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Fuzzy inference system to formulate compressive strength and ultimate strain of square concrete columns wrapped with fiber-reinforced polymer

Kourosh Nasrollahzadeh[1] · Ehsan Nouhi[1]

Abstract In this paper, fuzzy inference system (FIS) is employed to develop a more accurate approach to evaluate the strength and strain capacity of axially loaded concrete columns with the square section confined by fiber-reinforced polymer (FRP) wraps. To do so, an experimental database containing 261 test data on compressive strength and 112 test data on ultimate strain is collated from the literature. By using subtractive clustering algorithm to extract cluster centers from the experimental database, the structure of FIS model is identified. To select the best FIS model, several constant and linear (i.e., zeroth- and first-order) Takagi–Sugeno FIS models with different numbers of rules are developed and their performances in terms of the model output errors with respect to training data set as well as validation data set are compared. The finally proposed FIS models for calculation of strength and strain contain as few as three rules. Besides, the proposed FIS models are expressed as closed-form formulations, which can be conveniently used in practice. The outputs of the proposed FIS models agree favorably with the test data and outperform the existing models by providing more accurate prediction of both strength and strain capacity. In view of the FIS models, a parametric study is carried out to examine the influence of various variables including the section corner radius as well as the elastic modulus and tensile strength of FRP on the capacity of FRP-confined square columns.

1 Introduction

There is an increasing need for seismic strengthening of buildings and bridges worldwide. As for reinforced concrete (RC) structures, a major approach for strengthening is to increase confinement for column sections. This follows from the well-known fact that concrete confinement enhances strength and, more importantly, ductility of plain concrete. Among different techniques available for confinement of concrete, fiber-reinforced polymer (FRP) materials have received much attention thanks to their outstanding characteristics including high strength per weight, non-corrosive behavior, and construction ease. To facilitate the rapid application of FRP for confinement, many researchers have made efforts to evaluate the capacity of FRP-wrapped concrete columns since two decades ago. However, most of these studies have been devoted to circular columns, while very few investigations have dealt with square/rectangular sections. Besides, the majority of existing models have been aimed at predicting only strength capacity of FRP-wrapped concrete without getting into estimation of the strain capacity of that. The fact is that the efficiency of FRP confinement is more pronounced regarding the ultimate strain enhancement rather than strength increment. Therefore, evaluation of the ultimate strain as well as strength of axially loaded square columns wrapped with FRP is in high demand.

In order to arrive at a closed-form equation to estimate the capacity of FRP-confined concrete, the researchers have mostly used a regression analysis to fit a mathematical expression to the available experimental data. The basic

✉ Kourosh Nasrollahzadeh
nasrollahzadeh@kntu.ac.ir

[1] Department of Civil Engineering, K.N. Toosi University of Technology, P.O. Box 15875-4416, Tehran, Iran

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form of the regression equations used for FRP-confined concrete has been pretty much similar to what was initially developed for steel-confined concrete that was itself based on a proposal by Richart et al. [1] originally for the concrete cylinders subjected to uniform lateral stress. It is, however, obvious that FRP-confined concrete behaves very differently from steel-confined concrete since non-yielding FRP material induces a passive confinement, whereas steel exerts an active confinement beyond yielding. The researchers who developed such regression equations for the circular sections confined with FRP expressed the compressive strength (i.e., the maximum axial stress) of confined concrete as a function of the lateral stress provided by FRP wraps [2–13]. It is worth mentioning that at the ultimate lateral confining pressure, the hoop strain in the FRP wraps cannot reach the maximum tensile strain of FRP materials [14]. However, the current models differ in the way they evaluate the actual rupture strain for FRP hoops. As for square sections, the form of the regression equation presented by researchers is similar to that for circular sections except that an additional parameter called the shape factor is included in the model [15–22]. The shape factor is to take into account the non-uniform distribution of the confining pressure on the perimeter of non-circular sections. In the literature, the expressions presented for the shape factor vary greatly in the manner they incorporate different parameters. Although most of the previously mentioned investigations have employed the same list of the parameters affecting the strength/strain of FRP-confined square sections, their proposed predictive equations are much different as they contain these parameters with totally different magnitudes.

To overcome the problems inherent in the existing regression-based models, this paper employs fuzzy inference system (FIS) approach toward modeling the ultimate capacity of the square sections wrapped with FRP. Unlike the regression approach, which requires an initial assumption on the form of a mathematical expression containing an arbitrary combination of the parameters involved in the problem in hand, the FIS approach starts from scratch without any need to assume a particular form of equation in advance. In the FIS method similar to other soft computing techniques, knowing the list of influential parameters is the only information needed to train a model using an experimental database. The FIS technique has recently been applied by the author to another application of FRP composites [23]. Also, fuzzy logic has been employed by researchers to predict the concrete strength and/or strain [24, 25]. Other neural computing techniques including neural network, genetic algorithm, and ANFIS have already been applied for modeling the strength of FRP-confined columns with circular sections [26, 27]. Also, neural network has been used to predict the ultimate strain of circular specimens wrapped with FRP [28]. For non-circular columns confined by

FRP, the application of soft computing methods has been very limited as compared to circular sections. Pham and Hadi [29] employed artificial neural network (ANN) to estimate strength/strain of non-circular sections wrapped by FRP. Their proposed ANN strength model was trained, tested, and validated by the use of an experimental database containing 104 square/rectangular sections, whereas their ANN model for strain was based on 69 test data on only square columns. The ANN strength model had eight input parameters including length of two sides of the section, the corner radius, the FRP thickness, the tensile strength and elastic modulus of FRP, the unconfined compressive strength, and the corresponding axial strain. The input variables for their strain model were identical to the above-mentioned parameters, but length of only one side of section was considered since the strain model was for only square sections. They indicated that the proposed ANN models outperformed several existing confinement models suggested by other researchers. They also explicitly derived user-friendly equations from the ANN models in order to propose a simpler relationship to estimate the confined concrete strength/strain. Doran et al. [30] have recently used fuzzy logic approach to model the capacity of CFRP-wrapped square/rectangular concrete columns. However, their proposed model focused on only strength enhancement without dealing with the strain. They developed a Mamdani-type fuzzy inference system to predict the so-called lateral confinement coefficient, which was the ratio of the compressive strength of confined concrete to that of unconfined concrete. Five input variables were considered as follows: width and length of column section, total thickness and the elastic modulus of CFRP wrap and unconfined concrete compressive strength. Using 100 test data on FRP-wrapped column specimens collected from the literature, a FIS model containing 196 fuzzy rules with trapezoidal-shaped membership functions was established. The proposed model was then validated against 40 additional experimental data and was also compared with several nonlinear regression equations.

The current study utilizes a Takagi–Sugeno FIS approach to estimate the strength/strain capacity of the axially loaded FRP-wrapped concrete columns with square sections. To this aim, a large database containing the axial test results of square specimens wrapped with FRP is assembled from the literature. The database consists of 261 test data to be used for developing the strength model and 112 test data for the strain model. A novelty of the FIS model proposed in the current paper rests in presenting closed-form formulations for the problem in hand. This facilitates the application of FIS modeling in practice. In addition, the accuracy of the suggested FIS models is assessed against predictions by several existing models developed by other researchers. Furthermore, the effects of different parameters on strength/strain are addressed through a parametric study.

Table 1 Statistics of different variables in the experimental database of the compressive strength and ultimate axial strain for FRP-confined square columns

|	Compressive stress database (261 data)					Ultimate axial strain database (112 data)				
	Min	Max	Range	Mean	COV (%)	Min	Max	Range	Mean	COV (%)
b (mm)	100	458	358	159.64	26.70	108	458	350	159.89	27.66
r (mm)	0	60	60	27.21	61.64	5	60	55	30.92	50.72
f'_{co} (MPa)	6.8	56.7	49.9	35.92	31.83	20	54.7	34.7	36.86	27.15
t_f (mm)	0.117	5.04	4.923	0.547	129.16	0.117	5.04	4.923	0.813	120.22
f_f (MPa)	220	4519	4299	3389.21	37.95	220	4519	4299	2873.17	55.97
E_f (GPa)	10.5	640	629.5	196.92	81.83	10.5	257	246.5	162.31	55.18
$f'_{cc,exp}$ (MPa)/$\varepsilon_{cu,exp}$	11.1	95.2	84.1	49.76	29.77	0.001167	0.0395	0.038333	0.013903	53.49

b = section side length, r = corner radius, f'_{co} = compressive strength of unconfined concrete, t_f = thickness of FRP wrap, f_f = tensile strength of FRP, E_f = elastic modulus of FRP, $f'_{cc,exp}$ = experimental value for compressive strength of confined concrete, $\varepsilon_{cu,exp}$ = experimental value for ultimate axial strain of confined concrete

2 Experimental database and existing confinement models

In this paper, a large experimental database of concrete columns with square sections wrapped with FRP under uniaxial compression was compiled by an extensive survey of the literature. The current database includes 261 test results on the compressive strength while 112 test results for the ultimate strain. The test data for the strength were adopted from 18 various investigations [17, 18, 21, 31–45], and those for the strain were collated from 10 experimental programs [17, 18, 21, 31, 32, 34–36, 39, 45]. It should be mentioned that in the literature the number of experiments reporting results of the ultimate axial strain is much smaller than that of the compressive strength. All the specimens reported in the mentioned experimental campaigns were included in the current study. Nevertheless, in the case that an experimental campaign contained both rectangular and square sections, only the latter were considered since our study has been limited to the square columns. Besides, the column specimens reinforced with steel either longitudinally or transversely were omitted as the focus has been on the plain concrete sections without rebars. With these in mind, a careful investigation was conducted to extract the geometrical/mechanical properties along with the reported experimental values for the strength/strain capacity from the original papers published in open literature. To our knowledge, the test database collated herein is the largest one which has been ever used to study the FRP-confined square columns. The test specimens were rounded at the corners of the section and were wrapped with the unidirectional FRP composites whose fiber types were carbon, glass, or aramid. The height-to-width ratios of the specimens ranged between 1.6 and 4, and the section side length of the samples was in the range of 100–458 mm. The maximum value of the unconfined concrete cylindrical

strength for the column samples in the present database was about 57 MPa. Table 1 presents further details on the statistical descriptions for the mechanical and geometrical properties of the columns considered in the current test database. The geometric parameters of the column section are demonstrated in Fig. 1, which shows especially the radius at the section corner (i.e., r), which is a crucial parameter for the performance of the FRP-confined square column. All specimens in the database were made of plain concrete without any steel reinforcement.

