 19. Furthermore, utilizing the score function defined in Definition 14 in Sect. 3.2, we can obtain $E\left(\bar{a}_{13}^1\right) = 0.175$, $E\left(\bar{a}_{32}^1\right) = 0.100$, $E\left(\bar{a}_{12}^1\right) = 0.342$, and $E\left(\bar{a}_{11}^1\right) = 0.333$, such that $\bar{a}_{13}^1 \prec \bar{a}_{11}^1$, $\bar{a}_{32}^1 \prec \bar{a}_{11}^1$, and $\bar{a}_{12}^1 \prec \bar{a}_{11}^1$; therefore, according to Definition 20, $R_1$ is not acceptably consistent either..

As a result, the matrix $R_1$ must be modified based on Definitions 19 and 20 until it is completely consistent.

*Step 3* Improve the consistency degree of SVTNPR $\bar{R}$.

Assume that the experts are very sure about the first row of SVTNPR $\bar{R}$; then, based on Definition 18, we can see that the experts are also very sure about the first column of SVTNPR $\bar{R}$.

Let the elements in the second line be divided by the elements in the first line. The results are not a fixed value, but the three membership degrees satisfy the conditions in Definition 19; therefore, $\tilde{a}_{23}^1$ should be modified as $\tilde{a}_{23}^1 = \langle[0.3, 0.5, 0.5, 0.7], (0.7, 0, 0.3)\rangle$.

*Step 4* Obtain the completely consistent SVTNPR.

Check every element in the SVTNPR $R_1 = \left(\alpha_{ik}^1\right)_{m\times m}$ and modify all the elements; then, the modified results of $\overline{R}_1$ are as follows:

$$\bar{R}_1' = \begin{pmatrix} \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3,0.5,0.5,0.7],\\ (0.9,0.5,0.1)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.4,0.5,0.5,0.6],\\ (0.3,0.3,0.4)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.3,0.5,0.5,0.7],\\ (0.1,0.5,0.9)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3,0.5,0.5,0.7],\\ (0.7,0,0.3)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.3,0.5,0.5,0.7],\\ (0.4,0.3,0.3)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3,0.4,0.5,0.6],\\ (0.3,0,0.7)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle \end{pmatrix}.$$

*Step 5* Obtain the overall preference information.

Utilizing Eqs. (13) and (16), the overall preference value can be aggregated. It is denoted as matrices $T_{\mathrm{SVTNWAA}}$ and $T'_{\mathrm{SVTNWGA}}$ which are given as follows:

$$T_{SVTNWAA} = \begin{pmatrix} \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.295,0.4525,0.5475,0.705],\\ (0.6267,0.7248,0.695)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3825,0.4825,0.5175,0.6175],\\ (0.3704,0.7704,0.5815)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.295,0.4525,0.5475,0.705],\\ (0.305,0.7248,0.3733)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.27,0.4575,0.5425,0.73],\\ (0.6294,0.7881,0.7)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.3425,0.4825,0.5175,0.6575],\\ (0.4143,0.7543,0.6457)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.27,0.4175,0.5425,0.69],\\ (0.3,0.7881,0.3706)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle \end{pmatrix}$$

and

$$T'_{SVTNWGA} = \begin{pmatrix} \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.2925,0.4504,0.5458,0.7039],\\ (0.6325,0.3173,0.2944)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3817,0.4819,0.517,0.617],\\ (0.3565,0.2416,0.4267)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.2925,0.4504,0.5458,0.7039],\\ (0.2297,0.3173,0.7373)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.2689,0.4558,0.5411,0.7296],\\ (0.6382,0.1927,0.3)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.3402,0.4819,0.517,0.6563],\\ (0.4229,0.2416,0.3618)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.2689,0.4168,0.5411,0.686],\\ (0.3,0.1927,0.6435)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5,0.5,0.5,0.5],\\ (1,0,0)\end{matrix} \right\rangle \end{pmatrix}.$$

*Step 6* Calculate the ordering vector of each alternative.

