Neural Comput & Applic (2018) 29:187–203
https://doi.org/10.1007/s00521-016-2428-z

ORIGINAL ARTICLE

ivnpiv-Neutrosophic soft sets and their decision making based on similarity measure

Irfan Deli[1] · **Selim Eraslan**[2] · **Naim Çağman**[3]

Received: 10 March 2016 / Accepted: 8 June 2016 / Published online: 20 July 2016

Abstract In recent years, soft sets and neutrosophic sets have become a subject of great interest for researchers and have been widely studied based on decision-making problems. In this paper, we propose a new concept of the soft sets that is called interval-valued neutrosophic parameterized interval-valued neutrosophic soft sets (*ivnpivn*-soft sets). It is a generalization of the other soft sets such as fuzzy soft sets, intuitionistic fuzzy soft sets, neutrosophic soft sets, fuzzy parameterized soft sets, intuitionistic fuzzy parameterized soft sets, neutrosophic parameterized neutrosophic soft sets. Also, we proposed *ivnpivn*-soft matrices which are representative of the *ivnpivn*-soft sets. We then developed a decision-making method on the *ivnpivn*-soft sets and *ivnpivn*-soft matrices. Then, we proposed a numerical example to verify validity and feasibility of the developed method. Finally, the proposed method is compared with several different methods to verify its feasibility.

Keywords Soft sets · Neutrosophic sets · *ivnpivn*-soft sets · *ivnpivn*-soft matrices · Decision-making

✉ Irfan Deli
irfandeli@kilis.edu.tr

1 Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey

2 Kırıkkale Vocational School, Kırıkkale University, 71200 Kırıkkale, Turkey

3 Department of Mathematics, Gaziosmanpaşa University, 60250 Tokat, Turkey

1 Introduction

Smarandache [31, 32] introduced the notation of neutrosophic set to reflect the truth, indeterminate and false information simultaneously in real problems. Additionally, since neutrosophic sets were difficult to be applied in practical problems, single-valued neutrosophic sets which are an extension of intuitionistic fuzzy sets [4] and fuzzy sets [40] are introduced by Wang et al. [38]. The theory is characterized by a truth-membership, indeterminacy-membership and falsity-membership that describe by crisp numbers in real numbers. Then, interval neutrosophic sets defined by Wang et al. [37] which can be described by three real unit interval in [0, 1]. However, because of the ambiguity and complexity of in the real world, it is difficult for decision-makers to precisely express their preference by using these sets. Therefore, Molodtsov developed the concept of soft sets by using parameter set for the inadequacy of the parameterization tool of the theories. In the literature, many conclusions and propositions are obtained to use in decision-making problems; some of them are given in [1–3, 20, 29, 34–36, 39, 41].

After Molodtsov, many researchers combines the soft sets with the fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets and neutrosophic sets. For example, fuzzy soft sets [22], fuzzy parameterized soft sets [6, 8, 9], intuitionistic fuzzy soft sets [7, 23], intuitionistic fuzzy parameterized soft sets [12], interval-valued intuitionistic fuzzy parameterized soft sets [13], intuitionistic fuzzy parameterized fuzzy soft sets [16], neutrosophic soft sets [11, 21], interval-valued neutrosophic soft sets [14], interval-valued neutrosophic parameterized soft sets [10] and neutrosophic parameterized neutrosophic soft sets [15] have been studied by researchers. In these set theories, many conclusions and propositions are obtained to use in decision-making problems; some of them are given in [7, 11, 18, 26, 30, 33].

In this paper, we define a kind of soft sets called interval-valued neutrosophic parameterized interval-valued neutrosophic soft sets (*ivnpivn*-soft sets) which is generalization of *npn*-soft set [15]. The rest of paper is organized as follows. In Sect. 2, we review basic notions about neutrosophic sets, interval-valued neutrosophic sets, soft sets and neutrosophic parameterized neutrosophic soft sets. In Sect. 3, *ivnpivn*-soft sets and their operations are given. In Sect. 4, *ivnpivn*-soft matrices which are representative of the *ivnpivn*-soft sets have been introduced. In Sect. 5, we proposed a similarity measure and distance measures on *ivnpivn*-soft sets. In Sect. 6, we developed a decision-making method for *ivnpivn*-soft sets and give two illustrative example. In Sect. 7, the proposed method is compared with several extant methods to verify its feasibility. Finally, the conclusions are drawn.

2 Preliminary

In this section, we give the basic definitions and results of neutrosophic sets [31, 38], interval-valued neutrosophic sets [37], soft sets [28] and neutrosophic parameterized neutrosophic soft sets [15].

Definition 1 [38] Let U be a universe. A neutrosophic set $\widetilde{A}$ in U is characterized by a truth-membership function $T_{\widetilde{A}}$, an indeterminacy-membership function $I_{\widetilde{A}}$ and a falsity-membership function $F_{\widetilde{A}}$. $T_{\widetilde{A}}(u)$; $I_{\widetilde{A}}(u)$ and $F_{\widetilde{A}}(u)$ are real standard or nonstandard element of $^{-}[0,1]^{+}$. It can be written as

$$\widetilde{A} = \left\{ \langle u, (T_{\widetilde{A}}(u), I_{\widetilde{A}}(u), F_{\widetilde{A}}(u)) \rangle : u \in U \right\}$$

There is no restriction on the sum of $T_{\widetilde{A}}(u)$; $I_{\widetilde{A}}(u)$ and $F_{\widetilde{A}}(u)$, so $^{-}0 \leq T_{\widetilde{A}}(u) + I_{\widetilde{A}}(u) + F_{\widetilde{A}}(u) \leq 3^{+}$.

Example 1 Suppose that the universe of discourse $U = \{u_1, u_2, u_3\}$. It may be further assumed that the values of u_1, u_2 and u_3 are in $^{-}[0,1]^{+}$. Then, $\widetilde{A}$ is a neutrosophic set of U, such that,

$$\widetilde{A} = \{\langle u_1, (0.5, 0.6, 0.9)\rangle, \langle u_2, (0.4, 0.2, 0.7)\rangle, \langle u_3, (0.8, 0.3, 0.6)\rangle\}$$

Definition 2 [37] Let U be a universe. An interval value neutrosophic set A in U is characterized by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $u \in U$; T_A, I_A and $F_A \subseteq [0, 1]$.

Thus, an interval value neutrosophic set A over U can be represented by

$$A = \{\langle T_A(u), I_A(u), F_A(u)\rangle / u : u \in U\}$$

Here, $(T_A(u), I_A(u), F_A(u))$ is called interval value neutrosophic number for all $u \in U$ and all interval value neutrosophic numbers over U will be denoted by $IVN(U)$.

Example 2 Suppose that the universe of discourse $U = \{u_1, u_2\}$ where u_1 and characterizes the quality, u_2 indicates the prices of the objects. It may be further assumed that the values of u_1 and u_2 are subset of [0, 1] and they are obtained from a expert person. The expert construct an interval value neutrosophic set the characteristics of the objects as follows;

$$A = \{\langle [0.1, 1.0], [0.1, 0.4], [0.4, 0.7]\rangle / u_1, \langle [0.6, 0.9], [0.8, 1.0], [0.4, 0.6]\rangle / u_2\}$$

Definition 3 [37] Let A and B be two interval-valued neutrosophic sets. Then, for all $u \in U$,

1. A is empty, denoted by $A = \widehat{\emptyset}$, and is defined by $\widehat{\emptyset} = \{\langle [0,0], [1,1], [1,1]\rangle / u : u \in U\}$
2. A is universal, denoted by $A = \widehat{X}$, and is defined by $\widehat{X} = \{\langle [1,1], [0,0], [0,0]\rangle / u : u \in U\}$
3. The complement of A, denoted by A^c, is defined by

   $$A^{\widehat{c}} = \{\langle [\inf F_A(u), \sup F_A(u)], [1 - \sup I_A(u), 1 - \inf I_A(u)], [\inf T_A(u), \sup T_A(u)]\rangle / u : u \in U\}$$

4. $A \widehat{\subseteq} B \Leftrightarrow [\inf T_A(u) \leq \inf T_B(u),\ \sup T_A(u) \leq \sup T_B(u),\ \inf I_A(u) \geq \inf I_B(u),\ \sup I_A(u) \geq \sup I_B(u),\ \inf F_A(u) \geq \inf F_B(u),\ \sup F_A(u) \geq \sup F_B(u)]$.
5. Intersection of A and B, denoted by $A \widehat{\cap} B$, is defined by

   $$A \widehat{\cap} B = \{\langle [\min(\inf T_A(u), \inf T_B(u)), \min(\sup T_A(u), \sup T_B(u))], [\max(\inf I_A(u), \inf I_B(u)), \max(\sup IA(x), \sup I_B(u))], [\max(\inf F_A(u), \inf F_B(u)), \max(\sup F_A(u), \sup F_B(u))]\rangle / u : u \in U\}$$

6. Union of A and B, denoted by $A \widehat{\cup} B$, is defined by

   $$A \widehat{\cup} B = \{\langle [\max(\inf T_A(u), \inf T_B(u)), \max(\sup T_A(u), \sup T_B(u))], [\min(\inf I_A(u), \inf I_B(u)), \min(\sup I_A(u), \sup I_B(u))], [\min(\inf F_A(u), \inf F_B(u)), \min(\sup F_A(u), \sup F_B(u))]\rangle / u : u \in U\}$$

Definition 4 [17] t-norms are associative, monotonic and commutative two valued functions t that map from $[0, 1] \times [0, 1]$ into [0, 1]. These properties are formulated with the following conditions: $\forall a, b, c, d \in [0, 1]$,

1. $t(0, 0) = 0$ and $t(a, 1) = t(1, a) = a$,

2. If $a \le c$ and $b \le d$, then $t(a,b) \le t(c,d)$
3. $t(a,b) = t(b,a)$
4. $t(a,t(b,c)) = t(t(a,b),c)$

Definition 5 [17] t-conorms (s-norm) are associative, monotonic and commutative two placed functions s which map from $[0,1] \times [0,1]$ into [0, 1]. These properties are formulated with the following conditions: $\forall a,b,c,d \in [0,1]$,

1. $s(1,1) = 1$ and $s(a,0) = s(0,a) = a$,
2. if $a \le c$ and $b \le d$, then $s(a,b) \le s(c,d)$
3. $s(a,b) = s(b,a)$
4. $s(a,s(b,c)) = s(s(a,b),c)$

Definition 6 [28] Let U be a universe, X be a set of parameters that are describe the elements of U and $P(U)$ be the power set of U. Then, a soft set f over U is a function defined by

$$f : X \to P(U)$$

In other words, the soft set is a parameterized family of subsets of the set U. A soft set over U can be represented by the set of ordered pairs

$$f = \{(x, f(x)) : x \in X\}$$

Example 3 Suppose that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universe contains six house under consideration in a real agent and $X = \{x_1 = \text{cheap}, x_2 = \text{beatiful}, x_3 = \text{green surroundings}, x_4 = \text{costly}, x_5 = \text{large}\}$.

If a customer to select a house from the real agent, then he/she can construct a soft set f that describes the characteristic of houses according to own requests. Assume that $f(x_1) = \{u_1, u_2\}$, $f(x_2) = \{u_1\}$, $f(x_3) = \emptyset$, $f(x_4) = U$, $\{u_1, u_2, u_3, u_4, u_5\}$ then the soft set f is written by

$$f = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_4, u_5, u_6\}), (x_4, U), (x_5, \{u_1, u_2, u_3, u_4, u_5\})\}$$

Definition 7 [28] Let f and g be two soft sets over U. Then,

1. f is called an empty soft set, denoted by Φ_X, if $f(x) = \emptyset$, for all $x \in X$.
2. f is called a universal soft set, denoted by $f_{\tilde{X}}$, if $f(x) = U$, for all $x \in X$.
3. $\mathrm{Im}(f) = \{f(x) : x \in X\}$ is called image of f.
4. f is a soft subset of g, denoted by $f \tilde{\subseteq} g$, if $f(x) \subseteq g(x)$ for all $x \in X$.
5. f and g are soft equal, denoted by $f = g$, if and only if $f(x) = g(x)$ for all $x \in X$.
6. $(f \tilde{\cup} g)(x) = f(x) \cup g(x) \text{forallx} \in \text{X}$ is called union of f and g.
7. $(f \tilde{\cap} g)(x) = f(x) \cap g(x) \text{forallx} \in \text{X}$ is called intersection of f and g.
8. $f^{\tilde{c}}(x) = U \setminus f(x) \text{forallx} \in \text{X}$ is called complement of f.