In the current study, eight existing models [15–22] for estimation of the compressive strength for square/rectangular sections confined with FRP are considered. Four of these models [16, 18, 19, 21] have been developed to calculate the ultimate strain as well. All of these models
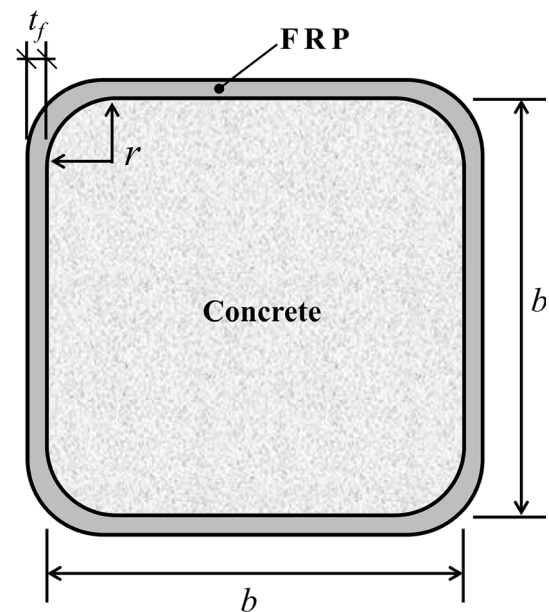
Fig. 1 Illustration of geometric parameters in square section

have been derived based on the regression of the experimental data. The basic form of the equation used in the existing models in the case of strength is as follows:

$$\frac{f'_{cc}}{f'_{co}} = 1 + k_1 k_s \left(\frac{f_l}{f'_{co}} \right) \qquad (1)$$

where f_l is the confining stress exerted by FRP, k_s is the shape factor to account for non-uniform distribution of the confining stress in a non-circular section, and k_1 is a coefficient to be tuned with respect to the experimental results. A similar equation has been employed in the available models for the strain capacity. Tables 2 and 3 present the equations for the existing models for the strength and strain, respectively. Some of these models could be used for both square and rectangular sections, while some others were developed only for square sections such as Campione and Miraglia [16], Al-Salloum [17], and Shehata et al. [21]. As given in Tables 2 and 3, different researchers have suggested various definitions for different

parameters in Eq. (1). For instance, the shape factor k_s in Mirmiran's model [15] was taken as $2r/D$, whereas in Lam and Teng's model [18], it was related to the effectively confined area of the section based on the arching action defined as the parabolas initiated from the rounded corners with the initial slope equal to the section diagonal. Besides, Mirmiran assumed that if a parameter called as modified confinement ratios (i.e., MCR) was <15%, the confinement effects could be negligible.

In an analogy with circular sections, the confining stress f_l as used in most of the existing models for rectangular/square sections is determined by the following expression:

$$f_l = \frac{2E_f \varepsilon_j t_f}{D} \qquad (2)$$

where ε_j is the ultimate tensile strain mobilized in the FRP wraps, and D for a rectangular/square section is assumed as the diameter of an equivalent circular section. There is a variety of definitions for D in different models. For

Table 2 Existing models for the compressive strength of FRP-confined square sections

Model	Expression
Mirmiran et al.	$\frac{f'_{cc}}{f'_{co}} = 1 + 6\left(\frac{2r}{h}\right)\left(\frac{f_l^{0.7}}{f'_{co}}\right)$ for MCR $= \left(\frac{2r}{h}\right)\left(\frac{f_l}{f'_{co}}\right) \geq 0.15$
Campione and Miraglia	$\frac{f'_{cc}}{f'_{co}} = 1 + 2\left[1 - \frac{2\left(1-\frac{2r}{b}\right)^2}{3\left[1-(4-\pi)\left(\frac{r}{b}\right)^2\right]}\right]\left(\frac{f_l}{f'_{co}}\right)$
Al-Salloum	$\frac{f'_{cc}}{f'_{co}} = 1 + 3.14\left(1 - \frac{2\left(1-\frac{2r}{b}\right)^2}{3\left[1-(4-\pi)\left(\frac{r}{b}\right)^2\right]}\right)\left(\frac{b}{\sqrt{2}b-2r\left(\sqrt{2}-1\right)}\right)\left(\frac{f_l}{f'_{co}}\right)$
Lam and Teng	$\frac{f'_{cc}}{f'_{co}} = 1 + 3.3\left(\frac{b}{h}\right)^2\left(\frac{1-\left(\left(\frac{b}{h}\right)(h-2r)^2+\left(\frac{h}{b}\right)(b-2r)^2/3\left(bh-(4-\pi)r^2\right)\right)-\rho_{sc}}{1-\rho_{sc}}\right)\frac{f_l}{f'_{co}}$
Ilki et al.	$\frac{f'_{cc}}{f'_{co}} = 1 + 2.54\frac{f_l}{f'_{co}}$
Wu and Wang	$\frac{f'_{cc}}{f'_{co}} = 1 + 2.16\left(\frac{2r}{b}\right)^{0.651}\left(\frac{f_l}{f'_{co}}\right)^{0.955}$
Shehata et al.	$\frac{f'_{cc}}{f'_{co}} = 1 + 0.85\frac{f_l}{f'_{co}}$
Kumutha et al.	$\frac{f'_{cc}}{f'_{co}} = 1 + 0.93\frac{f_l}{f'_{co}}$

ρ_{sc} = the ratio of the longitudinal reinforcement area to the gross area of the concrete section, b = length of the shorter side in rectangular section or the side length in square section, h = length of the longer side in rectangular section or the side length in square section

Table 3 Existing models for the ultimate axial strain of FRP-confined square sections

Model	Expression
Campione and Miraglia	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + \left[\frac{2[2(b-2r)+\pi r]t}{b^2-(4-\pi)r^2}\right]\left(f_f\left[0.85\left(\frac{2r}{b}\right)+0.15\right]\right)^2\frac{1}{\varepsilon_{co}E_f\left(f'_{co}+f_l\right)}$
Lam and Teng	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.75 + 12\left(\frac{h}{b}\right)^{0.5}\left(\frac{1-\left((b/h)(h-2r)^2+(h/b)(b-2r)^2/3(bh-(4-\pi)r^2)\right)-\rho_{sc}}{1-\rho_{sc}}\right)\frac{f_l}{f'_{co}}\left(\frac{\varepsilon_{h,rup}}{\varepsilon_{co}}\right)^{0.45}$
Ilki et al.	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 19.27\left(\frac{f_l}{f'_{co}}\right)^{0.53}$
Shehata et al.	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 13.5\frac{f_l}{f'_{co}}$

ε_{co} = the axial strain corresponding to the compressive strength for unconfined concrete (taken as 0.002), $\varepsilon_{h,rup}$ = the actual hoop strain of FRP wrap at rupture condition

Table 4 Expressions for the confining pressure in the existing strength/strain models

Model	Expression
Mirmiran et al.	$f_l = \frac{2 f_f t_f}{h}$
Campione and Miraglia	$f_l = \frac{2 f_f \left[1.7\left(\frac{r}{b}\right)+0.15\right] t_f}{b}$
Al-Salloum	$f_l = \frac{2 E_f \varepsilon_f t_f}{b\sqrt{2}-2r\left(\sqrt{2}-1\right)}$
Lam and Teng	$f_l = \frac{2 E_f \varepsilon_{h,rup} t_f}{\sqrt{h^2+b^2}}$
Ilki et al.	$f_l = 0.85\left(\frac{b+h}{bh}\right)\left[1 - \left(\frac{\left(\frac{b}{h}\right)(h-2r)^2+\left(\frac{h}{b}\right)(b-2r)^2}{3(bh-(4-\pi)r^2)}\right)\right]E_f \varepsilon_f t_f$
Wu and Wang	$f_l = \frac{2 E_f \varepsilon_f t_f}{b}$
Shehata et al.	$f_l = \frac{2 E_f \varepsilon_f t_f}{b}$
Kumutha et al.	$f_l = \frac{f_f t_f (b+h)}{bh}$

b = length of the shorter side in rectangular section or the side length in square section, h = length of the longer side in rectangular section or the side length in square section, ε_f = the maximum tensile strain of FRP material, f_f = the maximum tensile strength of FRP material, $\varepsilon_{h,rup}$ = the actual hoop strain of FRP wrap at rupture condition

instance, D was taken as length of the longer side [15] or even the shorter side [20] in rectangular sections and the side length in square sections [16, 21]. In some other models, D was assumed as length of the diagonal of the square section [17], or the diameter of the circle circumscribed about the rectangular section [18]. Various formulations for f_l are listed in Table 4. In most of the existing models, ε_j was taken equal to the maximum tensile strain of FRP material despite the fact that the FRP hoop strain at the actual rupture condition could be substantially less than the maximum strength of material. The model by Lam and Teng [18] was the only model which considered the actual hoop strain at rupture by assuming ε_j to be equal to almost half of the maximum tensile strain of CFRP material. Also, Campione and Miraglia [16] expressed ε_j as a function of the maximum strength of FRP material as well as the corner radius of the section.

