

A standard deviation based firefly algorithm for multi-objective optimization of WEDM process during machining of Indian RAFM steel

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Abstract Non-conventional machining processes always suffer due to their low productivity and high cost. However, a suitable machining process should improve its productivity without compromising product quality. This implies the necessity to use efficient multi-objective optimization algorithm in non-conventional machining processes. In this present paper, an effective standard deviation based multi-objective fire-fly algorithm is proposed to predict various process parameters for maximum productivity (without affecting product quality) during WEDM of Indian RAFM steel. The process parameters of WEDM considered for this study are: pulse current (I), pulse-on time (T_{on}), pulse-off time (T_{off}) and wire tension (WT). While, cutting speed (CS) and surface roughness (SR) were considered as machining performance parameters. Mathematical models relating the process and response parameters had been developed by linear regression analysis and standard deviation method was used to convert this multi objective into single objective by unifying the responses. The model was then implemented in firefly algorithm in order to optimize the process parameters. The computational results depict that the proposed method is well capable of giving optimal results in WEDM process and is fairly superior to the two most popular evolutionary algorithms (particle swarm optimization algorithm and differential evolution algorithm) available in the literature.

Keywords Multi-objective optimization · Standard deviation method · Firefly algorithm · Wire cut EDM · Reduced activation ferritic martensitic steel

List of symbols

I	Pulse current
T_{on}	Pulse-on time
T_{off}	Pulse-off time
WT	Wire tension
CS	Cutting speed
min CS	The minimum value of cutting speed
max CS	The maximum value of cutting speed
SR	Surface roughness
min SR	The minimum value of surface roughness
max SR	The maximum value of surface roughness
FA	Firefly algorithm
PSO	Particle swarm optimization algorithm
DE	Differential evolution algorithm
RAFM	Reduced activation ferritic martensitic steel
WEDM	Wire electrical discharge machining
r	Distance between two fire-fly
$I(r)$	Light intensity at distance (r)
I_0	Original light intensity at zero distance
γ	Light absorption coefficient
β	Attractiveness measure at distance (r)
β_0	Original attractiveness at zero distance
$x_i^*(k)$	Normalized value of output parameter ‘ i ’ at k th experiment
$x_i^{(o)}(k)$	Experimental value of output parameter ‘ i ’ at k th experiment
$\min x_i^{(o)}(k)$	The minimum value of output parameter ‘ i ’
$\max x_i^{(o)}(k)$	The maximum value of output parameter ‘ i ’
v_i	Variances of normalized output parameter ‘ i ’
μ_i	Mean of all normalized experimental values (n) for output parameter ‘ i ’

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n	Number of experiments
w_1, w_2	Individual response weight of cutting speed and surface roughness respectively
N	Population size/size of the swarm
T	Number of iteration

1 Introduction

After being introduced in 1960 [1] the wire-cut electro discharge machining (EDM) has come a long way to become one of the most sophisticated machining processes of modern era. It is basically used to cut very hard conductive material and very difficult shape with great accuracy with the help of electric spark. It does not require a special shaped electrode; instead it uses a continuous-travelling vertical wire under tension as the electrode. Electro discharge machining is basically a spark erosion process where with each spark a small amount of workpiece material melts or vaporizes and washed away by the dielectric [2]. This machining process is a stochastic in nature as sparking itself is a stochastic phenomenon [3]. In electrical discharge machining, selection of machining parameters is very important to achieve high machining performance. Generally based on experience or handbook values the desired machining parameters are chosen. But this method of selecting machining parameters does not ensure optimal or near optimal machining performance for particular electrical discharge machine and environment [4].

In any machining process machining time and surface finish are the most important performance characteristics. In the case of WEDM also it is same. But the problem is that cutting speed and surface roughness are two very contradictory parameters for choosing the optimal process parameters. To solve this problem many researchers had carried out extensive research using various optimization techniques. Tarn et al. [3] used feed forward neural network and simulated annealing to get optimized WEDM process parameters for better machining speed and surface roughness taking SUS-304 stainless steel as workpiece material. Equal weightage were given to both the responses. Spedding and Wang [2] incorporated response surface method to model the cutting speed and surface roughness of WEDM and tried to optimize the process parameters by artificial neural network (ANN) technique. A regression model and feasible direction method was used by Liao et al. [5] to reduce the machining time while not compromising the surface quality of WEDM process. Lin et al. [6] proposed a control strategy based fuzzy logic for better machining accuracy in WEDM. Tosun et al. [7] developed a mathematical model for kerf and material removal rate (MRR) by regression analysis and used analysis of variance