Definition 8 [15] Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, X be a set of parameters that describe the elements of U and K be a neutrosophic set over X. Then, a neutrosophic parameterized neutrosophic soft set (*npn*-soft set) N over U is a set defined by a set valued function f_N representing a mapping

$$f_N : K \to N(U)$$

where f_N is called approximate function of the *npn*-soft set N. For $x \in X$, the set $f_N(x)$ is called x-approximation of the *npn*-soft set N which may be arbitrary; some of them may be empty and some may have a nonempty intersection. It can be written a set of ordered pairs,

$$N = \big\{ (< x, T_N(x), I_N(x), F_N(x) > , \{ < u, T_{f_N(x)}(u), {}_{f_N(x)}(u), F_{f_N(x)}(u) > \ : u \in U\}) : x \in X \big\}$$

where

$$F_N(x), I_N(x), T_N(x), T_{f_N(x)}(u), I_{f_N(x)}(u), F_{f_N(x)}(u) \in [0,1]$$

Example 4 Let $U = \{u_1, u_2, u_3, u_4\}$, $X = \{x_1, x_2\}$. N be a *npn*-soft sets as

$$N = \Big\{ (\langle x_1, (0.1, 0.2, 0.3)\rangle, \{\langle u_1, (0.4, 0.6, 0.6)\rangle, \langle u_2, (0.6, 0.0, 0.1)\rangle, \langle u_3, (0.3, 0.4, 0.4)\rangle, \langle u_4, (0.5, 0.6, 0.2)\rangle\}), (\langle x_2, (0.9, 0.1, 0.5)\rangle, \langle u_1, (0.3, 0.8, 0.6)\rangle, \langle u_2, (0.7, 0.6, 0.5)\rangle, \langle u_3, (0.6, 0.6, 0.9)\rangle, \langle u_4, (0.4, 0.1, 0.5)\rangle, \}) \Big\}$$

Definition 9 [15] Let N, N_1 and N_2 be three *npn*-soft sets. Then,

1. Complement of an *npn*-soft set N, denoted by $N^{\tilde{c}}$, is defined by

$$N^{\tilde{c}} = \big\{ (\langle x, F_N(x), 1 - I_N(x), T_N(x)\rangle, \{\langle u, F_{f_N(x)}(u), 1 - I_{f_N(x)}(u), T_{f_N(x)}(u)\rangle : u \in U\}) : x \in X \big\}$$

2. Union of N_1 and N_2, denoted by $N_3 = N_1 \tilde{\cup} N_2$, is defined by

$$N_3 = \big\{ (\langle x, T_{N_3}(x), I_{N_3}(x), F_{N_3}(x)\rangle, \{\langle u, T_{f_{N_3(x)}}(u), I_{f_{N_3(x)}}(u), F_{f_{N_3(x)}}(u)\rangle : u \in U\}) : x \in X \big\}$$

where

$$T_{N_3}(x)=s(T_{N_1}(x),T_{N_2}(x)),\quad T_{f_{N_3(x)}}(u)=s(T_{f_{N_1(x)}}(u),T_{f_{N_2(x)}}(u)),$$
$$I_{N_3}(x)=t(I_{N_1}(x),I_{N_2}(x)),\quad I_{f_{N_3(x)}}(u)=t(I_{f_{N_1(x)}}(u),I_{f_{N_2(x)}}(u)),$$
$$F_{N_3}(x)=t(F_{N_1}(x),F_{N_2}(x)),\quad F_{f_{N_3(x)}}(u)=t(F_{f_{N_1(x)}}(u),F_{f_{N_2(x)}}(u))$$

3. Intersection of N_1 and N_2, denoted by $N_4=N_1\tilde{\cap}N_2$, is defined by

$$N_4=\{(\langle x,T_{N_4}(x),I_{N_4}(x),F_{N_4}(x)\rangle,\{\langle u,T_{f_{N_4(x)}}(u),I_{f_{N_4(x)}}(u),F_{f_{N_4(x)}}(u)\rangle : u\in U\}) : x\in X\}$$

where

$$T_{N_4}(x)=t(T_{N_1}(x),T_{N_2}(x)),\quad T_{f_{N_4(x)}}(u)=t(T_{f_{N_1(x)}}(u),T_{f_{N_2(x)}}(u)),$$
$$I_{N_4}(x)=s(I_{N_1}(x),I_{N_2}(x)),\quad I_{f_{N_4(x)}}(u)=s(I_{f_{N_1(x)}}(u),I_{f_{N_2(x)}}(u)),$$
$$F_{N_4}(x)=s(F_{N_1}(x),F_{N_2}(x)),\quad F_{f_{N_4(x)}}(u)=s(F_{f_{N_1(x)}}(u),F_{f_{N_2(x)}}(u))$$

3 *ivnpivn*-soft sets

In this section, we present concept of "interval-valued neutrosophic parameterized interval-valued neutrosophic soft sets" is abbreviated as "*ivnpivn*-soft sets". Then, we introduce some definitions and operations on *ivnpivn*-soft set and some properties of the sets which are connected to operations have been established.

In the following, some definitions and operations are defined on npn-soft set in [15] and on interval-valued neutrosophic soft set in [14] and we extended these definitions and operations to *ivnpivn*-soft sets.

Definition 10 Let U be a universe, $IVN(U)$ be a set of all interval-valued neutrosophic sets over U, X be a set of parameters that are describe the elements of U and K be a interval-valued neutrosophic set over X. Then, an *ivnpivn*-soft set $\mathcal{F}$ over U is a set defined by a set valued function f representing a mapping

$$f : K \to IVN(U)$$

where f is called approximate function of the *ivnpivn*-soft set $\mathcal{F}$. For $x\in X$, the set $f(x)$ is called x-approximation of the *ivnpivn*-soft set $\mathcal{F}$ which may be arbitrary; some of them may be empty and some may have a nonempty intersection. It can be written a set of ordered pairs,

$$\mathcal{F}=\{(\langle x,T_{\mathcal{F}}(x),I_{\mathcal{F}}(x),F_{\mathcal{F}}(x)\rangle,\{\langle T_{f_{\mathcal{F}}(x)}(u),I_{f_{\mathcal{F}}(x)}(u),F_{f_{\mathcal{F}}(x)}(u)\rangle/u : u\in U\}) : x\in X\}$$

where

$$T_{\mathcal{F}}(x),F_{\mathcal{F}}(x),I_{\mathcal{F}}(x),T_{f_{\mathcal{F}}(x)}(u),I_{f_{\mathcal{F}}(x)}(u),F_{f_{\mathcal{F}}(x)}(u)\subseteq[0,1]$$

and

$$T_{\mathcal{F}}(x)=[\inf T_{\mathcal{F}}(x),\sup T_{\mathcal{F}}(x)],$$
$$I_{\mathcal{F}}(x)=[\inf I_{\mathcal{F}}(x),\sup I_{\mathcal{F}}(x)],$$
$$F_{\mathcal{F}}(x)=[\inf F_{\mathcal{F}}(x),\sup F_{\mathcal{F}}(x)],$$
$$T_{f_{\mathcal{F}}(x)}(u)=[\inf T_{f_{\mathcal{F}}(x)}(u),\sup T_{f_{\mathcal{F}}(x)}(u)],$$
$$I_{f_{\mathcal{F}}(x)}(u)=[\inf I_{f_{\mathcal{F}}(x)}(u),\sup I_{f_{\mathcal{F}}(x)}(u)],$$
$$F_{f_{\mathcal{F}}(x)}(u)=[\inf F_{f_{\mathcal{F}}(x)}(u),\sup F_{f_{\mathcal{F}}(x)}(u)]$$

From now on, the set of all *ivnpivn*-soft sets over U will be denoted by $\tilde{\mathcal{F}}$.

Example 5 Assume that $U=\{u_1,u_2,u_3\}$ and $X=\{x_1,x_2,x_3\}$, then an *ivnpivn*-soft set can be written as

$$\mathcal{F}=\{(\langle x_1,[0.3,0.4],[0.5,0.6],[0.4,0.5]\rangle,\{\langle[0.5,0.6],[0.6,0.7],[0.3,0.4]\rangle/u_1,\langle[0.4,0.5],[0.7,0.8],[0.2,0.3]\rangle/u_2,\langle[0.6,0.7],[0.2,0.3],[0.3,0.5]\rangle/u_3\}),(\langle x_2,[0.1,0.2],[0.3,0.4],[0.6,0.7]\rangle,\{\langle[0.7,0.8],[0.3,0.4],[0.2,0.4]\rangle/u_1,\langle[0.8,0.4],[0.2,0.6],[0.3,0.4]\rangle/u_2,\langle[0.4,0.5],[0.1,0.3],[0.2,0.4]\rangle/u_3\}),(\langle x_3,[0.2,0.4],[0.4,0.5],[0.4,0.6]\rangle,\{\langle[0.2,0.3],[0.1,0.4],[0.3,0.6]\rangle/u_1,\langle[0.2,0.5],[0.1,0.6],[0.5,0.8]\rangle/u_2,\langle[0.3,0.7],[0.1,0.3],[0.6,0.7]\rangle/u_3\})\}$$

Definition 11 Let $\mathcal{F}\in\tilde{\mathcal{F}}$. Then, $\mathcal{F}$ is called

1. An empty *ivnpivn*-soft set, denoted by $\mathcal{O}$, is defined as:

$$\mathcal{O}=\{(\langle x,[0,0],[1,1],[1,1]\rangle,\{\langle[0,0],[1,1],[1,1]\rangle/u : u\in U\}) : x\in X\}$$

2. a universal *ivnpivn*-soft set, denoted by $\mathcal{U}$,

$$\mathcal{U}=\{(\langle x,[1,1],[0,0],[0,0]\rangle,\{\langle[1,1],[0,0],[0,0]\rangle/u : u\in U\}) : x\in X\}$$

Definition 12 Let $\mathcal{F},\mathcal{F}_1,\mathcal{F}_2\in\tilde{\mathcal{F}}$. Then,

1. $\mathcal{F}_1$ is a sub$-$*ivnpivn*-soft set of $\mathcal{F}_2$, denoted by $\mathcal{F}_1\tilde{\subseteq}\mathcal{F}_2$, if and only if

$$\inf T_{\mathcal{F}_1}(x)\le\inf T_{\mathcal{F}_2}(x),\qquad \sup T_{\mathcal{F}_1}(x)\le\sup T_{\mathcal{F}_2}(x),$$
$$\inf I_{\mathcal{F}_1}(x)\ge\inf I_{\mathcal{F}_2}(x),\qquad \sup I_{\mathcal{F}_1}(x)\ge\sup I_{\mathcal{F}_2}(x),$$
$$\inf F_{\mathcal{F}_1}(x)\ge\inf F_{\mathcal{F}_2}(x),\qquad \sup F_{\mathcal{F}_1}(x)\ge\sup F_{\mathcal{F}_2}(x),$$
$$\inf T_{f_{\mathcal{F}_1(x)}}(u)\le\inf T_{f_{\mathcal{F}_2(x)}}(u),\qquad \sup T_{f_{\mathcal{F}_1(x)}}(u)\le\sup T_{f_{\mathcal{F}_2(x)}}(u),$$
$$\inf I_{f_{\mathcal{F}_1(x)}}(u)\ge\inf I_{f_{\mathcal{F}_2(x)}}(u),\qquad \sup I_{f_{\mathcal{F}_1(x)}}(u)\ge\sup I_{f_{\mathcal{F}_2(x)}}(u),$$
$$\inf F_{f_{\mathcal{F}_1(x)}}(u)\ge\inf F_{f_{\mathcal{F}_2(x)}}(u),\qquad \sup F_{f_{\mathcal{F}_1(x)}}(u)\ge\sup F_{f_{\mathcal{F}_2(x)}}(u).$$

2. Complement of $\mathcal{F}$, denoted by $\mathcal{F}^{\tilde{c}}$, is defined by

$$\mathcal{F}^{\tilde{c}}=\{(\langle x,F_{\mathcal{F}}(x),1-I_{\mathcal{F}}(x),T_{\mathcal{F}}(x)\rangle,\{\langle F_{f_{\mathcal{F}}(x)}(u),1-I_{f_{\mathcal{F}}(x)}(u),T_{f_{\mathcal{F}}(x)}(u)\rangle/u : u\in U\}) : x\in X\}$$

where

$$T_{\mathcal{F}}(x) = [\inf T_{\mathcal{F}}(x), \sup T_{\mathcal{F}}(x)],$$
$$1 - I_{\mathcal{F}}(x) = [1 - \sup I_{\mathcal{F}}(x), 1 - \inf I_{\mathcal{F}}(x)],$$
$$F_{\mathcal{F}}(x) = [\inf F_{\mathcal{F}}(x), \sup F_{\mathcal{F}}(x)],$$
$$T_{f_{\mathcal{F}}(x)}(u) = [\inf T_{f_{\mathcal{F}}(x)}(u), \sup T_{f_{\mathcal{F}}(x)}(u)],$$
$$1 - I_{f_{\mathcal{F}}(x)}(u) = [1 - \sup I_{f_{\mathcal{F}}(x)}(u), 1 - \inf I_{f_{\mathcal{F}}(x)}(u)],$$
$$F_{f_{\mathcal{F}}(x)}(u) = [\inf F_{f_{\mathcal{F}}(x)}(u), \sup F_{f_{\mathcal{F}}(x)}(u)].$$

3. Union of $\mathcal{F}_1$ and $\mathcal{F}_2$, denoted by $\mathcal{F}_3 = \mathcal{F}_1 \tilde{\cup} \mathcal{F}_2$, is defined by

$$\mathcal{F}_3 = \{(\langle x, T_{\mathcal{F}_3}(x), I_{\mathcal{F}_3}(x), F_{\mathcal{F}_3}(x)\rangle, \{\langle T_{f_{\mathcal{F}_3(x)}}(u), I_{f_{\mathcal{F}_3(x)}}(u), F_{f_{\mathcal{F}_3(x)}}(u)\rangle/u : x \in U\}) : x \in X\}$$

where

$$T_{\mathcal{F}_3}(x) = [s(\inf T_{\mathcal{F}_1}(x), \inf T_{\mathcal{F}_2}(x)), s(\sup T_{\mathcal{F}_1}(x), \sup T_{\mathcal{F}_2}(x))],$$
$$I_{\mathcal{F}_3}(x) = [t(\inf I_{\mathcal{F}_1}(x), \inf I_{\mathcal{F}_2}(x)), t(\sup I_{\mathcal{F}_1}(x), \sup I_{\mathcal{F}_2}(x))],$$
$$F_{\mathcal{F}_3}(x) = [t(\inf F_{\mathcal{F}_1}(x), \inf F_{\mathcal{F}_2}(x)), t(\sup F_{\mathcal{F}_1}(x), \sup F_{\mathcal{F}_2}(x))],$$
$$T_{f_{\mathcal{F}_3(x)}}(u) = [s(\inf T_{f_{\mathcal{F}_1(x)}}(u), \inf T_{f_{\mathcal{F}_2(x)}}(u)), s(\sup T_{f_{\mathcal{F}_1(x)}}(u), \sup T_{f_{\mathcal{F}_2(x)}}(u))],$$
$$I_{f_{\mathcal{F}_3(x)}}(u) = [t(\inf I_{f_{\mathcal{F}_1(x)}}(u), \inf I_{f_{\mathcal{F}_2(x)}}(u)), t(\sup I_{f_{\mathcal{F}_1(x)}}(u), \sup I_{f_{\mathcal{F}_2(x)}}(u))],$$
$$F_{f_{\mathcal{F}_3(x)}}(u) = [t(\inf F_{f_{\mathcal{F}_1(x)}}(u), \inf F_{f_{\mathcal{F}_2(x)}}(u)), t(\sup F_{f_{\mathcal{F}_1(x)}}(u), \sup F_{f_{\mathcal{F}_2(x)}}(u))].$$