With reference to the mathematical expressions of the existing confinement models presented in Tables 2 and 3, it is concluded that a wide variety of forms for equations predicting strength/strain have been used in the literature so far. This is the main incentive for the authors to employ the FIS approach as it does not require a particular form of equation.

3 Overview of fuzzy inference system

Fuzzy inference system (FIS) is the method that employs fuzzy logic in order to derive conclusion (i.e., output) from known premises (i.e., given inputs). A FIS model is expressed by a list of logical statements called rules, which contain variables that are the members of given fuzzy sets. The fuzzy sets, which are the basis of fuzzy logic approach, were introduced by Zadeh [46]. A fuzzy set can admit variables with partial memberships in it. The membership function is a curve, which assigns a number in the range of 0–1 to a variable, showing the degree to which the variable is a member of a fuzzy set. The rules in a FIS model are IF–THEN statements as follows: IF x_1 is A_1 AND x_2 is A_2 AND … THEN y is B, where the antecedent, namely IF part, is composed of multiple parts interconnected with fuzzy logical operators such as AND. Once an input variable x_1 such as the unconfined concrete strength f'_{co} is presented to an IF–THEN rule, it is first fuzzified according to the corresponding membership function A_1. That means the value of f'_{co} is mapped to a value in 0–1 range, which is called the membership value. Then, the fuzzy operator AND, which may be defined as multiplication of membership values, is applied to all parts of the antecedent. As a result, the antecedent yields a value in range 0–1, indicating the rule's degree of truth. As for the consequent, namely THEN part of a rule, we here use a particular type of FIS called Takagi–Sugeno model [47] where the output membership function is a value calculated as a linear combination of all input variables, i.e., $y = a_0 + a_1 x_1 + a_2 x_2 + \cdots$. There are two kinds of Takagi–Sugeno FIS model: constant and linear. In the former, all coefficients except for a_0 are zero, whereas in the latter all coefficients have nonzero values. Finally, the effects of all rules are combined to give a single output by calculating weighted average of all rule outputs, in which each rule is weighted by its degree of truth.

3.1 Identification of FIS model by clustering of data

The idea employed in this paper is that clustering of data can serve as a basis to identify a FIS model. By clustering of numerical data, a number of cluster centers can be extracted from the database. Once a cluster center is determined, it implies a fuzzy IF–THEN rule in the form:

"IF a given input is near a cluster center, THEN the desired output is near the output corresponding to that cluster center." The truth degree for this fuzzy rule depends on how close the given input is to the considered cluster center. Among different algorithms available for data clustering, we here utilize subtractive clustering method proposed by Chiu [48]. A brief description of this method is presented herein.

Firstly, each test datum is considered as a vector whose coordinates include all input variables plus the corresponding output value. For instance, the ith test data for the compressive strength in the current study are presented as $\mathbf{x}_i = \left[b, r, f'_{co}, t_f, f_f, E_f, f'_{cc}\right]_i$, where various notations are defined in Table 1. Then, the coordinates are normalized with respect to their own range of values. For example, to normalize f'_{co}, it is divided by $(f'_{co,max} - f'_{co,min})$, in which $f'_{co,max}$ and $f'_{co,min}$ denote the maximum and minimum of f'_{co} in all test data, respectively. Secondly, the potential of being a cluster center is calculated for each test datum, as follows:

$$P_i = \sum_{j=1}^{n} \exp\left(\frac{-4}{r_c^2} \cdot \left\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_j\right\|^2\right) \tag{3}$$

where ‖ ‖ denotes the Euclidean distance between the normalized vectors of two test data $\hat{\mathbf{x}}_i$ and $\hat{\mathbf{x}}_j$, n is the number of all test data, r_c is a user-specified parameter that indicates the radius of influence for cluster centers, and P_i is the potential for the ith test data. Thirdly, the test data point that has the largest potential is chosen as the center of the first cluster. This is because the highest potential occurs for the point with the largest number of surrounding test data. Once a cluster is determined, the potential values of all test data are subtracted by the following amount:

$$P_i \leftarrow P_i - P_k^* \cdot \exp\left(\frac{-4}{\bar{r}_c^2} \cdot \left\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_k^*\right\|^2\right) \tag{4}$$

where $\hat{\mathbf{x}}_k^*$ and P_k^* are the center coordinates and the potential value of the kth cluster, respectively, and $\bar{r}_c$ is assumed to be equal to $1.25r_c$. Again, the next cluster center is the one with the highest subtracted potential. The procedure is repeated until the subtracted potentials fall below some user-specified thresholds. The method of subtraction from potentials ensures that the test data located very near a cluster center are not eligible to become another cluster center since their potentials are much reduced. Thereby, the cluster centers are distributed within the whole database. It is important to notice that the number of clusters obtained by subtractive clustering depends on the user-specified value of r_c. Later, the criteria for optimum selection of r_c will be described.

3.2 Optimization of the FIS model

Once the cluster centers of a test database are determined by use of subtractive clustering algorithm, a FIS model is identified following the previously mentioned explanations. It is obvious that the number of IF–THEN rules in the FIS model is equal to the number of clusters. The final output of the FIS model is derived as weighted average of outputs for all rules. For example, for the strength model in this paper and for a given input vector denoted by $[b, r, f'_{co}, t_f, f_f, E_f]$, the model output, i.e., f'_{cc}, is calculated as follows:

$$f'_{cc} = \frac{\sum_{i=1}^{c} w_i f'_{cci}}{\sum_{i=1}^{c} w_i} \tag{5}$$

$$f'_{cci} = a_{0i} + a_{1i}b + a_{2i}r + a_{3i}f'_{co} + a_{4i}t_f + a_{5i}f_f + a_{6i}E_f, \quad i = 1\ldots c \tag{6}$$

where c is the number of rules, and $a_{0i}, a_{1i}, \ldots$ are unknown constants, which are determined during training the model by minimizing the error between the model output and the corresponding experimental value using the least-squares optimization procedure. Other notations in Eq. (6) are explained in Table 1. It should be noted that Eq. (6), which expresses the consequent as a linear combination of all input variables, develops a particular type of FIS called Takagi–Sugeno model [47]. The degree of truth for the ith rule (i.e., w_i) is determined by an exponential function of the Euclidean distance from the ith cluster center to a given input data, as follows:

$$
\begin{aligned}
w_i = \exp\Bigg\{ \frac{-4}{r_c^2} \Bigg[& \left(\frac{b - b_i^*}{b_{max} - b_{min}}\right)^2 + \left(\frac{r - r_i^*}{r_{max} - r_{min}}\right)^2 \\
+ & \left(\frac{f'_{co} - f'^*_{coi}}{f'_{co,max} - f'_{co,min}}\right)^2 + \left(\frac{t_f - t_{fi}^*}{t_{f,max} - t_{f,min}}\right)^2 \\
+ & \left(\frac{f_f - f_{fi}^*}{f_{f,max} - f_{f,min}}\right)^2 + \left(\frac{E_f - E_{fi}^*}{E_{f,max} - E_{f,min}}\right)^2 \Bigg] \Bigg\}
\end{aligned} \tag{7}
$$

where the variables with a star indicate the cluster center's coordinates. The denominator of each term in Eq. (7) is the range of values in the test database for a given input variable. In the context of FIS formulations, Eq. (7) refers to the fuzzy rule antecedent with Gaussian membership functions. With reference to the above-mentioned equations, Appendices 1 and 2 present the closed-form relations for the FIS models proposed in this paper.

4 Development of FIS model for compressive strength and ultimate axial strain of FRP-confined square column

4.1 Selection of input variables

A crucial step in developing a FIS model is to identify all input variables affecting the problem in hand. As for FRP-wrapped concrete, the main parameters that are believed to be influential in evaluating the strength/strain capacity are as follows: b, r, f'_{co}, t_f, f_f, and E_f. It should be noted that all above-mentioned input variables are independent from each other. In other words, any other variable that is dependent on the considered variables should not be taken in the list of inputs for the prospective FIS model because its effect has already been captured through the existing variables. For instance, there is no need to include the maximum tensile strain of FRP as it can be derived from two other existing input variables, namely f_f divided by E_f.

It should be emphasized that in the FIS approach, there is no need to assume a pre-defined mathematical expression so as to incorporate the input variables into the model. Instead, the FIS approach itself captures each variable's effect on the output without requiring a priori knowledge. This is the difference between FIS modeling and regression-based equations as the latter need to assume an arbitrary format for the equation with a finite number of unknown constants, which are tuned with respect to the test results. Since the form of this equation is generally not known, the performance of regression models depends to some extent on making assumption about the form of the function being used. By looking at the available confinement models that are mostly based on regression, it is observed that a wide variety of forms for functions have been used so far by different researchers. Contrary to regression-based models, the FIS model starts from scratch by adopting only the list of input variables without assuming any function format for them.