Aggregate each row of SVTNPR $T_{\mathrm{SVTNWAA}} = (t_{ik})_{m\times m}$ and $T'_{\mathrm{SVTNWGA}} = \left(t'_{ik}\right)_{m\times m}$, yielding the matrices $U =$

$(u_i)_{m\times 1}$ and $U' = (u_i')_{m\times 1}$, respectively, which are composed of the ordering vector of each alternative.

$$U = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}\begin{pmatrix} \langle [1.1775, 1.435, 1.565, 1.8225], (0.5374, 0.2594, 0.3445) \rangle \\ \langle [1.065, 1.41, 1.59, 1.935], (0.4733, 0.2424, 0.4571) \rangle \\ \langle [1.1125, 1.4, 1.56, 1.8475], (0.3447, 0.2251, 0.5218) \rangle \end{pmatrix},$$

and

$$U' = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}\begin{pmatrix} \langle [1.1742, 1.4323, 1.5628, 1.821], (0.5365, 0.709, 0.6596) \rangle \\ \langle [1.0614, 1.4062, 1.5869, 1.9335], (0.4415, 0.7473, 0.4895) \rangle \\ \langle [1.1092, 1.3987, 1.558, 1.8422], (0.3484, 0.7881, 0.4673) \rangle \end{pmatrix}.$$

Then, the normalized matrix $V = (\tilde{u}_i)^{\mathrm{T}}_{m\times 1} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_m)$ from $U$ is as follows:

$$V = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}\begin{pmatrix} \langle [0.6085, 0.7416, 0.8088, 0.9419], (0.5374, 0.2594, 0.3445) \rangle \\ \langle [0.5504, 0.7287, 0.8217, 1.0], (0.4733, 0.2624, 0.4571) \rangle \\ \langle [0.5749, 0.7235, 0.8062, 0.9548], (0.3447, 0.2251, 0.5218) \rangle \end{pmatrix},$$

and the normalized matrix $V' = (u_i'')^{\mathrm{T}}_{m\times 1} = (u_1'', u_2'', \ldots, u_m'')$ from $U'$ is as follows:

$$V' = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}\begin{pmatrix} \langle [0.6073, 0.7408, 0.8083, 0.9418], (0.5365, 0.709, 0.6596) \rangle \\ \langle [0.549, 0.7273, 0.8207, 1.0], (0.4415, 0.7473, 0.4895) \rangle \\ \langle [0.5737, 0.7234, 0.8058, 0.9528], (0.3484, 0.7881, 0.4673) \rangle \end{pmatrix}.$$

*Step 7* Derive the score values of each alternative $A_i$.

The score values of each alternative can be obtained based on Definition 14.

When using the alternative information in $U$:

$$E(A_1) = 0.4996, \quad E(A_2) = 0.4532, \quad E(A_3) = 0.4072.$$

When using the alternative information in $U'$:

$$E(A_1) = 0.3015, \quad E(A_2) = 0.3109, \quad E(A_3) = 0.2781.$$

*Step 8* Gain the final ranking order and select the optimal alternative(s).

By comparing the values obtained in Step 8, including $E(A_1) > E(A_2) > E(A_3)$ obtained by $U$ and $E(A_2) > E(A_1) > E(A_3)$ obtained by $U'$, we can get the final ranking orders as $A_1 \succ A_2 \succ A_3$ and $A_2 \succ A_1 \succ A_3$, respectively. Because of the distinct inherent characteristic of these two operators, it is reasonable that the ultimate ranking results are different.

### 6.2 Comparative study and discussion

In order to illustrate the rationality and effectiveness of the proposed method, this subsection conducts a comparative study and discussion.

In solving the above example, the preference information of each alternative $A_i$ is integrated directly using the SVTNWAA and SVTNWGA operators, and consistency is not considered. The aggregated matrices can be calculated, respectively, as follows:

$$\dot{T}_{SVTNWAA} = \begin{pmatrix} \left\langle \begin{matrix}[0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.295, 0.4525, 0.5475, 0.705], \\ (0.6267, 0.7248, 0.695)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.3825, 0.4825, 0.5175, 0.6175], \\ (0.3704, 0.7704, 0.5815)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.295, 0.4525, 0.5475, 0.705], \\ (0.305, 0.7248, 0.3733)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.31, 0.4575, 0.5825, 0.73], \\ (0.6294, 0.7881, 0.7)\end{matrix} \right\rangle \\ \left\langle \begin{matrix}[0.3425, 0.4825, 0.5175, 0.6175], \\ (0.4185, 0.7704, 0.6296)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.27, 0.4175, 0.5425, 0.69], \\ (0.3, 0.7881, 0.3706)\end{matrix} \right\rangle & \left\langle \begin{matrix}[0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0)\end{matrix} \right\rangle \end{pmatrix},$$