(ANOVA) and signal to noise ratio (S/N ratio) to optimize the process parameters. Singh et al. [8] carried out a multi-objective optimization of EDM process parameters by using orthogonal array (OA) with Grey relational analysis. Sarkar et al. [9] investigated the wire cut electro discharge machining on γ -titanium aluminide alloy and modeled the process with additive model. They also tried to optimize the process parameters by constrain optimization and Pareto optimization algorithm. Kuriakose and Shunmugam [10] used multiple regression models to represent the relationship between input and output of WEDM. Optimization of the process parameters for cutting velocity and surface roughness was done by a multi-objective optimization method based on a non-dominated sorting genetic algorithm (NSGA). Chiang and Chang [11] deployed Taguchi based grey relational analysis to optimize the wire electric discharge machining (WEDM) process of Al_2O_3 particle reinforced material (6061 alloy) with multiple performance characteristics. Mahapatra and Patnaik [12] established the relationship between WEDM process parameters and responses like MRR, surface finish and kerf width by non-linear regression analysis. Genetic algorithm (GA) was used by them to optimize process parameters with multiple objectives. Artificial neural network (ANN) with back propagation algorithm with non-dominating sorting genetic algorithm-II was used by Mandal et al. [13] for multi-objective optimization of WEDM process. Kanagarajan et al. [14] studied the influence of various parameters of EDM of WC/CO composites on material removal rate and surface roughness. Process characteristics were modeled by second order polynomial equation and for optimization purpose non-dominated sorting genetic algorithm (NSGA-II) was used by them. Ramkrishnan and Karunamoorthy [15] used back propagation ANN algorithm to predict the response parameters like material removal rate and surface roughness for WEDM of Inconel 718 material. Sarkar et al. [16] correlated surface roughness, dimensional shift and cutting speed with various process parameters of WEDM of γ -TiAl alloy by response surface response surface methodology. In their study optimization of process parameters was done by desirability function approach and Pareto optimization algorithm. To predict the cutting speed and kerf width Saha et al. [17] deployed normalized radial basis function network (NRBFN) with enhanced k -means clustering technique for WEDM of 5 vol% TiC/Fe in situ metal matrix composite (MMC). Chen et al. [18] used back propagation neural network based simulated annealing algorithm to optimize the process parameters to get better cutting velocity and surface roughness for WEDM of pure tungsten. Neuro-Genetic technique; a combination of a radial basis function network (RBFN) and non-dominated sorting genetic algorithm (NSGAI) was deployed by Saha et al. [19] to optimize the process parameters for multi responses of WEDM

of WC/CO composites. Arindam Majumder [20] developed the relation between various process parameters with response parameter by response surface methodology (RSM) and then used genetic algorithm to minimize the electrode wear rate in EDM. Saha et al. [21] used neuro-genetic algorithm for multi-objective optimization of WEDM of 5 vol% titanium carbide (TiC) reinforced austenitic manganese steel metal matrix composite (MMC). The relation between multiple input and outputs of WEDM of AISI 316LN Stainless Steel was modeled by Majumder et al. [22] with the help of response surface methodology (RSM). They employed desirability based multi-objective particle swarm optimization (DMPSO) algorithm to optimize the process parameters for maximizing the MRR and Minimizing the EWR. From the literature it can be seen that various techniques is used to optimize the WEDM process parameters for multiple performance characteristics. However as per as machining is concerned WEDM does not depend upon the variety of material if they are conductive but optimum machining conditions varies with different workpiece material. In the other hand optimization techniques also can make a noticeable difference while finding the optimum machining conditions. Metaheuristic algorithms are very powerful in searching global optima for very difficult engineering and industrial problems. Especially they are proved to very efficient tool to solve multi-objective optimization problems [23, 24]. It has also been seen from the previous literature that the population base meta-heuristics algorithms performed better than the single point search meta-heuristics [25].

Currently firefly algorithm (FA) draws the attention of various researchers, working in different fields due to its flexibility to solve continuous problems, clustering and classifications, and combinatorial optimization problems very efficiently [26]. Apostolopoulos and Vlachos [27], proposed the firefly algorithm for multi-objective minimization problem of economic emissions load dispatch to minimize fuel cost and emission of generating units. The results show that firefly algorithm is much more accurate than other metaheuristic algorithms to find out the global optima with high success rates. Gandomi et al. [28] in their study to solve mixed variable structural optimization implemented firefly algorithm. The optimization results confirm the superiority firefly algorithm than other metaheuristic algorithms such as particle swarm optimization, genetic algorithm, simulated annealing and hunting search. Senthilnath et al. [29] used firefly algorithm for clustering and compared the results with artificial bee colony (ABC), particle swarm optimization (PSO) and other widely used algorithms. Their findings conclude that firefly algorithm is more worthy, efficient and successful to generate optimum result. Taleizadeh and Leopoldo [30] showed the applicability firefly algorithm in supply chain management

problems. Rao et al. [31] implemented firefly algorithm and bat algorithm for optimizing placement as well as sizing of static VAR compensator to enhance voltage stability. But yet a less effort has been given by the previous researchers in order to use this technique in the manufacturing field.

This paper investigated the optimum process parameters values for wire-electro discharge machining of Indian RAFM steel. RAFM steel is one of the newly developed materials profoundly used in nuclear power plant industry. To carry out the study, pulse current (I), pulse-on time (T_{on}), pulse-off time (T_{off}) and wire tension (WT) were taken as input process parameters, based upon the various literature available, as they are the very significant parameters for WEDM. These process parameters were than optimized with respect to the response parameters i.e. cutting speed and surface roughness. Mathematical models to relate the machining parameters with the response parameters had been developed by linear regression analysis. The adequacy of the mathematical models was checked by deploying analysis of variance (ANOVA). While, Standard deviation based technique was used to select individual weights for each responses during this study. Later, a firefly algorithm (FA) was employed to find out the optimum set of data for the described process parameters. The results thus obtained were then compared with the results obtained by particle swarm optimization algorithm (PSO) and differential evolution algorithm (DE) as well as the response parameters at the initial condition. Finally the optimized results were validated experimentally.

2 Regression analysis

Linear regression analysis is a statistical process. It estimates the relationships among variables. This method is extensively used in mathematical model building depending upon the relationship between the dependent and independent variables [32]. Linear regression technique consists of simple linear regression and multiple linear regression analysis. In simple linear regression only one independent variable can be used. But multiple linear regression analysis does not have such limits. That means, more than one independent variable can be used to explain the variation of dependent variable.