4.2 The optimum value for radius of influence of cluster centers

Another major step toward establishing a FIS model is to identify the cluster centers that are to be extracted from the experimental database. To do so, it is required to specify a value for r_c (i.e., the cluster's radius), and then, the cluster centers are readily determined by use of subtractive clustering algorithm. As stated earlier, the user-specified parameter r_c affects the number and coordinates of the cluster centers that will be identified during subtractive clustering process. Basically, a larger value of r_c leads to fewer number of clusters. It is recalled that in the FIS approach presented herein the number of clusters and number of rules are the same. Thereby, a larger r_c corresponds to a fewer number of fuzzy rules, resulting in a simpler FIS model. A model is, however, accepted if it can demonstrate low values of error in predicting the model output. To examine the effect of varying r_c, several FIS models corresponding to different values for r_c are constructed and compared in terms of various error measures listed in Tables 5 and 6 for strength and strain, respectively. The error measures considered here include mean absolute percentage error (MAPE) between model outputs and test results, average and coefficient of variation (COV) for the ratio of the test value to the model output. These error measures for each r_c are reported in three separate groups: training, validation, and total data sets as listed in Tables 5 and 6. It should be noted that the test data are partitioned into two groups: training and validation data sets. The training data contain those experimental data that are utilized for tuning the FIS model parameters during training stage, whereas the

Table 5 Comparison of error measures for FIS models with different values of cluster's radius (for the compressive strength)

Cluster's radius (r_c)	No. of rules	Order of FIS model	Training data set			Validation data set			Total data set		
			Mean[a]	MAPE	COV (%)	Mean[a]	MAPE	COV (%)	Mean[a]	MAPE	COV (%)
0.80	4	Linear	1.0043	8.41	12.06	1.0453	10.97	24.54	1.0166	9.17	17.12
0.85	*3*	*Linear*	*1.0027*	*8.81*	*11.55*	*1.0346*	*11.59*	*14.35*	*1.0122*	*9.64*	*12.58*
0.90	3	Linear	1.0079	9.89	14.09	1.0217	11.61	13.62	1.0121	10.41	13.96
0.95	3	Linear	1.0067	9.78	13.45	1.0049	13.27	15.07	1.0062	10.77	13.92
0.45	11	Constant	0.9997	14.96	19.98	1.0217	15.85	19.65	1.0064	15.24	19.90
0.60	7	Constant	0.9996	14.70	18.85	1.0043	19.13	22.77	1.0010	16.04	20.13
0.75	5	Constant	0.9998	18.14	22.39	1.0264	21.55	25.39	1.0080	19.18	23.42
0.80	4	Constant	0.9982	19.17	23.67	1.0299	22.27	26.65	1.0077	20.09	24.68
0.85	3	Constant	0.9991	23.69	24.38	1.0780	20.60	25.70	1.0227	22.77	25.09

The row in italics refers to the finally selected model

[a] For ($f'_{cc,exp}/f'_{cc,pred}$), that is the ratio of experimental to predicted value for the compressive strength of confined concrete

Table 6 Comparison of error measures for FIS models with different values of cluster's radius (for the ultimate axial strain)

Cluster's radius (r_c)	No. of rules	Order of FIS model	Training data set			Validation data set			Total data set		
			Mean[a]	MAPE	COV (%)	Mean[a]	MAPE	COV (%)	Mean[a]	MAPE	COV (%)
0.75	4	Linear	1.0477	22.12	46.50	1.1107	78.46	89.14	1.0657	38.22	62.98
0.76	4	Linear	1.0508	22.22	48.48	1.1048	79.22	88.09	1.0662	38.51	63.37
0.80	*3*	*Linear*	*1.0280*	*36.78*	*37.93*	*1.0363*	*39.99*	*44.14*	*1.0304*	*37.70*	*39.83*
0.85	3	Linear	1.0327	39.08	41.07	1.1725	35.82	54.24	1.0714	38.18	46.29
0.90	3	Linear	1.0366	39.27	43.30	1.1795	36.29	57.14	1.0762	38.45	48.78
0.41	12	Constant	1.0058	44.28	43.01	1.0795	52.71	54.58	1.0302	47.07	47.67
0.60	10	Constant	0.9981	45.06	38.21	1.0281	87.88	64.71	1.0070	57.68	47.95
0.70	7	Constant	0.9995	44.35	44.93	0.8572	123.70	61.38	0.9639	64.19	49.13
0.75	4	Constant	1.0000	55.74	46.40	1.0670	75.09	60.50	1.0191	61.27	51.34
0.80	3	Constant	1.0000	55.75	46.31	1.0671	75.36	60.45	1.0192	61.35	51.27

The row in italics refers to the finally selected model

[a] For ($\varepsilon_{cu,exp}/\varepsilon_{cu,pred}$), that is the ratio of experimental to predicted value for the ultimate axial strain of confined concrete

Table 7 Statistical comparison between the training and validation data sets used for the FIS model of the compressive strength

	Training data set					Validation data set				
	Min	Max	Range	Mean	COV (%)	Min	Max	Range	Mean	COV (%)
b (mm)	100	458	358	161.65	26.93	100	358	358	154.94	26.03
r (mm)	0	60	60	28.17	63.11	0	60	60	24.93	55.98
f'_{co} (MPa)	6.8	56.7	49.9	36.36	32.44	18.3	56.7	38.4	34.87	30.19
t_f (mm)	0.122	5.04	4.918	0.494	126.32	0.117	5.04	4.923	0.672	128.31
f_f (MPa)	220	4519	4299	3518.13	33.64	230	4519	4289	3086.73	47.43
E_f (GPa)	10.5	640	629.5	202.44	36.15	13.6	640	626.4	183.97	53.56
$f'_{cc,exp}$ (MPa)	11.1	90.9	79.8	49.48	30.49	25.4	95.2	69.8	50.42	28.22

Table 8 Statistical comparison between the training and validation data sets used for the FIS model of the ultimate axial strain

	Training data set					Validation data set				
	Min	Max	Range	Mean	COV %	Min	Max	Range	Mean	COV %
b (mm)	108	458	350	159.25	24.06	108	458	350	161.49	35.35
r (mm)	5	60	55	31.11	46.38	5	60	55	30.43	61.45
f'_{co} (MPa)	20	54.7	34.7	37.34	26.56	20	54.7	34.7	35.67	28.85
t_f (mm)	0.117	5.04	4.923	0.803	123.31	0.122	3.78	3.658	0.839	114.57
f_f (MPa)	230	4519	4289	2955.45	52.33	220	4519	4299	2667.47	66.07
E_f (GPa)	13.6	257	243.4	167.60	51.30	10.5	257	246.5	149.07	65.84
$\varepsilon_{cu,exp}$	0.00117	0.0395	0.03833	0.01364	48.20	0.0023	0.0387	0.0364	0.0146	64.14

validation data include new data that have not yet been introduced to the FIS model. In this study, almost 70% of all experimental data is used for training data set, and the rest is assigned to the validation data set. Tables 7 and 8 summarize the statistical information on both training and validation data for strength and strain, respectively.

The best value of r_c is the one whose corresponding FIS model yields the least error measures for the model output.

In this regard, several error measures including mean, MAPE, and COV, which are calculated based on the total experimental data, are evaluated in order to become optimum. Furthermore, the errors calculated for the validation data should not be much different from those obtained for the training data in order to maintain the capability of the proposed model to generalize the results. In other words, if the performance of a model is good on the training data

while being poor on the validation data, it is concluded that the model is so much fit to the training data (what is called as overfitting), and thus, it is no longer able to generalize the results. In the case that more than one FIS model perform similarly in terms of the error measures, the simplest model with less number of rules/clusters is preferred. Tables 5 and 6 compare the performance of several FIS models corresponding to different values of r_c. In each of Tables 5 and 6, two categories of FIS models are presented: linear and constant, which refer to the first-order and zeroth-order Takagi–Sugeno FIS models, respectively. According to the above-mentioned criteria and with reference to Tables 5 and 6, the final values of r_c are selected to be equal to 0.85 and 0.8 for the strength and strain FIS models, respectively. These selected values for r_c result in the FIS models whose performances are optimum in terms of minimizing all error measures simultaneously. Both of the proposed FIS models are linear and contain three rules/clusters.

5 Results and discussion

5.1 Proposal of closed-form equations for compressive strength/ultimate strain

A major drawback for most of soft computing models is that they do not provide closed-form equations for the problem in question. That is why soft computing paradigms are sometimes referred to as black-box models having invisible formulations. This may also limit their application in practice although their prediction accuracy is usually superior to the existing conventional models. To overcome such disadvantages associated with soft computing, an objective of this paper is to facilitate proposal of closed-form equations by employing subtractive clustering algorithm as a basis to develop FIS models. The form of the proposed equations in this work may not be as simple as what usually appears in the design codes, but instead they shed light on the underlying formulations of the proposed FIS model. Besides, the proposed equations can be easily programmed by design engineers to evaluate the strength and strain capacity of square concrete columns wrapped with FRP with an accuracy higher than that of the available confinement models.

As explained in the last section, the final values of r_c (i.e., cluster's radius) for the optimum FIS models were determined as 0.85 and 0.80 for the strength and strain FIS models, respectively. Given the value of r_c, subtractive clustering algorithm instantly identifies the cluster centers, which can be extracted from the available experimental data. Tables 9 and 10 list the input coordinates of the three cluster centers identified for the strength and strain FIS

Table 9 Parameters of the proposed FIS model for the compressive strength

Cluster center no.	Input coordinates of cluster center						Coefficients of consequents for the FIS rules						
	b (mm)	r (mm)	f'_{co} (MPa)	t_f (mm)	f_f (MPa)	E_f (GPa)	a_0	a_1	a_2	a_3	a_4	a_5	a_6
1	150	30	35.3	0.17	4192	229	14.98233	−0.19553	0.35756	0.85775	43.17073	4.50E−03	−0.03022
2	150	60	52	0.33	3788	226	−427.57088	0.55918	0.40019	−0.04986	152.02143	−1.61E−02	1.83023
3	150	25	31.8	1.2	935	75.1	25.03563	−0.06989	0.51825	0.45886	3.16580	−2.89E−03	0.06298

Table 10 Parameters of the proposed FIS model for the ultimate axial strain

Cluster center no.	Input coordinates of cluster center						Coefficients of consequents for the FIS rules						
	b (mm)	r (mm)	f'_{co} (MPa)	t_f (mm)	f_f (MPa)	E_f (GPa)	a_0	a_1	a_2	a_3	a_4	a_5	a_6
1	150	30	33	0.17	4364	219	0.01919	0.000164	−0.00019	−0.00161	0.00574	1.60E−05	−0.00021
2	152	25	42	0.9	1265	82.7	0.02724	−4.11E−05	3.34E−05	−1.66E−04	−0.00033	4.51E−06	−1.11E−04
3	150	45	53	0.17	3788	226	−0.04124	6.01E−04	−0.00036	−7.68E−04	0.00770	2.37E−05	−2.85E−04

models, respectively. Also, the consequent parameters (i.e., a_0, a_1, a_2, … in Eq. 6) for the proposed FIS models are reported in Tables 9 and 10. The consequent parameters were determined following a linear optimization procedure as explained earlier. Information provided in Tables 9 and 10 is sufficient to express the explicit formulation of the proposed FIS models, as presented in Appendices 1 and 2 for the strength and strain capacity of square columns wrapped with FRP, respectively. In addition, numerical examples are included in the Appendices 1 and 2. Moreover, Figs. 2 and 3 illustrate the membership functions for the strength and strain FIS models, respectively.