and

$$\dot{T'}_{SVTNWGA} = \begin{pmatrix} \left\langle \begin{matrix} [0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.2925, 0.4504, 0.5458, 0.7039], \\ (0.6325, 0.3173, 0.2944) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.3817, 0.4819, 0.517, 0.617], \\ (0.3565, 0.2416, 0.4267) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.2925, 0.4504, 0.5458, 0.7039], \\ (0.2297, 0.3173, 0.7373) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.3017, 0.4558, 0.582, 0.7296], \\ (0.6382, 0.1927, 0.3) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.3817, 0.4819, 0.517, 0.617], \\ (0.4229, 0.2416, 0.3618) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.2689, 0.4168, 0.5411, 0.686], \\ (0.3, 0.1927, 0.6435) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.5, 0.5, 0.5, 0.5], \\ (1, 0, 0) \end{matrix} \right\rangle \end{pmatrix}.$$

Then, the corresponding normalized matrices can be calculated, respectively, as follows:

$$\dot{V} = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{pmatrix} \langle [0.6085, 0.7416, 0.8088, 0.9419], (0.5374, 0.2594, 0.3445) \rangle \\ \langle [0.5711, 0.7287, 0.8424, 1.0], (0.4733, 0.2424, 0.4571) \rangle \\ \langle [0.5956, 0.7235, 0.8062, 0.9341], (0.3393, 0.2178, 0.5436) \rangle \end{pmatrix},$$

and

$$\dot{V'} = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{pmatrix} \langle [0.6073, 0.7408, 0.8083, 0.9418], (0.5365, 0.709, 0.6596) \rangle \\ \langle [0.5659, 0.7273, 0.8419, 1.0], (0.4431, 0.7478, 0.4911) \rangle \\ \langle [0.5951, 0.7234, 0.8058, 0.9325], (0.341, 0.791, 0.4503) \rangle \end{pmatrix}.$$

Therefore, we can obtain the score values based on Definition 14, and they are $\dot{E}(A_1) = 0.4996$, $\dot{E}(A_2) = 0.4645$, and $\dot{E}(A_3) = 0.4023$ in the normalized matrix $\dot{V}$, such that the ranking order is $A_1 \succ A_2 \succ A_3$; meanwhile, they are $\dot{E}(A_1) = 0.3015$, $\dot{E}(A_2) = 0.3145$, and $\dot{E}(A_3) = 0.2801$ in the normalized matrix $\dot{V'}$, such that the ranking order is $A_2 \succ A_1 \succ A_3$. Consequently, we can see that a different result is produced when the consistency of preference relations is not considered. This also indicates that our approach is reasonable.

## 7 Conclusion

This paper developed a novel single-valued trapezoidal neutrosophic MCDM method based on SVTNPRs. First of all, in order to overcome the disadvantages of existing operations and comparison methods for SVTNNs, which are not always in accordance with real MCDM situations, we defined some new operations and a new comparison method and explored their properties; second, we developed the SVTNWAA operator and SVTNWGA operator to aggregate decision information. And then, we constructed the SVTNPRs and defined both the complete consistency and acceptable consistency. Finally, we outlined a method for MCDM problem solving with SVTNPRs and applied a numerical example to verify the effectiveness of our method. Future research should take into account unknown weight information, which exists widely in real life; additionally, DMs' preferences on alternatives might be missing, and the problem of tackling incomplete preference information also represents an important issue for further study. Also, it is worthwhile to investigate SVNPR when the neutrosophic components are partially dependent and partially independent. Finally, it is worthwhile to propose new comparison methods and integrated methodologies to deal with MCDM problems with SVTNNs.

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**Compliance with ethical standards**

**Conflict of interest** The authors declare that there is no conflict of interest regarding the publication of this paper.

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