To describe a particular response or dependent variable using the independent ones general linear model for multiple regression analysis is used. General linear model and its assumption are shown below.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \epsilon \quad (1)$$

where y = response or independent variable; $\beta_0, \beta_1, \beta_2, \dots, \beta_n$ are unknown constants. x_1, x_2, \dots, x_n are independent variables; ϵ = random error.

Each value of y is deviate from the average y value by random error amount. Certain assumptions have to be taken into account.

1. ϵ values are independent.
2. ϵ values have a mean of 0 and a common variance σ^2 for any set x_1, x_2, \dots, x_n .
3. ϵ values are normally distributed.

When the above listed assumption met about random error (ϵ) the basic equation (deterministic general linear model) can be written as [33]

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \dots + \beta_nx_n. \tag{2}$$

3 Firefly algorithm (FA)

3.1 Basic foundation of firefly algorithm

The firefly algorithm was developed by Xin-She Yang. This is a meta-heuristic type algorithm, inspired by the social behavior of fireflies. Fireflies produce flash by virtue of bioluminescence [28]. The varied flashing light patterns are used to send courtship signal to other fireflies for mating. Firefly algorithm is based on the idealized behavior of the flashing characteristics of fireflies. To simplify, the flashing characteristics can be described by the following three rules:

1. As fireflies are unisex, so the attract each other regardless of their sex.
2. Attractiveness is defined by the brightness. With higher the brightness the firefly becomes more attractive. That means the lesser one attracts towards the brighter one.
3. With increase in distance the brightness and attractiveness decrease. If no one is brighter than a particular firefly, it will move randomly [34].

The pseudo code used to summarize the basic steps of firefly algorithm is shown below (Fig. 1).

3.2 Characteristic description of FA

Firefly algorithm basically depends upon the light intensity variation and formulation of attractiveness. In basic firefly algorithm the solution of the fitness function is defined by the light intensity. The light intensity varies with the distance (r) between two fireflies. The following equation shows the relation between light intensity and distance between two fireflies.

$$I(r) = I_0e^{-\gamma \times r^2} \tag{3}$$

where $I(r)$ is the light intensity at distance (r), I_0 is the original light intensity at $r = 0$ and γ is the light absorption

```

begin
  Call objective function  $f(\mathbf{X})$ ,  $\mathbf{X} = (x_1, \dots, x_d)^T$ 
  Initialize the population of fireflies  $\mathbf{X}_i (i=1, 2, \dots, n)$ 
  Determination of light intensity  $I_i$  at  $\mathbf{X}_i$  by  $f(\mathbf{X}_i)$ 
  Define light absorption coefficient  $\gamma$ 
  while ( $t < \text{Maximum Generation}$ )
    for  $i = 1 : n$  all  $n$  fireflies
      for  $j = 1 : n$  all  $n$  fireflies
        if ( $I_j > I_i$ )
          Move the firefly  $i$  towards  $j$  in  $d$ -dimension via Levy flights
        end if
        Attractiveness varies when distance  $r$  varies via  $\exp[-\gamma r^2]$ 
        Evaluate and update new solutions and light intensity respectively
      end for  $j$ 
    end for  $i$ 
    Ranking off the fireflies and find the current best
  end while
  Post-process results and visualization
End
    
```

Fig. 1 Pseudo code for firefly algorithm

coefficient. Inverse square law and approximation of absorption in Gaussian form jointly omit the singularity at $r = 0$ in the expression I/r^2 effectively.

On the other hand attractiveness and light intensity of fireflies are proportional to each other. So the attractiveness of fireflies also can be described by a similar equation to light intensity as follows.

$$\beta = \beta_0 e^{-\gamma \times r^2} \tag{4}$$

where β is the attractiveness measure at distance (r) and β_0 is the original attractiveness at zero distance.

From the above description it may be concluded that attractiveness and light intensity are synonyms. But there is a major difference between the two terms. Light intensity gives the absolute measure of light emitted by a firefly whereas attractiveness gives the relative measure of light that is seen by the other one [35].

Distance between two fireflies i and j at x_i and x_j can be expressed by a Euclidean equation as follows.

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^d (x_{ik} - x_{jk})^2} \tag{5}$$

where x_{ik} represents the component of the spatial coordinate x_i of the i th firefly and ‘ d ’ defines the number of dimensions [27].

Brighter firefly ‘ j ’ attracts the other firefly ‘ i ’. The movement of ‘ i ’ towards ‘ j ’ can be formulated by the following equation.

$$x_i = x_i + \beta_0 e^{-\gamma \times r_{ij}^2} (x_j - x_i) + \alpha \epsilon_i \tag{6}$$

where the second term is for attraction factor and the third term is for randomization. The randomness factor is denoted by ‘ α ’. ϵ_i is random vector quantity drawn from Gaussian distribution.