5.2 Comparison between the proposed FIS models and experimental data

Predictions by the proposed FIS models are compared against the test results in Figs. 4 and 5 for the strength and strain, respectively. It is evident that the accuracy of the proposed FIS models with as few as three rules is quite satisfactory as the model predictions agree with the test data. Besides, in Figs. 4 and 5, the scatter of data points around the diagonal line, where the experimental value equals the predicted value by the FIS model, for validation data is as good as that for training data, indicating the generalization ability of the proposed FIS models.

5.3 Comparison between the proposed FIS models and existing confinement models

Eight existing models for the compressive strength [15–22] and four models for the ultimate axial strain [16, 18, 19, 21] of FRP-confined square columns were considered herein for comparison with the FIS models proposed in this paper. Tables 2 and 3 present the equations for the existing models for the strength and strain, respectively, which relate strength/strain to the confining stress f_l exerted by FRP wraps. Different researchers have suggested various formulations for lateral confining pressure for square sections as listed in Table 4.

Tables 11 and 12 report several error measures for the proposed FIS models and the existing confinement models suggested by other researchers for the strength and strain, respectively. All models are evaluated by the database of FRP-confined square specimens containing 261 test results on the compressive strength and 112 test results for the ultimate strain. It is evident from Tables 11 and 12 that both of the proposed FIS models for strength/strain outperform all other existing models when taking into account all error measures simultaneously. In particular, the proposed FIS model for the ultimate axial strain demonstrates a higher accuracy against the other available models. The calculated mean for the proposed strain model is equal to
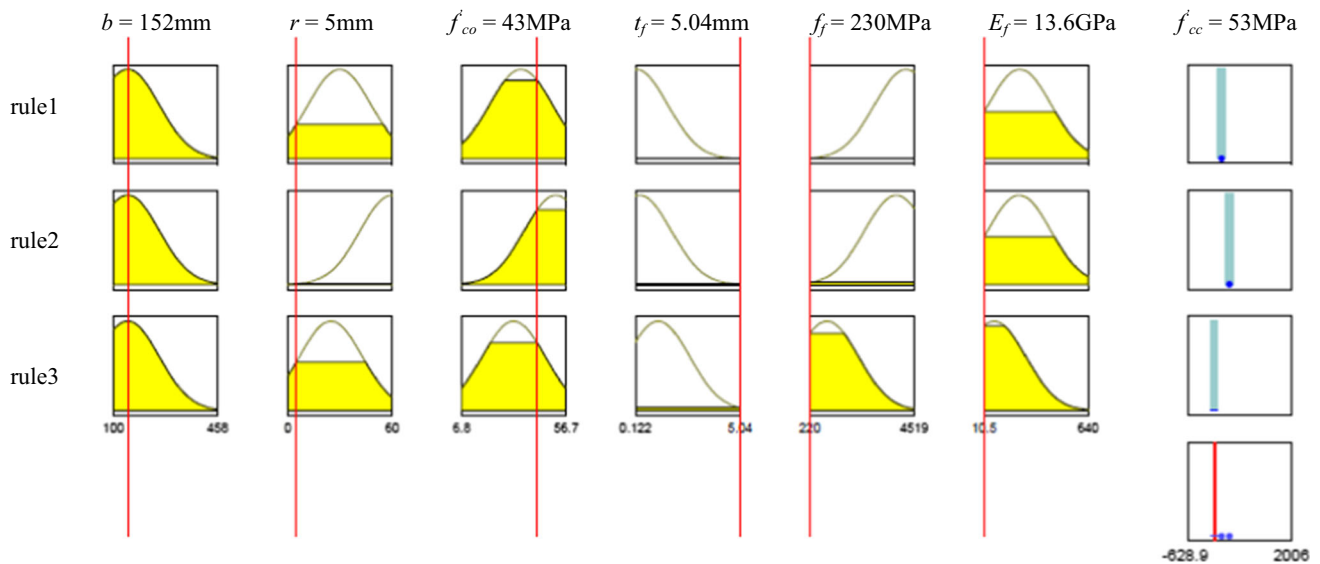
Fig. 2 A schematic diagram of the proposed FIS model for the compressive strength (a numerical example is also illustrated)
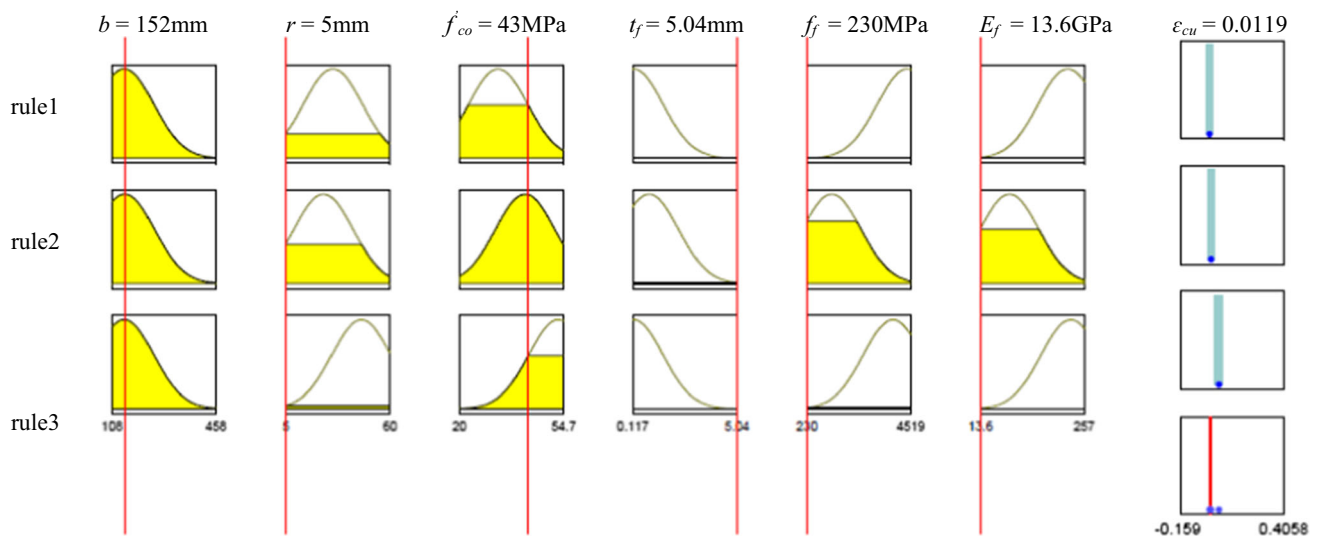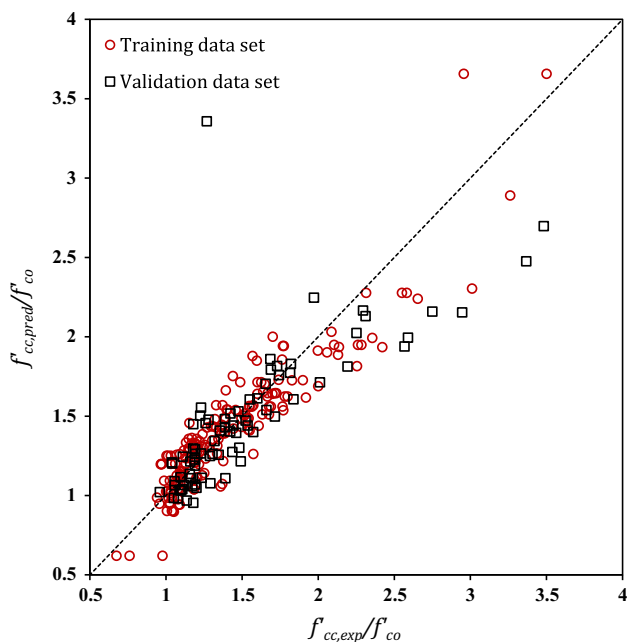

Fig. 3 A schematic diagram of the proposed FIS model for the ultimate axial strain (a numerical example is also illustrated)

1.03 with COV of about 40%, whereas mean of the best existing models ranges 1.17–1.47 with COV of approximately 60–67%. The predictions by the proposed strength/strain FIS models versus other existing models are depicted in Figs. 6 and 7. It is observed from Figs. 6 and 7 that the proposed FIS models develop a superior performance by demonstrating the least scatter of data points around the diagonal line as compared with the other models.