4. Intersection of $\mathcal{F}_1$ and $\mathcal{F}_2$, denoted by $\mathcal{F}_4 = \mathcal{F}_1 \tilde{\cap} \mathcal{F}_2$, is defined by

$$\mathcal{F}_4 = \{(\langle x, T_{\mathcal{F}_4}(x), I_{\mathcal{F}_4}(x), F_{\mathcal{F}_4}(x)\rangle, \{\langle T_{f_{\mathcal{F}_4(x)}}(u), I_{f_{\mathcal{F}_4(x)}}(u), F_{f_{\mathcal{F}_4(x)}}(u)\rangle/u : x \in U\}) : x \in X\}$$

where

$$T_{\mathcal{F}_4}(x) = [t(\inf T_{\mathcal{F}_1}(x), \inf T_{\mathcal{F}_2}(x)), t(\sup T_{\mathcal{F}_1}(x), \sup T_{\mathcal{F}_2}(x))],$$
$$I_{\mathcal{F}_4}(x) = [s(\inf I_{\mathcal{F}_1}(x), \inf I_{\mathcal{F}_2}(x)), s(\sup I_{\mathcal{F}_1}(x), \sup I_{\mathcal{F}_2}(x))],$$
$$F_{\mathcal{F}_4}(x) = [s(\inf F_{\mathcal{F}_1}(x), \inf F_{\mathcal{F}_2}(x)), s(\sup F_{\mathcal{F}_1}(x), \sup F_{\mathcal{F}_2}(x))],$$
$$T_{f_{\mathcal{F}_4(x)}}(u) = [t(\inf T_{f_{\mathcal{F}_1(x)}}(u), \inf T_{f_{\mathcal{F}_2(x)}}(u)), t(\sup T_{f_{\mathcal{F}_1(x)}}(u), \sup T_{f_{\mathcal{F}_2(x)}}(u))],$$
$$I_{f_{\mathcal{F}_4(x)}}(u) = [s(\inf I_{f_{\mathcal{F}_1(x)}}(u), \inf I_{f_{\mathcal{F}_2(x)}}(u)), s(\sup I_{f_{\mathcal{F}_1(x)}}(u), \sup I_{f_{\mathcal{F}_2(x)}}(u))],$$
$$F_{f_{\mathcal{F}_4(x)}}(u) = [s(\inf F_{f_{\mathcal{F}_1(x)}}(u), \inf F_{f_{\mathcal{F}_2(x)}}(u)), s(\sup F_{f_{\mathcal{F}_1(x)}}(u), \sup F_{f_{\mathcal{F}_2(x)}}(u))].$$

Example 6 Let $U = \{u_1, u_2, u_3\}$, $X = \{x_1, x_2, x_3\}$, $\mathcal{F}_1$ be given as in Example 5 and $\mathcal{F}_2$ be given as follows

$$\mathcal{F}_2 = \Big\{(\langle x_1, [0.6, 0.7], [0.2, 0.9], [0.7, 0.8]\rangle, \{\langle [0.1, 0.5], [0.7, 0.9], [0.4, 0.7]\rangle/u_1, \langle [0.1, 0.3], [0.2, 0.4], [0.8, 0.9]\rangle/u_2, \langle [0.3, 0.5], [0.7, 0.9], [0.6, 0.7]\rangle/u_3\}), (\langle x_2, [0.1, 0.4], [0.2, 0.4], [0.5, 0.8]\rangle, \{\langle [0.2, 0.3], [0.6, 0.8], [0.1, 0.2]\rangle/u_1, \langle [0.5, 0.9], [0.4, 0.6], [0.7, 0.9]\rangle/u_2, \langle [0.1, 0.2], [0.3, 0.4], [0.5, 0.6]\rangle/u_3\}), (\langle x_3, [0.3, 0.9], [0.5, 0.7], [0.3, 0.6]\rangle, \{\langle [0.3, 0.5], [0.2, 0.6], [0.4, 0.8]\rangle/u_1, \langle [0.3, 0.7], [0.2, 0.8], [0.6, 0.9]\rangle/u_2, \langle [0.4, 0.9], [0.2, 0.5], [0.6, 0.9]\rangle/u_3\})\Big\}$$

Then,

$$(\mathcal{F}_1)^{\tilde{c}} = \Big\{(\langle x_1, [0.4, 0.5], [0.5, 0.4], [0.3, 0.4]\rangle, \{\langle [0.3, 0.4], [0.4, 0.3], [0.5, 0.6]\rangle/u_1, \langle [0.2, 0.3], [0.3, 0.2], [0.4, 0.5]\rangle/u_2, \langle [0.3, 0.5], [0.8, 0.7], [0.6, 0.7]\rangle/u_3\}), (\langle x_2, [0.6, 0.7], [0.7, 0.6], [0.1, 0.2]\rangle, \{\langle [0.2, 0.4], [0.7, 0.6], [0.7, 0.8]\rangle/u_1, \langle [0.3, 0.4], [0.8, 0.4], [0.8, 0.4]\rangle/u_2, \langle [0.2, 0.4], [0.9, 0.7], [0.4, 0.5]\rangle/u_3\}), (\langle x_3, [0.4, 0.6], [0.6, 0.5], [0.2, 0.4]\rangle, \{\langle [0.3, 0.6], [0.9, 0.6], [0.2, 0.3]\rangle/u_1, \langle [0.5, 0.8], [0.9, 0.4], [0.2, 0.5]\rangle/u_2, \langle [0.6, 0.7], [0.9, 0.7], [0.3, 0.7]\rangle/u_3\})\Big\}$$

Let us consider the t-norm $min\{a, b\}$ and s-norm $max\{a, b\}$. Then,

$$\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2 = \Big\{(\langle x_1, [0.6, 0.7], [0.2, 0.6], [0.4, 0.5]\rangle, \{\langle [0.5, 0.6], [0.6, 0.7], [0.3, 0.4]\rangle/u_1, \langle [0.4, 0.5], [0.2, 0.4], [0.2, 0.3]\rangle/u_2, \langle [0.6, 0.7], [0.2, 0.3], [0.3, 0.5]\rangle/u_3\}), (\langle x_2, [0.1, 0.4], [0.2, 0.4], [0.5, 0.7]\rangle, \{\langle [0.7, 0.8], [0.3, 0.4], [0.1, 0.2]\rangle/u_1, \langle [0.8, 0.9], [0.2, 0.6], [0.3, 0.4]\rangle/u_2, \langle [0.4, 0.5], [0.1, 0.3], [0.2, 0.4]\rangle/u_3\}), (\langle x_3, [0.3, 0.9], [0.4, 0.5], [0.3, 0.6]\rangle, \{\langle [0.3, 0.5], [0.1, 0.4], [0.3, 0.6]\rangle/u_1, \langle [0.3, 0.7], [0.1, 0.6], [0.5, 0.8]\rangle/u_2, \langle [0.4, 0.9], [0.1, 0.3], [0.6, 0.7]\rangle/u_3\})\Big\}$$

and

$$\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2 = \Big\{(\langle x_1, [0.3,0.4], [0.5,0.9], [0.7,0.8]\rangle, \{\langle [0.1,0.5], [0.7,0.9], [0.4,0.7]\rangle/u_1, \langle [0.1,0.3], [0.7,0.8], [0.8,0.9]\rangle/u_2, \langle [0.3,0.5], [0.7,0.9], [0.6,0.7]\rangle/u_3\}), (\langle x_2, [0.1,0.2], [0.3,0.4], [0.5,0.8]\rangle, \{\langle [0.2,0.3], [0.6,0.8], [0.2,0.4]\rangle/u_1, \langle [0.5,0.4], [0.4,0.6], [0.7,0.9]\rangle/u_2, \langle [0.1,0.2], [0.3,0.4], [0.5,0.6]\rangle/u_3\}), (\langle x_3, [0.2,0.4], [0.5,0.7], [0.4,0.6]\rangle, \{\langle [0.2,0.3], [0.2,0.6], [0.4,0.8]\rangle/u_1, \langle [0.2,0.5], [0.2,0.8], [0.6,0.9]\rangle/u_2, \langle [0.3,0.7], [0.2,0.5], [0.6,0.9]\rangle/u_3\})\Big\}$$

Proposition 1 Let $\mathcal{F} \in \tilde{\mathcal{F}}$. Then,

1. $(\mathcal{F}^{\tilde{c}})^{\tilde{c}} = \mathcal{F}$
2. $\mathcal{O}^{\tilde{c}} = \mathcal{U}$
3. $\mathcal{F} \tilde{\subseteq} \mathcal{U}$
4. $\mathcal{O} \tilde{\subseteq} \mathcal{F}$
5. $\mathcal{F} \tilde{\subseteq} \mathcal{F}$

Proposition 2 Let $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3 \in \tilde{\mathcal{F}}$. Then,

1. $\mathcal{F}_1 \tilde{\subseteq} \mathcal{F}_2 \wedge \mathcal{F}_2 \tilde{\subseteq} \mathcal{F}_3 \Rightarrow \mathcal{F}_1 \tilde{\subseteq} \mathcal{F}_3$
2. $\mathcal{F}_1 = \mathcal{F}_2 \wedge \mathcal{F}_2 = \mathcal{F}_3 \Leftrightarrow \mathcal{F}_1 = \mathcal{F}_3$
3. $\mathcal{F}_1 \tilde{\subseteq} \mathcal{F}_2 \wedge \mathcal{F}_2 \tilde{\subseteq} \mathcal{F}_1 \Rightarrow \mathcal{F}_1 = \mathcal{F}_2$

Proposition 3 Let $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3 \in \tilde{\mathcal{F}}$. Then,

1. $\mathcal{F}_1 \tilde{\cup} \mathcal{F}_1 = \mathcal{F}_1$
2. $\mathcal{F}_1 \tilde{\cup} \mathcal{O} = \mathcal{F}_1$
3. $\mathcal{F}_1 \tilde{\cup} \mathcal{U} = \mathcal{U}$
4. $\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2 = \mathcal{F}_2 \tilde{\cup} \mathcal{F}_1$
5. $(\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2) \tilde{\cup} \mathcal{F}_3 = \mathcal{F}_1 \tilde{\cup} (\mathcal{F}_2 \tilde{\cup} \mathcal{F}_3)$

Proposition 4 Let $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3 \in \tilde{\mathcal{F}}$. Then,

1. $\mathcal{F}_1 \tilde{\cap} \mathcal{F}_1 = \mathcal{F}_1$
2. $\mathcal{F}_1 \tilde{\cap} \mathcal{O} = \mathcal{O}$
3. $\mathcal{F}_1 \tilde{\cap} \mathcal{U} = \mathcal{F}_1$
4. $\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2 = \mathcal{F}_2 \tilde{\cap} \mathcal{F}_1$
5. $(\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2) \tilde{\cap} \mathcal{F}_3 = \mathcal{F}_1 \tilde{\cap} (\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3)$

Proposition 5 Let $\mathcal{F}_1, \mathcal{F}_2 \in \tilde{\mathcal{F}}$. Then, De Morgan's laws are valid

1. $(\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2)^{\tilde{c}} = \mathcal{F}_1^{\tilde{c}} \tilde{\cap} \mathcal{F}_2^{\tilde{c}}$
2. $(\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2)^{\tilde{c}} = \mathcal{F}_1^{\tilde{c}} \tilde{\cup} \mathcal{F}_2^{\tilde{c}}$

Proposition 6 Let $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3 \in \tilde{\mathcal{F}}$. Then,

1. $\mathcal{F}_1 \tilde{\cap} (\mathcal{F}_2 \tilde{\cup} \mathcal{F}_3) = (\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2) \tilde{\cup} (\mathcal{F}_1 \tilde{\cap} \mathcal{F}_3)$
2. $\mathcal{F}_1 \tilde{\cup} (\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3) = (\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2) \tilde{\cap} (\mathcal{F}_1 \tilde{\cup} \mathcal{F}_3)$

4 *ivnpivn*-soft matrices

In this section, we presented *ivnpivn*-soft matrices which are representative of the *ivnpivn*-soft sets. The matrix is useful for storing an *ivnpivn*-soft sets in computer memory which are very useful and applicable. In the following, some definitions and operations on npn-soft sets are defined in [15]; we extend these definitions and operations to *ivnpivn*-soft sets.

Definition 13 Let $U = \{u_1, u_2, \ldots, u_m\}$, $X = \{x_1, x_2, \ldots, x_n\}$ and

$$\mathcal{F} = \big\{(\langle x_i, T_{\mathcal{F}}(x_i), I_{\mathcal{F}}(x_i), F_{\mathcal{F}}(x_i)\rangle, \{\langle T_{f_{\mathcal{F}(x_i)}}(u_j), I_{f_{\mathcal{F}(x_i)}}(u_j), F_{f_{\mathcal{F}(x_i)}}(u_j)\rangle/u_j : \ u_j \in U\}) : x_i \in X\big\}$$

be an *ivnpivn*-soft set over U. Where

$$T_{\mathcal{F}}(x_i) = [\inf T_{\mathcal{F}}(x_i), \sup T_{\mathcal{F}}(x_i)] \subseteq [0,1],$$
$$I_{\mathcal{F}}(x_i) = [\inf I_{\mathcal{F}}(x_i), \sup I_{\mathcal{F}}(x_i)] \subseteq [0,1],$$
$$F_{\mathcal{F}}(x_i) = [\inf F_{\mathcal{F}}(x_i), \sup F_{\mathcal{F}}(x_i)] \subseteq [0,1],$$
$$T_{f_{\mathcal{F}(x_i)}}(u_j) = [\inf T_{f_{\mathcal{F}(x_i)}}(u_j), \sup T_{f_{\mathcal{F}(x_i)}}(u_j)] \subseteq [0,1],$$
$$I_{f_{\mathcal{F}(x_i)}}(u_j) = [\inf I_{f_{\mathcal{F}(x_i)}}(u_j), \sup I_{f_{\mathcal{F}(x_i)}}(u_j)] \subseteq [0,1],$$
$$F_{f_{\mathcal{F}(x_i)}}(u_j) = [\inf F_{f_{\mathcal{F}(x_i)}}(u_j), \sup F_{f_{\mathcal{F}(x_i)}}(u_j)] \subseteq [0,1].$$

If $\alpha_i = \langle T_{\mathcal{F}}(x_i), I_{\mathcal{F}}(x_i), F_{\mathcal{F}}(x_i)\rangle$ and $V_{ij} = \langle T_{f_{\mathcal{F}(x_i)}}(u_j), I_{f_{\mathcal{F}}}(x_i)(u_j), F_{f_{\mathcal{F}(x_i)}}(u_j)\rangle$, then the $\mathcal{F}$ can be represented by a matrix as in the following form

$$\mathcal{F}[n \times m] = \left[\begin{array}{c|cccc} \alpha_1 & V_{11} & V_{12} & \cdots & V_{1m} \\ \alpha_2 & V_{21} & V_{22} & \cdots & V_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_n & V_{n1} & V_{n2} & \cdots & V_{nm} \end{array}\right]$$

which is called an $n \times m$ *ivnpivn*-soft matrix (or *ivnpivns*-matrix) of the *ivnpivn*-soft set $\mathcal{F}$ over U.