Table 1 Chemical properties of Indian RAFMS

Cr	C	Mn	V	W	Ta	N	O	P	S
8.8–9.2	0.10–0.12	0.40–0.60	0.20–0.24	0.9–1.1	0.006–0.008	0.002–0.004	<0.01	<0.02	<0.002
B	Ti	Nb	Mo	Ni	Cu	Al	Si	Co	AS + Sn + Sb
<0.001	<0.005	<0.001	<0.002	<0.005	<0.002	<0.005	<0.05	<0.005	<0.03

Table 2 Physical properties of Indian RAFMS

Hardness	405 ± 5 HV
Density	7760 kg/m ³
Thermal conductivity	33 W/m k (at room temperature)
Specific heat at constant pressure	622 J/kg k
Melting temperature	2032 k

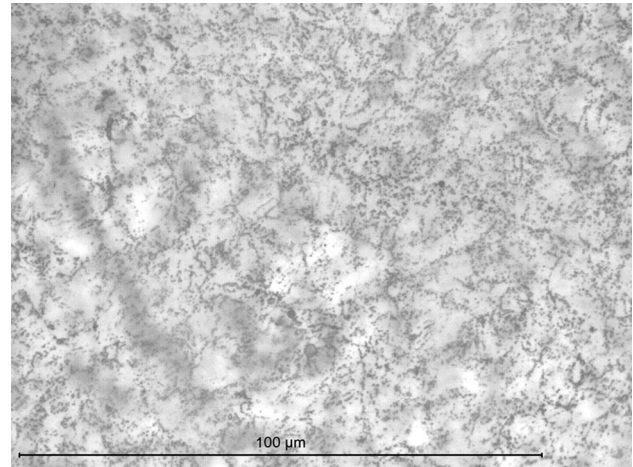
In most cases the values for β_0 can be taken as 1 and $\alpha \in [0, 1]$. Theoretically the value of the absorption coefficient $\gamma \in [0, \infty]$, but in most time the value of it typically varies from 0.1 to 10 [35].

4 Experimental procedure

For this study Indian RAFMS was used as workpiece material. The composition and physical properties of the material [36] are shown in Tables 1 and 2. Figure 2 shows the optical micro graph of Indian RAFMS. Electronica Sprintcut WEDM was used to perform the experiments. For conducting the experiments distilled water was used as dielectric medium. While cutting brass wire (CuZn37) electrode was used due to its favorable thermal property, easy availability and low cost.

During the experiment pulse current (I), pulse-on time (T_{on}), pulse-off time (T_{off}) and wire tension (WT) were taken as variable process parameters [3, 5] whereas the discharge voltage (V), flushing pressure (FP) and wire feed (WF) were taken as constant parameters due to machine constrains. The values taken for the constant parameters were 20 V for discharge voltage, 5 atm for flushing pressure and 4 mm/min for wire feed. For experimentation the ranges of process parameters were selected through trial and error method. Then three levels for each parameter were taken and the leveling of parameters was done accordingly. Table 3 shows the leveling of parameters.

The experiments started by selecting a suitable design of experiment. In this case Taguchi's L9 (3^4) orthogonal array was selected. After that the workpiece material (RAFMS) had been prepared by scaling it for nine number of cutting pass. The Scaling was done in such a way that the distance between two cuts remains 10 mm. The cutting length for the experiments was taken as 15 mm. Then the work

**Fig. 2** Micrographs of Indian RAFMS

material machined in the WEDM. Time taken by the machine to cut the 15 mm length for each pass was recorded very minutely with the help of a stopwatch.

There after the pieces of the workpiece material were cut down to find out the surface roughness of the Wire-EDMed faces. Figure 3 shows the work piece after WEDM. The surface roughness of the pieces was measured by Taylor Hobson surface profiler. The cutting speed and surface roughness values are shown in the Table 4.

5 Proposed methodology

The section deals with description of the proposed methodology for optimizing the process parameters during WEDM of RAFM. A brief description of this methodology is presented in the following subsections.

5.1 Mathematical model building

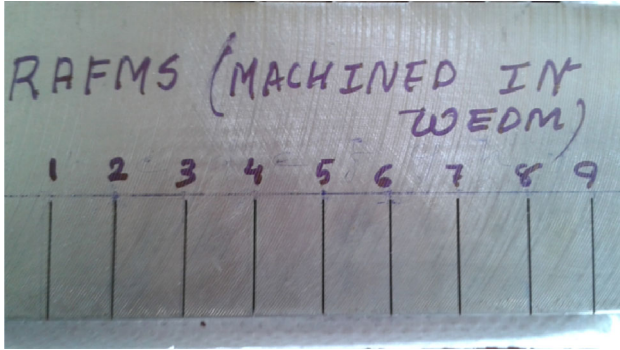
In this study with the help of multiple linear regression analysis mathematical models for cutting speed (CS) and surface roughness (SR) were developed. To find out the coefficients for the given four factors the deterministic general linear model of multiple linear regression analysis shown in Eq. 7 was used.

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 \quad (7)$$

Table 3 Input parameters and their level

Levels	Pulse-peak current (A)	T_{on} (machine unit)	T_{off} (machine unit)	Wire tension (machine unit)
1	210	110	50	4
2	220	115	55	6
3	230	120	60	8

For pulse on time, T_{on} machine unit = $0.05 \times T_{on} + 0.1 \mu s$; For pulse off time, T_{off} machine unit = $0.25 \times T_{off} + 2 \mu s$; wire tension, wt machine unit = $100 \times wt + 100 g$

**Fig. 3** RAFM steel after machined in WEDM

where y = Cutting speed or Surface roughness, x_1 = pulse-peak current (I), x_2 = pulse on time (T_{on}), x_3 = pulse of time (T_{off}) and x_4 = wire tension (WT).