5.4 Parametric study on influence of stiffness and strength of FRP wraps

A parametric investigation is carried out using the proposed FIS models to evaluate the effect of the elastic modulus, E_f, as well as the tensile strength, f_f, of the FRP material on the confined strength/strain capacity. To do so, the parameter under question is varied within its range of values, while other input parameters are kept constant. Figures 8 and 9 show the FIS model outputs along with the predictions by other models for varied values of the elastic modulus and tensile strength of FRP materials, respectively. With reference to Fig. 8, the proposed FIS model demonstrates that a larger elastic modulus of FRP enhances the strength but reduces the strain capacity of the FRP-wrapped concrete. This result agrees with the mechanics of confinement and can be explained as follows. By increasing the elastic modulus of FRP wraps, the lateral stiffness of confining device increases, leading to

Fig. 4 Prediction by the FIS model versus experimental values for the compressive strength

Fig. 5 Prediction by the FIS model versus experimental values for the ultimate axial strain

Table 11 Comparison of error measures for the proposed FIS model and existing models for the compressive strength

Model	MAPE (%)	For $f'_{cc,exp}/f'_{cc,pred}$	
		Mean	COV (%)
Mirmiran et al.	13.83	1.063	18.80
Campione and Miraglia	11.85	1.097	17.11
Al-Salloum	17.48	0.929	18.91
Lam and Teng	11.30	1.000	15.28
Ilki et al.	19.74	0.860	13.26
Wu and Wang	12.07	0.970	13.89
Shehata et al.	14.66	1.031	20.37
Kumutha et al.	14.82	1.005	19.93
Proposed FIS model	9.64	1.012	12.58

Table 12 Comparison of error measures for the proposed FIS model and existing models for the ultimate axial strain

Model	MAPE (%)	For $\varepsilon_{cu,exp}/\varepsilon_{cu,pred}$	
		Mean	COV (%)
Campione and Miraglia	58.64	3.036	63.72
Lam and Teng	47.44	1.465	59.58
Ilki et al.	104.57	0.714	59.69
Shehata et al.	66.81	1.169	67.32
Proposed FIS model	37.70	1.030	39.83

materials used for confinement. Therefore, the FRP materials having lower elastic modulus such as glass FRP (GFRP) are more effective in enhancing the ultimate axial strain while being less effective in improving the compressive strength of FRP-wrapped concrete. In the available models for strength, the FRP elastic modulus E_f is not independently incorporated, but it is used for calculating the FRP tensile strength f_f (or the effective ultimate strength) through multiplication by the FRP rupture strain (or the actual hoop strain). Since the FRP tensile strength f_f is kept constant in Fig. 8, the existing models do not vary with E_f despite the FIS model that is able to capture the impact of E_f on the compressive strength. As for the current strain models, only two of them take into account the effect of varying E_f, namely Lam and Teng [18] and Campione and Miraglia [16]. The FIS model predictions for the ultimate axial strain fall in between the two aforementioned existing models, as illustrated in Fig. 8. In regard to the effect of varying f_f in the proposed FIS models, Fig. 9 illustrates that the higher values of f_f result in improvement of strength as well as strain capacity. This conclusion is consistent with the general trends of the available models as depicted in Fig. 9.

exert a larger value of confining pressure on the concrete. As a result, the compressive strength is improved by changing the uniaxial stress state to a triaxial one. On the other hand, the higher lateral stiffness limits dilation of the concrete subjected to compression, and hence, the axial strain decreases owing to Poisson's effect. It is noted that the elastic modulus is an indicator of type of FRP
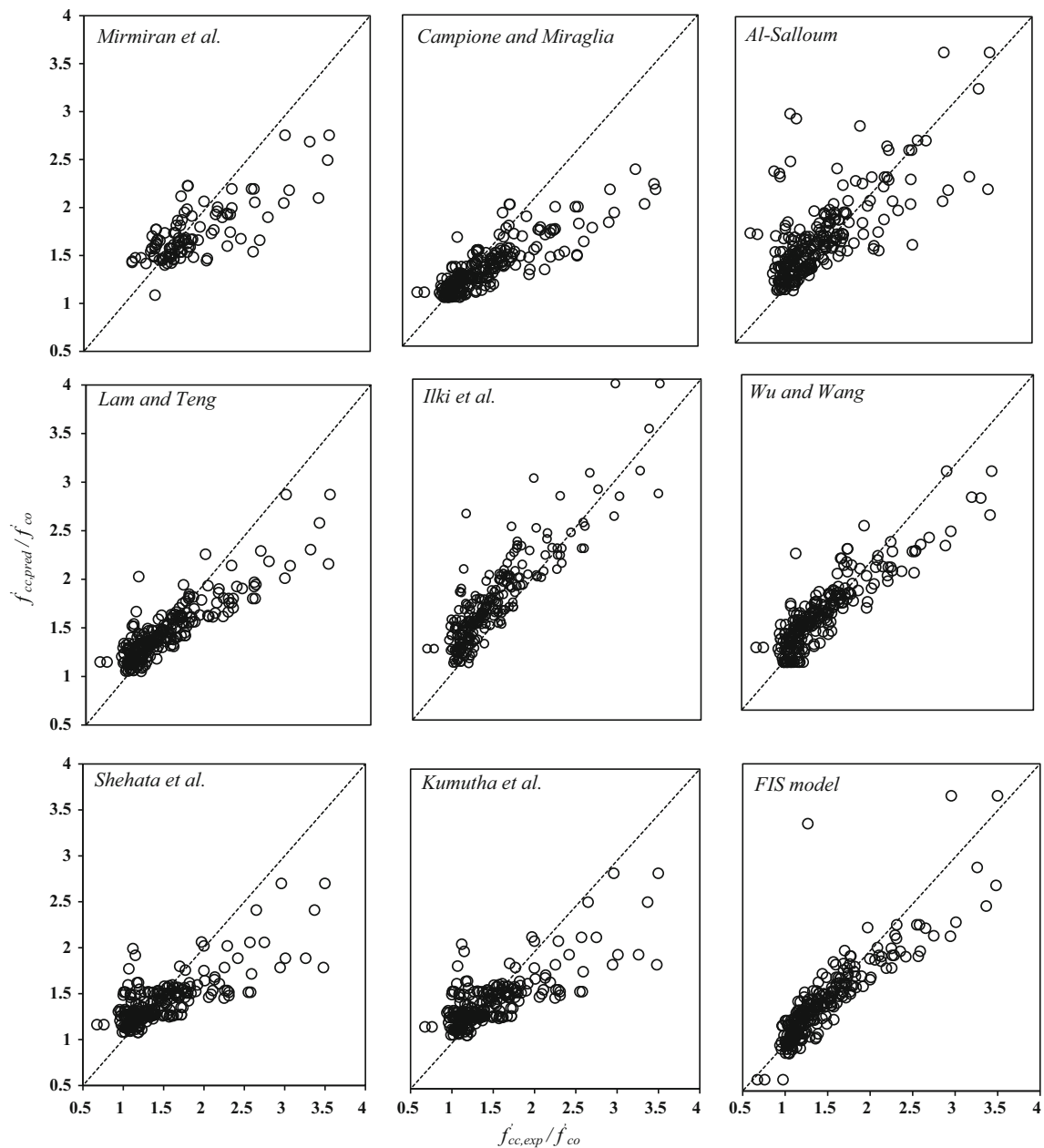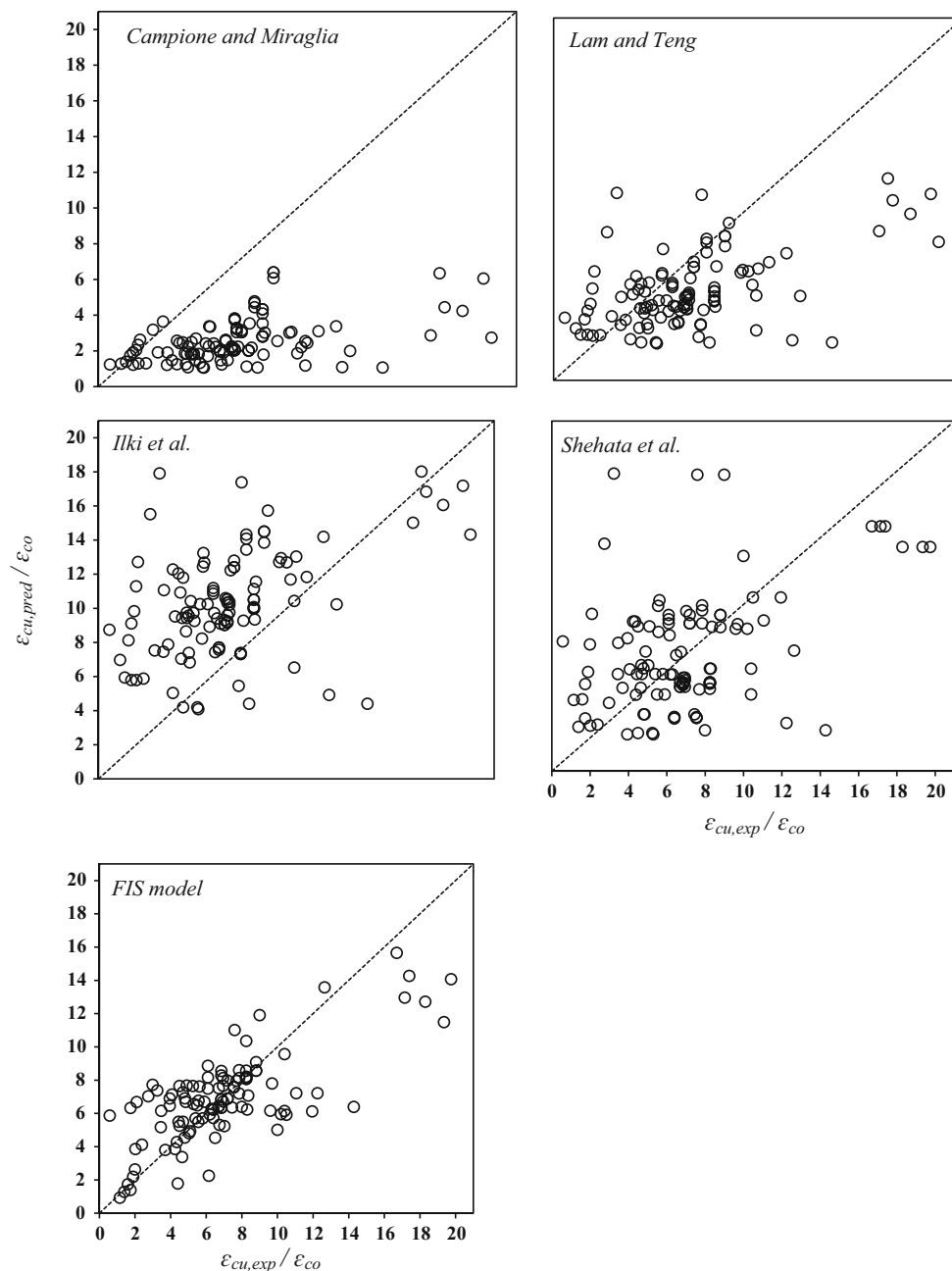
Fig. 6 Performance of the FIS model versus existing models for the compressive strength