According to this definition, an *ivnpivn*-soft set $\mathcal{F}$ is uniquely characterized by matrix $\mathcal{F}[n \times m]$. Therefore, we shall identify any *ivnpivn*-soft set with its *ivnpivns*-matrix and use these two concepts as interchangeable.

From now on, the set of all $n \times m$ *ivnpivns*-matrix over U will be denoted by $\tilde{\mathcal{F}}_{n \times m}$.

Example 7 Let $U = \{u_1, u_2, u_3\}$, $X = \{x_1, x_2, x_3\}$, $\mathcal{F}_1$ be given as in Example 5. Then, the *ivnpivns*-matrix of $\mathcal{F}_1$ is written by

$$\mathcal{F}_1[3\times 3]=\left[\begin{array}{c|ccc}\langle[.3,.4],[.5,.6],[.4,.5]\rangle & \langle[.5,.6],[.6,.7],[.3,.4]\rangle & \langle[.4,.5],[.7,.8],[.2,.3]\rangle & \langle[.6,.7],[.2,.3],[.3,.5]\rangle\\ \langle[.1,.2],[.3,.4],[.6,.7]\rangle & \langle[.7,.8],[.3,.4],[.2,.4]\rangle & \langle[.8,.4],[.2,.6],[.3,.4]\rangle & \langle[.4,.5],[.1,.3],[.2,.4]\rangle\\ \langle[.2,.4],[.4,.5],[.4,.6]\rangle & \langle[.2,.3],[.1,.4],[.3,.6]\rangle & \langle[.2,.5],[.1,.6],[.5,.8]\rangle & \langle[.3,.7],[.1,.3],[.6,.7]\rangle\end{array}\right]$$

Definition 14 Let $\mathcal{F}[n\times m]\in\widetilde{\mathcal{F}}_{n\times m}$. Then, $\mathcal{F}[n\times m]$ is called

1. A zero *ivnpivn*-soft matrix, denoted by $\mathcal{O}[n\times m]$, if $\alpha_i=\langle[0,0],[1,1],[1,1]\rangle$ and $V_{ij}=\langle[0,0],[1,1],[1,1]\rangle$ for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.
2. A universal *ivnpivn*-soft matrix, denoted by $\mathcal{U}[n\times m]$, if $\alpha_i=\langle[1,1],[0,0],[0,0]\rangle$ and $V_{ij}=\langle[1,1],[0,0],[0,0]\rangle$ for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

Example 8 Let $U=\{u_1,u_2,u_3\}, X=\{x_1,x_2,x_3\}$. Then, a zero *ivnpivn*-soft matrix is written by

$$\mathcal{O}[3\times 3]=\left[\begin{array}{c|ccc}\langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle\\ \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle\\ \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[0,0],[1,1],[1,1]\rangle\end{array}\right]$$

and a universal *ivnpivn*-soft matrix is written by

$$\mathcal{U}[3\times 3]=\left[\begin{array}{c|ccc}\langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle\\ \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle\\ \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & \langle[1,1],[0,0],[0,0]\rangle & <[1,1],[0,0],[0,0]\rangle\end{array}\right]$$

Definition 15 Let $\mathcal{F}_1[n\times m],\mathcal{F}_2[n\times m]\in\widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[n\times m]$ is a sub$-$*ivnpivn*-matrix of $\mathcal{F}_2[n\times m]$, denoted by $\mathcal{F}_1[n\times m]\widetilde{\subseteq}\mathcal{F}_2[n\times m]$, if

$$\begin{array}{ll}\inf T_{\mathcal{F}_1}(x_i)\le\inf T_{\mathcal{F}_2}(x_i) & \sup T_{\mathcal{F}_1}(x_i)\le\sup T_{\mathcal{F}_2}(x_i)\\ \inf I_{\mathcal{F}_1}(x_i)\ge\inf I_{\mathcal{F}_2}(x_i) & \sup I_{\mathcal{F}_1}(x_i)\ge\sup I_{\mathcal{F}_2}(x_i),\\ \inf F_{\mathcal{F}_1}(x_i)\ge\inf F_{\mathcal{F}_2}(x_i) & \sup F_{\mathcal{F}_1}(x_i)\ge\sup F_{\mathcal{F}_2}(x_i)\\ \inf T_{f_{\mathcal{F}_1(x_i)}}(u_j)\le\inf T_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup T_{f_{\mathcal{F}_1(x_i)}}(u_j)\le\sup T_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf I_{f_{\mathcal{F}_1(x_i)}}(u_j)\ge\inf I_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup I_{f_{\mathcal{F}_1(x_i)}}(u_j)\ge\sup I_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf F_{f_{\mathcal{F}_1(x_i)}}(u_j)\ge\inf F_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup F_{f_{\mathcal{F}_1(x_i)}}(u_j)\ge\sup F_{f_{\mathcal{F}_2(x_i)}}(u_j).\end{array}$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

2. $\mathcal{F}_1[n\times m]$ is a proper sub$-$*ivnpivn*-matrix of $\mathcal{F}_2[n\times m]$, denoted by $\mathcal{F}_1[n\times m]\widetilde{\subseteq}\mathcal{F}_2[n\times m]$, if

$$\begin{array}{ll}\inf T_{\mathcal{F}_1}(x_i)<\inf T_{\mathcal{F}_2}(x_i) & \sup T_{\mathcal{F}_1}(x_i)<\sup T_{\mathcal{F}_2}(x_i)\\ \inf I_{\mathcal{F}_1}(x_i)>\inf I_{\mathcal{F}_2}(x_i) & \sup I_{\mathcal{F}_1}(x_i)>\sup I_{\mathcal{F}_2}(x_i),\\ \inf F_{\mathcal{F}_1}(x_i)>\inf F_{\mathcal{F}_2}(x_i) & \sup F_{\mathcal{F}_1}(x_i)>\sup F_{\mathcal{F}_2}(x_i)\\ \inf T_{f_{\mathcal{F}_1(x_i)}}(u_j)<\inf T_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup T_{f_{\mathcal{F}_1(x_i)}}(u_j)<\sup T_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf I_{f_{\mathcal{F}_1(x_i)}}(u_j)>\inf I_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup I_{f_{\mathcal{F}_1(x_i)}}(u_j)>\sup I_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf F_{f_{\mathcal{F}_1(x_i)}}(u_j)>\inf F_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup F_{f_{\mathcal{F}_1(x_i)}}(u_j)>\sup F_{f_{\mathcal{F}_2(x_i)}}(u_j).\end{array}$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

3. $\mathcal{F}_1[n\times m]$ and $\mathcal{F}_2[n\times m]$ are equal $-$*ivnpivn*-matrix, denoted by $\mathcal{F}_1[n\times m]=\mathcal{F}_2[n\times m]$, if

$$\begin{array}{ll}\inf T_{\mathcal{F}_1}(x_i)=\inf T_{\mathcal{F}_2}(x_i) & \sup T_{\mathcal{F}_1}(x_i)=\sup T_{\mathcal{F}_2}(x_i)\\ \inf I_{\mathcal{F}_1}(x_i)=\inf I_{\mathcal{F}_2}(x_i) & \sup I_{\mathcal{F}_1}(x_i)=\sup I_{\mathcal{F}_2}(x_i),\\ \inf F_{\mathcal{F}_1}(x_i)=\inf F_{\mathcal{F}_2}(x_i) & \sup F_{\mathcal{F}_1}(x_i)=\sup F_{\mathcal{F}_2}(x_i)\\ \inf T_{f_{\mathcal{F}_1(x_i)}}(u_j)=\inf T_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup T_{f_{\mathcal{F}_1(x_i)}}(u_j)=\sup T_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf I_{f_{\mathcal{F}_1(x_i)}}(u_j)=\inf I_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup I_{f_{\mathcal{F}_1(x_i)}}(u_j)=\sup I_{f_{\mathcal{F}_2(x_i)}}(u_j),\\ \inf F_{f_{\mathcal{F}_1(x_i)}}(u_j)=\inf F_{f_{\mathcal{F}_2(x_i)}}(u_j) & \sup F_{f_{\mathcal{F}_1(x_i)}}(u_j)=\sup F_{f_{\mathcal{F}_2(x_i)}}(u_j).\end{array}$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

Definition 16 Let $\mathcal{F}[n\times m],\mathcal{F}_1[n\times m],\mathcal{F}_2[n\times m]\in\widetilde{\mathcal{F}}_{n\times m}$. Then

1 Union of $\mathcal{F}_1[n\times m]$ and $\mathcal{F}_2[n\times m]$, denoted by $\mathcal{F}_3[n\times m]=\mathcal{F}_1[n\times m]\widetilde{\cup}\mathcal{F}_2[n\times m]$, if

$$T_{\mathcal{F}_3}(x_i)=[s(\inf T_{\mathcal{F}_1}(x_i),\inf T_{\mathcal{F}_2}(x_i)),s(\sup T_{\mathcal{F}_1}(x_i),\sup T_{\mathcal{F}_2}(x_i))],$$
$$I_{\mathcal{F}_3}(x_i)=[t(\inf I_{\mathcal{F}_1}(x_i),\inf I_{\mathcal{F}_2}(x_i)),t(\sup I_{\mathcal{F}_1}(x_i),\sup I_{\mathcal{F}_2}(x_i))],$$
$$F_{\mathcal{F}_3}(x_i)=[t(\inf F_{\mathcal{F}_1}(x_i),\inf F_{\mathcal{F}_2}(x_i)),t(\sup F_{\mathcal{F}_1}(x_i),\sup F_{\mathcal{F}_2}(x_i))],$$
$$T_{f_{\mathcal{F}_3(x_i)}}(u_j)=[s(\inf T_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf T_{f_{\mathcal{F}_2(x_i)}}(u_j)),s(\sup T_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup T_{f_{\mathcal{F}_2(x_i)}}(u_j))],$$
$$I_{f_{\mathcal{F}_3(x_i)}}(u_j)=[t(\inf I_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf I_{f_{\mathcal{F}_2(x_i)}}(u_j)),t(\sup I_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup I_{f_{\mathcal{F}_2(x_i)}}(u_j))],$$
$$F_{f_{\mathcal{F}_3(x_i)}}(u_j)=[t(\inf F_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf F_{f_{\mathcal{F}_2(x_i)}}(u_j)),t(\sup F_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup F_{f_{\mathcal{F}_2(x_i)}}(u_j))].$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

2. Intersection of $\mathcal{F}_1[n\times m]$ and $\mathcal{F}_2[n\times m]$, denoted by $\mathcal{F}_4[n\times m]=\mathcal{F}_1[n\times m]\,\widetilde{\cap}\,\mathcal{F}_2[n\times m]$, if

$$T_{\widetilde{\mathcal{F}^c}}(x_i)=[\inf F_{\mathcal{F}}(x_i),\sup F_{\mathcal{F}}(x_i)],$$
$$I_{\widetilde{\mathcal{F}^c}}(x_i)=[1-\sup I_{\mathcal{F}}(x_i),1-\inf I_{\mathcal{F}}(x_i)],$$
$$F_{\widetilde{\mathcal{F}^c}}(x_i)=[\inf T_{\mathcal{F}}(x_i),\sup T_{\mathcal{F}}(x_i)],$$
$$T_{f_{\widetilde{\mathcal{F}^c}}(x_i)}(u_j)=[\inf F_{f_{\mathcal{F}}(x_i)}(u_j),\sup F_{f_{\mathcal{F}}(x_i)}(u_j)],$$
$$I_{f_{\widetilde{\mathcal{F}^c}}(x_i)}(u_j)=[1-\sup I_{f_{\mathcal{F}}(x_i)}(u_j),1-\inf I_{f_{\mathcal{F}}(x_i)}(u_j)],$$
$$F_{f_{\widetilde{\mathcal{F}^c}}(x_i)}(u_j)=[\inf T_{f_{\mathcal{F}}(x_i)}(u_j),\sup T_{f_{\widetilde{\mathcal{F}^c}}(x_i)}(u_j)].$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