After determining the coefficients square root transformation ($y^* = y^{(1/2)}$) was determined to develop mathematical models. The mathematical models thus obtained for cutting speed (CS) and surface roughness (SR) are as follow:

$$CS = (-6.08356 + 0.00970706 \times I + 0.056970 \times T_{on} - 0.024323 \times T_{off} + 0.015548 \times WT)^2 \quad (8)$$

$$SR = (-3.22606 + 0.012573 \times I + 0.016654 \times T_{on} - 0.000338626 \times T_{off} + 0.021980 \times WT)^2 \quad (9)$$

Table 4 Experimental results

Exp no.	I (A)	T_{on} (machine unit)	T_{off} (machine unit)	WT (machine unit)	CS (mm/min)	SR (μm)
1	210	110	50	4	1.11	1.66
2	210	115	55	6	1.61	2.14
3	210	120	60	8	2.08	2.37
4	220	110	55	8	1.26	2.54
5	220	115	60	4	1.6	2.41
6	220	120	50	6	3	2.67
7	230	110	60	6	0.99	2.41
8	230	115	50	8	2.71	3
9	230	120	55	4	2.91	3.1

5.2 Model adequacy checking

In this present study, to find out the adequacy of the developed mathematical models analysis of variance (ANOVA) was conducted. The analysis result values for cutting speed and surface roughness are shown in the Tables 5 and 6.

The cutting speed and surface roughness models have F value of 65.82 and 12.86 respectively which implies that the models are significant. The P values of the models also assure the significance of them. The determination coefficient (R^2) values of the models are 0.9850 and 0.9278 which indicates their capability to explain more than 98 % and 92 % of the total variations respectively. Further it has been observed that the adjusted R^2 value and predicted R^2 values of each regression model have reasonable agreement with each other. Moreover, for both the models the adequate precision, which measures the signal to noise ratio, were also calculated (Tables 5, 6). A ratio >4 is desirable. While in this present investigation the calculated adequate precision of CS and SR model are 19.511 and 10.569 respectively, which indicate adequate signal.

Additionally, for further illustration of accuracy the scatter plots between the predicted and actual values for cutting speed and surface roughness were drawn during this study. From the Figs. 4 and 5 it has been observed that the datas are spreaded closer to the 45° line, which shows adequacy of the developed models.

Table 5 Analysis of variance (ANOVA) results for cutting speed

Source	Sum of squares	df	Mean square	F value	P value Prob > F	Contribution (%)
Model	0.64	4	0.159	65.82	0.001	98.50
error	0.009	4	0.003			1.50
Total	0.648	8				
SD	0.049	R-squared	0.9850			
Mean	1.36	Adj R-squared	0.9783			
C.V. %	3.62	Pred R-squares	0.9183			
PRESS	0.053	Adeq precision	19.511			

Table 6 Analysis of variance (ANOVA) results for surface roughness

Source	Sum of squares	df	Mean square	F value	P value Prob > F	Contribution (%)
Model	0.148	4	0.037	12.86	0.0149	92.78
error	0.012	4	0.003			7.22
Total	0.16	8				
SD	0.054	R-squared	0.9278			
Mean	1.57	Adj R-squared	0.8557			
C.V. %	3.42	Pred R-squares	0.5079			
PRESS	0.079	Adeq precision	10.569			

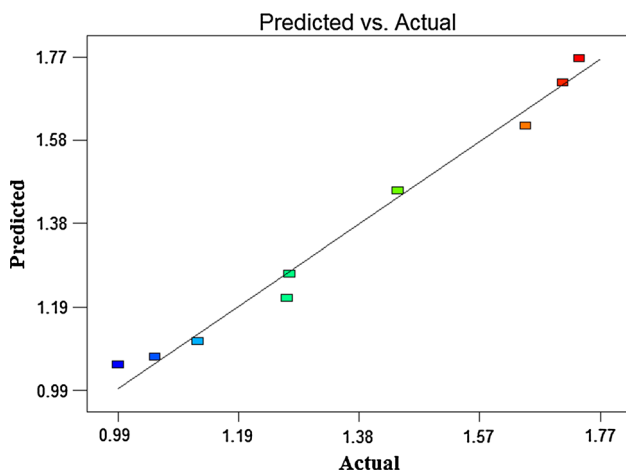


Fig. 4 Predicted versus actual plot for cutting speed

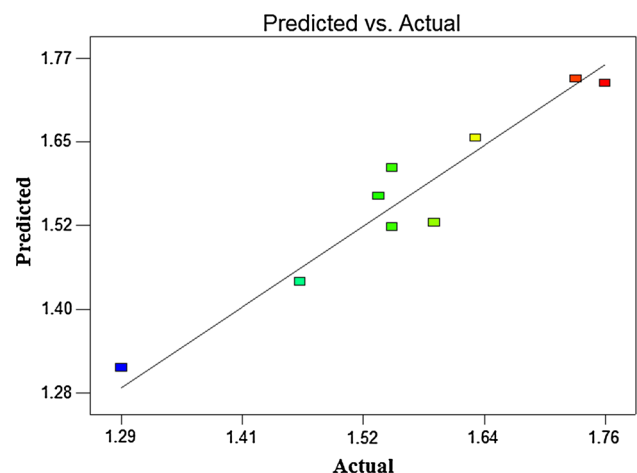


Fig. 5 Predicted versus actual plot for surface roughness

5.3 Calculation of individual weight using standard deviation method

In this study standard deviation method was used to give adequate individual weightage to the responses. The purpose for selecting this method over desirability method is to consider the degree of variability of responses with sequences during determining individual weight [37]. The individual weightages determined during this method is based upon the non-dimensional variation characteristic of the responses with experimental sequences. Figure 6 represents the box plot for linearly normalized values of

response variables which justifies the use of this standard deviation method. The box plot graphically summarizes the statistical distribution of each response during experimentation. From the figure it has clearly been seen that the 50 % normalized cutting speed values collected during experimentation were clustered between 0.097 and 0.905. While for surface roughness the 50 % data collected during experimentation were lying within the range 0.413–0.816. Therefore from the plot it can be conclude that the variation of cutting speed is more than surface roughness and it is unjust to give equal weightages to them. The various steps involved in this approach are as follow:

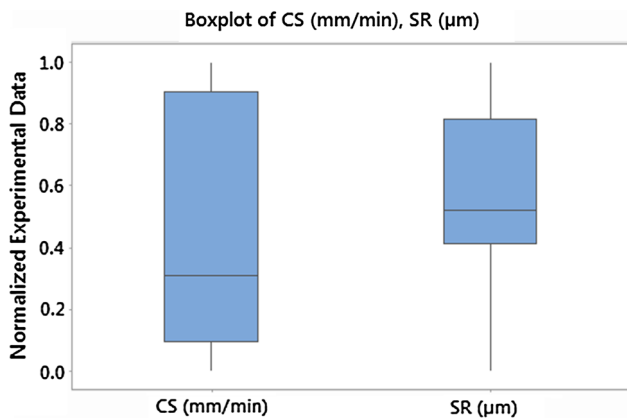


Fig. 6 Box plot for linearly normalized values of response variables

Table 7 Normalized result

Exp. no.	Pre-processed data	
	Linearly normalized response values	
	Cutting speed (higher the better)	Surface roughness (lower the better)
1	0.053701493	1
2	0.308457711	0.666666667
3	0.542288557	0.506944444
4	0.134328358	0.388888889
5	0.303482587	0.479666667
6	1	0.298611111
7	0	0.479166667
8	0.855721393	0.069444444
9	0.955223881	0
Mean value	0.462133776	0.432098765

Step 1 Initially the experimental values of responses (cutting speed and surface roughness) were normalized linearly using Eqs. 10 and 11 respectively. For cutting speed the normalization was carried out by “larger the better” scheme, while surface roughness were normalized using “lower the better” method (Table 7).

Larger the better:

$$x_i^*(k) = \frac{x_i^{(o)}(k) - \min x_i^{(o)}(k)}{\max x_i^{(o)}(k) - \min x_i^{(o)}(k)} \tag{10}$$

Lower the better:

$$x_i^*(k) = \frac{\max x_i^{(o)}(k) - x_i^{(o)}(k)}{\max x_i^{(o)}(k) - \min x_i^{(o)}(k)} \tag{11}$$

where: $x_i^{(o)}(k)$ = experimental value of output parameter ‘i’ at kth experiment, $\min x_i^{(o)}(k)$ = the minimum value of output parameter ‘i’ and $\max x_i^{(o)}(k)$ = the maximum value of output parameter ‘i’.

Table 8 Standard deviation and individual weight values of each response

Exp. no	Standard deviation values	
	Cutting Speed Variance	Surface Roughness Variance
1	0.161951923	0.32251155
2	0.023616402	0.05502199
3	0.006424753	0.00560184
4	0.107456539	0.00186711
5	0.025170271	0.00221537
6	0.289299834	0.01781902
7	0.213567834	0.00221537
8	0.154911036	0.13151833
9	0.24313763	0.18670955
Slandered deviation	0.369013132	0.28391707
Weightage	0.565251492	0.43490199

Step 2 During this step the variances of normalized cutting speed and surface roughness values were found by using Eq. 12. The standard deviation values were acquired by Eq. 13 (Table 8).

$$v_i = (x_i^*(k) - \mu_i)^2 \tag{12}$$

$$S = \sqrt{\frac{1}{n} \sum_{i=1}^n v_i} \tag{13}$$

where, v_i = variances of normalized output parameter ‘i’, μ_i = mean of all normalized experimental values (n) for output parameter ‘i’, n = number of experiments.

Step 3 In final step the ratio of the standard deviation value for each response to sum of the standard deviation values of both responses gives the individual weight for each response. Equations 14 and 15 represents the weightage equations for cutting speed and surface roughness respectively.

$$w_1 = \frac{S_1}{S_1 + S_2} \tag{14}$$

$$w_2 = \frac{S_2}{S_1 + S_2} \tag{15}$$

Thus by using the proposed approach, the calculated individual weight of cutting speed and surface roughness are 0.565 and 0.435 respectively. After calculating the individual weight, the two mathematical models as mentioned above (Eqs. 8, 9) were combined and a single mathematical model was generated shown in Eq. 16.

$$X = 0.565164748 \times \left(\frac{CS - \min CS}{\max CS - \min CS} \right) + 0.434835252 \times \left(\frac{\max SR - SR}{\max SR - \min SR} \right) \tag{16}$$

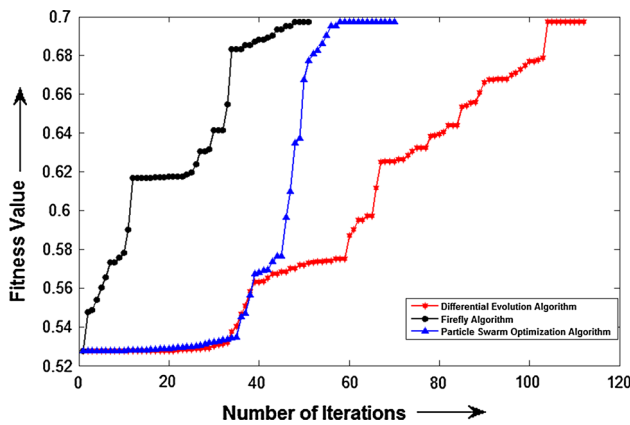


Fig. 7 Convergence characteristics of differential evolution algorithm (DE), firefly algorithm (FA) and partial swarm optimization Algorithm (PSO) for the optimization WEDM process parameters

5.4 Optimization through firefly algorithm

For the present study firefly algorithm was used to optimize the process parameters. In this regard the combined mathematical model was used as a fitness function for firefly algorithm. The population size for the algorithm was taken as 20. While the maximum number of iteration was taken as 200 in order to decide when to stop algorithm. The values for the tuning factors of FA like randomness factor (α), actual attractiveness (β_0) and absorption factor (γ) 0.2, 1 and 1 respectively [38]. The FA was coded in Matlab 2009a and executed in a PC with Intel i5-2450 M CPU with 4 GB RAM running at 2.50 GHz. To eliminate the effect of the stochastic behavior of this algorithm while finding the best optimal solution the algorithm was run thirty times. Figure 7 shows the best convergence rate of fitness value obtained when firefly algorithm.