5.5 Parametric study on influence of the corner radius of square section

It is well known that a larger corner radius improves the confinement effects in non-circular sections by increasing the effectively confined parts of section. In the existing strength/strain models, the influence of r is considered through introducing a coefficient called shape factor, which is the ratio of the area of effectively confined concrete to the total area of section. There are various

formulations proposed for shape factor by researchers so far. In our proposed FIS models, although no input variable is introduced as a shape factor, the effect of the corner radius can be captured through the FIS approach. Figure 10 presents the effect of different values of the corner radius on the strength/strain capacity as predicted by the proposed FIS models and other existing models. It is observed from Fig. 10 that the proposed FIS model is capable of taking into account the beneficial effect due to increasing the corner radius.

Fig. 7 Performance of the FIS model versus existing models for the ultimate axial strain

6 Conclusions

In this paper, fuzzy inference system (FIS) has been applied to propose two models for estimating the compressive strength and ultimate axial strain of square concrete columns wrapped with FRP. In order to identify FIS model, subtractive clustering algorithm has been used and performances of several Takagi–Sugeno FIS models with different numbers of rules have been examined. Predictions by the proposed FIS models have been compared with

those by the available experimental data and the existing confinement models. The conclusions are as follows:

1. Both of the proposed FIS models for strength and strain are expressed as closed-form formulations, which are simple to use in practice (see Appendices 1 and 2). The proposed FIS models with as few as three fuzzy rules offer a reliable alternative method to calculate the compressive strength and ultimate axial strain of FRP-wrapped square sections.
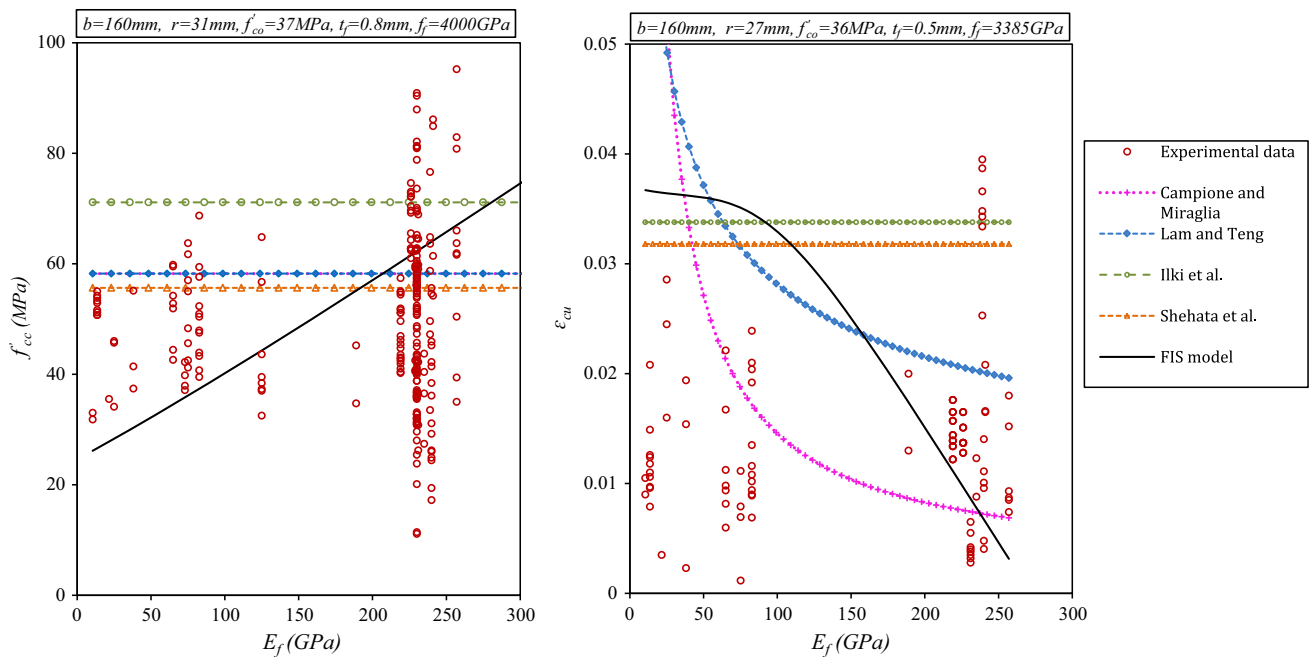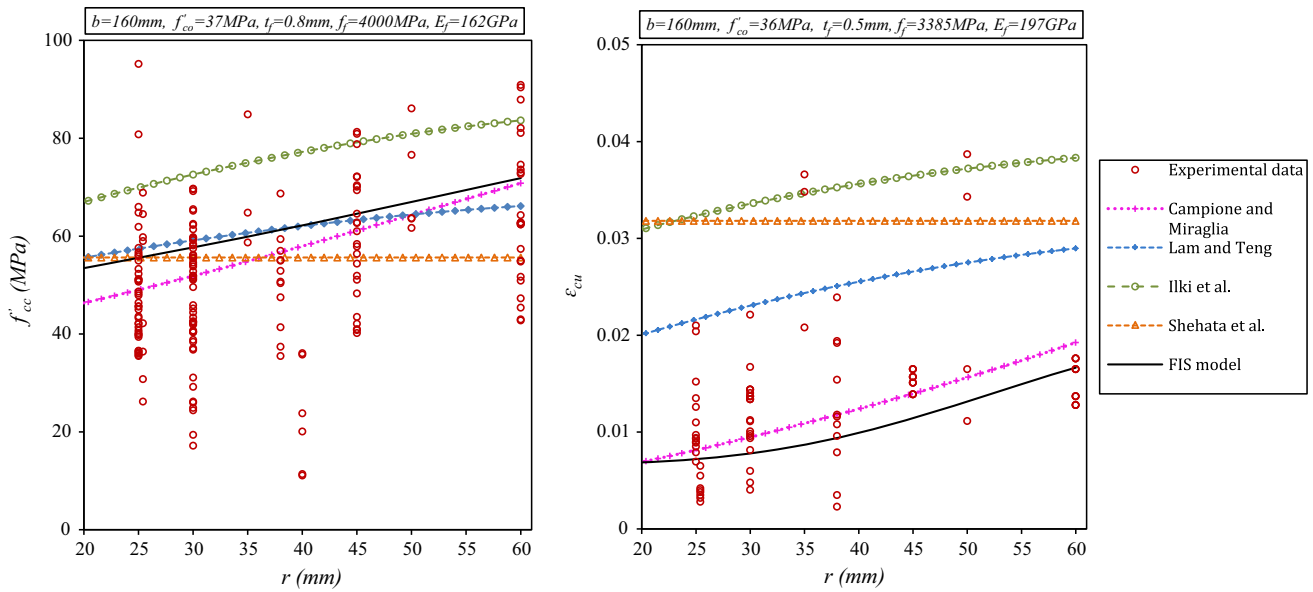
Fig. 8 Influence of different values of FRP elastic modulus on strength/strain of FRP-confined concrete
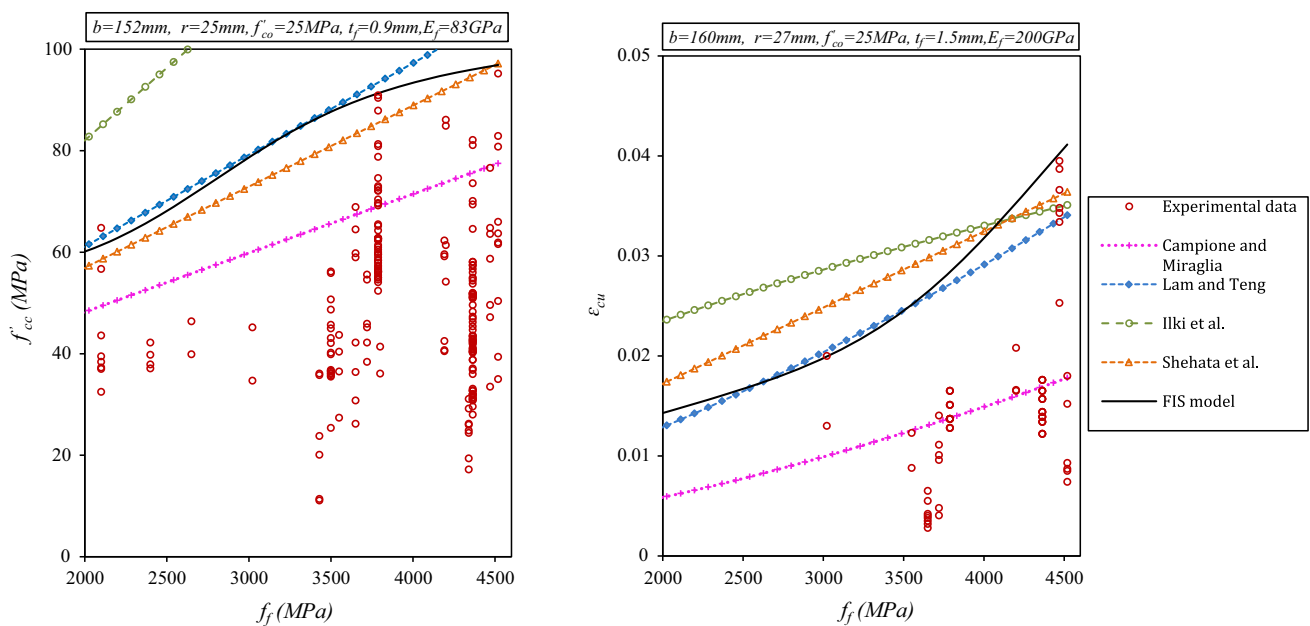
Fig. 9 Influence of different values of FRP tensile strength on strength/strain of FRP-confined concrete