Example 9 Consider Example 6. For t-norm and s-norm we use $min\{a,b\}$ and $max\{a,b\}$, respectively. Then, $\mathcal{F}_3[3\times 3]=\mathcal{F}_1[3\times 3]\,\widetilde{\cup}\,\mathcal{F}_2[3\times 3]$, $\mathcal{F}_4[3\times 3]=\mathcal{F}_1[3\times 3]\,\widetilde{\cap}\,\mathcal{F}_2[3\times 3]$ and $\widetilde{\mathcal{F}_1^c}[3\times 3]$ is given as

$$\mathcal{F}_3[3\times 3]=\left[\begin{array}{c|ccc}\langle[.6,.7],[.2,.6],[.4,.5]\rangle & \langle[.5,.6],[.6,.7],[.3,.4]\rangle & \langle[.4,.5],[.2,.4],[.2,.3]\rangle & \langle[.6,.7],[.2,.3],[.3,.5]\rangle\\ \langle[.1,.4],[.2,.4],[.5,.7]\rangle & \langle[.7,.8],[.3,.4],[.1,.2]\rangle & \langle[.8,.9],[.2,.6],[.3,.4]\rangle & \langle[.4,.5],[.1,.3],[.2,.4]\rangle\\ \langle[.3,.9],[.4,.5],[.3,.6]\rangle & \langle[.3,.5],[.1,.4],[.3,.6]\rangle & \langle[.3,.7],[.1,.6],[.5,.8]\rangle & \langle[.4,.9],[.1,.3],[.6,.7]\rangle\end{array}\right]$$
$$\mathcal{F}_4[3\times 3]=\left[\begin{array}{c|ccc}\langle[.3,.4],[.5,.9],[.7,.8]\rangle & \langle[.1,.5],[.7,.9],[.4,.7]\rangle & \langle[.1,.3],[.7,.8],[.8,.9]\rangle & \langle[.3,.5],[.7,.9],[.6,.7]\rangle\\ \langle[.1,.2],[.3,.4],[.5,.8]\rangle & \langle[.2,.3],[.6,.8],[.2,.4]\rangle & \langle[.5,.4],[.4,.6],[.7,.9]\rangle & \langle[.1,.2],[.3,.4],[.5,.6]\rangle\\ \langle[.2,.4],[.5,.7],[.4,.6]\rangle & \langle[.2,.3],[.2,.6],[.4,.8]\rangle & \langle[.2,.5],[.2,.8],[.6,.9]\rangle & \langle[.3,.7],[.2,.5],[.6,.9]\rangle\end{array}\right]$$

and

$$\widetilde{\mathcal{F}_1^c}[3\times 3]=\left[\begin{array}{c|ccc}\langle[.4,.5],[.4,.5],[.3,.4]\rangle & \langle[.3,.4],[.4,.3],[.5,.6]\rangle & \langle[.2,.3],[.3,.2],[.4,.5]\rangle & \langle[.3,.5],[.8,.7],[.6,.7]\rangle\\ \langle[.6,.7],[.6,.7],[.1,.2]\rangle & \langle[.2,.4],[.7,.6],[.7,.8]\rangle & \langle[.3,.4],[.8,.4],[.8,.4]\rangle & \langle[.2,.4],[.9,.7],[.4,.5]\rangle\\ \langle[.4,.6],[.5,.6],[.2,.4]\rangle & \langle[.3,.6],[.9,.6],[.2,.3]\rangle & \langle[.5,.8],[.9,.4],[.2,.5]\rangle & \langle[.6,.7],[.9,.7],[.3,.7]\rangle\end{array}\right].$$

$$T_{\mathcal{F}_4}(x_i)=[t(\inf T_{\mathcal{F}_1}(x_i),\inf T_{\mathcal{F}_2}(x_i)),t(\sup T_{\mathcal{F}_1}(x_i),\sup T_{\mathcal{F}_2}(x_i))],$$
$$I_{\mathcal{F}_4}(x_i)=[s(\inf I_{\mathcal{F}_1}(x_i),\inf I_{\mathcal{F}_2}(x_i)),s(\sup I_{\mathcal{F}_1}(x_i),\sup I_{\mathcal{F}_2}(x_i))],$$
$$F_{\mathcal{F}_4}(x_i)=[s(\inf F_{\mathcal{F}_1}(x_i),\inf F_{\mathcal{F}_2}(x_i)),s(\sup F_{\mathcal{F}_1}(x_i),\sup F_{\mathcal{F}_2}(x_i))],$$
$$T_{f_{\mathcal{F}_4(x_i)}}(u_j)=[t(\inf T_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf T_{f_{\mathcal{F}_2(x_i)}}(u_j)),t(\sup T_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup T_{f_{\mathcal{F}_2(x_i)}}(u_j))],$$
$$I_{f_{\mathcal{F}_4(x_i)}}(u_j)=[s(\inf I_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf I_{f_{\mathcal{F}_2(x_i)}}(u_j)),s(\sup I_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup I_{f_{\mathcal{F}_2(x_i)}}(u_j))],$$
$$F_{f_{\mathcal{F}_4(x_i)}}(u_j)=[s(\inf F_{f_{\mathcal{F}_1(x_i)}}(u_j),\inf F_{f_{\mathcal{F}_2(x_i)}}(u_j)),s(\sup F_{f_{\mathcal{F}_1(x_i)}}(u_j),\sup F_{f_{\mathcal{F}_2(x_i)}}(u_j))].$$

for $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$.

3. Complement of $\mathcal{F}[n\times m]$, denoted by $\widetilde{\mathcal{F}^c}[n\times m]$, if

Proposition 7 Let $\mathcal{F}\in\widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\left(\widetilde{\mathcal{F}^c}[m\times n]\right)^{\widetilde{c}}=\mathcal{F}[m\times n]$
2. $\widetilde{\mathcal{O}^c}[m\times n]=\mathcal{U}[m\times n]$

Proposition 8 Let $\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n],\mathcal{F}_3[m\times n]\in\widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[m\times n]\subseteq\mathcal{U}[m\times n]$
2. $\mathcal{O}[m\times n]\,\widetilde{\subseteq}\,\mathcal{F}_1[m\times n]$
3. $\mathcal{F}_1[m\times n]\,\widetilde{\subseteq}\,\mathcal{F}_1[m\times n]$
4. $\mathcal{F}_1[m\times n]\,\widetilde{\subseteq}\,\mathcal{F}_2[m\times n]\wedge\mathcal{F}_2[m\times n]\,\widetilde{\subseteq}\,\mathcal{F}_3[m\times n]\Rightarrow\mathcal{F}_1[m\times n]\,\widetilde{\subseteq}\,\mathcal{F}_3[m\times n]$

Proposition 9 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n], \mathcal{F}_3[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[m\times n] = \mathcal{F}_2[m\times n]$ and $\mathcal{F}_2[m\times n] = \mathcal{F}_3[m\times n] \Leftrightarrow \mathcal{F}_1[m\times n] = \mathcal{F}_3[m\times n]$
2. $\mathcal{F}_1[m\times n] \tilde{\subseteq} \mathcal{F}_2[m\times n]$ and $\mathcal{F}_2[m\times n] \tilde{\subseteq} \mathcal{F}_1[m\times n] \Leftrightarrow \mathcal{F}_1[m\times n] = \mathcal{F}_2[m\times n]$

Proposition 10 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n], \mathcal{F}_3[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_1[m\times n] = \mathcal{F}_1[m\times n]$
2. $\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{O}[m\times n] = \mathcal{F}_1[m\times n]$
3. $\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{U}[m\times n] = \mathcal{U}[m\times n]$
4. $\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_2[m\times n] = \mathcal{F}_2[m\times n] \tilde{\cup} \mathcal{F}_1[m\times n]$
5. $(\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_2[m\times n]) \tilde{\cup} \mathcal{F}_3[m\times n] = \mathcal{F}_1[m\times n] \tilde{\cup} (\mathcal{F}_2[m\times n] \tilde{\cup} \mathcal{F}_3[m\times n])$

Proposition 11 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n], \mathcal{F}_3[m\times n] \in \tilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_1[m\times n] = \mathcal{F}_1[m\times n]$
2. $\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{O}[m\times n] = \mathcal{O}[m\times n]$
3. $\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{U}[m\times n] = \mathcal{F}_1[m\times n]$
4. $\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_2[m\times n] = \mathcal{F}_2[m\times n] \tilde{\cap} \mathcal{F}_1[m\times n]$
5. $(\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_2[m\times n]) \tilde{\cap} \mathcal{F}_3[m\times n] = \mathcal{F}_1[m\times n] \tilde{\cap} (\mathcal{F}_2[m\times n] \tilde{\cap} \mathcal{F}_3[m\times n])$

Proposition 12 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then, De Morgan's laws are valid

1. $(\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_2[m\times n])^{\tilde{c}} = \mathcal{F}_1[m\times n]^{\tilde{c}} \tilde{\cap} \mathcal{F}_2[m\times n]^{\tilde{c}}$
2. $(\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_2[m\times n])^{\tilde{c}} = \mathcal{F}_1[m\times n]^{\tilde{c}} \tilde{\cup} \mathcal{F}_2[m\times n]^{\tilde{c}}$

Proposition 13 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n], \mathcal{F}_3[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then

1. $\mathcal{F}_1[m\times n] \tilde{\cap} (\mathcal{F}_2[m\times n] \tilde{\cup} \mathcal{F}_3[m\times n]) = (\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_2[m\times n]) \tilde{\cup} (\mathcal{F}_1[m\times n] \tilde{\cap} \mathcal{F}_3[m\times n])$
2. $\mathcal{F}_1[m\times n] \tilde{\cup} (\mathcal{F}_2[m\times n] \tilde{\cap} \mathcal{F}_3[m\times n]) = (\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_2[m\times n]) \tilde{\cap} (\mathcal{F}_1[m\times n] \tilde{\cup} \mathcal{F}_3[m\times n])$

5 Similarity measure on *ivnpivn*-soft sets

In the following, some definitions and operations on soft set is defined in [24], we extend these definitions and operations to *ivnpivn*-soft sets.

Definition 17 Let $U = \{u_1, u_2, \ldots, u_m\}$ be a universe, $E = \{x_1, x_2, \ldots, x_n\}$ be a set of parameters $\mathcal{F}_1[m\times n]$, $\mathcal{F}_2[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. If $\overrightarrow{\|T_{ij}\|}^2 \neq 0$ or $\overrightarrow{\|I_{ij}\|}^2 \neq 0$ or $\overrightarrow{\|F_{ij}\|}^2 \,/\!= 0$ for at least one $i \in \{1,2,\ldots,n\}$ and $j \in \{1,2,\ldots,m\}$, then similarity between $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ is defined by

$$S(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} \overrightarrow{\|V_i^2\|}\cdot\overrightarrow{\|T_{ij}\|}\cdot\overrightarrow{\|I_{ij}\|}\cdot\overrightarrow{\|F_{ij}\|}}{\sum_{i=1}^{n}\sum_{j=1}^{m} max\{\overrightarrow{\|T_{ij}}\|^2, \overrightarrow{\|I_{ij}}\|^2, \overrightarrow{\|F_{ij}}\|^2\}}$$

where

$$\begin{aligned}
\overrightarrow{V_i^2} &= \big(\inf T_i^1\cdot\inf T_i^2 + \sup T_i^1\cdot\sup T_i^2, \inf I_i^1\cdot\inf I_i^2 \\
&\quad +\sup I_i^1\cdot\sup I_i^2, \inf F_i^1\cdot\inf F_i^2 + \sup F_i^1\cdot\sup F_i^2\big)\\
\overrightarrow{T_{ij}} &= \big(\inf T_{i1}^1 + \sup T_{i1}^1 - \inf T_{i1}^2 - \sup T_{i1}^2, \inf T_{i2}^1 \\
&\quad +\sup T_{i2}^1 - \inf T_{i2}^2 - \sup T_{i2}^2, \ldots,\\
&\quad \inf T_{im}^1 + \sup T_{im}^1 - \inf T_{im}^2 - \sup T_{im}^2\big)\\
\overrightarrow{I_{ij}} &= \big(\inf I_{i1}^1 + \sup I_{i1}^1 - \inf I_{i1}^2 - \sup I_{i1}^2, \inf I_{i2}^1 \\
&\quad +\sup I_{i2}^1 - \inf I_{i2}^2 - \sup I_{i2}^2, \ldots,\\
&\quad \inf I_{im}^1 + \sup I_{im}^1 - \inf I_{im}^2 - \sup I_{im}^2\big)\\
\overrightarrow{F_{ij}} &= \big(\inf F_{i1}^1 + \sup F_{i1}^1 - \inf F_{i1}^2 - \sup F_{i1}^2, \inf F_{i2}^1 \\
&\quad +\sup F_{i2}^1 - \inf F_{i2}^2 - \sup F_{i2}^2, \ldots,\\
&\quad \inf F_{im}^1 + \sup F_{im}^1 - \inf F_{im}^2 - \sup F_{im}^2\big)
\end{aligned}$$

Note: If $\overrightarrow{\|T_{ij}\|}^2, \overrightarrow{\|I_{ij}\|}^2, \overrightarrow{F_{ij}\|}^2 = 0$ for all $i \in \{1,2,\ldots,n\}$ and $j \in \{1,2,\ldots,m\}$ or $\mathcal{F}_1[m\times n] = \mathcal{F}_2[m\times n]$, then $S(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) = 1$.

Definition 18 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then, $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ are said to be α-similar, denoted $\mathcal{F}_1[m\times n] \approx^{\alpha} \mathcal{F}_2[m\times n]$, if and only if $S(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) = \alpha$ for $\alpha \in (0,1)$.

We call the two *ivnpivn*-soft sets significantly similar if $S(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) > \frac{1}{2}$.