6 Results and discussion

In this section the optimal results (shown in table) obtained by the algorithm were compared with the results achieved by experiments and the existing algorithms (DE and PSO)

Table 9 Confirmation test result

Predicted condition	Observed condition	Error (%)	
Pulse current (A)	210	Pulse current (A) 210	
Ton (machine unit)	120	Ton (machine unit) 120	
Toff (machine unit)	50	Toff (machine unit) 50	
Wire tension (machine unit)	4	Wire tension (machine unit) 4	
Cutting speed (mm/min)	2.68	Cutting speed (mm/min) 2.70	0.75
Surface roughness (μm)	2.20	Surface roughness (μm) 2.23	1.36

Table 10 Input parameter setting for PSO and DE during 30 trial runs

Optimization algorithm	Process parameters
PSO	Cognitive parameters $c_1 = 2$ [39]
	Social parameters $c_2 = 2$ [39]
	Random number $r_1 =$ uniformly distributed random numbers in the range [0–1] [39]
	Random number $r_2 =$ uniformly distributed random numbers in the range [0–1] [39]
	Size of the swarm, $N = 20$
	Number of iterations, $T = 200$
DE	Weight function [39], $w^N = w_{\max} - \frac{(w_{\max} - w_{\min}) \times T}{T_{\max}}$
	where, initial weight, $w_{\max} = 1$ [39]
	Final weight, $w_{\min} = 0.01$ [39]
	Population size (N) = 20
	Number of iterations, $T = 200$
	Mutation factor (F) = 0.8 [40]
	Crossover factor (CR) = 0.8 [40]

to find the effectiveness of the proposed approach. The detailed discussions are given below.

6.1 Confirmative test

The present investigation consist a confirmation test to validate the optimized parameters. The confirmation test results are shown in Table 9.

The percentage of errors between the predicted and experimental values of cutting speed and surface roughness can be seen in Table 9. It can be observed from the table that the percentages of errors are very small. Thus excellent reproducibility of the experimental conclusions is confirmed by the confirmation test results.

6.2 Comparison of FA with PSO, DE and experimental results

In this section the performance of the proposed approach was investigated. For such investigation initially the computational performance of the applied Firefly algorithm (FA) was compared with the existing particle swarm

Table 11 Comparison between the predicted optimal condition and initial condition

Initial condition		Predicted condition	
Pulse current (A)	210	Pulse current (A)	210
T_{on} (machine unit)	110	T_{on} (machine unit)	120
T_{off} (machine unit)	50	T_{off} (machine unit)	50
Wire tension (machine unit)	4	Wire tension (machine unit)	4
Cutting speed (mm/min)	1.11	Cutting speed (mm/min)	2.68
Surface roughness (μm)	1.66	Surface roughness (μm)	2.20

optimization algorithm (PSO) [39] and differential evolution optimization algorithm (DE) [40]. During this comparison the achieved maximum fitness value, number of iterations and computational time were considered as performance parameters. The process parameters used for existing PSO and DE in this study were taken from previous literature, Majumder [39] and Roque et al. [40] respectively and are shown in Table 10. However, the maximum number of iteration and population size for PSO and DE were taken as 100 and 20 respectively to make the performance comparison more appropriate. Both of these existing meta-heuristics (PSO and DE) were coded in MATLAB 2009a programming environment. Figure 7 represents the comparison between the convergence characteristic of applied firefly algorithm (FA) and existing particle swarm optimization algorithm (PSO) and differential evolution optimization algorithm (DE) during one of the test run.

In addition to that the performance of the developed approach was checked by comparing the achieved optimal condition with the initial condition. The initial experiment results and predicted results are shown in the Table 11.

Table 12 Best, worst, mean and standard deviation of solutions achieved by FA, PSO and DE during 30 trial runs

Solution	FA	PSO	DE
Best	0.697356877	0.697357	0.697357
Worst	0.697281469	0.6832	0.532316
Mean	0.69735185	0.691678	0.637061
Standard Deviation	1.91316E-05	0.007041	0.062612

Table 13 t Test statistics of paired differences of solution for each evolutionary algorithm

MSE-test	Mean	DF	Standard deviation	Standard error mean	95 % confidence interval of the difference		t	P value
					Lower	Upper		
PSO vs DE	0.054618	29	0.062344	0.011382	0.035278	0	4.7984	0.000
FA vs DE	0.060291	29	0.062605	0.011430	0.040870	0	5.2748	0.000
FA vs PSO	0.0056736	29	0.0070401	0.0012853	0.0034896	0	4.4141	0.000

6.3 Discussion of results

The comparison of applied FA with existing PSO and DE was carried out by running each of these algorithms thirty times during this study. Table 12 presents the best, worst, mean and standard deviation of the optimal solutions achieved from thirty trial runs. From the results it has been observed that each of these three algorithms achieved a similar best optimum solution. However, standard deviation of these test runs indicates a consistency in performance of FA as compared to the other two. For further illustration pair-wise t test between the three algorithms (Table 13) were performed with 95 % confidence interval. The hypotheses, used for this paired t test are as follows: Null hypothesis: [mean (term 1) – mean (term 2) ≤ 0], Alternative hypothesis: [mean (term 1) – mean (term 2) > 0]. If the calculated t value and P value of pair difference is positive and <0.05 respectively, then term 1 performed significantly better term 2. In this case, the results of this analysis indicate that FA significantly outperformed PSO and DE. This is expected due to two major advantageous characteristics of FA, namely automatic subdivision and the ability of controlling the randomness with the progression of convergence [26].