2.   Comparison between the two proposed FIS models and a large database of axially tested FRP-wrapped square specimens containing 261 test results on the compressive strength and 112 test results for the ultimate strain indicates that predictions using the proposed FIS models quite agree with the test data. Besides, the performance of the FIS models on an unfamiliar data set is shown to be as good as that on the familiar data set used for model training, confirming the generalization ability of the proposed FIS models.

3.   The proposed FIS models outperform the existing models for the strength and strain capacity of square columns confined by FRP. The experimental-to-predicted ratio of the strength or strain for the proposed FIS models has an average value of about unity. Furthermore, the coefficient of variation for the proposed FIS models is less than that of the other existing models.

4.   In light of the proposed FIS models, the influences of elastic modulus along with tensile strength of FRP

Fig. 10 Influence of different values of section corner radius on strength/strain of FRP-confined concrete

materials used for confinement are examined through parametric studies. The results reveal that the proposed FIS models are capable of capturing the underlying mechanics of FRP confinement. According to the proposed FIS approach, a higher FRP elastic modulus enhances the compressive strength but reduces the ultimate axial strain. Also, the predictions of the FIS models for the effect of FRP tensile strength on the strength/strain are consistent with the general trends of the existing models. Besides, the proposed FIS models for strength and strain are capable of predicting the beneficial effects of increasing the corner radius of square sections confined with FRP wraps although the commonly used shape factor is not introduced as an input variable in the proposed model, meaning that the FIS approach itself can account for such an effect.

Based on the attained results, the FIS approach is successfully applied to model the confinement effects in square concrete columns wrapped by FRP. It is noteworthy that applicability of the FIS models is limited to the numerical domain of input parameters used in this study. It is, however, unlikely that the FIS models are capable of extrapolating beyond this domain. Further research may be focused on extending application of FIS modeling to other fields of structural use of FRP composites.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix 1: Closed-form formulation of the proposed FIS model for compressive strength of FRP-confined square columns

$$w_1 = \exp\left(\frac{-4}{0.85^2}\left\{\left(\frac{b-150}{358}\right)^2 + \left(\frac{r-30}{60}\right)^2 + \left(\frac{f'_{co}-35.3}{49.9}\right)^2 \right.\right.$$
$$\left.\left. + \left(\frac{t_f-0.17}{4.923}\right)^2 + \left(\frac{f_f-4192}{4299}\right)^2 + \left(\frac{E_f-229}{629.5}\right)^2\right\}\right)$$

$$w_2 = \exp\left(\frac{-4}{0.85^2}\left\{\left(\frac{b-150}{358}\right)^2 + \left(\frac{r-60}{60}\right)^2 + \left(\frac{f'_{co}-52}{49.9}\right)^2 \right.\right.$$
$$\left.\left. + \left(\frac{t_f-0.33}{4.923}\right)^2 + \left(\frac{f_f-3788}{4299}\right)^2 + \left(\frac{E_f-226}{629.5}\right)^2\right\}\right)$$

$$w_3 = \exp\left(\frac{-4}{0.85^2}\left\{\left(\frac{b-150}{358}\right)^2 + \left(\frac{r-25}{60}\right)^2 + \left(\frac{f'_{co}-31.8}{49.9}\right)^2 \right.\right.$$
$$\left.\left. + \left(\frac{t_f-1.2}{4.923}\right)^2 + \left(\frac{f_f-935}{4299}\right)^2 + \left(\frac{E_f-75.1}{629.5}\right)^2\right\}\right)$$

$$f'_{cc1} = -0.19553b + 0.35756r + 0.85775f'_{co}$$
$$+ 43.17073t_f + 0.0045f_f - 0.03022E_f$$
$$+ 14.98233$$

$$f'_{cc2} = 0.55918b + 0.40019r - 0.04986f'_{co}$$
$$+ 152.02143t_f - 0.0161f_f + 1.83023E_f$$
$$- 427.57088$$

$$f'_{cc3} = -0.06989b + 0.51825r + 0.45886f'_{co}$$
$$+ 3.16580t_f - 0.0029f_f + 0.06298E_f$$
$$+ 25.03563$$

$$f'_{cc} = \frac{\sum_{i=1}^{3} w_i f'_{cci}}{\sum_{i=1}^{3} w_i}.$$

Example Let us consider a column with $b = 152$ mm, $r = 5$ mm, $f'_{co} = 43$ MPa, $t_f = 5.04$ mm, $f_f = 230$ MPa, $E_f = 13.6$ GPa. This gives: $w_1 = 7.1e-6$, $w_2 = 6e-7$, $w_3 = 0.01151$, $f'_{cc1} = 242$ MPa, $f'_{cc2} = 445$ MPa, $f'_{cc3} = 52.9$ MPa. Then, the confined compressive strength by the FIS model is calculated as $f'_{cc} = 53$ MPa (the experimental value: $f'_{cc,exp} = 54.2$ MPa).

Appendix 2: Closed-form formulation of the proposed FIS model for ultimate axial strain of FRP-confined square columns

$$w_1 = \exp\left(\frac{-4}{0.80^2}\left\{\left(\frac{b-150}{350}\right)^2 + \left(\frac{r-30}{55}\right)^2 + \left(\frac{f'_{co}-33}{34.7}\right)^2\right.\right.$$
$$\left.\left.+ \left(\frac{t_f-0.17}{4.923}\right)^2 + \left(\frac{f_f-4364}{4299}\right)^2 + \left(\frac{E_f-219}{246.5}\right)^2\right\}\right)$$

$$w_2 = \exp\left(\frac{-4}{0.80^2}\left\{\left(\frac{b-152}{350}\right)^2 + \left(\frac{r-25}{55}\right)^2 + \left(\frac{f'_{co}-42}{34.7}\right)^2\right.\right.$$
$$\left.\left.+ \left(\frac{t_f-0.9}{4.923}\right)^2 + \left(\frac{f_f-1265}{4299}\right)^2 + \left(\frac{E_f-82.7}{246.5}\right)^2\right\}\right)$$

$$w_3 = \exp\left(\frac{-4}{0.80^2}\left\{\left(\frac{b-150}{350}\right)^2 + \left(\frac{r-45}{55}\right)^2 + \left(\frac{f'_{co}-53}{34.7}\right)^2\right.\right.$$
$$\left.\left.+ \left(\frac{t_f-0.17}{4.923}\right)^2 + \left(\frac{f_f-3788}{4299}\right)^2 + \left(\frac{E_f-226}{246.5}\right)^2\right\}\right)$$

$$\varepsilon_{cu1} = 0.000164b - 0.00019r - 0.00161f'_{co}$$
$$+ 0.00574t_f + 1.60 \times 10^{-5}f_f - 0.00021E_f$$
$$+ 0.01919$$

$$\varepsilon_{cu2} = -4.11 \times 10^{-5}b + 3.34 \times 10^{-5}r - 1.66 \times 10^{-4}f'_{co}$$
$$- 0.00033t_f + 4.51 \times 10^{-6}f_f - 1.11 \times 10^{-4}E_f$$
$$+ 0.02724$$

$$\varepsilon_{cu3} = 6.01 \times 10^{-4}b - 0.00036r - 7.68 \times 10^{-4}f'_{co}$$
$$+ 0.00770t_f + 2.37 \times 10^{-5}f_f - 2.85 \times 10^{-4}E_f$$
$$- 0.04124$$

$$\varepsilon_{cu} = \frac{\sum_{i=1}^{3} w_i \varepsilon_{cui}}{\sum_{i=1}^{3} w_i}.$$

Example Let us consider a column with $b = 152$ mm, $r = 5$ mm, $f'_{co} = 43$ MPa, $t_f = 5.04$ mm, $f_f = 230$ MPa, $E_f = 13.6$ GPa. This gives: $w_1 = 1.5e-8$, $w_2 = 0.00223$, $w_3 = 6.4e-9$, $\varepsilon_{cu1} = 0.00379$, $\varepsilon_{cu2} = 0.01187$, $\varepsilon_{cu3} = 0.05567$. Then, the confined ultimate axial strain by the FIS model is calculated as $\varepsilon_{cu} = 0.0119$ (the experimental value: $\varepsilon_{cu,exp} = 0.0124$).

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