Example 10 Consider Example 6. If two *ivnpivn*-matrices $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ are written by

$$\mathcal{F}_1[3\times 3] = \left[\begin{array}{c|ccc}
\langle[.3,.4],[.5,.6],[.5,.6]\rangle & \langle[.5,.6],[.6,.7],[.3,.4]\rangle & \langle[.4,.5],[.7,.8],[.2,.3]\rangle & \langle[.6,.7],[.2,.3],[.3,.5]\rangle\\
\langle[.1,.2],[.3,.4],[.6,.7]\rangle & \langle[.7,.8],[.3,.4],[.2,.4]\rangle & \langle[.4,.8],[.2,.6],[.3,.4]\rangle & \langle[.4,.5],[.1,.3],[.2,.4]\rangle\\
\langle[.2,.4],[.4,.5],[.4,.6]\rangle & \langle[.2,.3],[.1,.4],[.3,.6]\rangle & \langle[.2,.5],[.1,.6],[.5,.8]\rangle & \langle[.3,.7],[.1,.3],[.6,.7]\rangle
\end{array}\right],$$

and

$$\mathcal{F}_2[3\times 3]=\left[\begin{array}{l|lll}\langle[.6,.7],[.2,.9],[.7,.8]\rangle & \langle[.1,.5],[.7,.9],[.4,.7]\rangle & \langle[.1,.3],[.2,.4],[.8,.9]\rangle & \langle[.3,.5],[.7,.9],[.6,.7]\rangle\\ \langle[.1,.4],[.2,.4],[.5,.8]\rangle & \langle[.2,.3],[.6,.8],[.1,.2]\rangle & \langle[.5,.9],[.4,.6],[.7,.9]\rangle & \langle[.1,.2],[.3,.4],[.5,.6]\rangle\\ \langle[.3,.9],[.5,.7],[.3,.6]\rangle & \langle[.3,.5],[.2,.6],[.4,.8]\rangle & \langle[.3,.7],[.2,.8],[.6,.9]\rangle & \langle[.4,.9],[.2,.5],[.6,.9]\rangle\end{array}\right]$$

Then, we can obtain

$$\begin{aligned}
&i=1,\quad (0.5,0.5,0.5)\quad\Rightarrow\quad \|T_{1j}\|=\sqrt{0.75}\,,\\
&i=2,\quad (1.0,-0.2,0.6)\quad\Rightarrow\quad \|T_{2j}\|=\sqrt{1.4},\\
&i=3,\quad (-0.3,-0.3,-0.3)\quad\Rightarrow\quad \|T_{3j}\|=\sqrt{0.27},\\
&i=1,\quad (-0.3,0.9,-1.1)\quad\Rightarrow\quad \|I_{1j}\|=\sqrt{2.11}\,,\\
&i=2,\quad (-0.7,-0.2,-0.3)\quad\Rightarrow\quad \|I_{2j}\|=\sqrt{0.62},\\
&i=3,\quad (-0.3,-0.3,-0.3)\quad\Rightarrow\quad \|I_{3j}\|=\sqrt{0.27},\\
&i=1,\quad (-0.4,-1.2,-0.5)\quad\Rightarrow\quad \|F_{1j}\|=\sqrt{1.85},\\
&i=2,\quad (0.3,-0.9,-0.5)\quad\Rightarrow\quad \|F_{2j}\|=\sqrt{1.15},\\
&i=3,\quad (-0.3,-0.2,-0.2)\quad\Rightarrow\quad \|F_{3j}\|=\sqrt{0.17}.
\end{aligned}$$

and

$$\begin{aligned}
&i=1,\quad V_1^2=(0.46,0.64,0.82)\quad\Rightarrow\quad \|V_1^2\|=\sqrt{1.2936},\\
&i=2,\quad V_2^2=(0.09,0.22,0.86)\quad\Rightarrow\quad \|V_2^2\|=\sqrt{0.7961},\\
&i=3,\quad V_3^2=(0.38,0.55,0.48)\quad\Rightarrow\quad \|V_3^2\|=\sqrt{0.6773},
\end{aligned}$$

Now the similarity between $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ is calculated as

Definition 19 Let $\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n]\in\widetilde{\mathcal{F}}_{n\times m}$. Then, the distances between $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ are defined as,

1. Hamming measure,

$$d(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])=\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}|V_i.V_{ij}|$$

where

$$\begin{aligned}\widetilde{V}_i=&\inf T_i^1+\sup T_i^1-\inf T_i^2-\sup T_i^2+\inf I_i^1\\&+\sup I_i^1-\inf I_i^2-\sup I_i^2+\inf F_i^1+\sup F_i^1\\&-\inf F_i^2-\sup F_i^2\end{aligned}$$

and

$$\begin{aligned}\widetilde{V_{ij}}=&\inf T_{ij}^1+\sup T_{ij}^1-\inf T_{ij}^2-\sup T_{ij}^2+\inf I_{ij}^1\\&+\sup I_{ij}^1-\inf I_{ij}^2-\sup I_{ij}^2+\inf F_{ij}^1+\sup F_{ij}^1\\&-\inf F_{ij}^2-\sup F_{ij}^2\end{aligned}$$

$$\begin{aligned}S(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3])&=\frac{\sqrt{1.2936}\sqrt{0.75}\sqrt{2.11}\sqrt{1.85}\;+\sqrt{0.7961}\sqrt{1.4}\sqrt{0.62}\sqrt{1.15}\;+\sqrt{0.6773}\sqrt{0.27}\sqrt{0.27}\sqrt{0.17}}{\max\{0.75,2.11,1.85\}+\max\{1.4,0.63,1.15\}+\max\{0.27,0.27,0.17\}}\\&=\frac{1.95+0.89+0.09}{3.78}=0.78\end{aligned}$$

Proposition 14 Let $\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n],\mathcal{F}_3[m\times n]\in\widetilde{\mathcal{F}}_{n\times m}$. Then, the followings hold;

(i.) $S(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])=S(\mathcal{F}_2[m\times n],\mathcal{F}_1[m\times n])$,

(ii.) $0\le S(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])\le 1$,

(iii.) $S(\mathcal{F}_1[m\times n],\mathcal{F}_1[m\times n])=1$.

Proof Proof easily can be made by using Definition 10.

Now we give distance measures between $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ with propositions by using the study of Jiang et al. [19]. □

2. Normalized Hamming measure,

$$l(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])=\frac{1}{mn}d(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])$$

3. Euclidean distance,

$$e(\mathcal{F}_1[m\times n],\mathcal{F}_2[m\times n])=\left(\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{V}_i.\overline{V}_{ij}|\right)^{\frac{1}{2}}$$

where

$$\overline{V}_i = \left(\inf T_i^1 + \sup T_i^1 - \inf T_i^2 - \sup T_i^2\right)^2 + \left(\inf I_i^1 + \sup I_i^1 - \inf I_i^2 - \sup I_i^2\right)^2 + \left(\inf F_i^1 + \sup F_i^1 - \inf F_i^2 - \sup F_i^2\right)^2$$

and

$$\overline{V}_{ij} = \left(\inf T_{ij}^1 + \sup T_{ij}^1 - \inf T_{ij}^2 - \sup T_{ij}^2\right)^2 + \left(\inf I_{ij}^1 + \sup I_{ij}^1 - \mathit{inf} I_{ij}^2 - \sup I_{ij}^2\right)^2 + \left(\inf F_{ij}^1 + \sup F_{ij}^1 - \inf F_{ij}^2 - \sup F_{ij}^2\right)^2$$

4. Normalized Euclidean distance,

$$q(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) = \frac{1}{mn} e(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n])$$

Example 11 Consider Example 10. Now we give distance measures between $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ as,

1. Hamming measure,

$$\widetilde{V_1} = -0.6 + 0 - 0.4 = -1$$
$$\widetilde{V_2} = -0.2 + 0.1 + 0 = -0.1$$
$$\widetilde{V_3} = -0.6 - 0.3 + 0.1 = -0.8$$

and

$$\widetilde{V_{11}} = 0.5 - 0.3 - 0.4 = -0.2$$
$$\widetilde{V_{12}} = 0.5 + 0.9 - 0.1 = 0.2$$
$$\widetilde{V_{13}} = 0.5 - 1.1 - 0.5 = -1.1$$

$$\widetilde{V_{21}} = 1 - 0.7 + 0.3 = 0.6$$
$$\widetilde{V_{22}} = -0.2 - 0.2 - 0.9 = -1.3$$
$$\widetilde{V_{23}} = 0.6 - 0.3 - 0.5 = -0.2$$

$$\widetilde{V_{31}} = -0.3 - 0.3 - 0.3 = -0.9$$
$$\widetilde{V_{32}} = -0.3 - 0.3 - 0.2 = -0.8$$
$$\widetilde{V_{31}} = -0.3 - 0.3 - 0.2 = -0.8$$

Then, we can obtain Hamming measure between $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ as,

$$d(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}|V_i.V_{ij}|$$
$$= \frac{1}{2}(0.2 + 0.2 + 1.1 + 0.6 + 0.13 + 0.02 + 0.72 + 0.64 + 0.64) = 1.855$$

2. Normalized Hamming measure,

$$l(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \frac{1}{3.3} d(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \frac{1}{9} 1.855 \cong 0.2061$$

3. Euclidean distance,

$$\overline{V}_1 = 0.36 + 0 + 0.16 = 0.52$$
$$\overline{V}_2 = 0.04 + 0.01 + 0 = 0.05$$
$$\overline{V}_3 = 0.36 + 0.09 + 0.01 = 0.46$$

and

$$\overline{V}_{11} = 0.25 + 0.09 + 0.16 = 0.50$$
$$\overline{V}_{12} = 0.25 + 0.81 + 1.44 = 2.50$$
$$\overline{V}_{13} = 0.25 + 1.21 + 0.25 = 1.71$$

$$\overline{V}_{21} = 1 + 0.49 + 0.09 = 1.58$$
$$\overline{V}_{22} = 0.04 + 0.04 + 0.81 = 0.89$$
$$\overline{V}_{23} = 0.36 + 0.09 + 0.25 = 0.70$$

$$\overline{V}_{31} = 0.09 + 0.09 + 0.09 = 0.27$$
$$\overline{V}_{32} = 0.09 + 0.09 + 0.04 = 0.22$$
$$\overline{V}_{33} = 0.09 + 0.09 + 0.04 = 0.22$$

Then, we can obtain Euclidean distance between $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ as,

$$e(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \left(\sum_{i=1}^{3}\sum_{j=1}^{3}|\overline{V}_i.\overline{V}_{ij}|\right)^{\frac{1}{2}} = (2.4492 + 0.1585 + 0.3266)^{\frac{1}{2}} = (2.9343)^{\frac{1}{2}} \cong 1.713$$

4. Normalized Euclidean distance,

$$q(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \frac{1}{3.3} e(\mathcal{F}_1[3\times 3], \mathcal{F}_2[3\times 3]) = \frac{1}{9} 1.713 \cong 0.19$$

Theorem 1 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then, the followings hold;

(i.) $d(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) \leq mn$
(ii.) $l(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) \leq 1$
(iii.) $e(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) \leq \sqrt{mn}$
(iv.) $q(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) \leq 1$

Proof Proof easily can be made by using Definition 6. □

Definition 20 Let $\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n] \in \widetilde{\mathcal{F}}_{n\times m}$. Then, by using the distances, similarity measure of $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ is defined as,

$$S_K(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n]) = \frac{1}{1 + K(\mathcal{F}_1[m\times n], \mathcal{F}_2[m\times n])}$$

where $K \in \{d, l, e, q\}$.

Example 12 Consider Example 11. Now we give similarity measure of $\mathcal{F}_1[3\times 3]$ and $\mathcal{F}_2[3\times 3]$ by using the distances of Example 11 as,

$$S_d(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3]) = \frac{1}{1+d(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3])} = \frac{1}{1+1.855} \cong 0.35$$

$$S_l(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3]) = \frac{1}{1+l(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3])} = \frac{1}{1+0.2061} \cong 0.83$$

$$S_e(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3]) = \frac{1}{1+e(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3])} = \frac{1}{1+1.713} \cong 0.37$$

$$S_q(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3]) = \frac{1}{1+q(\mathcal{F}_1[3\times 3],\mathcal{F}_2[3\times 3])} = \frac{1}{1+0.19} \cong 0.84$$

6 Decision-making method

In this section, we construct a decision-making method that is based on the similarity measure of two *ivnpivn*-soft sets. The algorithm of decision-making method can be given as:

Step 1. Construct an *ivnpivn*-soft set $\mathcal{F}_1$ over U for problem with the help of a expert,

Step 2. Construct an *ivnpivn*-soft set $\mathcal{F}_2$ based on a responsible person for the problem,

Step 3. Write *ivnpivn*-matrices $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ for $\mathcal{F}_1$ and $\mathcal{F}_2$ according to Definition 13, respectively,

Step 4. Calculate the similarity between $\mathcal{F}_1[m\times n]$ and $\mathcal{F}_2[m\times n]$ according to Definition 17,

Step 5. Determine result by using the similarity.

Now, we can give an application for the decision-making method. The similarity measure can be applied to detect whether an ill person is suffering from a certain disease or not.

6.1 Application

Let us consider the decision-making problem adopted from [24]. In this applications, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from cancer. For this, we first construct an *ivnpivn*-soft set for the illness and an *ivnpivn*-soft set for the ill person. We then find the similarity measure of these two *ivnpivn*-soft sets. If they are significantly similar, then we conclude that the person is possibly suffering from cancer.

Example 13 Assume that our universal set contain only two elements cancer and not cancer, i.e. $U=\{u_1,u_2\}$. Here the set of parameters X is the set of certain visible symptoms, let us say, $X=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$ where $x_1=$ jaundice, $x_2=$ bone pain, $x_3=$ headache, $x_4=$ loss of appetite, $x_5=$ weight loss, $x_6=$ heal wounds , $x_7=$ handle and shoulder pain, $x_8=$ lump anywhere on the body for no reason and $x_9=$ chest pain.