Table 14 Best, worst, mean and standard deviation of iterations required by FA, PSO and DE during 30 trial runs

Solution	FA	PSO	DE
Best	40	53	100
Worst	50	59	120
Mean	45.16667	55.83333	109.9333
Standard deviation	3.052227	1.78274	5.936058

Table 15 *t* Test statistics of paired differences of number of iteration for each evolutionary algorithm

MSE-test	Mean	DF	Standard deviation	Standard error mean	95 % confidence interval of the difference		<i>t</i>	<i>P</i> value
					Lower	Upper		
PSO vs DE	−54.100	29	5.886	1.0746	−52.274	0	−50.343	0.000
FA vs DE	−64.767	29	7.389	1.3491	−62.474	0	−48.009	0.000
FA vs PSO	−10.667	29	3.585	0.65449	−9.5546	0	−16.298	0.000

Table 16 Best, worst, mean and standard deviation of computational time required by FA, PSO and DE during 30 trial runs

Solution	FA	PSO	DE
Best	0.156	0.0468	0.468
Worst	0.3744	0.0936	0.9828
Mean	0.226667	0.06526	0.57096
Standard deviation	0.037004	0.011392	0.100983

The calculated values of mean, best, worst and standard deviation of number of iterations required for convergence of FA, PSO and DE during thirty number of trail runs were reported in Table 14. The results shown in this table indicates that the applied firefly algorithm (FA) requires significantly less iteration as compare to particle swarm optimization algorithm (PSO) and differential evolution optimization algorithm (DE). This has also been verified by paired *t* tests of iterations (Table 15) used by FA, PSO and DE during convergence. The following hypotheses are used for this *t* test study: Null hypothesis: [mean (term 1) − mean (term 2) ≥ 0], Alternative hypothesis: [mean (term 1) − mean (term 2) < 0].

Based on the results of Table 16 it has been seen that the PSO requires less computational time for convergence as compare to the applied FA and existing DE. While for more elaboration of analysis a paired *t* test between the CPU time taken by FA, PSO and DE for convergence were conducted and the results were shown in Table 17. The results reflect that PSO converges quickly followed by FA and DE. It is because of the fact that the complexity of PSO (O(NT)) is lower than FA (O(N²T)) and is same as required by DE (O(NT)). However in case of PSO the

number of iteration taken for convergence is much lesser than DE and moderately higher than FA.

Moreover from Table 11 it can be clearly seen that with optimized parameters the cutting speed increases along with the surface roughness. The increase in cutting speed is 141.44 % and increase in surface roughness is 32.53 %. As the cutting speed increased by a very high rate, the slight increment in surface roughness can be neglected.

Thus it can be conclude that the proposed approach is the best performed technique for optimizing WEDM process parameters.

7 Conclusion

The present study investigated the application of Standard Deviation Method based multi-objective firefly algorithm to predict the optimal process parameters during wire electro discharge machining (WEDM) of Indian reduced activation ferritic martensitic (RAFM) steel. The study includes pulse current (*I*), pulse-on time (*T_{on}*), pulse-off time (*T_{off}*) and wire tension (WT) as the process parameters. However, the response parameters considered in this study are: cutting speed and surface roughness. Based on the results the following conclusions can be drawn:

1. The solution quality achieved by applied firefly algorithm (FA) is significantly better than the two efficient algorithms (PSO and DE) found in literature.
2. The applied firefly algorithm (FA) has faster convergence rate as compare to DE in terms of CPU time. While comparing with PSO the proposed FA requires more CPU time for convergence.

Table 17 *t* Test statistics of paired differences of computational time for each evolutionary algorithm

MSE-test	Mean	DF	Standard deviation	Standard error mean	95 % confidence interval of the difference		<i>t</i>	<i>P</i> value
					Lower	Upper		
PSO vs DE	−0.50570	29	0.10108	0.018455	−0.47434	0	−27.402	0.000
FA vs DE	−0.34429	29	0.09004	0.016439	−0.31636	0	−20.944	0.000
PSO vs FA	−0.16141	29	0.03335	0.0060880	−0.17175	0	−26.512	0.000

3. By using standard deviation method multiple criterions can be logically aggregated to convert a single performance index (MPI). Single performance index (MPI) can easily be optimized to determine the optimal machining environment which facilitates it for mass production and consequently product quality improvement.
4. The proposed integrated approach was found to be capable to optimize the process parameters during WEDM of Indian RAFMS.

Even though, the proposed technique is efficient enough to solve the multi-attribute optimization problem. But the major disadvantage of this approach is its dependency on the collected experimental data set during individual weight selection. Therefore, it is required to overcome such uncertainty in individual weight selection by combining some other approach such as fuzzy logic and principal component analysis with standard deviation method. This can be taken as a future scope of this research. Aside from this in future, the proposed approach can also be used in other real life multi-objective optimization problems.

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