Step 1. We construct an *ivnpivn*-soft set $\mathcal{F}_1$ over U for cancer with the help of a medical person as:

$$\begin{aligned}\mathcal{F}_1=\Big\{&(\langle x_1,[0.5,0.7],[0.1,0.2],[0.7,0.8]\rangle,\{\langle[0.5,0.6],[0.1,0.3],[0.8,0.9]\rangle/u_1,\langle[0.4,0.6],[0.1,0.3],[0.4,0.5]\rangle/u_2\}),\\
&(\langle x_2,[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle,\{\langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle/u_1,\langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle/u_2\}),\\
&(\langle x_3,[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle,\{\langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle/u_1,\langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle/u_2\}),\\
&(\langle x_4,[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle,\{\langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle/u_1,\langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle/u_2\}),\\
&(\langle x_5,[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle,\{\langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle/u_1,\langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle/u_2\}),\\
&(\langle x_6,[0.3,0.5],[0.7,0.8],[0.2,0.6]\rangle,\{\langle[0.5,0.7],[0.3,0.5],[0.4,0.8]\rangle/u_1,\langle[0.2,0.6],[0.5,0.6],[0.3,0.7]\rangle/u_2\}),\\
&(\langle x_7,[0.3,0.8],[0.6,0.7],[0.5,0.9]\rangle,\{\langle[0.5,0.9],[0.5,0.8],[0.7,0.9]\rangle/u_1,\langle[0.3,0.7],[0.8,0.9],[0.4,0.5]\rangle/u_2\}),\\
&(\langle x_8,[0.2,0.6],[0.3,0.4],[0.5,0.7]\rangle,\{\langle[0.4,0.7],[0.7,0.9],[0.3,0.6]\rangle/u_1,\langle[0.6,0.7],[0.2,0.4],[0.1,0.5]\rangle/u_2\}),\\
&(\langle x_9,[0.1,0.2],[0.5,0.6],[0.1,0.6]\rangle,\{\langle[0.3,0.4],[0.2,0.3],[0.4,0.6]\rangle/u_1,\langle[0.6,0.9],[0.4,0.7],[0.6,0.8]\rangle/u_2\})\Big\}\end{aligned}$$

Step 2. We construct an *ivnpivn*-soft sets $\mathcal{F}_2$ based on data of ill person as:

$$\mathcal{F}_2 = \Big\{ (\langle x_1, [0.3,0.4],[0.5,0.6],[0.4,0.5]\rangle, \{\langle [0.1,0.9],[0.1,0.5],[0.2,0.6]\rangle/u_1, \langle [0.0,0.9],[0.1,0.2],[0.0,0.1]\rangle/u_2\}),$$
$$(\langle x_2, [0.1,0.2],[0.3,0.4],[0.6,0.7]\rangle, \{\langle [0.0,0.1],[0.5,0.7],[0.8,0.9]\rangle/u_1, \langle [0.1,0.3],[0.0,0.2],[0.8,0.9]\rangle/u_2\}),$$
$$(\langle x_3, [0.2,0.4],[0.4,0.5],[0.4,0.6]\rangle, \{\langle [0.1,0.3],[0.4,0.6],[0.6,0.7]\rangle/u_1, \langle [0.8,0.9],[0.9,1.0],[0.6,0.7]\rangle/u_2\}),$$
$$(\langle x_4, [0.5,0.6],[0.6,0.7],[0.3,0.4]\rangle, \{\langle [0.0,0.9],[0.2,0.3],[0.0,0.1]\rangle/u_1, \langle [0.5,0.8],[0.1,0.4],[0.7,0.9]\rangle/u_2\}),$$
$$(\langle x_5, [0.3,0.5],[0.7,0.8],[0.2,0.3]\rangle, \{\langle [0.9,0.1],[0.5,0.8],[0.1,0.2]\rangle/u_1, \langle [0.8,0.9],[0.1,0.3],[0.2,0.4]\rangle/u_2\}),$$
$$(\langle x_6, [0.6,0.7],[0.2,0.3],[0.3,0.5]\rangle, \{\langle [0.1,0.3],[0.8,0.9],[0.9,1.0]\rangle/u_1, \langle [0.8,1.0],[0.8,0.9],[0.1,0.2]\rangle/u_2\}),$$
$$(\langle x_7, [0.7,0.8],[0.3,0.4],[0.2,0.4]\rangle, \{\langle [0.8,1.0],[0.7,0.8],[0.0,0.1]\rangle/u_1, \langle [0.6,0.7],[0.0,0.1],[0.6,0.9]\rangle/u_2\}),$$
$$(\langle x_8, [0.8,0.9],[0.2,0.6],[0.3,0.4]\rangle, \{\langle [0.6,0.9],[0.8,1.0],[0.3,0.4]\rangle/u_1, \langle [0.0,0.1],[0.0,0.2],[0.9,1.0]\rangle/u_2\}),$$
$$(\langle x_9, [0.3,0.4],[0.7,0.9],[0.1,0.2]\rangle, \{\langle [0.8,0.9],[0.7,0.8],[0.5,0.6]\rangle/u_1, \langle [0.0,1.0],[0.0,0.3],[0.7,0.9]\rangle/u_2\}) \Big\}$$

Step 3. We construct *ivnpivn*-matrices $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_2[9 \times 2]$ for $\mathcal{F}_1$ and $\mathcal{F}_2$, respectively as:

$$\mathcal{F}_1[9\times 2] = \left[\begin{array}{l|ll}
\langle [0.5,0.7],[0.1,0.2],[0.7,0.8]\rangle & \langle [0.5,0.6],[0.1,0.3],[0.8,0.9]\rangle & \langle [0.4,0.6],[0.1,0.3],[0.4,0.5]\rangle \\
\langle [0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle [0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle [0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \\
\langle [0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle [0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle [0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\
\langle [0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle [0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle [0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\
\langle [0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle [0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle [0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\
\langle [0.3,0.5],[0.7,0.8],[0.2,0.6]\rangle & \langle [0.5,0.7],[0.3,0.5],[0.4,0.8]\rangle & \langle [0.2,0.6],[0.5,0.6],[0.3,0.7]\rangle \\
\langle [0.3,0.8],[0.6,0.7],[0.5,0.9]\rangle & \langle [0.5,0.9],[0.5,0.8],[0.7,0.9]\rangle & \langle [0.3,0.7],[0.8,0.9],[0.4,0.5]\rangle \\
\langle [0.2,0.6],[0.3,0.4],[0.5,0.7]\rangle & \langle [0.4,0.7],[0.7,0.9],[0.3,0.6]\rangle & \langle [0.6,0.7],[0.2,0.4],[0.1,0.5]\rangle \\
\langle [0.1,0.2],[0.5,0.6],[0.1,0.6]\rangle & \langle [0.3,0.4],[0.2,0.3],[0.4,0.6]\rangle & \langle [0.6,0.9],[0.4,0.7],[0.6,0.8]\rangle
\end{array}\right],$$

$$\mathcal{F}_2[9\times 2] = \left[\begin{array}{l|ll}
\langle [0.3,0.4],[0.5,0.6],[0.4,0.5]\rangle & \langle [0.1,0.9],[0.1,0.5],[0.2,0.6]\rangle & \langle [0.0,0.9],[0.1,0.2],[0.0,0.1]\rangle \\
\langle [0.1,0.2],[0.3,0.4],[0.6,0.7]\rangle & \langle [0.0,0.1],[0.5,0.7],[0.8,0.9]\rangle & \langle [0.1,0.3],[0.0,0.2],[0.8,0.9]\rangle \\
\langle [0.2,0.4],[0.4,0.5],[0.4,0.6]\rangle & \langle [0.1,0.3],[0.4,0.6],[0.6,0.7]\rangle & \langle [0.8,0.9],[0.9,1.0],[0.6,0.7]\rangle \\
\langle [0.5,0.6],[0.6,0.7],[0.3,0.4]\rangle & \langle [0.0,0.9],[0.2,0.3],[0.0,0.1]\rangle & \langle [0.5,0.8],[0.1,0.4],[0.7,0.9]\rangle \\
\langle [0.3,0.5],[0.7,0.8],[0.2,0.3]\rangle & \langle [0.9,0.1],[0.5,0.8],[0.1,0.2]\rangle & \langle [0.8,0.9],[0.1,0.3],[0.2,0.4]\rangle \\
\langle [0.6,0.7],[0.2,0.3],[0.3,0.5]\rangle & \langle [0.1,0.3],[0.8,0.9],[0.9,1.0]\rangle & \langle [0.8,1.0],[0.8,0.9],[0.1,0.2]\rangle \\
\langle [0.7,0.8],[0.3,0.4],[0.2,0.4]\rangle & \langle [0.8,1.0],[0.7,0.8],[0.0,0.1]\rangle & \langle [0.6,0.7],[0.0,0.1],[0.6,0.9]\rangle \\
\langle [0.8,0.9],[0.2,0.6],[0.3,0.4]\rangle & \langle [0.6,0.9],[0.8,1.0],[0.3,0.4]\rangle & \langle [0.0,0.1],[0.0,0.2],[0.9,1.0]\rangle \\
\langle [0.3,0.4],[0.7,0.9],[0.1,0.2]\rangle & \langle [0.8,0.9],[0.7,0.8],[0.5,0.6]\rangle & \langle [0.0,1.0],[0.0,0.3],[0.7,0.9]\rangle
\end{array}\right]$$

Step 4. We calculated the similarity between $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_2[9 \times 2]$ as:

$$S(\mathcal{F}_1[9 \times 2], \mathcal{F}_2[9 \times 2]) = 0.29\langle\frac{1}{2}$$

Step 5. The $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_2[9 \times 2]$ are not significantly similar. Therefore, we conclude that the person is not possibly suffering from cancer.

Example 14 Let us consider Example 13 with different ill person.

Step 1. We construct an *ivnpivn*-soft set $\mathcal{F}_1$ over U for cancer with the help of a medical person as

$$\begin{aligned}\mathcal{F}_1 = \Big\{ & (\langle x_1, [0.5, 0.7], [0.1, 0.2], [0.7, 0.8]\rangle, \{\langle [0.5, 0.6], [0.1, 0.3], [0.8, 0.9]\rangle/u_1, \langle [0.4, 0.6], [0.1, 0.3], [0.4, 0.5]\rangle/u_2\}),\\
& (\langle x_2, [0.7, 0.8], [0.0, 0.1], [0.1, 0.2]\rangle, \{\langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3]\rangle/u_1, \langle [0.6, 0.7], [0.3, 0.4], [0.8, 0.9]\rangle/u_2\}),\\
& (\langle x_3, [0.3, 0.6], [0.2, 0.3], [0.3, 0.4]\rangle, \{\langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4]\rangle/u_1, \langle [0.4, 0.5], [0.2, 0.4], [0.7, 0.9]\rangle/u_2\}),\\
& (\langle x_4, [0.6, 0.7], [0.1, 0.2], [0.2, 0.3]\rangle, \{\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3]\rangle/u_1, \langle [0.3, 0.6], [0.3, 0.5], [0.8, 0.9]\rangle/u_2\}),\\
& (\langle x_5, [0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\rangle, \{\langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4]\rangle/u_1, \langle [0.7, 0.9], [0.2, 0.3], [0.4, 0.5]\rangle/u_2\}),\\
& (\langle x_6, [0.3, 0.5], [0.7, 0.8], [0.2, 0.6]\rangle, \{\langle [0.5, 0.7], [0.3, 0.5], [0.4, 0.8]\rangle/u_1, \langle [0.2, 0.6], [0.5, 0.6], [0.3, 0.7]\rangle/u_2\}),\\
& (\langle x_7, [0.3, 0.8], [0.6, 0.7], [0.5, 0.9]\rangle, \{\langle [0.5, 0.9], [0.5, 0.8], [0.7, 0.9]\rangle/u_1, \langle [0.3, 0.7], [0.8, 0.9], [0.4, 0.5]\rangle/u_2\}),\\
& (\langle x_8, [0.2, 0.6], [0.3, 0.4], [0.5, 0.7]\rangle, \{\langle [0.4, 0.7], [0.7, 0.9], [0.3, 0.6]\rangle/u_1, \langle [0.6, 0.7], [0.2, 0.4], [0.1, 0.5]\rangle/u_2\}),\\
& (\langle x_9, [0.1, 0.2], [0.5, 0.6], [0.1, 0.6]\rangle, \{\langle [0.3, 0.4], [0.2, 0.3], [0.4, 0.6]\rangle/u_1, \langle [0.6, 0.9], [0.4, 0.7], [0.6, 0.8]\rangle/u_2\}) \Big\}\end{aligned}$$

Step 2. We construct an *ivnpivn*-soft sets $\mathcal{F}_3$ based on data of ill person as

$$\begin{aligned}\mathcal{F}_3 = \Big\{ & (\langle x_1, [0.3, 0.4], [0.5, 0.6], [0.4, 0.5]\rangle, \{\langle [0.1, 0.2], [0.7, 0.8], [0.0, 0.1]\rangle/u_1, \langle [0.0, 0.1], [0.9, 1.0], [0.9, 1.0]\rangle/u_2\}),\\
& (\langle x_2, [0.1, 0.2], [0.3, 0.4], [0.6, 0.7]\rangle, \{\langle [0.0, 0.1], [0.5, 0.6], [0.8, 0.9]\rangle/u_1, \langle [0.1, 0.2], [0.0, 0.1], [0.3, 0.4]\rangle/u_2\}),\\
& (\langle x_3, [0.2, 0.3], [0.4, 0.5], [0.6, 0.7]\rangle, \{\langle [0.1, 0.2], [0.0, 0.1], [0.6, 0.7]\rangle/u_1, \langle [0.8, 0.9], [0.9, 1.0], [0.0, 0.1]\rangle/u_2\}),\\
& (\langle x_4, [0.5, 0.6], [0.6, 0.7], [0.3, 0.4]\rangle, \{\langle [0.0, 0.1], [0.6, 0.7], [0.1, 0.2]\rangle/u_1, \langle [0.4, 0.5], [0.7, 0.8], [0.2, 0.3]\rangle/u_2\}),\\
& (\langle x_5, [0.3, 0.4], [0.7, 0.8], [0.9, 1.0]\rangle, \{\langle [0.9, 1.0], [0.6, 0.7], [0.7, 0.8]\rangle/u_1, \langle [0.8, 0.9], [0.4, 0.5], [0.2, 0.3]\rangle/u_2\}),\\
& (\langle x_6, [0.6, 0.7], [0.4, 0.5], [0.5, 0.6]\rangle, \{\langle [0.1, 0.2], [0.8, 0.9], [0.9, 1.0]\rangle/u_1, \langle [0.3, 0.4], [0.4, 0.5], [0.1, 0.2]\rangle/u_2\}),\\
& (\langle x_7, [0.7, 0.8], [0.9, 1.0], [0.1, 0.2]\rangle, \{\langle [0.1, 0, 2], [0.7, 0.8], [0.0, 0.1]\rangle/u_1, \langle [0.9, 1.0], [0.0, 0.1], [0.6, 0.7]\rangle/u_2\}),\\
& (\langle x_8, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]\rangle, \{\langle [0.6, 0.7], [0.2, 0.3], [0.3, 0.4]\rangle/u_1, \langle [0.0, 0.1], [0.1, 0.2], [0.9, 1.0]\rangle/u_2\}),\\
& (\langle x_9, [0.0, 0.1], [0.7, 0.8], [0.8, 0.9]\rangle, \{\langle [0.8, 0.9], [0.7, 0.8], [0.2, 0.3]\rangle/u_1, \langle [0.0, 0.1], [0.4, 0.5], [0.2, 0.3]\rangle/u_2\}) \Big\}\end{aligned}$$

Step 3. We construct *ivnpivn*-matrices $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_3[9 \times 2]$ for $\mathcal{F}_1$ and $\mathcal{F}_3$, respectively as

$$\mathcal{F}_1[9\times 2] = \left[\begin{array}{c|cc}
\langle[0.5,0.7],[0.1,0.2],[0.7,0.8]\rangle & \langle[0.5,0.6],[0.1,0.3],[0.8,0.9]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.4,0.5]\rangle \\
\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \\
\langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\
\langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\
\langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\
\langle[0.3,0.5],[0.7,0.8],[0.2,0.6]\rangle & \langle[0.5,0.7],[0.3,0.5],[0.4,0.8]\rangle & \langle[0.2,0.6],[0.5,0.6],[0.3,0.7]\rangle \\
\langle[0.3,0.8],[0.6,0.7],[0.5,0.9]\rangle & \langle[0.5,0.9],[0.5,0.8],[0.7,0.9]\rangle & \langle[0.3,0.7],[0.8,0.9],[0.4,0.5]\rangle \\
\langle[0.2,0.6],[0.3,0.4],[0.5,0.7]\rangle & \langle[0.4,0.7],[0.7,0.9],[0.3,0.6]\rangle & \langle[0.6,0.7],[0.2,0.4],[0.1,0.5]\rangle \\
\langle[0.1,0.2],[0.5,0.6],[0.1,0.6]\rangle & \langle[0.3,0.4],[0.2,0.3],[0.4,0.6]\rangle & \langle[0.6,0.9],[0.4,0.7],[0.6,0.8]\rangle
\end{array}\right],$$

$$\mathcal{F}_3[9\times 2] = \left[\begin{array}{c|cc}
\langle[0.3,0.4],[0.5,0.6],[0.4,0.5]\rangle & \langle[0.1,0.2],[0.7,0.8],[0.0,0.1]\rangle & \langle[0.0,0.1],[0.9,1.0],[0.9,1.0]\rangle \\
\langle[0.1,0.2],[0.3,0.4],[0.6,0.7]\rangle & \langle[0.0,0.1],[0.5,0.6],[0.8,0.9]\rangle & \langle[0.1,0.2],[0.0,0.1],[0.3,0.4]\rangle \\
\langle[0.2,0.3],[0.4,0.5],[0.6,0.7]\rangle & \langle[0.1,0.2],[0.0,0.1],[0.6,0.7]\rangle & \langle[0.8,0.9],[0.9,1.0],[0.0,0.1]\rangle \\
\langle[0.5,0.6],[0.6,0.7],[0.3,0.4]\rangle & \langle[0.0,0.1],[0.6,0.7],[0.1,0.2]\rangle & \langle[0.4,0.5],[0.7,0.8],[0.2,0.3]\rangle \\
\langle[0.3,0.4],[0.7,0.8],[0.9,1.0]\rangle & \langle[0.9,1.0],[0.6,0.7],[0.7,0.8]\rangle & \langle[0.8,0.9],[0.4,0.5],[0.2,0.3]\rangle \\
\langle[0.6,0.7],[0.4,0.5],[0.5,0.6]\rangle & \langle[0.1,0.2],[0.8,0.9],[0.9,1.0]\rangle & \langle[0.3,0.4],[0.4,0.5],[0.1,0.2]\rangle \\
\langle[0.7,0.8],[0.9,1.0],[0.1,0.2]\rangle & \langle[0.1,0.2],[0.7,0.8],[0.0,0.1]\rangle & \langle[0.9,1.0],[0.0,0.1],[0.6,0.7]\rangle \\
\langle[0.8,0.9],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.6,0.7],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.0,0.1],[0.1,0.2],[0.9,1.0]\rangle \\
\langle[0.0,0.1],[0.7,0.8],[0.8,0.9]\rangle & \langle[0.8,0.9],[0.7,0.8],[0.2,0.3]\rangle & \langle[0.0,1.0],[0.4,0.5],[0.2,0.3]\rangle
\end{array}\right]$$

Step 4. We calculated the similarity between $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_3[9 \times 2]$ as

$$S(\mathcal{F}_1[9 \times 2], \mathcal{F}_3[9 \times 2]) = 0.94$$

Step 5. Here the $\mathcal{F}_1[9 \times 2]$ and $\mathcal{F}_3[9 \times 2]$ are significantly similar. Therefore, we conclude that the person is possibly suffering from cancer.

7 Comparison analysis and discussion

In this section, we present a comparative analysis aiming to certify the feasibility of the proposed method based on similarity measures. The comparative analysis compares the proposed method with four other methods which use similarity measure based on distance measures under *ivnpivn*-soft set environments.

Firstly, proposed method, the method 1 based on Hamming distance measure, method 2 based on normalized Hamming distance measure, method 3 based on Euclidean distance measure and method 4 based on normalized Euclidean distance measure between two *ivnpivn*-soft set are compared. The results from the different methods used to resolve the decision-making problem in Example 13 are shown in Table 1.

From Table 1, similarity measure between two *ivnpivn*-soft set are significantly similar in method 2 and 4. Therefore, we conclude that the person is possibly suffering from cancer in the methods. Similarity measure between two *ivnpivn*-soft set are not significantly similar in the method 1 and method 3. Therefore, we conclude that the person is not possibly suffering from cancer in the methods.

Secondly, the method 1 based on Hamming distance measure, method 2 based on normalized Hamming distance measure, method 3 based on Euclidean distance measure and method 4 based on normalized Euclidean distance measure between two *ivnpivn*-soft set are compared. The results from the different methods used to resolve the decision-making problem in Example 14 are shown in Table 2.

From Table 2, similarity measure between two *ivnpivn*-soft set are significantly similar in proposed method, method 2 and 4. Therefore, we conclude that the person is possibly suffering from cancer in the methods. Similarity measure between two *ivnpivn*-soft set are not significantly similar in the method 1 and method 3. Therefore, we

Table 1 Results for different similarity measures

Methods	Similarity measures	Measure values	The person suffering from cancer
Method 1	$S_d(\mathcal{F}_1[9\times 2], \mathcal{F}_2[9\times 2])$	$\cong$ 0.2	No
Method 2	$S_l(\mathcal{F}_1[9\times 2], \mathcal{F}_2[9\times 2])$	$\cong$0.818	Yes
Method 3	$S_e(\mathcal{F}_1[9\times 2], \mathcal{F}_2[9\times 2])$	$\cong$0.144	No
Method 4	$S_q(\mathcal{F}_1[9\times 2], \mathcal{F}_2[9\times 2])$	$\cong$0.752	Yes
Proposed method	$S(\mathcal{F}_1[9\times 2], \mathcal{F}_2[9\times 2])$	$\cong$0.29	No

Table 2 Results for different similarity measures

Methods	Similarity measures	Measure values	The person suffering from cancer
Method 1	$S_d(\mathcal{F}_1[9\times 2], \mathcal{F}_3[9\times 2])$	$\cong$0.1244	No
Method 2	$S_l(\mathcal{F}_1[9\times 2], \mathcal{F}_3[9\times 2])$	$\cong$0.719	Yes
Method 3	$S_e(\mathcal{F}_1[9\times 2], \mathcal{F}_3[9\times 2])$	$\cong$0.1094	No
Method 4	$S_q(\mathcal{F}_1[9\times 2], \mathcal{F}_3[9\times 2])$	$\cong$0.6887	Yes
Proposed Method	$S(\mathcal{F}_1[9\times 2], \mathcal{F}_3[9\times 2])$	$\cong$0.94	Yes

conclude that the person is not possibly suffering from cancer in the methods.

The reasons why the differences exist is given as. The some methods does not consider the normalized values of distance measures while the other methods does. In addition, the methods use both distance measure and similarity measure while the proposed method used the only similarity measure and the results of these methods may be different with the change in distance measures. Also the distance measures cannot take into account the included angle between two *ivnpivn*-soft set while the proposed similarity measure can. Consequently, the methods and the proposed method may have different results. Generally speaking, the proposed method can effectively tackle the decision-making problems under *ivnpivn*-soft set environments including medical diagnosis.

8 Conclusion

In this paper, we define the notion of interval-valued neutrosophic parameterized interval-valued neutrosophic soft set, called *ivnpivn*-soft set, in a new way by using interval-valued neutrosophic set and soft set. Furthermore, we proposed some definitions and operations on *ivnpivn*-soft set and constructed *ivnpivn*-soft matrix which are more functional to make theoretical studies in the *ivnpivn*-soft set theory. Also, *ivnpivn*-soft set can be expanding with new research subjects such as algebraic structures, graph, soft computing techniques and game theory.

Acknowledgments The authors thank the editors and anonymous reviewers for their helpful comments and suggestions.

References

1. Ali M, Smarandache F (2016) Complex neutrosophic set. Neural Comput Appl 25:1–18
2. Ali M, Deli I, Smarandache F (2016) The theory of neutrosophic cubic sets and their applications in pattern recognition. J Intell Fuzzy Syst 30(4):1957–1963. doi:10.3233/IFS-151906
3. Ali M, Smarandache F, Wang J (2016) δ-equalities of neutrosophic sets. In: IEEE World congress on computational intelligence, Vancouver, Canada
4. Atanassov KT (1999) Intuitionistic fuzzy sets. Springer, New York
5. Cağman N, Çıtak F, Enginoğlu S (2010) Fuzzy parameterized fuzzy soft set theory and its applications. Turkish J Fuzzy Syst 1(1):21–35
6. Cağman N, Çıtak F, Enginoğlu S (2011) FP-soft set theory and its applications. Ann Fuzzy Math Inf 2(2):219–226
7. Cağman N, Karataş S (2013) Intuitionistic fuzzy soft set theory and its decision making. J Intell Fuzzy Syst 24(4):829–836
8. Cağman N, Deli I (2012) Means of FP-soft sets and its applications. Hacet J Math Stat 41(5):615–625
9. Cağman N, Deli I (2012) Product of FP-soft sets and its applications. Hacet JMathStat 41(3):365–374
10. Broumi S, Deli I, Smarandache F (2014) Interval valued neutrosophic parameterized soft set theory and its decision making. J New Result Sci 7:58–71
11. Deli I, Broumi S (2015) Neutrosophic soft matrices and NSM-decision making. J Intell Fuzzy Syst 28(5):2233–2241
12. Deli I, Cağman N (2015) Intuitionistic fuzzy parameterized soft set theory and its decision making. Appl Soft Comput 28:109–113
13. Deli I, Karataş S (2016) Interval valued intuitionistic fuzzy parameterized soft set theory and its decision making. J Intell Fuzzy Syst 30(3):2073–2082

14. Deli I. Interval-valued neutrosophic soft sets and its decision making. Int J Mach Learn Cybern. doi:10.1007/s13042-015-0461-3
15. Deli I (2015) npn-Soft sets theory and applications. Ann Fuzzy Math Inf 10(6):847–862
16. Yagubi EE, Salleh AR (2013) Intuitionistic fuzzy parameterized fuzzy soft set. J Qual Meas Anal 9(2):73–81
17. Dubois D, Prade H, Fuzzy H (1980) Set and systems: theory and applications. Academic Press, New York
18. Feng F, Jun YB, Liu X, Li L (2010) An adjustable approach to fuzzy soft sets based decision making. J Comput Appl Math 234:10–20
19. Jiang Y, Tang Y, Liu H, Chen Z (2013) Entropy on intuitionistic fuzzy soft sets and on interval-valued fuzzy soft sets. Inf Sci 240(10):95–114
20. Ma YX, Wang JQ, Wang J, Wu XH (2016) An interval neutrosophic linguistic multi-criteria group decision-making method and its application in selecting medical treatment options. Neural Comput Appl. doi:10.1007/s00521-016-2203-1
21. Maji PK (2013) Neutrosophic soft sets. Comput Math Appl 45:555–562
22. Maji PK, Biswas R, Roy AR (2001) Fuzzy soft sets. J Fuzzy Math 9(3):589–602
23. Maji PK, Biswas R, Roy AR (2001) Intuitionistic fuzzy soft sets. J Fuzzy Math 9(3):677–692
24. Majumdar P, Samanta SK (2008) Similarity measure of soft sets. New Math Nat Comput 4(1):1–12
25. Mondal JI, Roy TK (2013) Some properties on intuitionistic fuzzy soft matrices. Int J Math Res 5(2):267–276
26. Mondal JI, Roy TK (2014) Intuitionistic fuzzy soft matrix theory and multi criteria in decision making based on T-norm operators. Math Stat 2(2):55–61
27. Mondal JI, Roy TK (2013) Theory of fuzzy soft matrix and its multi criteria in decision making based on three basic t-norm operators. Int J Innov Res Sci Eng Technol 2(10):5715–5723
28. Molodtsov DA (1999) Soft set theory-first results. Comput Math Appl 37:19–31
29. Peng J et al (2014) An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Appl Soft Comput 25:336–346
30. Rajarajeswari P, Dhanalakshmi TP (2013) Intuitionistic fuzzy soft matrix theory and its application in decision making. Int J Eng Res Technol 2(4):1100–1111
31. Smarandache F (1998) A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth
32. Smarandache F (2005) Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. Int J Pure Appl Math 24:287–297
33. Saikia BK, Boruah H, Das PK (2013) Application of intuitionistic fuzzy soft matrices in decision making problems. Int J Math Trends Technol 4(11):254–265
34. Tian ZP, Wang J, Wang JQ, Zhang HY (2016) Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. Group Decis Negot. doi:10.1007/s10726-016-9479-5
35. Tian ZP, Wang J, Wang JQ, Zhang HY (2016) A likelihood-based qualitative flexible approach with hesitant fuzzy linguistic information. Cogn Comput. doi:10.1007/s12559-016-9400-1
36. Tian ZP, Zhanga HY, Wang J, Wang JQ, Chen XH (2015) Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets. Int J Syst Sci. doi:10.1080/00207721.1102359
37. Wang H, Smarandache F, Zhang YQ, Sunderraman R (2005) Interval neutrosophic sets and logic: theory and applications in computing. Hexis; Neutrosophic book series, No 5
38. Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistructure 4:410–413
39. Wu XH, Ju WJQ, Peng J, Chen XH (2016) Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. Int J Fuzzy Syst. doi:10.1007/s40815-016-0180-2
40. Zadeh LA (1965) Fuzzy sets. Inform Control 8:338–353
41. Zhang HY, Ji P, Wang JQ, Chen XH (2016) A neutrosophic normal cloud and its application in decision-making. Cogn Comput. doi:10.1007/s12559-016-9394-8